Hindawi
Journal of Advanced Transportation
Volume 2023, Article ID 8815106, 16 pages
https://doi.org/10.1155/2023/8815106

Research Article

Optimal Parking Slots Reservation and Allocation Problem for Periodic Parking Platforms with Preference Constraints

Xianmin Song,[1] **Tianshu Zhan**,[1] **Haitao Li**,[1] **Bo Liu**,[1] **Yunxiang Zhang**,[1] **and Xin Liu**[2]

[1]*School of Transportation, Jilin University, Changchun 130022, China*
[2]*Big Data and Network Management Center, Jilin University, Changchun 130022, China*

Correspondence should be addressed to Haitao Li; lihait@jlu.edu.cn

Received 10 June 2023; Revised 18 August 2023; Accepted 23 August 2023; Published 30 August 2023

Academic Editor: Yajie Zou

Various solutions, such as parking reservation systems, have been proposed to alleviate the difficulty in finding parking slots. In such systems, parking requests are submitted in advance by drivers, and the systems will reserve appropriate parking spots for drivers if their requests are accepted. However, the parking slots may be allocated unreasonably, which may lead to a waste of space and time resources. In addition, there is a game relationship between operator's profit (OP) and users' benefits (UB), which may affect the sustainable development of the system, if balanced improperly. Given the drivers' arrival and departure time and their parking preference, the paper proposes a periodic reservation and allocation mode (PRAM) and establishes a dual-objective binary integer linear model to solve the reservation and allocation problem. The model aims to maximize the comprehensive benefits of the operator and users and to take full advantage of parking resources. We proposed a TOPSIS-SA algorithm (Technique for Order Preference by Similarity to an Ideal Solution and Simulated Annealing algorithm) to solve our model. Numerical experiments show that our model performs better than the baseline models on all performance metrics such as total operating profit, users' average walking distance, acceptance rate, and utilization of parking slots.

1. Introduction

With the acceleration of urbanization, car ownership in metropolises has increased rapidly, resulting in a growing shortage of parking slots in large cities. In addition, the opacity of parking information and the low utilization of parking slots also contribute to the parking problem. As a result, drivers often spend a lot of time searching for parking slots that meet their needs, which makes cruising traffic increase. The desperate scramble for parking slots also adds to traffic congestion and environmental pollution. As found in a global parking survey by IBM, it took drivers more than 20 minutes on average to find a satisfactory parking slot [1]. Shoup found that 30% of traffic congestion in road networks is caused when people are circulating around to find a parking spot, and vehicles in a small Los Angeles neighbourhood burn 47,000 gallons of gasoline looking for parking spots, producing 730 tons of carbon dioxide and a total of 945,000 miles extra travel within a year [2].

Due to the limitation of urban scale, parking problems are difficult to solve by continually building new parking facilities. Thus, from the perspective of parking demand management, various methods have been proposed including levying some road toll to the vehicle [3–5] on the basis of traffic flow studies [6–8], giving priority to the development of public transport [9, 10] and developing high-occupancy vehicle lanes [11, 12]. However, with the continuous growth of car ownership, the above methods gradually show limitations. Therefore, the parking reservation systems, which can effectively improve the parking efficiency and reduce parking cruising, have become an effective strategy to alleviate parking problems. Parking reservation was proposed and studied as a subsystem of the parking guidance system by Inabak [13]. Parking reservation systems provide drivers with information inquiry and parking slots reservation services through SMS [14] or online network [15–17]. Some reservation systems support recommending appropriate parking slots for drivers, making parking more convenient and efficient. Yang and Liu found out that traffic congestion at the bottleneck can be temporally relieved by combining reserved and unreserved parking spots [18, 19].

Parking allocation plays a decisive role in optimizing the utilization of parking slots and improving the operating efficiency of parking lots. In this way, parking difficulties can be effectively alleviated through reasonable parking allocation. Many studies have focused on how to properly allocate parking slots. The studies can be categorized into two categories. The first category of studies focuses on allocating slots based on user optimum. Shin proposed a smart parking guidance algorithm which supports drivers to find the most appropriate parking facility [20]. To suggest the most suitable parking facilities, he considered several factors, such as driving distance and walking distance. Then, he proposed a real-time parking reservation service and formulated a mixed-integer programming model to minimize the total travel cost of all users [21]. In order to determine the optimal sites of street parking facilities in a working area and minimize the total queue delay, Du presented a method of mathematical programs with equilibrium constraints [22]. Said used game theory to model the parking solution of the proposed reservation system and solved the main problems faced by car drivers in looking for available parking slots, such as parking fees, the amount of walking, parking duration, and so on [23].

The second category of studies focuses on allocating slots based on system optimum. Operator's profit (OP) and the utilization of parking slots are considered most in the parking allocation models. Zhang and Liu considered operator's income and expenditure in the parking allocation models [24, 25]. Xue et al. further considered the impact of rejecting requests on the platform and added the penalty fee into their objective functions to avoid high rejection rate [26–28]. To reduce cruising traffic, Gao et al. integrated shared parking into the ride-sourcing platform and proposed a platform profit maximization novel business model [29]. Xiao and Xu considered a parking market with several kinds of parking players and established a model to pursue the balance between the profits of parking platforms [30]. Zhao et al. and Tang et al. defined the utilization of parking slots as the ratio of the total occupied duration to the total available time of all the parking slots and maximized the utilization of parking slots in parking allocation [31, 32]. Similarly, Wang maximized the utilization of parking slots in allocation by considering the expectation of the total number of occupied parking hours in his model. Some scholars also studied how to achieve the idea of system optimum from other angles [33]. In order to maximize the number of vehicles that can be accommodated in the area, a shared allocation model for night parking between residential area and nearby business district was established by Hu [34]. Aiming at balancing the parking demand among multiple public facilities, Kim established a parking allocation model to alleviate the overloading caused by the imbalance of parking demands among multiple parking lots [35]. Similarly, to solve the spatiotemporal imbalance in parking space utilization, Su considered the shared parking plan in the era of autonomous vehicles with parking autonomy [36]. Considering the dynamic parking demand and drivers' probabilistic choice of each parking lot, Wang established a parking allocation model under dynamic parking fee [37]. To minimize the total travel cost of the system, Zhang et al. examined optimal capacity allocation strategies under system optimum and user optimum, respectively [38].

The real-time reservation and allocation mode (RRAM) is used in most user-optimum studies. In such a real-time mode, a user can submit his/her request at any time, and when the request is received, a parking slot will be allocated immediately. The timeliness of allocation is guaranteed in such a mode. However, the platform will lose the global view and the parking resources will not be fully utilized. In particular, the problem of demand is getting worse in commuting situations. The system needs enough time to develop a reasonable allocation scheme. Compared with RRAM, the periodic reservation and allocation mode (PRAM) which can guide the travel modes of citizens and adjust the distribution of parking demand is commonly required. More periodic allocation models for parking reservation services, such as same-day parking and day-ahead allocation, have been developed, which guarantee the full utilization of parking resources and require users to submit requests in advance.

Moreover, on the one hand, the studies of user optimum ignored that each user has his/her own expectation and the allocation results may not meet users' expectations. As a result, users may cancel their parking reservations after receiving the allocation results. On the other hand, the allocation of both private shared parking slots and public parking slots considered only the user optimum or the system optimum in the aforementioned studies. They ignored that it is equally important to ensure the benefits of both the e-parking operators and the users in the current situation. Ensuring the OP is conducive to the long-term service provided by the platform, users' benefits (UB) should also be guaranteed so that users will use the parking allocation platform for a long time.

Therefore, to address the above limitations, in this paper, we investigate how to fulfill the periodic service, i.e., the prebooked allocation services, by proposing a PRAM. Then, to avoid cancellations of user reservations due to dissatisfaction with the allocated parking slots, we establish the constraints on users' parking preference. Considering the game relationship between OP and UB, a dual-objective binary integer model is established to solve the parking resource optimization problem. However, we combine the TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) method and simulated annealing algorithm and propose a TOPSIS-SA algorithm which allows multiple objectives to approach their optimal levels to solve the model. The algorithm allows multiple objectives to approach their optimal levels. At the same time, falling into local optima can also be effectively avoided. The superiority of our model is validated by comparing it with two baseline models. In addition, sensitivity analyses are conducted and the matching of parking supply and demand is studied. We also provide recommendations for the optimal penalty factor, the optimal supply-demand ratio, and the optimal minimum utilization threshold for platform operators that can be employed to alleviate parking problems and develop smart parking platforms.

The remainder of the paper is organized as follows. Section 2 presents the reservation mechanism and the allocation problem and establishes a dual-objective binary integer model to solve the problem. In addition, we provide two baseline models for comparison. Section 3 presents the solution algorithm for the optimal allocation model. Performance metrics and numerical experiments are presented in Section 4, and conclusions and suggestions for future research are given in Section 5. The list of nomenclature in this paper can be found in Table 1.

2. Mathematical Model

2.1. Reservation Mechanism and Variables. Figure 1 is used to describe the reservation mechanism. The problem concerns a travel zone with multiple user destinations and heterogeneous parking lots. Suppose the platform provides a certain number of available parking slots in several parking lots. Parking slots have the same available time window, but the charge rate is different in different parking lots. Parking requests are submitted by users in advance (within the period provided by the platform, e.g., before 22:00 the previous night) through the parking reservation platform supported by map platforms. Users can search for their destinations and submit parking preferences. After receiving a request, the reservation and allocation system judges whether the user's demand for the parking duration and parking preference can be satisfied. If the demand cannot be satisfied, the system will inform the user on the application interface that there is no parking slot that meets the requirements. If the demand can be satisfied, the system will notify the user that the application has been submitted successfully and inform the user of the request delivery time. The request will then be placed in the allocation pool. The system will complete the parking allocation and update the time window supply of the parking slots. Finally, the allocation results will be presented to users within the specified period so that those who are rejected have enough time to plan for other alternatives.

