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Research Article

Line Planning under the Operation Mode of Line Sharing between Metro and Suburban Railway

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Due to high transport efficiency, reduced transfer time, and various other advantages, the joint operation of different rail transit systems emerges as the optimal choice for rail transit systems. This article mainly studies the line-planning problem under the line-sharing operation mode between metro and suburban railway. First, a complex multiobjective programming model is established to maximize the net profit of two operating companies and the time savings of passengers. The constraints of this model encompass passenger flow, available vehicles, line carrying capacity, station capacity, cross-line configuration, departure frequency, and variable value range. Second, the linear weighted sum method is introduced to consolidate three objective functions into a single one, while utilizing the improved artificial bee colony (IABC) algorithm to address the line-planning problem. Besides, the traditional artificial bee colony (TABC) algorithm and the simulated annealing (SA) algorithm are provided as comparison groups to solve the same numerical example problem. The results demonstrate significant reductions in travel time by adopting the line-sharing operation mode. In addition, the IABC algorithm exhibits better solution quality and higher efficiency than both the TABC and SA algorithms. The proposed method proves to be valuable in formulating and optimizing the line plan.

1. Introduction

By the end of May 2023, 54 cities across 31 provinces in China built and operated metro, suburban railway, and other types of rail transit lines, resulting in a total of 292 lines. In May 2023, the total operating mileage of rail transit reached 9652.6 kilometers, capable of running 3.22 million trains and transporting 2.49 billion passengers.

At present, most of the rail transit lines in China still use the operation mode of different rail transit systems operated independently, which offers the advantages of simplicity and noninterference between systems. Nevertheless, with an increase in travel demand, the limitations of independent operation become more apparent. Taking the Beijing subway as an example, it comprises 475 stations, including 81 transfer stations. The transfer passenger volume on weekdays has climbed to 5.8 million, which accounts for about 90% of the total volume. Table 1 illustrates the significant passenger flow and congestion at transfer stations. Moreover, many transfer stations have long and narrow transfer passageway, leading to occasional difficulty in boarding trains due to overcrowded platforms. All the above factors may result in a longer transfer time. The transfer issue has a detrimental impact on passenger travel efficiency and the appeal of the subway as a transportation mode.

In addition, under the independent operation mode of each system, vehicle resources of different lines cannot be shared. As a result, lines with surplus transportation capacity are unable to support those with insufficient capacity. This poor matching between demand and supply and inadequate

Table 1: Transfer volume of Beijing subway in the morning peak.

No.	Station name	Passenger flow in the morning peak (×10000)	Daily transfer passenger flow (×10000)
1	Songjiazhuang	5.3	30.2
2	Xizhimen	4.2	33.5
3	Hujialou	3.3	26.5
4	Huixin Xijie Nankou	3.5	22.7
5	Sihuidong (E)	3.1	19.9
6	Jianguomen	2.6	22.6

resource sharing results in low resource utilization and hinders the timely and effective response to emergencies. Statistics reveal that certain sections of the Beijing rail transit network experience overcrowding during the morning rush hour, with a load factor of nearly 150% observed in areas like the Life Science Park-Xi'erqi section of the Changping Line. On the other hand, there are sections with very low load factors, such as the Yancun East-Zicaowu section of the Yanfang line, which is only 8%. It can be seen that the imbalance of passenger distribution is serious in the morning rush hour and that there is a large waste of transportation capacity in the sections with less passenger flows.

With the rapid development of rail transit systems, travel distances and frequencies gradually increase, and passengers pay more and more attention to the comfort and convenience of travel while enjoying punctuality and safety. A conventional single and independent transportation system is hard to meet the travel needs brought by the increasingly expansion of the transit network scale. As a result, joint operation of multimodal rail transit systems is the shape of things to come. Joint operation can not only reduce the number of transfer passengers and provide more convenient service but also promote the utilization rate of trains and other facilities, reducing operating cost.

Joint operation refers to a mode in which trains can run through lines belonging to different railway systems, allowing them to share all or part of the line with trains originally running on it. Joint operation can be classified into three operation modes: vehicle renting, line leasing, and line sharing, with line sharing being the most commonly adopted operation mode. The earliest instances of the line-sharing operation can be traced back to Karlsruhe, Germany [1]. In this case, the city tram line could share track with a German Federal Railways freight line. Drechsler [2] pointed out that this operation mode created more convenient travel conditions between Kraichgau and Karlsruhe. Compared to the previous operation mode, transportation demand has grown rapidly, and the number of passengers has increased by approximately quintuple. In addition to delivering ferocious success in Germany, the line-sharing operation mode has also yielded favorable outcomes in other European cities as well as Japan. For example, after the Tokyu Corporation adopted the line-sharing operation mode, the number of passengers crossing lines between Shibuya and Yokohama increased by a huge 13.1%. As a result, the annual financial revenue of the Tokyu Corporation increased by 2 billion yen [3]. The aforementioned successful cases in various countries illustrate the significant advantages of the line-sharing operation mode in actual transportation, which can facilitate passenger travel and generate considerable operational benefits.

For the sake of fast and convenient travel, alleviating the pressure on transfer stations during rush hours, and improving the competitiveness of the rail transit travel mode, it is necessary to adopt the line-sharing approach. However, the line-sharing operation mode needs to meet certain conditions, and many scholars have conducted a series of explorations on the technical requirements it must satisfy. Griffin [4] took the Sunderland Metro and light rail in the United Kingdom as an example, elaborated on the technical problems that need to be solved in the joint operation, and introduced the characteristics of a dual-voltage system and its application prospect. He also emphasized that the line sharing operation was perhaps the most effective way of meeting the travel needs of people living in urban and suburban areas. Novales et al. [5] analyzed the solutions to technical problems such as traction power, supply system, wheel/rail design, structural strength, communication system, and passenger access (platform height, gap between platform, and train) under the line-sharing operation of trams and state-owned railways. Naegeli et al. [6] surveyed existing systems to identify key requirements for the successful case of the line-sharing operation. These requirements included network design, city layout, population density, and existing technical standards. Ito [7] introduced the developments and current situation of the line-sharing operation in Tokyo and indicated that running direct trains between different operators required close coordination and full negotiation. Kurosaki [8] analyzed the striking differences in the organization mode and management system of the line-sharing operation between Europe and Tokyo. He pointed out that it was crucial for other countries to choose the appropriate style of railway operation while introducing the line sharing operation.

The above studies have technically proved the feasibility and necessity of the line-sharing operation. However, the theoretical and practical research on the line-sharing operation in China is still in the exploration stage. For example, some scholars discussed under what situation the line-sharing operation mode will obtain good performance. Zheng [9] believed that the line-sharing operation mode should be adopted under the condition of uneven spatio-temporal distribution of passenger flows, or the number of passengers decreasing progressively with distance. Li [10] held that the line-sharing operation mode could be adopted between the two rail transit systems when the transfer passenger flow is large and the transfer time is long. Furthermore, some scholars pay attention to the macrolayout [11, 12]; others study the plan and design of the line-sharing operation with realistic cases [13–15]. There is limited research on transport organization under the line-sharing operation condition. Considering limitations in this area, we need to conduct research on the line-sharing operation mode at the transport organization level.

Line-planning problem (LPP) is the most crucial phase in railway transport organization. It mostly deals with stopping schemes, optimal paths, and service frequencies for demand cover [16–18]. There are many existing studies focused on subprocesses, such as those mainly concerned halting patterns [19, 20], train frequency [21, 22], and optimal routes [23, 24]. There have also been numerous studies on the combinatorial problem involving the aforementioned subprocesses. Feng et al. [25] constructed a mixed-integer nonlinear program model to develop an optimal train service plan and determined the number of service routes, frequency, and train stopping patterns for a real-life case study in Chengdu, China. Szeto and Jiang [26] proposed a bilevel programming model to find the optimal routes and service frequencies. Meng et al. [27] transferred LPP into a complex network with the typical characteristics of small-world and degree free. Fu et al. [28] used an integrated hierarchical method to determine train frequencies as well as various halting patterns. Schöbel [29] combined the line-planning problem with other transport organization issues and established a generic model to describe the problem. Canca et al. [30] considered maximizing the net profit as the objective and used an adaptive large neighborhood search metaheuristic algorithm to solve the line-planning problem. Zhao et al. [31] applied Stackelberg game theory to deal with the combined optimization problem that balances the tradeoffs between operating costs and passenger travel cost by incorporating the passenger flow assignment into line planning. Micco et al. [32] presented an approach to design better line plans for realistic scenarios and proposed a new metric to compare different transit networks. Zhang et al. [33] proposed a unified integer linear programming model for the integrated optimization of line planning and train timetabling.

Through the above analysis of the development and research status of the line-sharing operation mode, the advantages of the line-sharing operation mode are demonstrated, and the feasibility of implementing the line-sharing operation mode is explained. However, research on the line-sharing operation in China is still in its early stages, and there is limited focus on the transport organization level in existing studies. Since LPP serves as the cornerstone of transport organization, it is imperative to conduct research on LPP within the context of the line-sharing operation mode.

After reviewing the existing literature on LPP, we provide a comprehensive overview of its key aspects and summarize previous research conducted on this problem. Previous research on LPP has provided useful references for this paper from different aspects, but most of the existing studies only concern a single rail transit system, such as the high-speed rail network and urban railways. The aforementioned studies do not take into account LPP under the conditions of joint operation in different rail transit systems. Unlike previous research, we establish a multiobjective comprehensive optimization model for LPP under the operation mode of line sharing between metro and suburban railway.

The detailed contributions are as follows:

(1) The line-sharing operation mode is studied from the perspective of line plan. A novel line plan model under the operation mode of line sharing between metro and suburban railway is built. In addition, all kinds of transfer passengers and fare clearing rules under the line-sharing mode are described in detail.

(2) The artificial bee colony algorithm is improved to solve the line plan problem. An actual case is used to verify the reliability of the proposed model and algorithm, which inspire the realization of the line-sharing operation between different rail transit systems.

The remainder of the paper is organized as follows. Section 2 uses a mathematical approach to describe LPP in detail. Section 3 designs the improved artificial bee colony (IABC) algorithm to address the line plan problem. Numerical experiments and comparative analysis are shown in Section 4 to verify the effectiveness of the proposed method. Section 5 draws conclusions and puts forward the future study aspects.

2. Methodologies

2.1. Problem Description. Since the traditional operation mode fails to meet the travel needs of passengers adequately, we propose a scenario of joint operation between the metro and suburban railway systems. Cross-line metro (or suburban) trains could operate seamlessly between the metro and suburban lines, and the two operating companies adopt the line-sharing operation mode.

The fact is that the metro line and suburban railway line interconnect at the cross-line station S_b, as shown in Figure 1. $S_{Met} = \{S_1, \cdots, S_a, \cdots, S_b\}$ is the set of metro stations, $S_{Sub} = \{S_b, \cdots, S_c, \cdots, S_N\}$ is the set of suburban railway stations, and N is the total number of stations. The direction from S_1 to S_N is considered the upward direction, while the direction from S_N to S_1 is considered the downward direction. E_1 is the running section of metro trains without a cross-line operation. In other words, metro trains depart from the station S_1 then run to the station S_b and turn back. Similarly, E_2 is the running section of cross-line metro trains, E_3 is the running section of suburban trains without a cross-line operation, and E_4 is the running section of cross-line suburban trains. The station S_a is the departure station of cross-line suburban trains in the upward direction, while the station S_c is the terminal station of cross-line metro trains in the upward direction.

Line zoning is shown in Figure 2, and the subsequent description of travel time savings is based on this division. Generally, the passenger volume is small in sections I and IV, which are located at ends of the line, while it is large in sections II and III.

Taking the upward direction as an example, as shown in Figure 3, passengers traveling in the upward direction can be divided into 10 categories: Both the origin and destination of passengers belonging to Class 11-up (where “up” refers to the upward direction) are within section I. The passengers

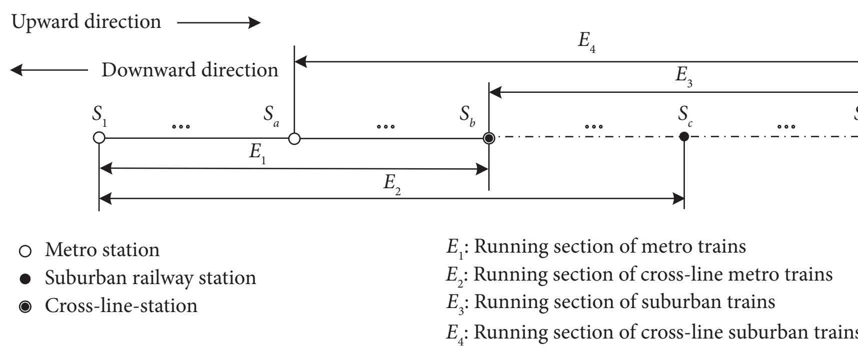

Figure 1: Joint operation between metro and suburban railway.

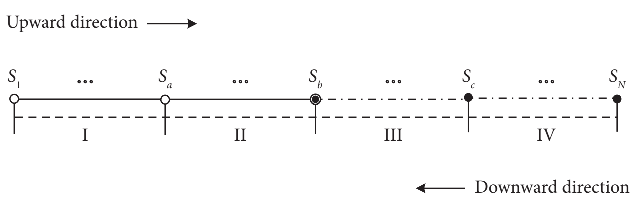

Figure 2: Line zoning.

belonging to Class 12-up have their travel path originating in section I and their destination in section II. The travel path for Class 13-up passengers begins in section I and concludes in section III. Similarly, we can get the origin and destination for other passenger categories.

Under the line-sharing operation mode, all expenses arising from the operation of cross-line trains are borne by the enterprises to which the cross-line trains belong. The ticket income brought by passengers taking cross-line trains belongs to the enterprises at which the cross-line section is located. Detailed fare clearing rules are shown in Figure 4.

In this mode, the tracks of metro and suburban lines are integrated and shared, allowing trains from different lines to operate on the overlapping sections. Passengers can travel from suburban railway line to metro line without transferring. Line planning under the line-sharing operation mode requires coordinating train frequency, resource utilization, and ticket income clearing to ensure smooth and conflict-free operations at shared segments.

2.2. Assumptions. Before constructing a mathematical model, certain necessary assumptions are made as follows:

(1) The conditions for operating cross-line trains between different types of lines are satisfied.

(2) Both metro and suburban railway adopt the all-stop mode. Cross-line trains and noncross-line trains are composed of the same type of vehicles.

(3) Passengers can only accept a single transfer, and they always prefer a direct train between two stations compared to a transfer train. The arrival of passengers follows a uniform distribution.

(4) All the stations can be regarded as turn back stations. S_a can be any station between S_1 and S_b, and S_c can be any station between S_b and S_N. Passengers can transfer at S_a, S_b, or S_c.

2.3. Symbols of the Line Plan Model. The meaning of all the symbols of the LPP model is listed in Table 2.

2.4. Objective Functions of the Line Plan Model. The quality of a line plan is often judged by the travel time from the passengers' point of view and operation costs from the perspective of operators. Therefore, the objective function is considered from three parts: net profit of the subway operating company, net profit of the suburban railway operating company, and passengers travel time savings.

The net profit of the metro operating company can be expressed as fare revenue minus the operating costs:

$$
\begin{aligned}
W^{Met} &= W^{Met}_{ticket} - W^{Met}_{run},\\
W^{Met}_{ticket} &= \sum_{c=b+1}^{N} x_c \cdot \omega^{Met}_{ticket} \cdot l^{Met} \cdot \left[\sum_{i=1}^{b-1}\sum_{j=i+1}^{b} q_{ij}\cdot (j-i) + \sum_{i=1}^{b}\sum_{j=b+1}^{c} q_{ij}\cdot (b-i) + \sum_{i=2}^{b}\sum_{j=1}^{i-1} q_{ij}\cdot (i-j)\right.\\
&\quad \left. + \sum_{i=b+1}^{c}\sum_{j=1}^{b} q_{ij}\cdot (b-j)\right] + \sum_{a=1}^{b-1}\sum_{c=b+1}^{N} y_a \cdot x_c \cdot \omega^{Met}_{ticket}\cdot l^{Met}\cdot \left[\sum_{i=a}^{b-1}\sum_{j=c}^{N} q_{ij}\cdot (b-i) + \sum_{i=c}^{N}\sum_{j=a}^{b-1} q_{ij}\cdot (b-j)\right],\\
W^{Met}_{run} &= \omega^{Met}_{run}\cdot m^{Met}\cdot \left\{2f^{Met}\cdot (b-1)\cdot l^{Met} + 2f^{Met}_{cross}\cdot \left[(b-1)\cdot l^{Met} + \sum_{c=b+1}^{N} x_c \cdot (c-b)\cdot l^{Sub}\right]\right\}.
\end{aligned}
\tag{1}
$$

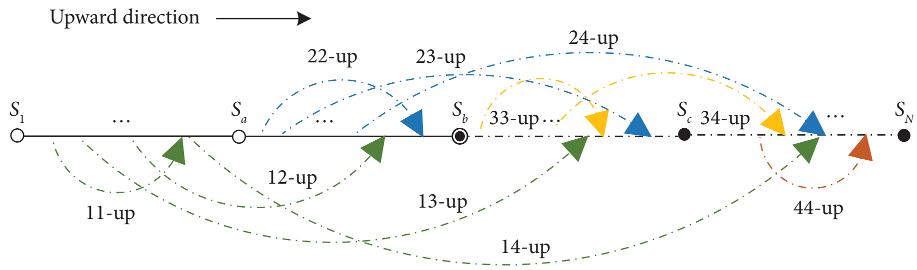

FIGURE 3: Passenger distribution in the upward direction.

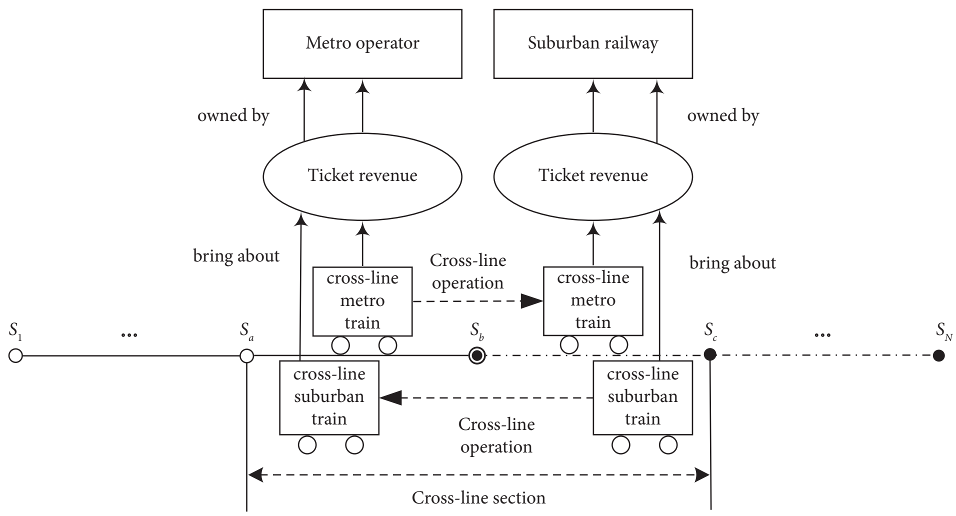

FIGURE 4: Fare clearing rules under line-sharing mode.

TABLE 2: The symbols used in the LPP model.

Symbols	Description	Role
W^{Met}	Net profit of the metro operating company	Objective
W^{Sub}	Net profit of the suburban railway operating company	Objective
T	Total travel time savings of passengers compared with that before joint operation	Objective
W^t	Cost savings converted from travel time savings	Objective
$W_{\text{run}}^{\text{Met}}$	Operating costs of the metro operating company	Intermediate variable
$W_{\text{run}}^{\text{Sub}}$	Operating costs of the suburban railway operating company	Intermediate variable
$W_{\text{ticket}}^{\text{Met}}$	Fare revenue of the metro operating company	Intermediate variable
$W_{\text{ticket}}^{\text{Sub}}$	Fare revenue of the suburban railway operating company	Intermediate variable
$T_{o,d}$	Total travel time savings of passengers from the section o to the section d, $o, d \in \{\text{I}, \text{II}, \text{III}, \text{IV}\}$ and $o \leq d$	Intermediate variable
$T_{\text{od}}^{\text{up}}$	Total travel time savings of passengers from the section o to the section d in the upward direction	Intermediate variable
$T_{\text{od}}^{\text{down}}$	Total travel time savings of passengers from the section d to the section o in the downward direction	Intermediate variable
$f^{\text{Met}'}$	Service frequency of metro trains before joint operation	Intermediate variable
$f^{\text{Sub}'}$	Service frequency of suburban trains before joint operation	Intermediate variable
$q_{e_\pi}^{\text{Met}}$	The number of passengers taking metro trains in section e_π	Intermediate variable
$q_{\text{cross},e_\pi}^{\text{Met}}$	The number of passengers taking cross-line metro trains in the section e_π	Intermediate variable
$q_{e_\pi}^{\text{Sub}}$	The number of passengers taking suburban trains in the section e_π	Intermediate variable
$q_{\text{cross},e_\pi}^{\text{Sub}}$	The number of passengers taking cross-line suburban trains in the section e_π	Intermediate variable
f^{Met}	Service frequency of metro trains after joint operation	Decision variable
f^{Sub}	Service frequency of suburban trains after joint operation	Decision variable
f_{cross}^{Met}	Service frequency of cross-line metro trains	Decision variable
$f_{\text{cross}}^{\text{Sub}}$	Service frequency of cross-line suburban trains	Decision variable
x_c	0-1 variable. If S_c is the terminal station of the section E_2 (running section of cross-line metro trains), $x_c = 1$, else $x_c = 0$	Decision variable

Table 2: Continued.

Symbols	Description	Role
y_a	0-1 variable. If S_a is the terminal station of the section E_4 (running section of cross-line suburban trains), $y_a = 1$, else $y_a = 0$	Decision variable
$\omega_{\text{run}}^{\text{Met}}$	Cost of each metro vehicle running per kilometer	Parameter
$\omega_{\text{run}}^{\text{Sub}}$	Cost of each suburban vehicle running per kilometer	Parameter
$\omega_{\text{ticket}}^{\text{Met}}$	Metro fares per kilometer of each person	Parameter
$\omega_{\text{ticket}}^{\text{Sub}}$	Suburban railway fares per kilometer of each person	Parameter
m^{Met}	Marshalling number of metro trains	Parameter
m^{Sub}	Marshalling number of suburban trains	Parameter
l^{Met}	Average station spacing of metro lines	Parameter
l^{Sub}	Average station spacing of suburban lines	Parameter
q_{ij}	Passenger volumes from the station S_i to the station S_j	Parameter
ω_{time}	Passengers' nonworking time value coefficient	Parameter
t_{transfer}	Time taken by each passenger for a single transfer	Parameter
$q_{\max}^{\text{Met}\,\prime}$	Passenger volumes of the section with the largest passenger flow in the metro line	Parameter
$q_{\max}^{\text{Sub}\,\prime}$	Passenger volumes of the section with the largest passenger flow in the suburban line	Parameter
A^{Met}	Capacity of a metro (cross-line metro) train	Parameter
A^{Sub}	Capacity of a suburban (cross-line suburban) train	Parameter
η^{Met}	Capacity utilization of the metro (cross-line metro) trains	Parameter
η^{Sub}	Capacity utilization of the suburban (cross-line suburban) trains	Parameter
M^{Met}	Number of available metro vehicles	Parameter
M^{Sub}	Number of available suburban vehicles	Parameter
$C_{\max}^{\text{Met}}$	Maximal carrying capacity of metro line	Parameter
$C_{\max}^{\text{Sub}}$	Maximal carrying capacity of suburban line	Parameter
$F_{\min}^{\text{Met}}$	Minimal departing frequency of metro and cross-line metro trains	Parameter
$F_{\min}^{\text{Sub}}$	Minimal departing frequency of suburban and cross-line suburban trains	Parameter
C_{S_i}	Through capacity of the station S_i	Parameter