Our model is based on the following assumptions:

(1) Users submit their parking duration based on the time interval given by the platform and they strictly abide by the parking duration they submitted

(2) Users will not cancel the reservation due to personal factors such as itinerary change

For simplicity, we divide the daily available time period T into a number of intervals (t min each interval) in our parking system. Let K be the total number of intervals. As a result, $k = 1$ is the first time interval (e.g., 8:00–8:30) and $k = K$ is the last time interval (e.g., 21:30–22:00). Let J be the total number of the parking lots rented by the platform. Let N be the total number of the parking slots, and we introduce a binary indicator a_{nj} which is defined to be 1 if parking slot n belongs to parking lot j and 0 otherwise. Thus, we have the parking slot distribution matrix $A_{N\times J} = [a_{nj}]$, where $n \in [1,N]$ and $j \in [1,J]$. Similarly, s_{nk} which is defined to be 1 if parking slot n is available in time interval k and 0 otherwise is introduced and the parking supply matrix $S_{N\times K} = [s_{nk}]$, where $n \in [1,N]$ and $k \in [1,K]$, can be defined.

Let M' denote the total number of parking requests and let $t_{m'}^{\text{start}}$ and $t_{m'}^{\text{end}}$ be the start and end time interval of request m', $t_{m'}^{\text{start}}, t_{m'}^{\text{end}} \in [1,K]$, where $m' \in [1,M']$ and $k \in [1,K]$. Thus, the duration for request m' is $dur_{m'} = t_{m'}^{\text{end}} - t_{m'}^{\text{start}} + 1$. We further introduce a binary indicator $d_{m'k}$ which is defined to be 1 if parking request m' includes time interval k and 0 otherwise. Thus, we have the initial parking demand matrix $D_{M'\times K} = [d_{m'k}]$, where $m' \in [1,M']$ and $k \in [1,K]$.

Let (x_j, y_j) be the central coordinate of parking lot j and let $(x_{m'}, y_{m'})$ be the central coordinate of the destination of request m'. For simplicity, we take the linear distance between the parking lot j and the destination of request m' as the walking distance of request m' after parking. Let $l_{m'j}$ to be the walking distance of request m' which can be calculated as follows:

$$l_{m'j} = \sqrt{\left(x_{m'} - x_j\right)^2 + \left(y_{m'} - y_j\right)^2}. \tag{1}$$

Let f_j (yuan/h) and p_j (yuan/day) denote the charge rate of parking lot j and purchase price of parking lot j, respectively. We select the walking distance after parking and charge rate, which are the most concerned indicators of parking users, as the parking preferences of users. Users can submit the maximum acceptable walking distance $l_{m'}^{\max}$ and the maximum acceptable charge rate $f_{m'}^{\max}$ when they use the platform. Thus, the constraint of users' parking preferences can be expressed as follows:

$$l_{m'j} \le l_{m'}^{\max}, \tag{2}$$

$$f_j \le f_{m'}^{\max}. \tag{3}$$

Since there are situations where the existing parking resources cannot satisfy the parking duration and parking preferences of some users, these requests will not be put into the allocation pool. Let M be the total number of requests in the allocation pool and update the index number of the requests in the allocation pool to m. Thus, we have the final parking demand matrix $D_{M\times K} = [d_{mk}]$, where $m \in [1,M]$ and $k \in [1,K]$.

We then introduced a decision variable x_{mn} which is defined to be 1 if request m is allocated to parking slot n and 0 otherwise. Thus, we have the parking slot allocation matrix $X_{M\times N} = [x_{mn}]$, where $m \in [1,M]$ and $n \in [1,N]$. Then, based on the parking slot distribution matrix $A_{N\times J}$, we have the following parking lot allocation matrix, $C_{M\times J} = [c_{mj}] = X_{M\times N} \times A_{N\times J}$, or the following equation:

$$c_{mj} = x_{mn} \bullet a_{nj}. \tag{4}$$

Clearly, $c_{mj} = 1$ if request m is allocated to parking lot j; otherwise, $c_{mj} = 0$.

Table 1: The meaning of notations in this paper.

Notations	Meaning of notations
T	The daily available time period, h
K	Total number of time intervals
t	The length of each time interval, h
J	The total number of parking lots provided by the platform
N	The total number of parking slots provided by the platform
a_{nj}	Parking slot distribution status, when parking slot n belongs to parking lot j, $a_{nj} = 1$; otherwise, $a_{nj} = 0$
$A_{N\times J}$	The parking slot distribution matrix, $A_{N\times J} = [a_{nj}]$, where $n = 1, 2, 3, \ldots, N$ and $j = 1, 2, 3, \ldots, J$
s_{nk}	Parking supply status, when parking slot n is available in time interval k, $s_{nk} = 1$; otherwise, $s_{nk} = 0$
$S_{N\times K}$	The parking supply matrix, $S_{N\times K} = [s_{nk}]$, where $n = 1, 2, 3, \ldots, N$ and $k = 1, 2, 3, \ldots, K$
M'	The initial total number of the parking requests
m'	The initial index number of the parking requests
m	The index number of the parking requests in allocation pool
$t_{m'}^{\text{start}}$ (t_m^{start})	The start time interval of request m' (m)
$t_{m'}^{\text{end}}$ (t_m^{end})	The end time interval of request m' (m)
$d_{m'k}$ (d_{mk})	Parking demand status, if parking request m' (m) includes time interval k, $d_{m'k} = 1$ $(d_{mk} = 1)$; otherwise, $d_{m'k} = 0$ $(d_{mk} = 0)$
$D_{M'\times K}$	The initial parking demand matrix, $D_{M'\times K} = [d_{m'k}]$, where $m' = 1, 2, 3, \ldots, M'$ and $k = 1, 2, 3, \ldots, K$
$D_{M\times K}$	The final parking demand matrix, $D_{M\times K} = [d_{mk}]$, and $m = 1, 2, 3, \ldots, M$; $k = 1, 2, 3, \ldots, K$
$[x_j, y_j]$	The coordinate of parking lot j
$[x_{m'}, y_{m'}]$	The coordinate of the destination of request m'
$[x_m, y_m]$	The coordinate of the destination of request m
$l_{m'j}$ (l_{mj})	The walking distance of request m' (m) after parking
p_j	The purchase price of parking lot j, yuan/ da y
f_j	The selling price of parking lot j, yuan/h
$l_{m'}^{\max}$ $(l_m^{\max})$	The maximum acceptable walking distance of request m' (m), m
$f_{m'}^{\max}$ $(f_m^{\max})$	The maximum acceptable charge rate of request m' (m), yuan/h
x_{mn}	Parking slot allocation result, when request m is allocated to parking slot n, $x_{mn} = 1$; otherwise, $x_{mn} = 0$
$X_{M\times N}$	The parking slot allocation matrix, $X_{M\times N} = [x_{mn}]$, where $m = 1, 2, 3, \ldots, M$ and $n = 1, 2, 3, \ldots, N$
$C_{M\times J}$	The parking lot allocation matrix, $C_{M\times J} = [c_{mj}] = X_{M\times N} \times A_{N\times J}$
c_{mj}	The parking lot allocation result, when request m is allocated to parking lot j, $c_{mj} = 1$; otherwise, $c_{mj} = 0$
μ	Rejection penalty factor, yuan/request
ω_1	The total operating profit, yuan
ω_1'	The actual operating profit, yuan
ω_2	The average walking distance of users, m.
U	Utilization of parking slots
A	Acceptance rate

2.2. Model Methodology

2.2.1. Operating Profit Optimum (PO). From the perspective of OP, the platform operator aims to maximize the operating profit. However, the platform's blind pursuit of profits may lead to high rejection rates and low service levels. Therefore, we introduce a rejection penalty factor μ (yuan/request) to avoid a low service level. We further assume that the impact of the platform investment at the early stage and the expenses incurred in the operation management process on the OP can be negligible. Thus, from the perspective of OP, we can formulate the reservation and allocation problem as follows:

$$\max \omega_1 = \sum_{m=1}^{M}\sum_{j=1}^{J}\left(f_j \bullet c_{mj} \bullet dur_m\right) - \sum_{j=1}^{J}\sum_{n=1}^{N}\left(p_j \bullet a_{nj}\right) - \mu \bullet \left(M - \sum_{m=1}^{M}\sum_{n=1}^{N} x_{mn}\right), \tag{5}$$