Similarly, the net profit of the suburban railway operating company can be expressed as:

$$\begin{aligned}
W^{Sub} &= W_{ticket}^{Sub} - W_{run}^{Sub},\\
W_{ticket}^{Sub} &= \sum_{c=b+1}^{N} x_c \cdot \omega_{ticket}^{Sub} \cdot l^{Sub} \cdot \left[\sum_{i=1}^{b}\sum_{j=b+1}^{c} q_{ij} \cdot (j-b) + \sum_{i=b+1}^{c}\sum_{j=1}^{b} q_{ij} \cdot (i-b) + \sum_{i=b}^{N-1}\sum_{j=i+1}^{N} q_{ij} \cdot (j-i)\right.\\
&\quad \left. + \sum_{i=b+1}^{N}\sum_{j=b}^{i-1} q_{ij} \cdot (i-j)\right] + \sum_{a=1}^{b-1}\sum_{c=b+1}^{N} y_a \cdot x_c \cdot \omega_{ticket}^{Sub} \cdot l^{Sub} \cdot \left[\sum_{i=a}^{b-1}\sum_{j=c}^{N} q_{ij} \cdot (j-b) + \sum_{i=c}^{N}\sum_{j=a}^{b-1} q_{ij} \cdot (i-b)\right],\\
W_{run}^{Sub} &= \omega_{run}^{Sub} \cdot m^{Sub} \cdot \left\{2f^{Sub} \cdot (N-b) \cdot l^{Sub} + 2f_{cross}^{Sub} \cdot \left[(N-b) \cdot l^{Sub} + \sum_{a=1}^{b-1} y_a \cdot (b-a) \cdot l^{Met}\right]\right\}.
\end{aligned} \tag{2}$$

The total travel time savings can be expressed as the sum of the time savings for ten types of passenger flows. Taking the travel time savings of Class 11-up passenger flows as an example, $T_{1,1}$ consists of two parts: T_{11}^{up} and T_{11}^{down}. Total travel time includes waiting time, time on the train, and transfer time. Time on the train mainly depends on the technical speed of vehicles and the travel distance. Joint operation has little impact on this kind of time, so we only consider waiting time and transfer time. Under the condition of the short departure interval, passenger waiting time is close to half of the departure interval [34, 35]. In China, upward and downward trains typically operate in pairs, meaning that the service frequency and train types are the same in both directions. Therefore, the waiting time of passengers before joint operation can be expressed as $1/2f^{Met'}$. As for $f^{Met'}$, it can be calculated by maximum passenger volumes of each section in the metro line, which is shown in formula (6). The waiting time of passengers after joint operation can be expressed as $1/2(f^{Met} + f_{cross}^{Met})$, and there is no transfer time. Therefore, the total travel time savings of Class 11-up passenger flows are $q_{ij} \cdot [1/2f^{Met'} - 1/2(f^{Met} + f_{cross}^{Met})]$. Similarly, we can derive the travel time saving formulas of other types of passenger flows:

$$T = \sum_{o=1}^{4}\sum_{d=o}^{4} T_{o,d}, \tag{3}$$

$$\begin{aligned} T_{1,1} = T_{11}^{up} + T_{11}^{down} &= \sum_{a=1}^{b-1} y_a \cdot \sum_{i=1}^{a-1}\sum_{j=i+1}^{a} q_{ij} \cdot \left[\frac{1}{2f^{Met'}} - \frac{1}{2\left(f^{Met} + f_{cross}^{Met}\right)}\right] \\ &+ \sum_{a=1}^{b-1} y_a \cdot \sum_{i=2}^{a}\sum_{j=1}^{i-1} q_{ij} \cdot \left[\frac{1}{2f^{Met'}} - \frac{1}{2\left(f^{Met} + f_{cross}^{Met}\right)}\right], \end{aligned} \tag{4}$$

$$f^{Met'} = \frac{q_{\max}^{Met'}}{A^{Met} \cdot \eta^{Met}}, \tag{5}$$

$$\begin{aligned} T_{1,2} = T_{12}^{up} + T_{12}^{down} &= \sum_{a=1}^{b-1} y_a \cdot \sum_{i=1}^{a-1}\sum_{j=a+1}^{b} q_{ij} \cdot \left[\frac{1}{2f^{Met'}} - \frac{1}{2\left(f^{Met} + f_{cross}^{Met}\right)}\right] \\ &+ \sum_{a=1}^{b-1} y_a \cdot \sum_{i=a+1}^{b}\sum_{j=1}^{a-1} q_{ij} \cdot \left[\frac{1}{2f^{Met'}} - \frac{1}{2\left(f^{Met} + f_{cross}^{Met}\right)}\right], \end{aligned} \tag{6}$$

$$\begin{aligned} T_{1,3} = T_{13}^{up} + T_{13}^{down} &= \sum_{a=1}^{b-1}\sum_{c=b+1}^{N} y_a \cdot x_c \cdot \sum_{i=1}^{a-1}\sum_{j=b+1}^{c} q_{ij} \cdot \left[\left(\frac{1}{2f^{Met'}} + t_{transfer} + \frac{1}{2f^{Sub'}}\right) - \frac{1}{2f_{cross}^{Met}}\right] \\ &+ \sum_{a=1}^{b-1}\sum_{c=b+1}^{N} y_a \cdot x_c \cdot \sum_{i=b+1}^{c}\sum_{j=1}^{a-1} q_{ij} \cdot \left[\left(\frac{1}{2f^{Met'}} + t_{transfer} + \frac{1}{2f^{Sub'}}\right) - \frac{1}{2f_{cross}^{Met}}\right], \end{aligned} \tag{7}$$

$$f^{Sub'} = \frac{q_{\max}^{Sub\,\prime}}{A^{Sub} \cdot \eta^{Sub}}, \tag{8}$$

$$\begin{aligned} T_{1,4} = T_{14}^{up} + T_{14}^{down} &= \sum_{a=1}^{b-1}\sum_{c=b+1}^{N} y_a \cdot x_c \cdot \sum_{i=1}^{a-1}\sum_{j=c+1}^{N} q_{ij} \cdot \left\{\left(\frac{1}{2f^{Met'}} + \frac{1}{2f^{Sub'}}\right) - \left[\frac{1}{2\left(f_{cross}^{Met} + f^{Met}\right)} + \frac{1}{2\left(f_{cross}^{Sub} + f^{Sub}\right)}\right]\right\} \\ &+ \sum_{a=1}^{b-1}\sum_{c=b+1}^{N} y_a \cdot x_c \cdot \sum_{i=c+1}^{N}\sum_{j=1}^{a-1} q_{ij} \cdot \left\{\left(\frac{1}{2f^{Met'}} + \frac{1}{2f^{Sub'}}\right) - \left[\frac{1}{2\left(f_{cross}^{Met} + f^{Met}\right)} + \frac{1}{2\left(f_{cross}^{Sub} + f^{Sub}\right)}\right]\right\}, \end{aligned} \tag{9}$$

$$\begin{aligned} T_{2,2} = T_{22}^{up} + T_{22}^{down} &= \sum_{a=1}^{b-1} y_a \cdot \sum_{i=a}^{b-1}\sum_{j=i+1}^{b} q_{ij} \cdot \left[\frac{1}{2f^{Met'}} - \frac{1}{2\left(f_{cross}^{Met} + f^{Met} + f_{cross}^{Sub}\right)}\right] \\ &+ \sum_{a=1}^{b-1} y_a \cdot \sum_{i=a+1}^{b}\sum_{j=a}^{i-1} q_{ij} \cdot \left[\frac{1}{2f^{Met'}} - \frac{1}{2\left(f_{cross}^{Met} + f^{Met} + f_{cross}^{Sub}\right)}\right], \end{aligned} \tag{10}$$

$$\begin{aligned} T_{2,3} = T_{23}^{up} + T_{23}^{down} &= \sum_{a=1}^{b-1}\sum_{c=b+1}^{N} y_a \cdot x_c \cdot \sum_{i=a}^{b-1}\sum_{j=b+1}^{c} q_{ij} \cdot \left[\left(\frac{1}{2f^{Met'}} + t_{transfer} + \frac{1}{2f^{Sub'}}\right) - \frac{1}{2\left(f_{cross}^{Met} + f_{cross}^{Sub}\right)}\right] \\ &+ \sum_{a=1}^{b-1}\sum_{c=b+1}^{N} y_a \cdot x_c \cdot \sum_{i=b+1}^{c}\sum_{j=a}^{b-1} q_{ij} \cdot \left[\left(\frac{1}{2f^{Met'}} + t_{transfer} + \frac{1}{2f^{Sub'}}\right) - \frac{1}{2\left(f_{cross}^{Met} + f_{cross}^{Sub}\right)}\right], \end{aligned} \tag{11}$$

$$T_{2,4} = T_{24}^{up} + T_{24}^{down} = \sum_{a=1}^{b-1}\sum_{c=b+1}^{N} y_a \cdot x_c \cdot \sum_{i=a}^{b-1}\sum_{j=c+1}^{N} q_{ij} \cdot \left[\left(\frac{1}{2f^{Met'}} + t_{transfer} + \frac{1}{2f^{Sub'}}\right) - \frac{1}{2f_{cross}^{Sub}}\right] + \sum_{a=1}^{b-1}\sum_{c=b+1}^{N} y_a \cdot x_c \cdot \sum_{i=c+1}^{N}\sum_{j=a}^{b-1} q_{ij} \cdot \left[\left(\frac{1}{2f^{Met'}} + t_{transfer} + \frac{1}{2f^{Sub'}}\right) - \frac{1}{2f_{cross}^{Sub}}\right], \tag{12}$$

$$T_{3,3} = T_{33}^{up} + T_{33}^{down} = \sum_{c=b+1}^{N} x_c \cdot \sum_{i=b}^{c-1}\sum_{j=i+1}^{c} q_{ij} \cdot \left[\frac{1}{2f^{Sub'}} - \frac{1}{2\left(f_{cross}^{Met} + f^{Sub} + f_{cross}^{Sub}\right)}\right] + \sum_{c=b+1}^{N} x_c \cdot \sum_{i=b+1}^{c}\sum_{j=b}^{i-1} q_{ij} \cdot \left[\frac{1}{2f^{Sub'}} - \frac{1}{2\left(f_{cross}^{Met} + f^{Sub} + f_{cross}^{Sub}\right)}\right], \tag{13}$$

$$T_{3,4} = T_{34}^{up} + T_{34}^{down} = \sum_{c=b+1}^{N} x_c \cdot \sum_{i=b}^{c-1}\sum_{j=c+1}^{N} q_{ij} \cdot \left[\frac{1}{2f^{Sub'}} - \frac{1}{2\left(f^{Sub} + f_{cross}^{Sub}\right)}\right] + \sum_{c=b+1}^{N} x_c \cdot \sum_{i=c+1}^{N}\sum_{j=b}^{c-1} q_{ij} \cdot \left[\frac{1}{2f^{Sub'}} - \frac{1}{2\left(f^{Sub} + f_{cross}^{Sub}\right)}\right], \tag{14}$$

$$T_{4,4} = T_{44}^{up} + T_{44}^{down} = \sum_{c=b+1}^{N} x_c \cdot \sum_{i=c}^{N-1}\sum_{j=i+1}^{N} q_{ij} \cdot \left[\frac{1}{2f^{Sub'}} - \frac{1}{2\left(f^{Sub} + f_{cross}^{Sub}\right)}\right] + \sum_{c=b+1}^{N} x_c \cdot \sum_{i=c+1}^{N}\sum_{j=c}^{i-1} q_{ij} \cdot \left[\frac{1}{2f^{Sub'}} - \frac{1}{2\left(f^{Sub} + f_{cross}^{Sub}\right)}\right]. \tag{15}$$

2.5. Integration of Objective Functions. Since the measuring unit and the order of magnitude of T are different from those of W^{Met} and W^{Sub}, we introduce the time value coefficient ω_{time} to convert time into cost:

$$W^t = \omega_{time} \cdot T. \tag{16}$$

Owing to the difficulty of solving a multiobjective problem directly, we intend to use the linear weighted sum method to set different weights for the objective functions, which can merge multiple objectives into one. Φ_1 and Φ_2 are weight coefficients of optimization objectives W^{Met} and W^{Sub}, respectively. The values of two coefficients are between (0, 1). $(1 - \Phi_1 - \Phi_2)$ is the weight coefficient of the optimization objective W^t. The minimum and maximum values of three objective functions are calculated in advance; then, the min-max normalization method is used to unify the order of magnitude. W is the general objective after transformation:

$$\max W = \Phi_1 \cdot \frac{W^{Met} - W_{\min}^{Met}}{W_{\max}^{Met} - W_{\min}^{Met}} + \Phi_2 \cdot \frac{W^{Sub} - W_{\min}^{Sub}}{W_{\max}^{Sub} - W_{\min}^{Sub}} + \left(1 - \Phi_1 - \Phi_2\right) \cdot \frac{W^t - W_{\min}^t}{W_{\max}^t - W_{\min}^t}. \tag{17}$$

2.6. Constraints of the Line-Planning Model

2.6.1. Constraint of Passenger Flow. The design of line planning should follow the principle of aligning train operations with passenger flow volume. Consequently, meeting the needs of passengers is the primary condition. For metro trains, the following formulas shall be met:

$$
q_{e_\pi}^{Met} = \begin{cases}
\sum_{1\le i\le \pi}\sum_{\pi<j\le b}\dfrac{f^{Met}}{f^{Met}+f_{cross}^{Met}}\cdot q_{ij}+\sum_{1\le i\le \pi}\sum_{c<j\le N}\dfrac{3f^{Met}}{3f^{Met}+5f_{cross}^{Met}}\cdot q_{ij} & e_\pi\in \text{I, upward direction,}\\[2ex]
\sum_{\pi<i\le b}\sum_{1\le j\le \pi}\dfrac{f^{Met}}{f^{Met}+f_{cross}^{Met}}\cdot q_{ij}+\sum_{c<i\le N}\sum_{1\le j\le \pi}\dfrac{3f^{Met}}{3f^{Met}+5f_{cross}^{Met}}\cdot q_{ij} & e_\pi\in \text{I, downward direction,}\\[2ex]
\sum_{1\le i<a}\sum_{\pi<j\le b}\dfrac{f^{Met}}{f^{Met}+f_{cross}^{Met}}\cdot q_{ij}+\sum_{a\le i\le \pi}\sum_{\pi<j\le b}\dfrac{f^{Met}}{f^{Met}+f_{cross}^{Met}+f_{cross}^{Sub}}\cdot q_{ij} & \\[2ex]
+\sum_{1\le i<a}\sum_{c<j\le N}\dfrac{f^{Met}}{f^{Met}+2f_{cross}^{Met}+f_{cross}^{Sub}}\cdot q_{ij} & e_\pi\in \text{II, upward direction,}\\[2ex]
\sum_{\pi<i\le b}\sum_{1\le j<a}\dfrac{f^{Met}}{f^{Met}+f_{cross}^{Met}}\cdot q_{ij}+\sum_{\pi<i\le b}\sum_{a\le j\le \pi}\dfrac{f^{Met}}{f^{Met}+f_{cross}^{Met}+f_{cross}^{Sub}}\cdot q_{ij} & \\[2ex]
+\sum_{c<i\le N}\sum_{1\le j<a}\dfrac{f^{Met}}{f^{Met}+2f_{cross}^{Met}+f_{cross}^{Sub}}\cdot q_{ij} & e_\pi\in \text{II, downward direction,}
\end{cases} \tag{18}
$$

$$
q_{e_\pi}^{Met}\le f^{Met}\cdot A^{Met}\cdot \eta^{Met}, e_\pi\in\{\text{I, II}\}. \tag{19}
$$

Taking $e_\pi \in$ II, the upward direction is taken as an example to explain the above formula. The passenger flows taking subway trains in this section are shown in Figure 5, including ① passenger flows from a station in section I to a station (the position of this station is located at station $S_{\pi+1}$ or behind $S_{\pi+1}$) in section II, ② passenger flows from a station (the position of this station is located at station S_π or before S_π) in section II to another station (the position of this station is located at station $S_{\pi+1}$ or behind $S_{\pi+1}$) in section II, and ③ passenger flows from a station in section I to a station in section IV. According to the assumption (3), a direct train between two stations is always the first priority for passengers comparing to a transfer train between two stations. It can be seen that the passenger flows from a station in section I to a station in section III, from a station in section II to a station in section III, and from a station in section II to a station in section IV will only choose to take cross-line trains. Therefore, these passenger flows are not included in $q_{e_\pi}^{Met}$.

The assumptions indicate that cross-line trains and noncross-line trains are composed of the same type of vehicles. Passengers only have a preference for choosing between direct or transfer options and do not show a preference for any specific train itself. During the study period, passengers arrive at the station evenly, and they typically opt to board the first train that arrives after they reach the station, as long as it can take them to their destination. Therefore, we can calculate the amount of passenger flows taking metro trains or cross-line metro trains based on the ratio of operating frequency.

Class ① passenger flows may choose to take metro trains or cross-line metro trains. Therefore, Class ① passenger flows taking metro trains in the section e_π are $\sum_{1\le i<a}\sum_{\pi<j\le b}[f^{Met}/(f^{Met}+f_{cross}^{Met})]\cdot q_{ij}$. Class ② passenger flows can choose to take metro trains, cross-line metro trains, or cross-line suburban trains. Therefore, Class ② passenger flows taking metro train in the section e_π are $\sum_{a\le i\le \pi}\sum_{\pi<j\le b}[f^{Met}/(f^{Met}+f_{cross}^{Met}+f_{cross}^{Sub})]\cdot q_{ij}$. There are three different transfer schemes for Class ③ passenger flows, namely, transfer at the station S_a, transfer at the station S_b, and transfer at the station S_c.

Passengers who transfer at the station S_a have the option to either take regular metro trains or cross-line metro trains to reach the station S_a and then transfer to cross-line suburban trains to reach their destination. The passenger transfer scheme diagram at the station S_a is shown in Figure 6; for the sake of contrast, cross-line trains and noncross-line trains in the figure are represented by different icons, but they use the same type of vehicles in practical terms.

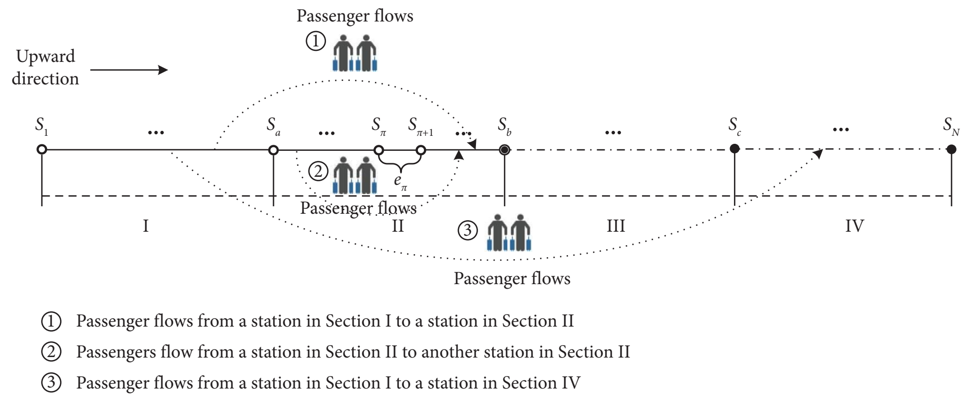

Figure 5: Schematic diagram of the passenger flow composition.

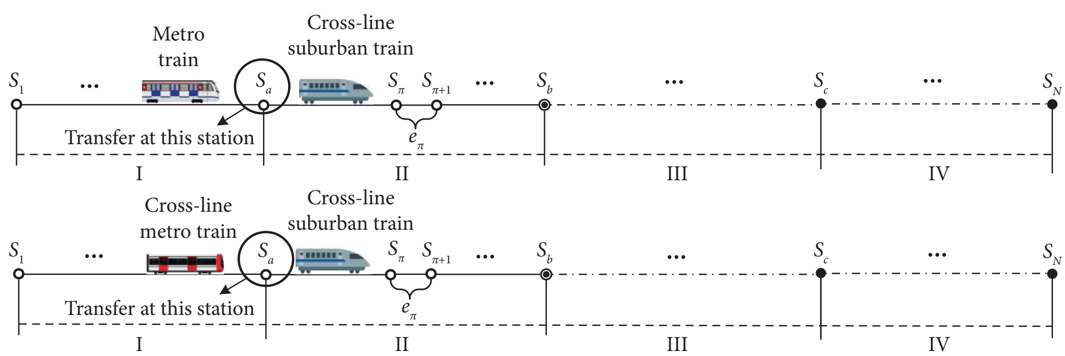

Figure 6: Transfer scheme diagram of station S_a.

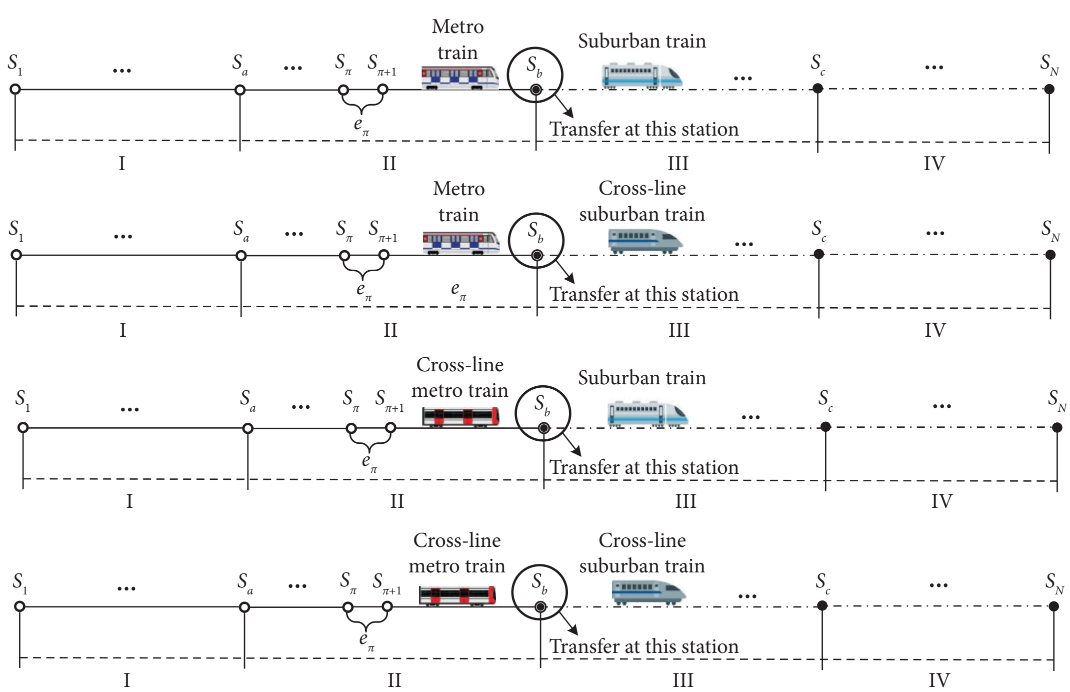

Figure 7: Transfer scheme diagram of station S_b.

Those who transfer at the station S_b may take metro trains or cross-line metro trains to the transfer station S_b and then board suburban trains or cross-line suburban trains to their destination. The transfer scheme diagram of the station S_b is shown in Figure 7.

We can clearly see the transfer scheme of the station S_c from Figure 8. Passengers who transfer at the station S_c can only board cross-line metro trains in the section e_π, and they will subsequently transfer to suburban trains or cross-line suburban trains at the station S_c.