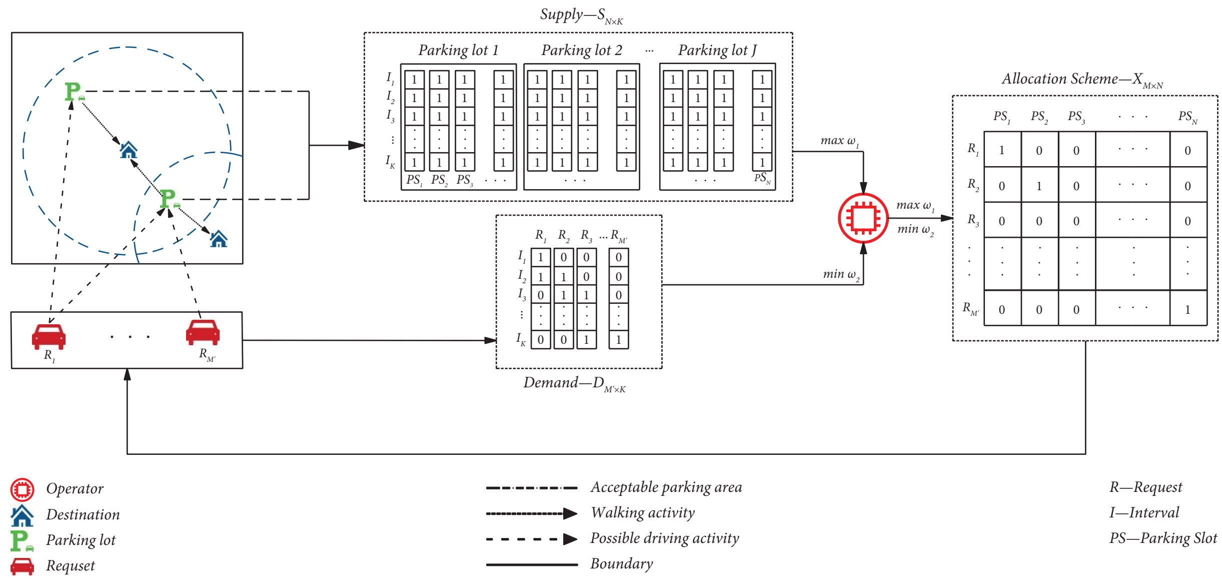


Figure 1: The problem scenario.

subject to

$$l_{mj} \le l_m^{\max}, \tag{6}$$

$$f_j \le f_m^{\max}, \tag{7}$$

$$\left[t_m^{\text{start}}, t_m^{\text{end}}\right] \in [1, K], \tag{8}$$

$$x_{nm} \bullet d_{mk} \le s_{nk}, \tag{9}$$

$$\sum_{n=1}^{N} x_{mn} \le 1, \tag{10}$$

$$\sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{k=1}^{K} \left(x_{nm} \bullet d_{mk}\right) \le N, \tag{11}$$

$$x_{mn}, d_{mk}, s_{nk}, a_{nj} \in \{0, 1\}. \tag{12}$$

In equation (5), the first term is the sum of parking fees paid by the users assigned to the parking slots, representing the total revenue of the platform; the second term is the total cost for purchasing the parking slots; and the third term is the product of the number of rejected users and the rejection penalty factor, representing the long-term loss due to request rejection, where $(M - \sum_{m=1}^{M}\sum_{n=1}^{N} x_{mn})$ is the total number of rejected requests. In the constraint set, equations (6) and (7) are the constraints of users' parking preferences; equation (8) simply indicates that the parking duration of users must be within the system available interval; the time window constraint must be met in the process of parking allocation, which means the parking slots must be idle during the parking duration in the allocation scheme; therefore, equation (9) guarantees that any parking slot should accommodate at most one car in each time interval; our problem is a special assignment problem in which each request can only be accepted or rejected; hence, equation (10) guarantees that each request can be allocated to at most one parking lot; similarly, the parking slots constraint must be met in the process of parking allocation, which means the number of requests accepted in each time interval cannot exceed the total number of the parking slots in the allocation scheme, as is shown in equation (11); equation (12) simply implies x_{mn}, d_{mk}, s_{nk}, and a_{nj} are binary variables.

2.2.2. Walking Distance Optimum (WO). From the perspective of UB, we choose to minimize the average walking distance, which can be formulated as follows:

$$\min \omega_2 = \frac{\sum_{m=1}^{M} \sum_{j=1}^{J} \left(c_{mj} \bullet l_{mj}\right)}{\sum_{m=1}^{M} \sum_{n=1}^{N} x_{mn}}. \tag{13}$$

The constraint set of equation (13) is the same as equation (5).

2.2.3. The Optimal Allocation Model (OM). We aim to pursue comprehensive benefits of operator and users under the game relationship between OP and UB. Thus, the optimal allocation model can be formulated as follows:

$$\begin{cases} \max \omega_1 = \sum_{m=1}^{M} \sum_{j=1}^{J} \left(f_j \bullet c_{mj} \bullet dur_m \right. \\ \\ \sum_{j=1}^{J} \sum_{n=1}^{N} \left(p_j \bullet a_{nj} \right) - \mu \bullet \left(M - \sum_{m=1}^{M} \sum_{n=1}^{N} x_{mn} \right) \\ \\ \min \omega_2 = \dfrac{\sum_{m=1}^{M} \sum_{j=1}^{J} \left(c_{mj} \bullet l_{mj} \right)}{\sum_{m=1}^{M} \sum_{n=1}^{N} x_{mn}} \end{cases}$$

$$\begin{aligned} \text{s.t. } & l_{mj} \leq l_m^{\max} \\ & f_j \leq f_m^{\max} \\ & \left[t_m^{\text{start}}, t_m^{\text{end}} \right] \in [1, K] \\ & \sum_{n=1}^{N} x_{mn} \leq 1 \\ & x_{nm} \bullet d_{mk} \leq s_{nk} \\ & \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{k=1}^{K} \left(x_{nm} \bullet d_{mk} \right) \leq N \\ & x_{mn}, d_{mk}, s_{nk}, a_{nj} \in \{0, 1\}. \end{aligned} \tag{14}$$

3. Baseline Models

For the purpose of comparison, we consider two baseline models of parking allocation which are first-come-first-serve (FCFS) and first-book-first-serve (FBFS).

FCFS means that the platform processes requests according to the actual arrival order of the users. The allocation algorithm for FCFS is shown in Algorithm 1.

To minimize the individual-level waiting time, the FBFS applies a myopic strategy. When a parking request is submitted by a user, the platform immediately allocates a parking spot. The allocation algorithm for FBFS is shown in Algorithm 2.

3.1. Solution Algorithm for the Optimal Allocation Model. The optimal allocation model established in this paper belongs to the dual-objective multidimensional combinatorial optimization problems. We pursue the maximization of bilateral benefits under the game relationship between OP and UB. Therefore, the TOPSIS method, which pursues multiple objectives to approach their optimal level, is chosen to normalize our dual-objective optimal parking allocation model. The idea of normalization can be described as follows.

We first solve the two single-objective problems which are PO and WO separately. Then, we assume that the optimal solutions of the two problems are $\max \omega_1 = \omega_1^*$ and $\min \omega_2 = \omega_2^*$. $\omega^* = (\omega_1^*, \omega_2^*)$ is called an ideal point in the solution set because it is always hard to reach. Thus, we assume ω which is the closest to ω^* as an approximate optimal solution under the measure that we expect. One of the most direct methods is the shortest TOPSIS method, so we establish the following equation:

$$\varphi(\omega) = \sqrt{\left(\omega_1 - \omega_1^*\right)^2 + \left(\omega_2 - \omega_2^*\right)^2}. \tag{15}$$

Then, we need to find the solution that is closest to the ideal point in the solution set. Thus, $\varphi[\omega(x_{mn})]$ needs to be minimized, which can be expressed by equation (16). The optimal solution of equation (16) is taken as the optimal solution of OM.

$$\min \varphi[\omega(x_{mn})] = \sqrt{\left(\omega_1 - \omega_1^*\right)^2 + \left(\omega_2 - \omega_2^*\right)^2}. \tag{16}$$

As a result, the dual-objective multidimensional combinatorial optimization problem is transformed into three single-objective optimization problems. When solving the single-objective optimization problems, precise analytical algorithms such as enumeration method, dynamic programming method, and branch and bound method are effective methods in the case of small-dimensional problems. However, the time consumed will increase exponentially as the size of solving scale grows and the operational efficiency will decrease rapidly, making it difficult to quickly obtain the optimal solution and easy to fall into the local optimal solution. Thus, we choose the simulated annealing algorithm, which is more efficient, as our solution algorithm.

The simulated annealing algorithm is a kind of serial structure optimization algorithm that eventually tends to global optimal. By simulating the annealing process of solid in physics, the algorithm gives a time-varying probability jump that eventually tends to zero for the search process, thus effectively avoiding falling into local optimal. The algorithm starts from a certain high initial temperature. With the continuous decrease of temperature parameters, it randomly finds the global optimal solution of the objective function in the solution set combined with the probabilistic jump characteristics. The basic process of TOPSIS-SA is shown in Algorithm 3.

4. Numerical Experiments

In this section, we conduct numerical experiments to conduct quantitative studies and illustrate the models and results.

4.1. Performance Metrics. To evaluate the superiority of OM, four performance metrics including operating profit, average walking distance of users, utilization of parking slots, and acceptance rate are used to evaluate the model.

Operating profit and acceptance rate are the performance metrics that e-parking platform operators concern most. There are two kinds of operating profit which are total operating profit which has been given in equation (5) and actual operating profit ω_1' which is the total operating profit without taking the loss of rejection into account.

Input: $D_{M\times K}$, $S_{N\times K}$, $A_{N\times J}$.
Output: $X_{M\times N}$.
Initialize: sort all parking requests according to their arrival time t_m^{arr}.
for $m \in [1, M]$ **do**
  if the request can be accepted **then**
    $x_{mn} = 1$.
    update $S_{N\times K}$.
  else
    $x_{mn} = 0$.
end

Algorithm 1: Allocation algorithm of the FCFS.

Input: $D_{M\times K}$, $S_{N\times K}$, $A_{N\times J}$.
Output: $X_{M\times N}$.
Initialize: sort all parking requests according to their arrival time t_m^{book}.
for $m \in [1, M]$ **do**
  if the request can be accepted **then**
    $x_{mn} = 1$.
    update $S_{N\times K}$.
  else
    $x_{mn} = 0$.
end

Algorithm 2: Allocation algorithm of the FBFS.

(1) Set the objective function as $\max \omega_1$.
(2) **Input:** $D_{M\times K}$, $S_{N\times K}$, $A_{N\times J}$.
(3) **Output:** $X_{M\times N}$.
(4) Set initial temperature T_{start}, termination temperature T_{end}, temperature attenuation coefficient α, iteration number maxgen.
(5) **Initialize:** $T = T_{\text{start}}$; randomly generate the initial solution and assign it as the optimal solution, which is $S_{\text{best}} = S_0$.
(6) **for** $T > T_{\text{end}}$ **do**
(7)   gen = 1.
(8)   **for** gen $\in [1, \text{maxgen}]$ **do**
(9)   Perturb S_{best} to produce a new solution which is S'.
(10)   **if** S' is better than S_{best} **then**
(11) $S_{\text{best}} = S'$.
(12)   gen = gen + 1.
(13)   **else if** $\text{rando m}[0,1] < \exp[-(S_0 - S')/T]$
(14) $S_{\text{best}} = S'$.
(15)   **else**
(16)   gen = gen + 1.
(17)   **end**
(18) $T = T \bullet \alpha$.
(19) **end**
(20) Set the objective function as $\min \omega_2$.
(21) Repeat Step 2 to Step 19 and the solution of $\min \omega_2$ is ω_2^*.
(22) Set the objective function as min $\varphi[\omega(x_{mn})] = \sqrt{(\omega_1 - \omega_1^*)^2 + (\omega_2 - \omega_2^*)^2}$.
(23) Repeat Step 2 to Step 19.
(24) The algorithm terminates and the optimal allocation scheme is output.

Algorithm 3: TOPSIS-SA algorithm for OM.

$$\omega_1 = \sum_{m=1}^{M}\sum_{j=1}^{J}\left(f_j \bullet c_{mj} \bullet dur_m\right) - \sum_{j=1}^{J}\sum_{n=1}^{N}\left(p_j \bullet a_{nj}\right) - \mu \bullet \left(M - \sum_{m=1}^{M}\sum_{n=1}^{N} x_{mn}\right),$$
$$\omega_1' = \sum_{m=1}^{M}\sum_{j=1}^{J}\left(f_j \bullet c_{mj} \bullet dur_m\right) - \sum_{j=1}^{J}\sum_{n=1}^{N}\left(p_j \bullet a_{nj}\right). \quad (17)$$

The average walking distance of users is used to measure whether the users' benefits are fully considered, which can be calculated as follows:

$$\omega_2 = \frac{\sum_{m=1}^{M}\sum_{j=1}^{J}\left(c_{mj} \bullet l_{mj}\right)}{\sum_{m=1}^{M}\sum_{n=1}^{N} x_{mn}}. \quad (18)$$

The utilization of parking slots is an important metric to measure whether the parking slots are fully utilized, which means the ratio of the total occupied duration to the total supply duration. It can be calculated as follows:

$$U = \frac{\sum_{n=1}^{N}\sum_{k=1}^{K}\left(\sum_{m=1}^{M} x_{nm} \bullet d_{mk}\right)}{N \bullet K}. \quad (19)$$

The acceptance rate reflects the service level of the parking reservation platform. It is the ratio of the number of requests received to the number of requests in the allocation pool. It can be calculated as follows:

$$A = \frac{\sum_{m=1}^{M}\sum_{n=1}^{N} x_{mn}}{M}. \quad (20)$$

4.2. Computation Results on a Basic Example. Suppose the time interval k is 0.5 h, and the daily available time period is $T = 14$ h (8 : 00–22 : 00). Thus, the total number of the time interval is $K = 28$. For simplicity, we suppose that a reservation and allocation cycle is 24 h, which means that users submit requests at least before 22 : 00 the previous night, and the system will allocate parking slots for all requests at once. The allocation results will be presented to the users by 23 : 00 the previous night, giving rejected users enough time to plan for alternative options. In the base case, we suppose that our study area is a 500 m × 500 m region from (0, 0) to (500, 500) in the two-dimensional coordinate system. Suppose the platform buys 25 parking slots from parking lot 1 and 25 parking slots from parking lot 2 and receives 500 parking requests. Therefore, $N = 50$ and $M' = 500$. The parking fee is assumed to be $i_1 = 8$ (yuan/h) per vehicle and $i_2 = 4$ (yuan/h) per vehicle. The purchase cost is $p_1 = 40$ (yuan/day) per slot and $p_2 = 20$ (yuan/day) per slot, and the penalty factor is $\mu = 4$ (yuan/request). Furthermore, we suppose that the arrival time of users follows a Poisson distribution, and the parking duration follows an exponential distribution, as usually considered in the literature [39]. The average parking duration is assumed to be 3 h. For simplicity, suppose that 50% of the users whose maximum acceptable walking distance is 300 m and the rest of users' maximum acceptable walking distance is 500 m. Suppose that 50% of the users whose maximum acceptable parking fee is 6 (yuan/h) and the rest of the users' maximum acceptable parking fee is 10 (yuan/h). The central coordinates of parking lot 1 and parking lot 2 are assumed to be (100, 100) and (400, 400), respectively. The travel destinations of users are randomly distributed in the study areas.

We use MATLAB R2016a to generate the experimental data including arrival time, parking duration, travel destination, and parking preference, as shown in Figures 2 and 3:

PO and WO are also chosen to compare with OM. Figure 4 shows the assignment results of a basic example. In Figure 4, the x-axis and the y-axis represent the time interval and the parking slots, respectively. Occupied parking slots are indicated by coloured blocks and are labeled with "1" and free parking slots are indicated by white blocks and are labeled with "0." Table 2 presents the computation results of the five strategies.

It can be clearly seen from the allocation results that the allocation scheme of OM significantly outperforms that of FCFS and FBFS on all four performance metrics. Compared with PO, OM can shorten the average walking distance by 21.90% while sacrificing 10.44% of the profit. In terms of acceptance rate and utilization of parking slots, the performance of PO and OM is basically the same. Compared with WO, OM can increase the operating profit by 66.53% while increasing average walking distance by 24.40%. Meanwhile, OM's acceptance rate is 14% higher than WO's. Thus, OM can take both OP and users' walking distance into consideration and realize the comprehensive optimal of OP and UB.

4.3. Extended Experiments. To further test the capabilities of OM and conduct sensitivity analysis, we expand the number of parking slots in each parking lot to 50 and vary the number of parking requests from 0 to 2000. The rest of the experimental setup is consistent with the basic experiment.

From Figure 5, we can see that the performance of OM in the extended experiment is basically consistent with that in the basic experiment. OM's total operating profit is second only to PO's and significantly better than that of the other three methods; the acceptance rate of OM is basically consistent with that of PO, and obviously better than that of the other three methods; OM's average walking distance is second only to WO's, and obviously less than that of the other three methods; in terms of parking slots utilization, OM performs slightly better than the other four methods.

From Figure 6(a), we can see that the operator is at a deficit when there are a few parking requests. As the number of requests increases, the total operating profit increases linearly, and the total operating profit gradually turns from a loss to a profit when the number of requests is

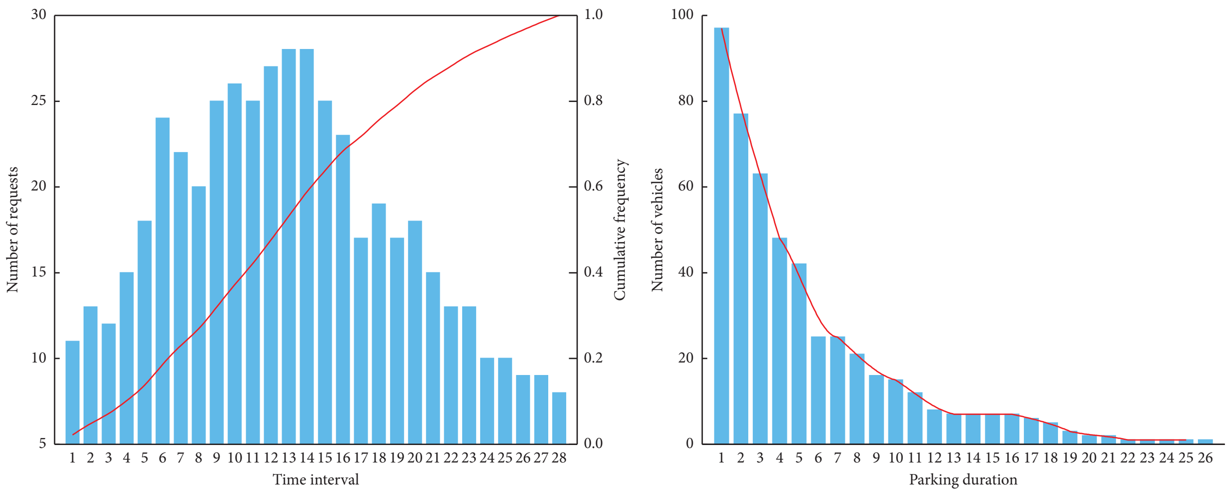


FIGURE 2: The distribution of users' arrival time and parking time.