Therefore, Class ③ passenger flows taking metro trains in the section e_π are $\sum_{1\leq i<a}\sum_{c<j\leq N}[2f^{Met}/(2f^{Met}+4f^{Met}_{cross}+2f^{Sub}_{cross})]\cdot q_{ij}$. After the reduction of a fraction, we can get $\sum_{1\leq i<a}\sum_{c<j\leq N}[f^{Met}/(f^{Met}+2f^{Met}_{cross}+f^{Sub}_{cross})]\cdot q_{ij}$.

Similarly, the following formulas shall be met, respectively, as shown in equations (24)~(29):

$$q^{Met}_{cross,e_\pi}=\begin{cases}\sum\limits_{1\leq i\leq\pi}\sum\limits_{\pi<j\leq b}\dfrac{f^{Met}_{cross}}{f^{Met}+f^{Met}_{cross}}\cdot q_{ij}+\sum\limits_{1\leq i\leq\pi}\sum\limits_{b<j\leq c}q_{ij}+\sum\limits_{1\leq i\leq\pi}\sum\limits_{c<j\leq N}\dfrac{5f^{Met}_{cross}}{3f^{Met}+5f^{Met}_{cross}}\cdot q_{ij} & e_\pi\in\text{I, upward direction,}\\[2ex]
\sum\limits_{\pi<i\leq b}\sum\limits_{1\leq j\leq\pi}\dfrac{f^{Met}_{cross}}{f^{Met}+f^{Met}_{cross}}\cdot q_{ij}+\sum\limits_{b<i\leq c}\sum\limits_{1\leq j\leq\pi}q_{ij}+\sum\limits_{c<i\leq N}\sum\limits_{1\leq j\leq\pi}\dfrac{5f^{Met}_{cross}}{3f^{Met}+5f^{Met}_{cross}}\cdot q_{ij} & e_\pi\in\text{I, downward direction,}\\[2ex]
\sum\limits_{1\leq i<a}\sum\limits_{\pi<j\leq b}\dfrac{f^{Met}_{cross}}{f^{Met}+f^{Met}_{cross}}\cdot q_{ij}\sum\limits_{1\leq i<a}\sum\limits_{b<j\leq c}q_{ij}+\sum\limits_{1\leq i<a}\sum\limits_{c<j\leq N}\dfrac{2f^{Met}_{cross}}{f^{Met}+2f^{Met}_{cross}+f^{Sub}_{cross}}\cdot q_{ij} & \\[2ex]
+\sum\limits_{a\leq i\leq\pi}\sum\limits_{\pi<j\leq b}\dfrac{f^{Met}_{cross}}{f^{Met}+f^{Met}_{cross}+f^{Sub}_{cross}}\cdot q_{ij}+\sum\limits_{a\leq i\leq\pi}\sum\limits_{b<j\leq c}\dfrac{f^{Met}_{cross}}{f^{Met}_{cross}+f^{Sub}_{cross}}\cdot q_{ij} & e_\pi\in\text{II, upward direction,}\\[2ex]
\sum\limits_{\pi<i\leq b}\sum\limits_{1\leq j<a}\dfrac{f^{Met}_{cross}}{f^{Met}+f^{Met}_{cross}}\cdot q_{ij}+\sum\limits_{b<i\leq c}\sum\limits_{1\leq j<a}q_{ij}+\sum\limits_{c<i\leq N}\sum\limits_{1\leq j<a}\dfrac{2f^{Met}_{cross}}{f^{Met}+2f^{Met}_{cross}+f^{Sub}_{cross}}\cdot q_{ij} & \\[2ex]
+\sum\limits_{\pi<i\leq b}\sum\limits_{a\leq j\leq\pi}\dfrac{f^{Met}_{cross}}{f^{Met}+f^{Met}_{cross}+f^{Sub}_{cross}}\cdot q_{ij}+\sum\limits_{b<i\leq c}\sum\limits_{a\leq j\leq\pi}\dfrac{f^{Met}_{cross}}{f^{Met}_{cross}+f^{Sub}_{cross}}\cdot q_{ij} & e_\pi\in\text{II, downward direction,}\\[2ex]
\sum\limits_{1\leq i<a}\sum\limits_{\pi<j\leq c}q_{ij}+\sum\limits_{1\leq i<a}\sum\limits_{c<j\leq N}\dfrac{f^{Met}_{cross}}{f^{Met}_{cross}+f^{Sub}+2f^{Sub}_{cross}}\cdot q_{ij}+\sum\limits_{a\leq i<b}\sum\limits_{\pi<j\leq c}\dfrac{f^{Met}_{cross}}{f^{Met}_{cross}+f^{Sub}_{cross}}\cdot q_{ij} & \\[2ex]
+\sum\limits_{b\leq i\leq\pi}\sum\limits_{\pi<j\leq c}\dfrac{f^{Met}_{cross}}{f^{Met}_{cross}+f^{Sub}+f^{Sub}_{cross}}\cdot q_{ij} & e_\pi\in\text{III, upward direction,}\\[2ex]
\sum\limits_{\pi<i\leq c}\sum\limits_{1\leq j<a}q_{ij}+\sum\limits_{c<i\leq N}\sum\limits_{1\leq j<a}\dfrac{f^{Met}_{cross}}{f^{Met}_{cross}+f^{Sub}+2f^{Sub}_{cross}}\cdot q_{ij}+\sum\limits_{\pi<i\leq c}\sum\limits_{a\leq j<b}\dfrac{f^{Met}_{cross}}{f^{Met}_{cross}+f^{Sub}_{cross}}\cdot q_{ij} & \\[2ex]
+\sum\limits_{\pi<i\leq c}\sum\limits_{b\leq j\leq\pi}\dfrac{f^{Met}_{cross}}{f^{Met}_{cross}+f^{Sub}+f^{Sub}_{cross}}\cdot q_{ij} & e_\pi\in\text{III, downward direction,}\end{cases}\tag{20}$$

$$q^{Met}_{cross,e_\pi}\leq f^{Met}_{cross}\cdot A^{Met}\cdot\eta^{Met},e_\pi\in\{\text{I, II, III}\}\tag{21}$$

$$q_{e_\pi}^{Sub} = \begin{cases} \sum\limits_{1\le i<a}\sum\limits_{c<j\le N}\dfrac{f^{Sub}}{f^{Sub}+2f_{cross}^{Sub}+f_{cross}^{Met}}\cdot q_{ij}+\sum\limits_{b\le i\le\pi}\sum\limits_{\pi<j\le c}\dfrac{f^{Sub}}{f^{Sub}+f_{cross}^{Sub}+f_{cross}^{Met}}\cdot q_{ij} \\ +\sum\limits_{b\le i\le\pi}\sum\limits_{c<j\le N}\dfrac{f^{Sub}}{f^{Sub}+f_{cross}^{Sub}}\cdot q_{ij} \quad e_\pi\in \text{III, upward direction,} \\ \sum\limits_{c<i\le N}\sum\limits_{1\le j<a}\dfrac{f^{Sub}}{f^{Sub}+2f_{cross}^{Sub}+f_{cross}^{Met}}\cdot q_{ij}+\sum\limits_{\pi<i\le c}\sum\limits_{b\le j\le\pi}\dfrac{f^{Sub}}{f^{Sub}+f_{cross}^{Sub}+f_{cross}^{Met}}\cdot q_{ij} \\ +\sum\limits_{c<i\le N}\sum\limits_{b\le j\le\pi}\dfrac{f^{Sub}}{f^{Sub}+f_{cross}^{Sub}}\cdot q_{ij} \quad e_\pi\in \text{III, downward direction,} \\ \sum\limits_{1\le i<a}\sum\limits_{\pi<j\le N}\dfrac{3f^{Sub}}{3f^{Sub}+5f_{cross}^{Sub}}\cdot q_{ij}+\sum\limits_{b\le i\le\pi}\sum\limits_{\pi<j\le N}\dfrac{f^{Sub}}{f^{Sub}+f_{cross}^{Sub}}\cdot q_{ij} \quad e_\pi\in \text{IV, upward direction,} \\ \sum\limits_{\pi<i\le N}\sum\limits_{1\le j<a}\dfrac{3f^{Sub}}{3f^{Sub}+5f_{cross}^{Sub}}\cdot q_{ij}+\sum\limits_{\pi<i\le N}\sum\limits_{b\le j\le\pi}\dfrac{f^{Sub}}{f^{Sub}+f_{cross}^{Sub}}\cdot q_{ij} \quad e_\pi\in \text{IV, downward direction,} \end{cases} \tag{22}$$

$$q_{e_\pi}^{Sub} \le f^{Sub}\cdot A^{Sub}\cdot \eta^{Sub}, e_\pi\in\{\text{III, IV}\},$$

$$q_{cross,e_\pi}^{Sub} = \begin{cases} \sum\limits_{1\le i<a}\sum\limits_{c<j\le N}\dfrac{f_{cross}^{Sub}}{f^{Met}+2f_{cross}^{Met}+f_{cross}^{Sub}}\cdot q_{ij}+\sum\limits_{a\le i\le\pi}\sum\limits_{\pi<j\le b}\dfrac{f_{cross}^{Sub}}{f^{Met}+f_{cross}^{Met}+f_{cross}^{Sub}}\cdot q_{ij} \\ +\sum\limits_{a\le i\le\pi}\sum\limits_{b<j\le c}\dfrac{f_{cross}^{Sub}}{f_{cross}^{Sub}+f_{cross}^{Met}}\cdot q_{ij}+\sum\limits_{a\le i\le\pi}\sum\limits_{c<j\le N}q_{ij} \quad e_\pi\in \text{II, upward direction,} \\ \sum\limits_{c<i\le N}\sum\limits_{1\le j<a}\dfrac{f_{cross}^{Sub}}{f^{Met}+2f_{cross}^{Met}+f_{cross}^{Sub}}\cdot q_{ij}+\sum\limits_{\pi<i\le b}\sum\limits_{a\le j\le\pi}\dfrac{f_{cross}^{Sub}}{f^{Met}+f_{cross}^{Met}+f_{cross}^{Sub}}\cdot q_{ij} \\ +\sum\limits_{b<i\le c}\sum\limits_{a\le j\le\pi}\dfrac{f_{cross}^{Sub}}{f_{cross}^{Sub}+f_{cross}^{Met}}\cdot q_{ij}+\sum\limits_{c<i\le N}\sum\limits_{a\le j\le\pi}q_{ij} \quad e_\pi\in \text{II, downward direction,} \\ \sum\limits_{1\le i<a}\sum\limits_{c<j\le N}\dfrac{2f_{cross}^{Sub}}{f^{Sub}+2f_{cross}^{Sub}+f_{cross}^{Met}}\cdot q_{ij}+\sum\limits_{a\le i<b}\sum\limits_{\pi<j\le c}\dfrac{f_{cross}^{Sub}}{f_{cross}^{Sub}+f_{cross}^{Met}}\cdot q_{ij}+\sum\limits_{a\le i<b}\sum\limits_{c<j\le N}q_{ij} \\ +\sum\limits_{b\le i\le\pi}\sum\limits_{\pi<j\le c}\dfrac{f_{cross}^{Sub}}{f^{Sub}+f_{cross}^{Met}+f_{cross}^{Sub}}\cdot q_{ij}+\sum\limits_{b\le i\le\pi}\sum\limits_{c<j\le N}\dfrac{f_{cross}^{Sub}}{f^{Sub}+f_{cross}^{Sub}}\cdot q_{ij} \quad e_\pi\in \text{III, upward direction,} \\ \sum\limits_{c<i\le N}\sum\limits_{1\le j<a}\dfrac{2f_{cross}^{Sub}}{f^{Sub}+2f_{cross}^{Sub}+f_{cross}^{Met}}\cdot q_{ij}+\sum\limits_{\pi<i\le c}\sum\limits_{a\le j<b}\dfrac{f_{cross}^{Sub}}{f_{cross}^{Sub}+f_{cross}^{Met}}\cdot q_{ij}+\sum\limits_{c<i\le N}\sum\limits_{a\le j<b}q_{ij} \\ +\sum\limits_{\pi<i\le c}\sum\limits_{b\le j\le\pi}\dfrac{f_{cross}^{Sub}}{f^{Sub}+f_{cross}^{Met}+f_{cross}^{Sub}}\cdot q_{ij}+\sum\limits_{c<i\le N}\sum\limits_{b\le j\le\pi}\dfrac{f_{cross}^{Sub}}{f^{Sub}+f_{cross}^{Sub}}\cdot q_{ij} \quad e_\pi\in \text{III, downward direction,} \\ \sum\limits_{1\le i<a}\sum\limits_{\pi<j\le N}\dfrac{5f_{cross}^{Sub}}{3f^{Sub}+5f_{cross}^{Sub}}\cdot q_{ij}+\sum\limits_{a\le i<b}\sum\limits_{\pi<j\le N}q_{ij}+\sum\limits_{b\le i\le\pi}\sum\limits_{\pi<j\le N}\dfrac{f_{cross}^{Sub}}{f^{Sub}+f_{cross}^{Sub}}\cdot q_{ij} \quad e_\pi\in \text{IV, upward direction,} \\ \sum\limits_{\pi<i\le N}\sum\limits_{1\le j<a}\dfrac{5f_{cross}^{Sub}}{3f^{Sub}+5f_{cross}^{Sub}}\cdot q_{ij}+\sum\limits_{\pi<i\le N}\sum\limits_{a\le j<b}q_{ij}+\sum\limits_{\pi<i\le N}\sum\limits_{b\le j\le\pi}\dfrac{f_{cross}^{Sub}}{f^{Sub}+f_{cross}^{Sub}}\cdot q_{ij} \quad e_\pi\in \text{IV, downward direction,} \end{cases} \tag{24}$$