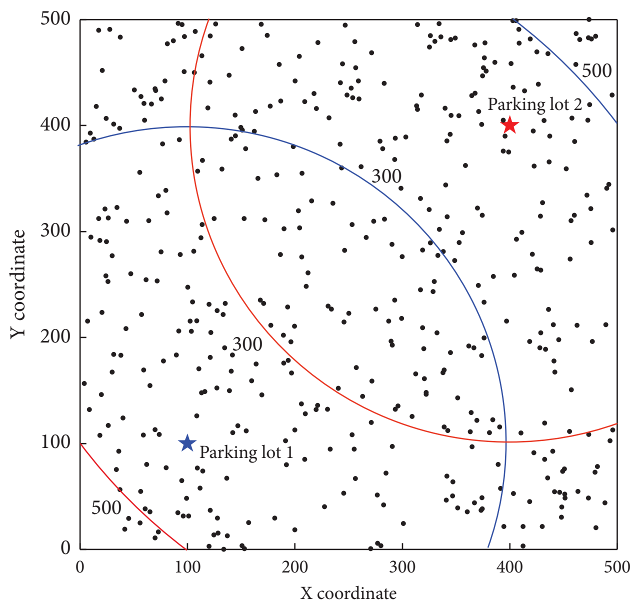


FIGURE 3: The distribution of parking lots and destinations.

around 300. When the number of requests exceeds 500, the total operating profit presents different changing rules under $\mu = 0$ and $\mu = 4$, which means the effect of the penalty factor is gradually manifested. The total operating profit starts to grow at a slower speed under $\mu = 4$. The total operating profit reaches its maximum when the number of requests approaches 800. The total operating profit starts to decrease as the number of requests continues to increase because the parking supply is less than the parking demand, resulting in more and more users being rejected by the platform. The penalty factor thus directly leads to a decrease in the total operating profit. However, in the absence of penalty term ($\mu = 0$), the operating profit starts to grow at a slower speed and approaches the maximum when the number of requests is over 800, which means that it is hard to increase the OP by accepting more requests when the parking demand exceeds the parking supply.

From Figure 6(b), we can also see that the parking supply can meet the parking demand when the number of parking requests is below 800 and the inhibitory effect of the penalty factor is inconspicuous. However, as the number of parking requests continues to increase, the parking supply cannot meet the parking demand. As a result, the acceptance rate keeps decreasing under $\mu = 0$ and the acceptance rate starts to decrease slower under $\mu = 4$, which means the inhibitory effect of punishment factor is manifested.

From Figure 7, we can see that the average walking distance and the utilization of parking slots show the same trend under $\mu = 0$ and $\mu = 4$, which means that the two performance metrics are not affected by the penalty factor. Meanwhile, from Figure 7(a), we can see that the change of average walking distance tends to be stable after the number of requests reaches 1000, indicating that the average walking

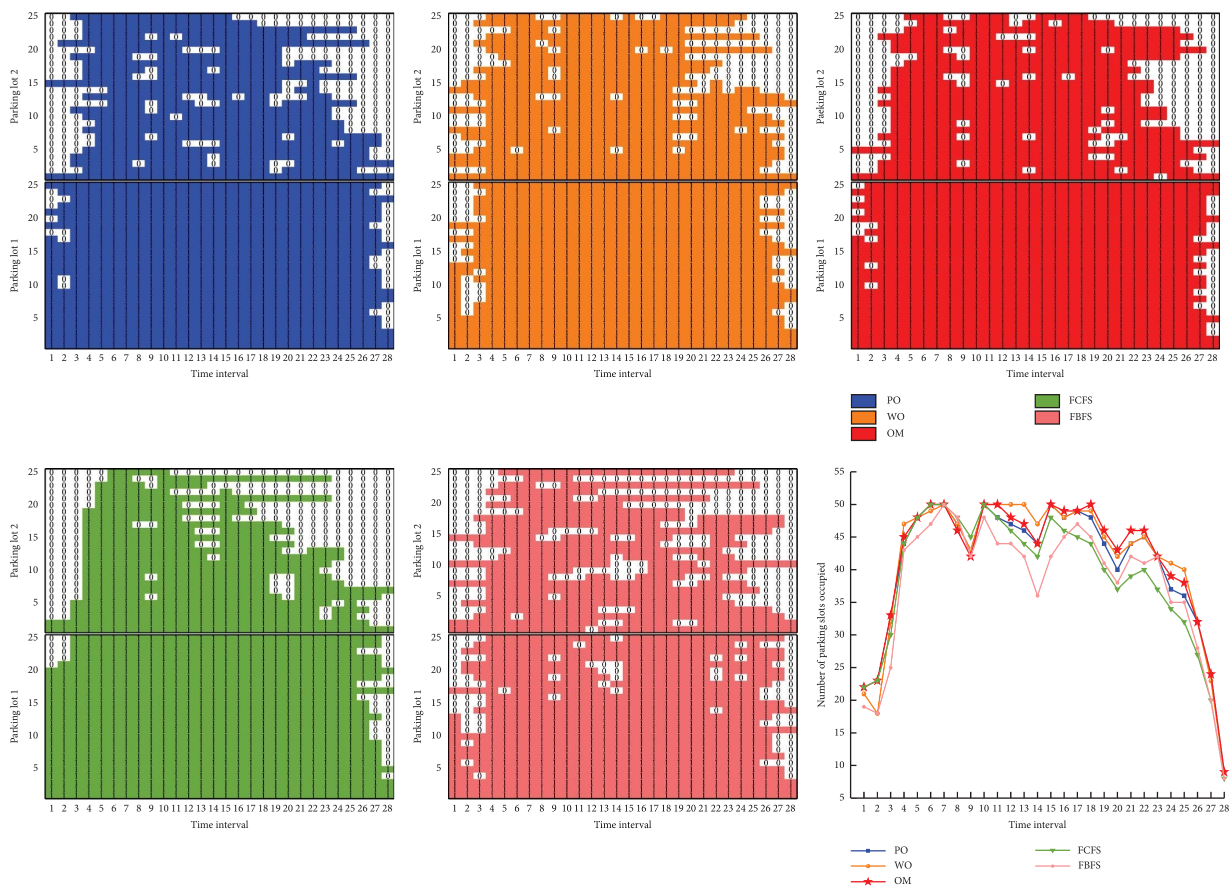


Figure 4: Allocation schemes of different strategies.

Table 2: Computation results of different strategies.

Allocation method	Performance metrics			
	ω_1 (yuan)	ω_2 (m)	U	A
PO	900	154.78	0.81	0.81
WO	484	97.17	0.80	0.66
OM	806	120.88	0.83	0.80
FCFS	648	144.47	0.78	0.67
FBFS	500	134.54	0.76	0.71

distance has reached the optimal level under the current supply and demand condition, which is difficult to be further optimized.

To study the effect of penalty factor on the platform, we vary the penalty factor from 0 to 5 in four situations of short supply ($M' = 800$, 1200, 1600, and 2000) and observe the change in total operating profit and acceptance rate which is shown in Figure 8. In Figure 8, it is obvious that the total operating profit decreases with the increase of penalty factor and the acceptance rate increases with the increase of penalty factor, which means that with the increase of the penalty factor, the restriction effect on the model's rejection of requests in pursuit of profit will also significantly increase. However, when the penalty is higher than 3.0, the acceptance rate starts to increase more slowly, while the total operating profit decreases linearly, indicating that it is difficult to continuously improve the restriction effect by increasing the penalty factor. Thus, we suggest that the value of penalty factor should be set within the range of 2.0 to 3.0 (yuan/request).

For each combination of demand or supply, we can find the maximum profit by solving OM. Figure 9 plots the change of ω_1 in the two-dimensional space of parking demand and supply under different penalty factors, respectively. From Figure 9, the optimal ratio of N/M' can be found. This is significant because it suggests the number of parking slots that the platform operator should purchase for a given or predicted number of parking requests. It also suggests the number of parking requests that the platform operator should accept for a given number of parking slots. Comparing Figure 9 ($\mu = 0$) and ($\mu > 0$), we can see that the optimal ratio changes slightly, indicating that the penalty

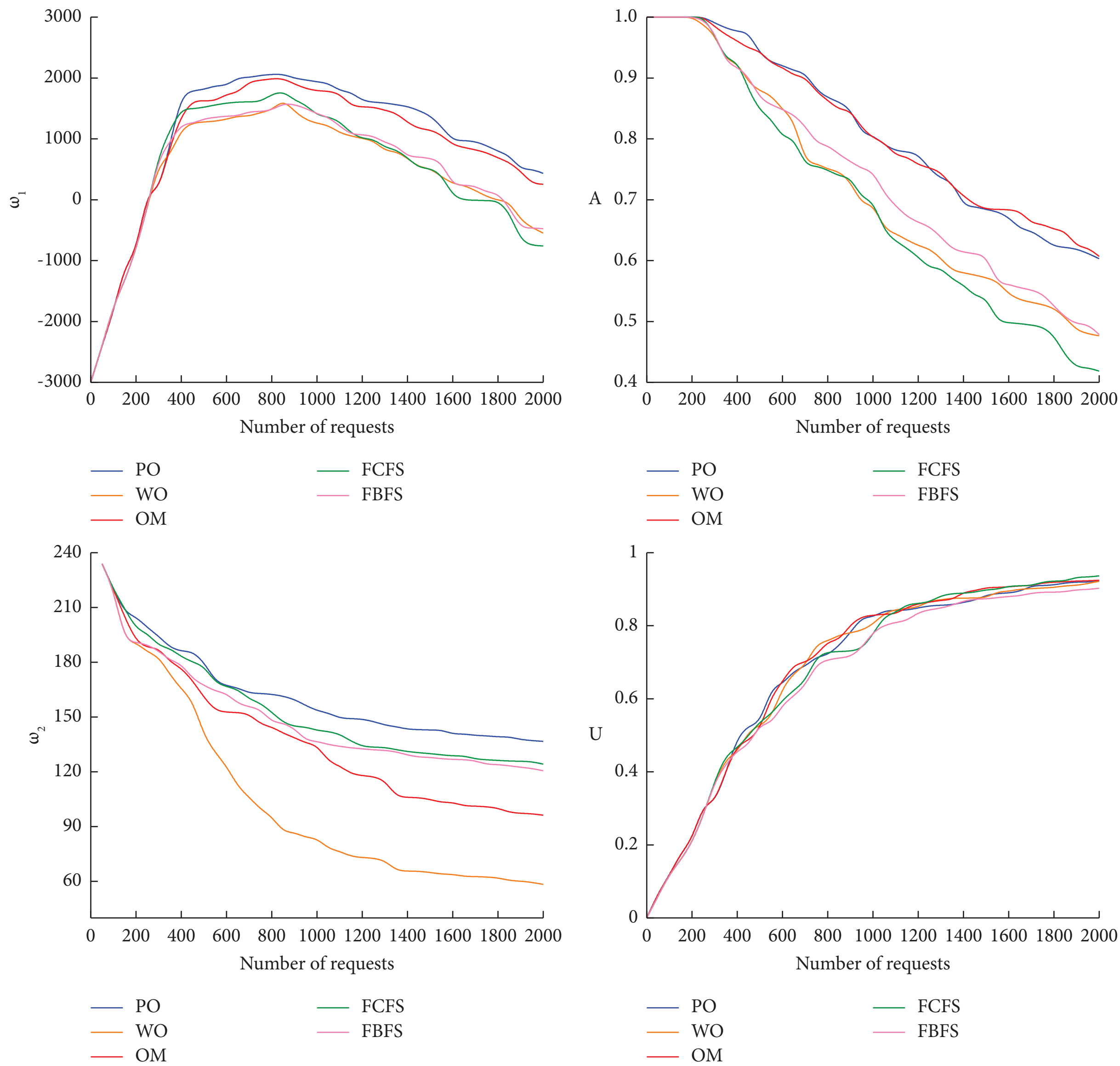
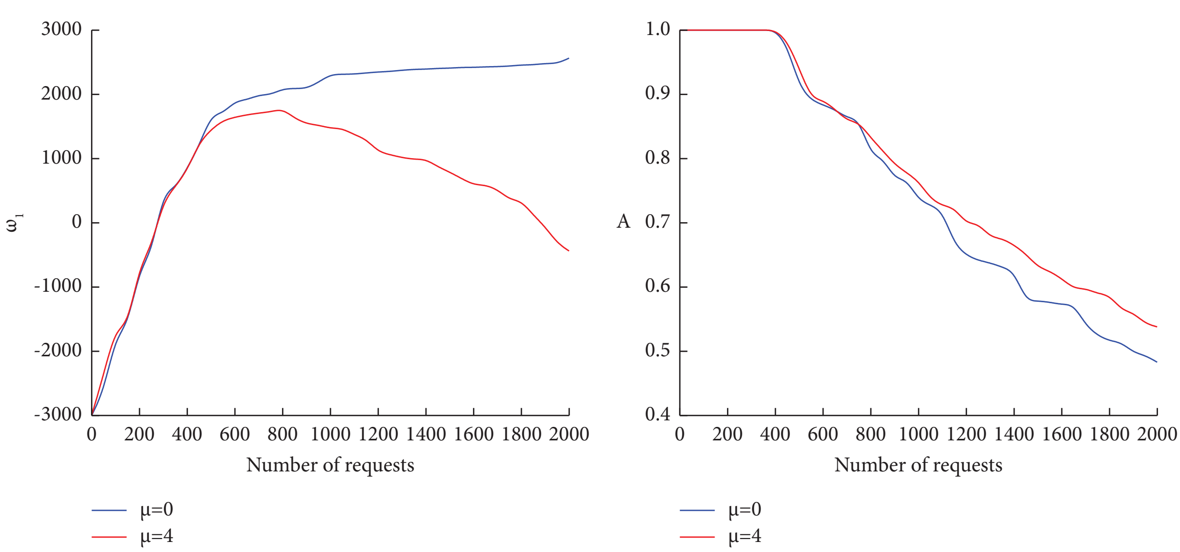


Figure 5: Change in performance metrics with the number of requests.



(a) (b)

Figure 6: Change in ω_1 (a) and A (b) with the number of requests.

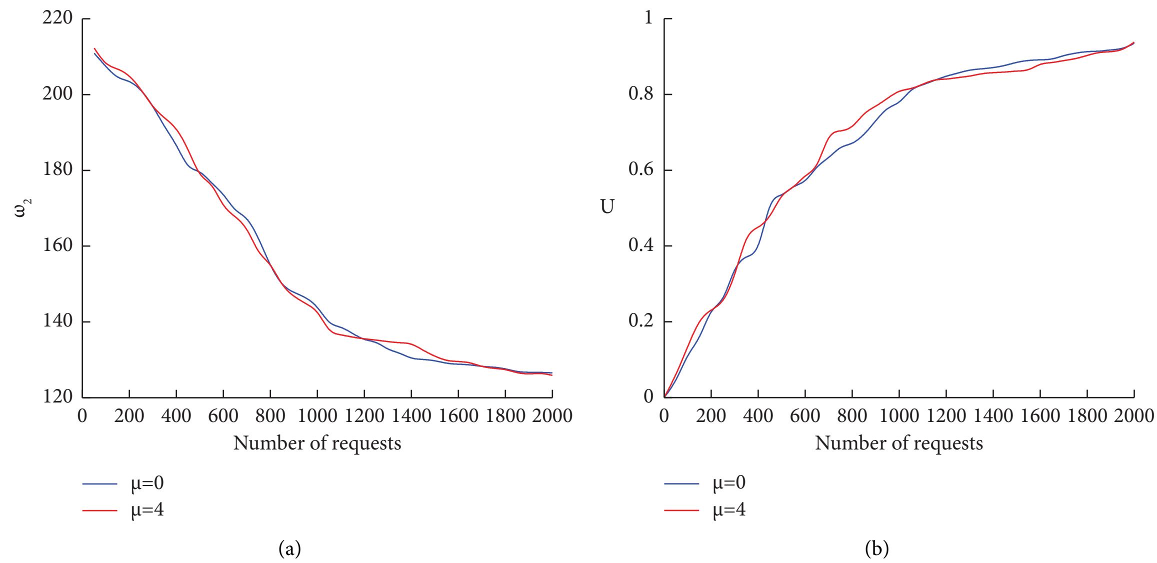


Figure 7: Change in ω_2 (a) and U (b) with the number of requests.