$$q_{cross,e_\pi}^{Sub} \le f_{cross}^{Sub}\cdot A^{Sub}\cdot \eta^{Sub}, e_\pi\in\{\text{II, III, IV}\}. \tag{25}$$

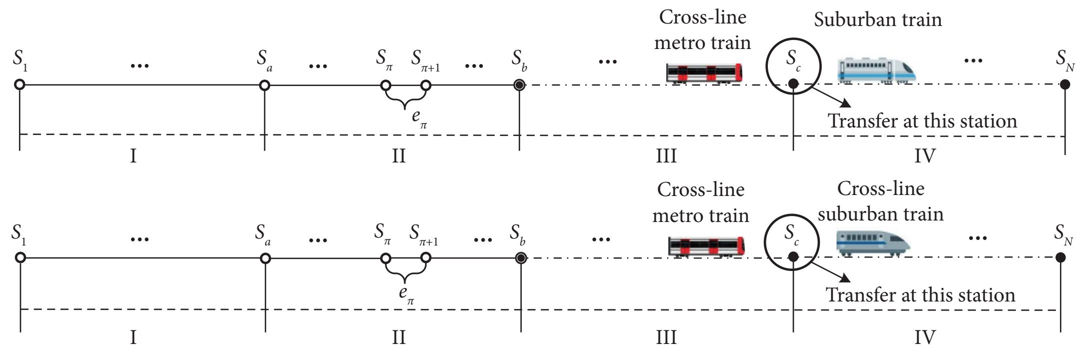

FIGURE 8: Transfer scheme diagram of the station S_c.

2.6.2. Constraint of Available Vehicles. The operated vehicles should be less than the total available vehicles:

$$\left(f^{Met} + f^{Met}_{cross}\right) \cdot m^{Met} \le M^{Met}, \tag{26}$$

$$\left(f^{Sub} + f^{Sub}_{cross}\right) \cdot m^{Sub} \le M^{Sub}. \tag{27}$$

2.6.3. Constraint of Line-Carrying Capacity. During the considered period (one hour), the service frequency of trains must not exceed the line-carrying capacity:

$$f^{Met} + f^{Met}_{cross} + f^{Sub}_{cross} \le C^{Met}_{\max}, \tag{28}$$

$$f^{Sub} + f^{Sub}_{cross} + f^{Met}_{cross} \le C^{Sub}_{\max}. \tag{29}$$

2.6.4. Constraint of Train Service Frequency. According to the relevant specifications, service frequency of different kinds of trains shall not be lower than the minimum departure frequency [36]:

$$f^{Met} + f^{Met}_{cross} \ge F^{Met}_{\min}, \tag{30}$$

$$f^{Sub} + f^{Sub}_{cross} \ge F^{Sub}_{\min}. \tag{31}$$

2.6.5. Constraint of the Station Capacity. The passing capacity of stations cannot be exceeded:

$$f^{Met} + f^{Met}_{cross} \le C_{S_i}, \quad i \in \{1, \cdots, a-1\}, \tag{32}$$

$$f^{Met} + f^{Met}_{cross} + f^{Sub}_{cross} \le C_{S_i}, \quad i \in \{a, \cdots, b-1\}, \tag{33}$$

$$f^{Sub} + f^{Sub}_{cross} + f^{Met}_{cross} \le C_{S_i}, \quad i \in \{b, \cdots, c-1\}, \tag{34}$$

$$f^{Sub} + f^{Sub}_{cross} \le C_{S_i}, \quad i \in \{c, \cdots, N\}. \tag{35}$$

2.6.6. Constraint of Cross-Line Setting. Location values of the station S_a and station S_c are unique:

$$\sum_{c=b+1}^{N} x_c = 1, \tag{36}$$

$$x_c \in \{0, 1\}, \quad \forall c \in \{b+1, \cdots, N\}, \tag{37}$$

$$\sum_{a=1}^{b-1} y_a = 1, \tag{38}$$

$$y_a \in \{0, 1\}, \quad \forall a \in \{1, \cdots, b-1\}. \tag{39}$$

2.6.7. Constraint of the Variable Value. The values of the decision variables are natural numbers:

$$f^{Met}, f^{Sub}, f^{Met}_{cross}, f^{Sub}_{cross} \in \mathbf{N}. \tag{40}$$

3. Algorithm Design

LPP has been proven to be an NP-hard problem, even for a single rail line [37–39]. The above mathematical model is nonlinear and hard to be solved in a short time using the existing software. Therefore, we employ a heuristic algorithm to tackle the problem.

The traditional artificial bee colony (TABC) algorithm, initially proposed by Karaboga in 2005, simulates the behavior of bees in honey collection. It offers advantages such as simple implementation, wide applicability, and strong robustness [40]. The TABC algorithm demonstrates superior efficiency compared to other swarm intelligence algorithms, and it is less prone to becoming trapped at locally optimal values [41]. Therefore, we select the artificial bee colony algorithm to solve the line-planning problem and improve the algorithm to achieve higher efficiency.

3.1. Algorithm Fundamentals. The traditional artificial bee colony (TABC) algorithm mainly involves four elements: scout bees, employed bees, onlooker bees, and food sources. The food source represents the feasible solution, and the quality of the food source can be assessed by the fitness function value in the algorithm. Employed bees are in charge of the food source search in the neighborhood of every food source domain and the comparison between the current food source and the previous one with a greedy criterion, which means accepting the better one all the time. There is some information exchange between employed bees and onlooker bees after the neighborhood search. Onlooker bees can either choose to follow the employed bees with a certain probability to search the new food source in the domain, where they will convert to employed bees, or they can stay unchanged. When a food source cannot be improved after multiple continuous searches by the employed bees, scout bees intervene. Scout bees will search new food sources to replace the old ones outside the searching neighborhood.

The fitness function designed in this paper is

$$G = W - \sigma \cdot P^2, \tag{41}$$

where W represents the general objective, σ represents the penalty coefficient, P represents the penalty term when violating constraint formulas (18)–(40), and G is the fitness value. The larger the fitness value, the better the solution.

3.2. Initialization of Food Sources. A set of solution vectors, denoted as a food source $V = [f^{Met}, f^{Sub}, f^{Met}_{cross}, f^{Sub}_{cross}, a, c]$, is randomly generated. These vectors consist of the train service frequency and the location of stations S_a and S_c and must comply with constraint formulas (18)–(40). Suppose there are $nPop$ food sources, the specific location of the food source V is determined according to the following equation, where V^i_d represents the d-th solution component of the i-th food source and U_d and L_d represent the upper and lower bounds of the traversal, respectively, that is, the maximum and minimum values of the d-th solution component:

$$V^i_d = L_d + rand(0,1) \cdot (U_d - L_d). \tag{42}$$

3.3. Renewal and Improvement Strategy of IABC. Employed bees search for new food sources around the i-th food source according to formula (43), where a is the acceleration coefficient, θ is the random number uniformly distributed in the interval [−1, 1], and V^j is the neighborhood food source of V^i. According to the greedy criterion, the one with better fitness value will always be selected. Otherwise, the original one will not be replaced:

$$V^{new}_d = V^i_d + a \cdot \theta \cdot \left(V^i_d - V^j_d\right), \quad j \neq i. \tag{43}$$

The improved roulette strategy in formula (44) is used to calculate the probability that onlooker bees follow employed bees to gather honey. After the onlooker bee selects a certain food source, it adopts the same neighborhood search strategy as the employed bee. When the onlooker bee finds a better food source, it swaps roles with the employed bee, and the original food source is replaced with the new one:

$$p_i = 1 - \frac{G_i}{\sum_{i=1}^{nPop} G_i}. \tag{44}$$

In the TABC algorithm, if the iterative number of times $trial(i)$ reaches the threshold $I_{\max}$ and the food source has not been updated yet, then this food source V^i is abandoned and the scout bee looks for a new food source. The formula for the location of the new food source is

$$V^{new}_d = \begin{cases} L_d + rand(0,1) \cdot (U_d - L_d) & trial(i) \geq I_{\max}, \\ V^i_d \quad trial(i) < I_{\max}. \end{cases} \tag{45}$$

In order to improve the efficiency of global search, the current global optimal solution can be used to guide the generation of new solutions, thereby effectively developing the solution space information near the new solution. When $trial(i) \geq I_{\max}$, a food source V^{new} is generated according to formula (45). Then, the scout bees begin search for the new food source $V^{new'}$ using the information from the current globally optimal food source V^{best} with a probability p_0. Afterward, the fitness values of the newly discovered food source $V^{new'}$ and V^{new} are compared, and the better solution is retained:

$$V^{new'}_d = V^{best}_d + \theta \cdot \left(V^{best}_d - V^{new}_d\right). \tag{46}$$

3.4. Solving Process of IABC. The solution steps are as follows, and a more visual description is provided in Figure 9:

Step 1: We import the basic data and set the parameter values of the algorithm, which include the number of food sources $nPop$, current search times of each food source $trial(i)$, maximum search number for a single food source $I_{\max}$, maximum iteration number $R_{\max}$, current iteration number $Cycle$, and the probability p_0 of the global optimal solution guiding the generation of new solutions. Let $trial(i) = 0$ and $Cycle = 0$.

Step 2: We initialize the food source using formula (42), calculate the fitness value of the solution, and then record the initial global optimal solution.

Step 3: Employed bees search for new food sources using formula (43). Once a new solution has been found, its fitness value is calculated. If the new solution has a better fitness value, then it replaces the original food source, and we make $trial(i) = 0$ after the replacement; otherwise, there is no replacement, and let $trial(i) = trial(i) + 1$.

Step 4: Onlooker bees follow employed bees using the improved roulette strategy in search of new food sources. Onlooker bees choose to follow employed bees according to the improved roulette strategy in search of new food sources. If a new solution has a better fitness value, the original food source is replaced by the new one, and we make $trial(i) = 0$ after the replacement; otherwise, there is no replacement, and let $trial(i) = trial(i) + 1$.

Step 5: If the fitness values of a given food source have not been improved during $I_{\max}$ searches, this food source is abandoned. At this point, scout bees are dispatched to search for new food sources randomly. New solutions are generated using formula (45), and then learned from the global optimal solution with probability p_0. If the fitness value has been improved within $I_{\max}$ search cycles, it continues searching around this food source.

Step 6: We update the global optimal solution, make $Cycle = Cycle + 1$, and judge whether $Cycle \geq R_{\max}$. If it reaches the maximum iteration number, then it terminates the cycle and outputs the optimal solution. Otherwise, we jump to Step3.

4. Computing Case and Result Analysis

4.1. Basic Data. The line schematic is shown in Figure 10. The average station spacing of the metro line is 1.5 km, with a total length of 22.5 km. For the suburban line, the average station spacing is 3 km, with a total length of 30 km. The set of metro stations is $S_{Met} = \{S_1, S_2, \cdots, S_{16}\}$, and the set of suburban railway stations is $S_{Sub} = \{S_{16}, S_{13}, \cdots, S_{26}\}$. S_{16} is the cross-line-station, which connects two lines.

Most parameter values are given based on the practical experience or referring to the data in the existing studies [9, 10, 36, 40, 41]. The parameters of the model are shown in Table 3.

The forecasted passenger OD (origination and destination) flows are shown in Table 4. After a large number of experiments and calculation, we determine the optimal parameters for the algorithm: the number of food sources $nPop = 30$, the maximum search number for a single food source $I_{\max} = 10$, the maximum iteration number of the algorithm $R_{\max} = 2000$, and the probability of using the global optimal solution to guide the generation of new solutions $p_0 = 0.4$.

4.2. Sensitivity Analysis of Objective Function Weight. To conduct a detailed sensitivity analysis on the weight of the objective function, we enumerate all possible weights for each of the three objective functions at intervals of 0.1, and the algorithm is implemented in Visual C#. The comparative analysis results are presented in Table 5. Taking (1, 1, 8) as an example, the meaning of this weight allocation is as follows: $\Phi_1 = 0.1$, $\Phi_2 = 0.1$, and $1 - \Phi_1 - \Phi_2 = 0.8$. As can be seen from Table 5, the line-sharing operation mode can lead to significant savings in travel time. Furthermore, when the weight coefficient of one optimization goal increases, the value of this objective function shows an increasing trend. However, due to the limited solution space of the problem and many constraints, the objective function will no longer change once it reaches a certain value, and this is further described in Figure 11.

4.3. Analysis of Cross-Line's Terminal Station Setting. To analyze the influence of the cross-line's terminal station setting, we standardized the weight of the objective functions using $\Phi_1 = 0.3$, $\Phi_2 = 0.3$, and $(1 - \Phi_1 - \Phi_2) = 0.4$ as an example. From Table 6, we can readily conclude that different terminal stations of the cross-line exert significant influence on the objective function value. Therefore, it is essential to reasonably set the location of the cross-line's terminal station in advance.

When $\Phi_1 = 0.3$, $\Phi_2 = 0.3$, and $(1 - \Phi_1 - \Phi_2) = 0.4$, the optimal crossing-line routing scheme diagram can be described in Figure 12. In the figure, different line shapes represent the service routings of different types of trains, and the operating frequency of each service route is also shown in Figure 12.

4.4. Calculation Results of Different Algorithms. We use the improved artificial bee colony (IABC) algorithm to solve the line-planning problem in this paper. IABC is derived from improving the updating strategy of food sources based on the traditional artificial bee colony (TABC) algorithm. To validate the effectiveness of the proposed improvement measures in addressing the line-planning problems within the research scenario presented in this paper, the TABC algorithm is employed as a comparative benchmark. In addition, the simulated annealing (SA) algorithm is a very classic heuristic algorithm, and it is widely used in solving the line-planning problems [31, 34]. Therefore, the SA algorithm is also used as the comparison group, see Section 3.1. Algorithm Fundamentals for the basic principle of TABC. The basic principle of SA is as follows.

In 1982, Kirkpatrick introduced the idea of annealing to combinatorial optimization problems and proposed the simulated annealing (SA) algorithm as a solution for tackling large-scale combinatorial optimization problems [42]. Temperature is used as a control parameter in the algorithm. The internal energy E is analogous to the value of the objective function. In the process of cooling and annealing, a solution exists for every temperature. As temperature continuously declines, the value of the objective function also changes, and the local optimal solution is constantly searched in this process. Finally, as the internal energy decreases to the minimum, the global optimal solution for the problem is found.

The concrete ideas of SA can be summarized as follows. First, we set any feasible solution as the initial solution for the problem. Then, we generate the neighborhood solution according to certain criteria. In this paper, a new neighborhood solution is generated by randomly perturbing the current solution vector. Finally, the algorithm decides whether to accept the current or neighborhood solution based on the Metropolis criterion.

The Metropolis criterion is an important sampling method proposed by Metropolis in 1953. Suppose that E_i represents the internal energy of the current state i, E_j represents the internal energy of the updated new state j, and if $E_j < E_i$, then we accept the new state j as the current state; otherwise, we accept the new state j with the probability p. The probability p can be calculated according to the following equation:

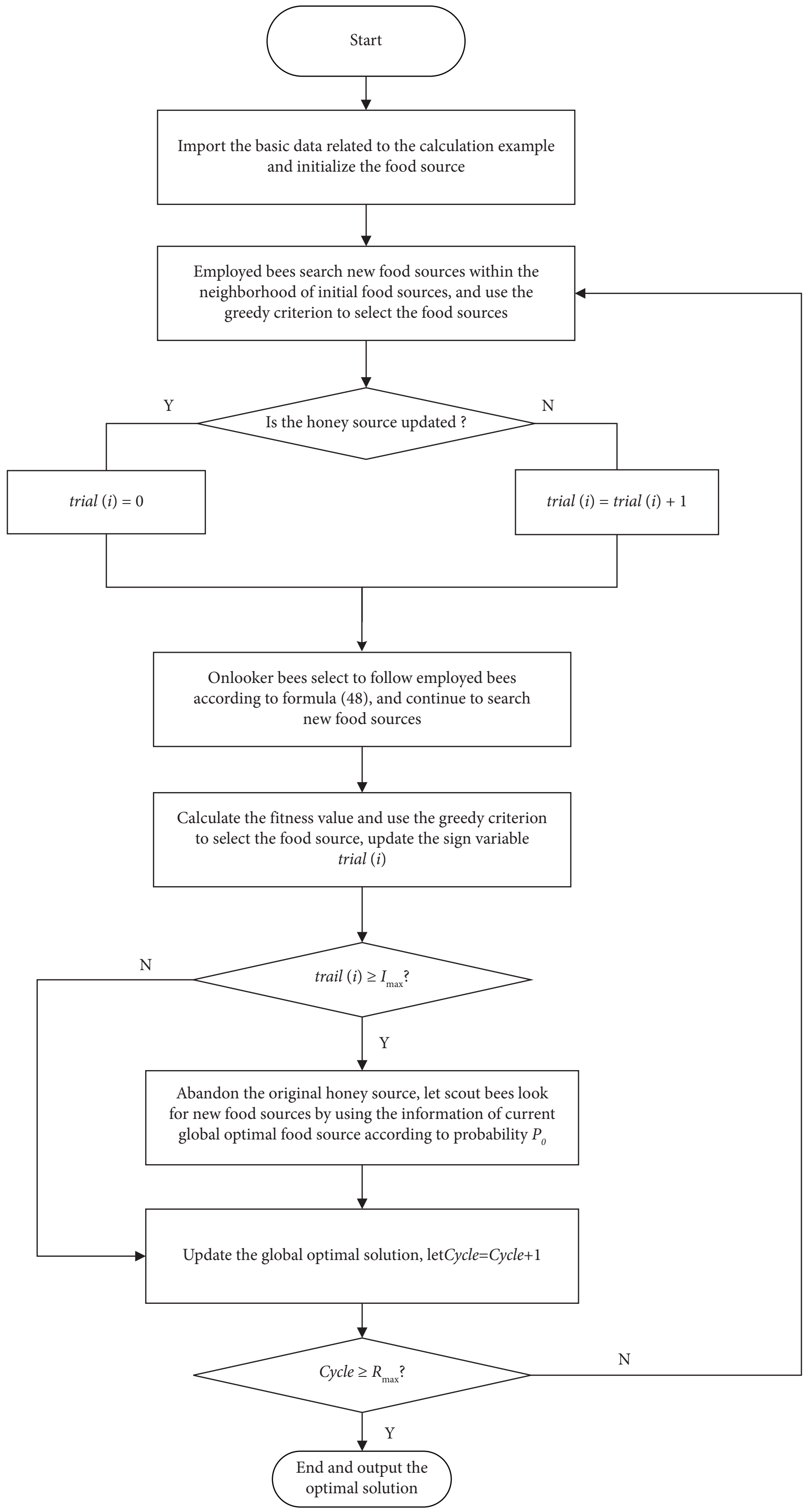

Figure 9: Flowchart of IABC.

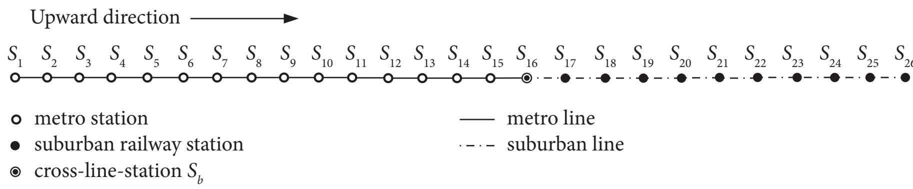

Figure 10: Line schematic in the example.

Table 3: Parameters of the model.

Parameters	Value	Measurement unit
ω_{time}	10	Yuan/person·hour
ω_{run}^{Met}	30	Yuan/vehicle·km
ω_{ticket}^{Met}	0.3	Yuan/person·km
ω_{run}^{Sub}	45	Yuan/vehicle·km
ω_{ticket}^{Sub}	0.4	Yuan/person·km
m^{Met}	6	Vehicle
m^{Sub}	6	Vehicle
A^{Met}	1860	Person
A^{Sub}	1460	Person
η^{Met}	90%	—
η^{Sub}	75%	—
M^{Met}	180	Vehicle
M^{Sub}	160	Vehicle
$t_{transfer}$	8	min
$C_{\max}^{Met}$	30	Pair/hour
$C_{\max}^{Sub}$	24	Pair/hour
$F_{\min}^{Met}$	6	Pair/hour
$F_{\min}^{Sub}$	6	Pair/hour
C_{S_i}	30	Pair/hour

Table 4: The passenger OD matrix (upward direction) during the study period.

OD	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	—	0	1	1	4	3	9	12	10	34	64	39	41	54	51	13	19	21	24	16	19	18	8	13	12	15
2	—	—	6	3	6	18	15	52	43	91	195	161	191	208	119	104	243	228	154	130	164	240	225	121	127	106
3	—	—	—	3	9	11	15	24	20	60	116	75	273	276	274	180	132	85	125	176	147	97	137	215	191	217
4	—	—	—	—	0	3	14	0	6	25	34	21	29	18	15	13	13	17	21	8	5	6	10	5	8	16
5	—	—	—	—	—	6	2	1	7	26	36	75	72	54	51	6	25	38	32	64	38	28	12	40	34	40
6	—	—	—	—	—	—	1	9	11	45	72	64	56	33	35	27	39	46	48	41	48	41	45	44	38	45
7	—	—	—	—	—	—	—	7	22	57	97	108	140	144	151	144	151	97	121	131	152	151	122	104	96	117
8	—	—	—	—	—	—	—	—	2	25	52	52	148	158	153	188	113	146	135	104	166	88	135	126	79	102
9	—	—	—	—	—	—	—	—	—	9	43	74	47	45	42	96	92	105	105	125	84	104	85	106	108	92
10	—	—	—	—	—	—	—	—	—	—	35	118	267	197	159	116	139	135	136	215	104	94	72	96	86	96
11	—	—	—	—	—	—	—	—	—	—	—	49	271	179	275	122	240	235	160	137	205	103	97	105	77	110
12	—	—	—	—	—	—	—	—	—	—	—	—	7	105	157	202	85	192	155	189	194	199	115	39	55	87
13	—	—	—	—	—	—	—	—	—	—	—	—	—	5	7	0	16	109	89	66	93	43	128	5	4	18
14	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6	25	4	12	0	12	55	25	81	4	5	10
15	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10	2	10	6	8	7	4	15	31	21	55
16	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8	0	45	30	70	39	106	3	4	8
17	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6	7	3	4	0	4	17	15	54
18	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	24	11	19	12	31	3	4	10
19	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4	5	2	6	6	30	18
20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3	1	13	1	1	2
21	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0	24	0	0	4
22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	27	1	2	2
23	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0	7	17
24	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3	5
25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

Table 5: Comparative analyses of different weights.