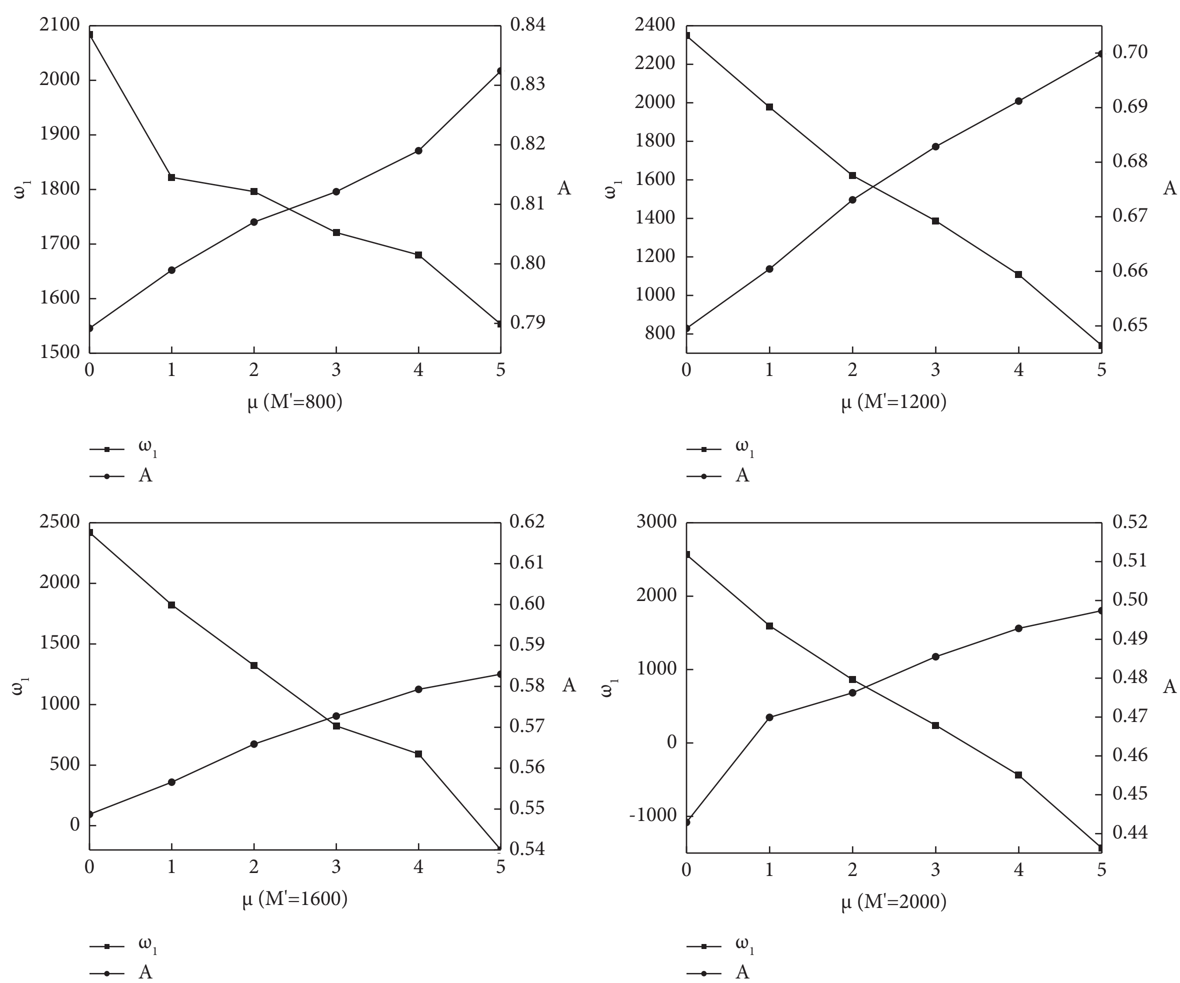


Figure 8: Change in ω_1 and A with the value of μ under different number of requests.

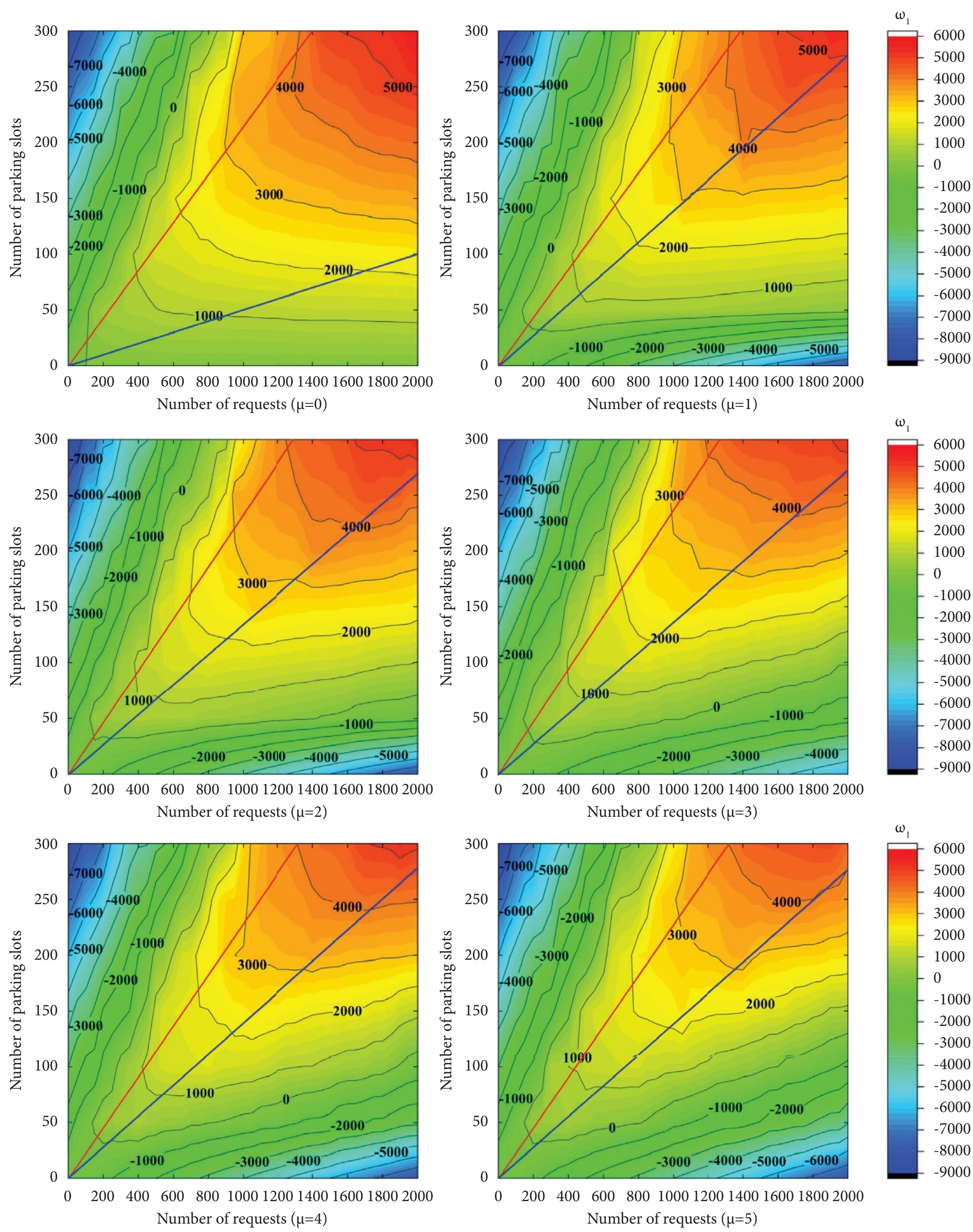


FIGURE 9: Change of ω_1 (10^3) in the two-dimensional space of parking demand and supply.

factor has little impact on the optimal number of parking slots for a given demand. However, when the rejection loss is neglected, the slope will be moderate for a given supply.

The minimum utilization threshold $U_{\min}$ for parking slots is introduced to study the impact of improving the utilization of parking slots on the platform. From Figure 10, we can see that after $U_{\min}$ is above 70%, with the increase of $U_{\min}$, the actual operating profit, the average walking distance, and the acceptance rate decrease. The acceptance rate decreases more rapidly and the slope of average walking distance gets gentle when $U_{\min}$ is above 80%. However, the acceptance rate decreases significantly when $U_{\min}$ is above 90%. Thus, we can find out that more requests received by the platform can improve the actual operating profit, the utilization of parking slots, and the average walking distance, since an increase in the number of requests favors better

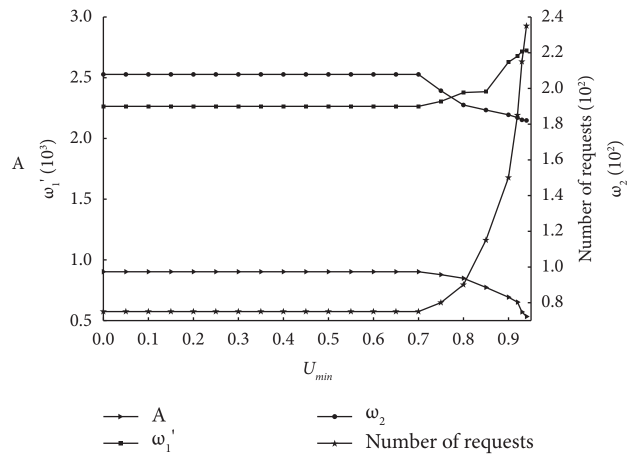


FIGURE 10: Impact analysis on $U_{\min}$.

selection in parking allocation. However, at the same time, the acceptance rate will decrease, leading to a decrease in service level, which will decrease faster with the increase of demands. Therefore, we suggest that the value of $U_{\min}$ should be set within the range of 70% to 80%.

5. Conclusions and Future Research

Considering the parking problems caused by the imbalance between parking supply and parking demand, we examined the regional parking allocation problem through an e-parking-platform. A dual-objective binary integer linear programming model is proposed to allocate certain parking requests to specific parking slots in order to pursue full use of parking resources and comprehensive optimal of OP and UB. We then propose a TOPSIS-SA algorithm to solve the model. The OM method is compared with FCFS, FBFS, PO, and WO methods in numerical experiments. The results show that the OM model significantly outperforms FCFS and FBFS on all four performance metrics. In addition, the OM method can decrease the average walking distance by 21.90% compared with the PO model and it can improve the operating profit by 66.53% compared with the WO model which means the OM model realize the comprehensive optimal of OP and UB. At the same time, the sensitivity analyses are conducted and the matching of parking supply and demand is studied. We find that the optimal penalty factor should be set within 2.0 to 3.0 and the optimal combination of supply and demand can be found in the contour plot (Figure 9). Moreover, the contour plot demonstrates that the penalty factor has little impact on the optimal number of parking slots for a given demand. In addition, the optimal minimum utilization threshold for parking slots is suggested to be within 70% to 80% so that the service level is guaranteed while the utilization of parking slots is improved.

In addition, this study considers that balancing the parking demand among multiple parking lots is conducive to alleviate the traffic congestion and improve the operating efficiency of parking lots [35, 36]. It is of our interest to incorporate balancing the parking demand among multiple parking lots in our future research so that the traffic congestion caused by parking problems can be further alleviated and the parking efficiency can be further improved. Moreover, the uncertainty in drivers' arrival/departure time is not taken into account in this study. However, the driver may arrive earlier or depart later, which may cause service failure [33]. In this context, we consider pursuing the comprehensive optimal of OP and UB while addressing the parking unpunctuality. However, the optimal operation decisions under different operation objectives will be affected by such a trade-off. Thus, this idea should be examined in the further studies. In addition, case verification should be carried out in future research to realize the practical application of the parking reservation and allocation system.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

Acknowledgments

This study was supported by the National Natural Science Foundation of China (no. 52131202).