Weight	S_a	S_c	f^{Met}	f^{Sub}	f^{Met}_{cross}	f^{Sub}_{cross}	W^{Met}	W^{Sub}	T	W
(1, 1, 8)	7	26	3	1	9	5	102398.4	190625.4	211507.1	0.948324
(1, 2, 7)	8	26	4	5	10	1	74097.0	212927.4	211451.8	0.944113
(1, 3, 6)	8	26	4	5	10	1	74097.0	212927.4	211451.8	0.943053
(1, 4, 5)	8	26	4	5	10	1	74097.0	212927.4	211451.8	0.941993
(1, 5, 4)	8	26	4	5	10	1	74097.0	212927.4	211451.8	0.940934
(1, 6, 3)	8	26	4	5	10	1	74097.0	212927.4	211451.8	0.939874
(1, 7, 2)	8	26	4	5	10	1	74097.0	212927.4	211451.8	0.938814
(1, 8, 1)	8	26	4	5	10	1	74097.0	212927.4	211451.8	0.937754
(2, 1, 7)	7	26	6	1	7	6	115898.4	169745.4	211473.2	0.913511
(2, 2, 6)	7	26	3	1	9	5	102398.4	190625.4	211507.1	0.905349
(2, 3, 5)	7	26	3	1	9	5	102398.4	190625.4	211507.1	0.899916
(2, 4, 4)	8	26	4	5	10	1	74097.0	212927.4	211451.8	0.897224
(2, 5, 3)	8	26	4	5	10	1	74097.0	212927.4	211451.8	0.896164
(2, 6, 2)	8	26	4	5	10	1	74097.0	212927.4	211451.8	0.895104
(2, 7, 1)	8	26	4	5	10	1	74097.0	212927.4	211451.8	0.894044
(3, 1, 6)	1	23	5	1	1	21	247157.4	−224384.4	212989.4	0.908556
(3, 2, 5)	7	26	6	1	7	6	115898.4	169745.4	211473.2	0.869967
(3, 3, 4)	7	26	3	1	9	5	102398.4	190625.4	211507.1	0.862374
(3, 4, 3)	7	26	3	1	9	5	102398.4	190625.4	211507.1	0.856941
(3, 5, 2)	7	26	3	1	9	5	102398.4	190625.4	211507.1	0.851508
(3, 6, 1)	8	26	4	5	10	1	74097.0	212927.4	211451.8	0.850335
(4, 1, 5)	1	23	5	1	1	21	247157.4	−224384.4	212989.4	0.907335
(4, 2, 4)	7	26	6	1	7	6	115898.4	169745.4	211473.2	0.835905
(4, 3, 3)	7	26	6	1	7	6	115898.4	169745.4	211473.2	0.826424
(4, 4, 2)	7	26	3	1	9	5	102398.4	190625.4	211507.1	0.819398
(4, 5, 1)	7	26	3	1	9	5	102398.4	190625.4	211507.1	0.813966
(5, 1, 4)	1	23	5	1	1	21	247157.4	−224384.4	212989.4	0.906113
(5, 2, 3)	1	23	5	1	1	21	247157.4	−224384.4	212989.4	0.819060
(5, 3, 2)	7	26	6	1	7	6	115898.4	169745.4	211473.2	0.792361
(5, 4, 1)	7	26	6	1	7	6	115898.4	169745.4	211473.2	0.782880
(6, 1, 3)	1	23	5	1	1	21	247157.4	−224384.4	212989.4	0.904891
(6, 2, 2)	1	23	5	1	1	21	247157.4	−224384.4	212989.4	0.817838
(6, 3, 1)	7	26	6	1	7	6	115898.4	169745.4	211473.2	0.758298
(7, 1, 2)	1	23	5	1	1	21	247157.4	−224384.4	212989.4	0.903669
(7, 2, 1)	1	23	5	1	1	21	247157.4	−224384.4	212989.4	0.816616
(8, 1, 1)	1	19	5	1	1	21	250292.4	−254138.4	212975.6	0.903076

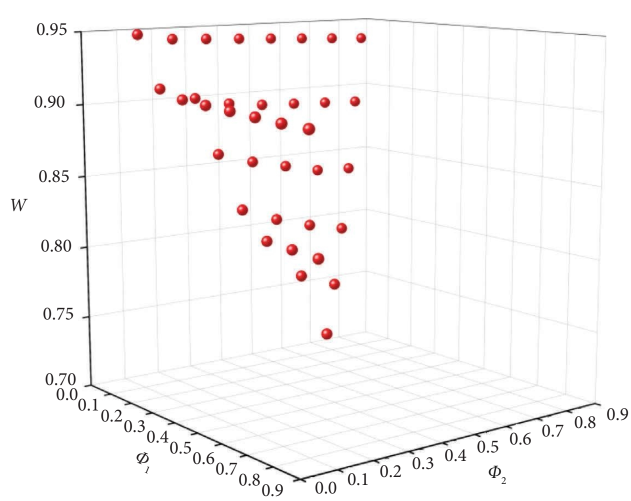

Figure 11: Diagram of objective function changing with weight values.

Table 6: Computing results under different terminal station settings.

S_a	S_c	f^{Met}	f^{Sub}	f^{Met}_{cross}	f^{Sub}_{cross}	W^{Met}	W^{Sub}	T	W
7	22	5	1	7	10	129800.4	23997.6	185918.0	0.731032
7	23	1	1	10	8	115571.4	90023.4	194272.3	0.776640
7	24	8	1	6	8	120873.6	93753.0	200127.7	0.795499
7	25	6	1	7	7	118104.6	131463.0	206277.7	0.828636
7	26	3	1	9	5	102398.4	190625.4	211507.1	0.862374
8	26	8	4	8	2	79497.0	207167.4	211480.6	0.854365
9	26	2	5	13	1	32320.2	208859.4	211845.9	0.819849
10	26	6	5	12	1	18379.2	207689.4	212038.3	0.808855
11	26	4	5	13	1	14883.6	204431.4	212025.1	0.804235

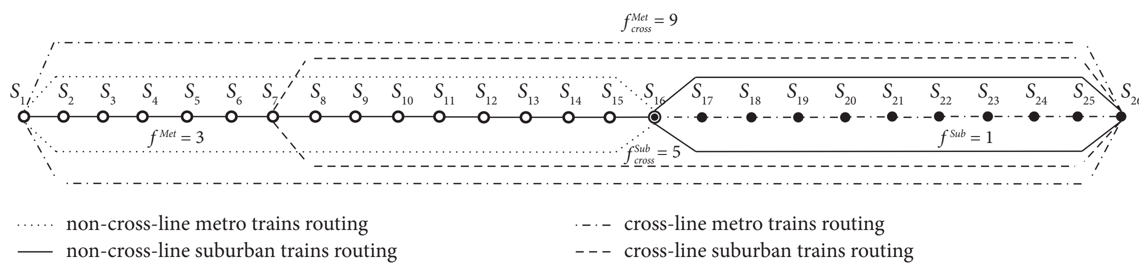

Figure 12: The optimal routing scheme.

Table 7: Comparative analysis of different algorithms.

Algorithms	Iteration times	S_a	S_c	f^{Met}	f^{Sub}	f^{Met}_{cross}	f^{Sub}_{cross}	W^{Met}	W^{Sub}	T	W
IABC	700	7	26	3	1	9	5	102398.4	190625.4	211507.1	0.862374
TABC	880	7	26	6	1	7	6	115898.4	169745.4	211473.2	0.860486
SA	1020	8	26	8	4	8	2	79497.0	207167.4	211480.6	0.854365

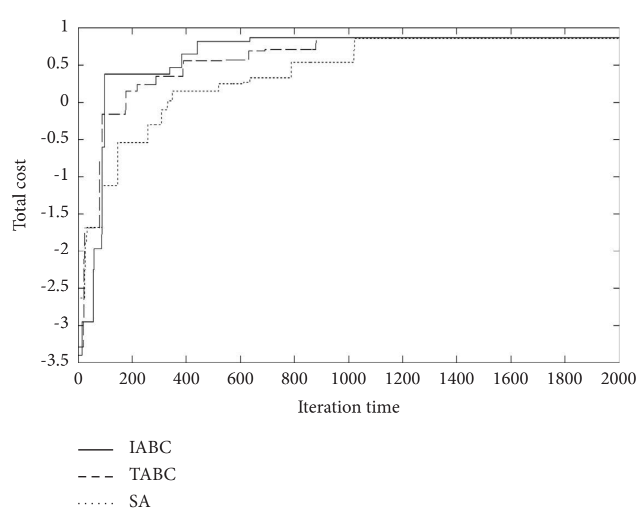

Figure 13: Iterative process diagram of three algorithms.

$$p = \exp\left[\frac{-\left(E_j - E_i\right)}{k \cdot \text{Temperature}}\right], \quad (47)$$

where k is the Boltzmann constant and ε is randomly generated on the interval (0, 1). If $p > \varepsilon$, the new state j is accepted; otherwise, it is discarded.

In addition, other related parameters required in SA can be obtained through repeated calculations: (1) initial temperature $\text{Temperature}_0 = 10^6$, (2) decreasing function of temperature $\text{Temperature}_{g+1} = \beta \cdot \text{Temperature}_g$, let $\beta = 0.95$, (3) iterations limit at each temperature limit = 10, and (4) final temperature $\text{Temperature}_e = 10^{-6}$. The inner loop is repeatedly executed until the number of iterations at the current temperature reaches a certain number of times, after which the temperature is reduced according to the formula $\text{Temperature}_{g+1} = \beta \cdot T\text{emperature}_g$. These steps are repeated until the temperature drops to Temperature_e, and the optimal solution is output.

TABC and SA are implemented using C# programming on the same computer to solve the model in this paper. We also take the weight (3, 3, 4) as an example, and the calculation results are shown in Table 7.

As it can be seen from Table 7, IABC has higher solution efficiency and quality than TABC and SA. Compared with TABC, W increases from 0.860486 to 0.862374 and solution efficiency increases by 20.45%. Compared with SA, W increases from 0.854365 to 0.862374 and solution efficiency increases by 31.37%. The iterative processes of the three algorithms are shown in Figure 13. This figure demonstrates that IABC has a better solution quality and convergence rate. IABC can better adapt to the characteristics of this line plan model, and the improvement strategy for TABC is effective.

However, the above analysis of solution efficiency and quality is limited to the model of this paper, as it depends on the algorithm implementations and problems tackled. With this study, it cannot be shown that the artificial bee colony algorithm is always better than the simulated annealing algorithm in any study scenario.

5. Conclusion

Line planning of rail transit systems is a critical issue in transportation management, affecting key operational elements such as train timetabling, rolling stock planning, and crew planning. As joint operations between different rail transit systems become more prevalent, preparing line plans under joint operating conditions and allocating transportation resources efficiently have become pressing issues for transit enterprises to address at present.

The proposed approach in this paper can solve the line-planning problem under the operation mode of line sharing between metro and suburban railway, distribute the OD flow on the metro line and suburban line, and determine the location of cross-line stations. The objective and constraints of the line plan model describe the problem precisely, and the improved artificial bee colony algorithm can solve the model efficiently. The proposed approach may be helpful to solve the problem of transportation resources sharing and improve transportation efficiency and service quality. However, we only consider the line-sharing operation between metro and suburban railway lines, and passengers are unidirectional and static. The line plan under other different rail transit systems joint operation mode with networked transportation routes and dynamic passenger flows can be further studied in the future.

Data Availability

The data that support the findings of this study are available from the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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