References

[1] P. Newswire, *IBM Global Parking Survey: Drivers Share Worldwide Parking Woes*, PR Newswire US, Chicago, IL, USA, 2011.

[2] D. C. Shoup, "Cruising for parking," *Transport Policy*, vol. 13, no. 6, pp. 479–486, 2006.

[3] C. Chu, C. Wang, and S. Hu, "Application of E-tag in pricing road tolls and parking fees for traffic congestion mitigation," *Transportation Research Procedia*, vol. 25, pp. 2913–2922, 2017.

[4] D. Triantafyllos, C. Illera, T. Djukic, and J. Casas, "Dynamic congestion toll pricing strategies to evaluate the potential of route-demand diversion on toll facilities," *Transportation Research Procedia*, vol. 41, pp. 731–740, 2019.

[5] A. F. Lentzakis, R. Seshadri, and M. Ben-Akiva, "Predictive distance-based road pricing—designing tolling zones through unsupervised learning," *Transportation Research Part A: Policy and Practice*, vol. 170, Article ID 103611, 2023.

[6] X. Yang, Y. Zou, and L. Chen, "Operation analysis of freeway mixed traffic flow based on catch-up coordination platoon," *Accident Analysis & Prevention*, vol. 175, Article ID 106780, 2022.

[7] S. Wu, Y. Zou, L. Wu, and Y. Zhang, "Application of Bayesian model averaging for modeling time headway distribution," *Physica A: Statistical Mechanics and its Applications*, vol. 620, Article ID 128747, 2023.

[8] X. Yang, Y. Zou, H. Zhang, X. Qu, and L. Chen, "Improved deep reinforcement learning for car-following decision-making," *Physica A: Statistical Mechanics and Its Applications*, vol. 624, Article ID 128912, 2023.

[9] H. Haitao, K. Yang, H. Liang, M. Menendez, and S. I. Guler, "Providing public transport priority in the perimeter of urban networks: a bimodal strategy," *Transportation Research Part C: Emerging Technologies*, vol. 107, pp. 171–192, 2019.

[10] T. Borchers and R. A. Ribeiro, "A framework for selecting bus priority system locations in medium-sized cities: case study in Araraquara, Brazil," *Case Studies on Transport Policy*, vol. 10, no. 4, pp. 2053–2063, 2022.

[11] L. Zhong, K. Zhang, Y. Marco Nie, and J. Xu, "Dynamic carpool in morning commute: role of high-occupancy-vehicle (HOV) and high-occupancy-toll (HOT) lanes," *Transportation Research Part B: Methodological*, vol. 135, pp. 98–119, 2020.

[12] M. C. Cohen, A. Jacquillat, A. Ratzon, and R. Sasson, "The impact of high-occupancy vehicle lanes on carpooling," *Transportation Research Part A: Policy and Practice*, vol. 165, pp. 186–206, 2022.

[13] K. Inaba, M. Shibui, T. Naganawa, M. Ogiwara, and N. Yoshikai, "Intelligent parking reservation service on the internet," in *Proceedings 2001 Symposium on Applications and the Internet Workshops*, San Diego, CA, USA, January 2001.

[14] N. H. H. M. Hanif, M. H. Badiozaman, and H. Daud, "Smart parking reservation system using short message services (SMS)," in *2010 International Conference on Intelligent and Advanced Systems*, Manila, June 2010.

[15] M. Karthi and P. Harris, "Smart parking with reservation in cloud based environment," in *2016 IEEE International Conference on Cloud Computing in Emerging Markets (CCEM)*, Bangalore, India, October 2016.

[16] W. Hong, Z. You, B. Xiao, J. Ma, and K. Hong, "Intelligent parking space sharing system," in *2021 3rd International Conference on Artificial Intelligence and Advanced Manufacture (AIAM)*, Manchester, UK, October 2021.

[17] C. C. How, I. F. Kamsin, N. K. Zainal, H. A. A. H. Sithiq, and N. A. A. Rahman, "Smart parking reservation mobile application," in *2022 IEEE International Conference on Distributed Computing and Electrical Circuits and Electronics (ICDCECE)*, Ballari, India, April 2022.

[18] H. Yang, W. Liu, X. Wang, and X. Zhang, "On the morning commute problem with bottleneck congestion and parking space constraints," *Transportation Research Part B: Methodological*, vol. 58, pp. 106–118, 2013.

[19] W. Liu, H. Yang, Y. Yin, and F. Zhang, "A novel permit scheme for managing parking competition and bottleneck congestion," *Transportation Research Part C: Emerging Technologies*, vol. 44, pp. 265–281, 2014.

[20] J. Shin and H. Jun, "A study on smart parking guidance algorithm," *Transportation Research Part C: Emerging Technologies*, vol. 44, pp. 299–317, 2014.

[21] H. He, Z. Zhang, and P. Yan, "A real-time reservation service for smart parking system," in *2018 15th International Conference on Service Systems and Service Management (ICSSSM)*, pp. 1–6, Hangzhou, China, July 2018.

[22] Y. Du, S. Yu, Q. Meng, and S. Jiang, "Allocation of street parking facilities in a capacitated network with equilibrium constraints on drivers' traveling and cruising for parking," *Transportation Research Part C: Emerging Technologies*, vol. 101, pp. 181–207, 2019.

[23] A. M. Said, A. E. Kamal, and H. Afifi, "An intelligent parking sharing system for green and smart cities based IoT," *Computer Communications*, vol. 172, pp. 10–18, 2021.

[24] F. Zhang, W. Liu, X. Wang, and H. Yang, "Parking sharing problem with spatially distributed parking supplies," *Transportation Research Part C: Emerging Technologies*, vol. 117, Article ID 102676, 2020.

[25] W. Liu, F. Zhang, and H. Yang, "Modeling and managing the joint equilibrium of destination and parking choices under hybrid supply of curbside and shared parking," *Transportation Research Part C: Emerging Technologies*, vol. 130, Article ID 103301, 2021.

[26] Z. Xue, J. Cheng, and K. Ji, "An allocation model of shared parking with continuous time windows," in *2021 IEEE 8th International Conference on Industrial Engineering and Applications (ICIEA)*, pp. 616–620, Chengdu, China, April 2021.

[27] C. Shao, H. Yang, Y. Zhang, and J. Ke, "A simple reservation and allocation model of shared parking lots," *Transportation Research Part C: Emerging Technologies*, vol. 71, pp. 303–312, 2016.

[28] M. Xie, X. Zhang, Z. Wu, S. Wei, Y. Gao, and Y. Wang, "A shared parking optimization framework based on dynamic resource allocation and path planning," *Physica A: Statistical Mechanics and Its Applications*, vol. 616, Article ID 128649, 2023.

[29] J. Gao, S. Li, and H. Yang, "Shared parking for ride-sourcing platforms to reduce cruising traffic," *Transportation Research Part C: Emerging Technologies*, vol. 137, Article ID 103562, 2022.

[30] H. Xiao and M. Xu, "Competition among parking platforms in the presence of indirect network effects," *Transportation Research Part C: Emerging Technologies*, vol. 150, Article ID 104111, 2023.

[31] P. Zhao, H. Guan, H. Wei, and S. Liu, "Mathematical modeling and heuristic approaches to optimize shared parking resources: a case study of Beijing, China," *Transportation Research Interdisciplinary Perspectives*, vol. 9, Article ID 100317, 2021.

[32] Z. Tang, Y. Jiang, and F. Yang, "An efficient Lagrangian relaxation algorithm for the shared parking problem," *Computers & Industrial Engineering*, vol. 176, Article ID 108860, 2023.

[33] S. Wang, Z. Li, and N. Xie, "A reservation and allocation model for shared-parking addressing the uncertainty in drivers' arrival/departure time," *Transportation Research Part C: Emerging Technologies*, vol. 135, Article ID 103484, 2022.

[34] X. Liu, H. Hu, and Z. Deng, "The shared allocation model of night parking spaces between residential areas and adjacent business districts," in *2019 International Conference on Artificial Intelligence and Advanced Manufacturing (AIAM)*, pp. 527–532, Dublin, Ireland, October 2019.

[35] O. Tran Thi Kim, N. H. Tran, C. Pham, T. LeAnh, M. T. Thai, and C. Hong, "Parking assignment: minimizing parking expenses and balancing parking demand among multiple parking lots," *IEEE Transactions on Automation Science and Engineering*, vol. 17, no. 3, pp. 1320–1331, 2020.

[36] Q. Su and D. Z. W. Wang, "On the morning commute problem with distant parking options in the era of autonomous vehicles," *Transportation Research Part C: Emerging Technologies*, vol. 120, Article ID 102799, 2020.

[37] Y. Wang and Q. Chen, "Dynamic parking allocation model in a multidestination multiple parking lot system," *IEEE Intelligent Transportation Systems Magazine*, vol. 14, no. 5, pp. 195–208, 2022.

[38] X. Zhang, W. Liu, M. Levin, and S. Travis Waller, "Equilibrium analysis of morning commuting and parking under spatial capacity allocation in the autonomous vehicle environment," *Transportation Research Part E: Logistics and Transportation Review*, vol. 172, Article ID 103071, 2023.

[39] A. Richardson, "An improved parking duration study method," in *Proceedings of the 7th Australian Road Research Board Conference*, pp. 397–413, Adelaide, Australia, August 1974.