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*Research Article*

# Vertical Equity Analysis of Parking Reservation Based on the Auction Strategy

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As an on-demand mobility service, parking reservations can greatly alleviate the issue of parking challenges. There are currently three primary strategies for parking reservation: first-come-first-served, permit reservation, and auction. In contrast to the first-come-first-served and permit-reserved strategies, the auction strategy uses dynamic pricing to allocate parking supplies efficiently based on the auction, which attracts more scholars for the research. However, parking reservations based on the auction process may have an inequity issue because drivers' age, gender, income level, and location of residence fluctuate. This inequity may limit the growth of reserved parking by influencing parking drivers' acceptance of reserved parking. But currently, very few scholars focus on the issue of reserved parking equity, and even fewer measure this nebulous and personal issue. In consideration of this, the Lorenz curve of parking reservation and the vertical equity index of parking reservation are proposed in this paper along with the calculation method for the index, which enables the problem of reserved parking vertical equity to be visualized and made concrete. The numerical experimental method is used to analyze the vertical equity of drivers with varying income levels, utilizing the Vickrey–Clarke–Groves (VCG) auction process as an example. According to the research, loss-averse drivers are more than gain-neutral and gain-seeking drivers when the income levels of the drivers using reserved parking are the same. With the increasing number of high-income drivers involved in the parking reservation, medium to low-income drivers would lose their chances of successful reservations because of their uncompetitive bid price which leads to inequity issues when the number is less than the number of parking spaces. In contrast, the vertical equity index changes more for gain-seeking drivers while being generally steady when loss-averse and gain-neutral drivers participate. For instance, when the profit and loss coefficient is 3 and 15% of drivers with high-income levels use the parking reservation platform, the vertical equity index rises from 0.09128 to 0.45434. The reference price has a moderating influence on the vertical equity index when the number of driver participants at high-income levels remains constant. In general, within a reasonable range, the higher the reference price, the more equitable the parking reservation procedure and the lower the vertical equity index.

## 1. Introduction

The rate of urbanization has been accelerating as China's economy continues to strengthen. Based on the statistical data, China's rate of urbanization increased by 31.94% between 2008 and 2022, from 33.28% [1] to 65.22% [2]. The notable progress in urbanization has resulted in a substantial surge in automobile ownership. Research indicates that an additional 1.2 to 1.5 parking spaces are required for each new automobile to accommodate the increased demand for parking in cities [3]. Based on this forecast, China may be 80 million parking spaces short, which is the root cause of the problem of parking difficulties. Furthermore, the problem of parking difficulty is made worse by the fact that car parks and parking drivers fail to share information, making it challenging for drivers to locate suitable and empty parking spaces quickly [4]. Transportation congestion [5, 6] caused by parking issues also contributes to environmental pollution [7].

Reducing carbon emissions has emerged as a major area of transportation-related scientific interest in recent years [8–10]. Afterward, intelligent parking emerged as a solution to lessen carbon emissions and ease parking issues. On the one hand, academics suggested using the sharing theory to make private parking spaces public and increase the effectiveness of how residential areas' parking facilities are used. This was accomplished by offering a staggered, paid shared parking service, which initially solved the issue of parking challenges [11–15]. On the other hand, an increasing number of academics approach the problem of parking difficulties from the standpoint of parking reservation, with the goal of reducing its severity through optimal parking resource allocation and raising the rate at which parking spaces in public parking lots are filled [16]. Inaba et al. introduced the idea of parking reservation, and the first parking reservation system was put forward [17]. Subsequently, scholars have conducted comprehensive studies on the construction of intelligent parking systems [18–21], the response to demand disturbances [22–25], and the behavior of reserved parking [26–31], which enables safe and efficient reserved parking.

The effectiveness and equity of the parking reservation process are determined by the parking reservation strategy. The first-come-first-served strategy served as the foundation of early research studies on reserved parking strategies [32, 33]. "First-come-first-served" is a strategy that allocates parking spaces according to the order of drivers' reservation, with the first reserved first served. The classic parking lot charging system, in which the parking fee is set and decided by the parking lot itself, is preserved with the first-reserved-first-served reservation system. Parking attendants are thus limited to passively accepting the car park's pricing structure and are unable to negotiate a better rate for their spaces. Meanwhile, the first-come-first-served strategy, which does not take into account the individual characteristics or actions of drivers when it comes to parking, does not guarantee that drivers who have an immediate need for a space will be granted priority in securing that space, nor does it meet the needs of various drivers for parking spaces.

Furthermore, numerous academics have examined the topic of parking permit management [34–37] since the suggestion of a tradable parking credit management model [38]. Nevertheless, the issue of exorbitant parking rates has not been resolved. To solve the parking cost issue, Xu and Sun developed an enhanced parking permit reservation model that does away with the drawbacks of the conventional fee-based and permit reservation models, such as low parking space utilization, lengthy search times, and parking uncertainty [39]. Nevertheless, the model has implementation issues and ignores the effect that pricing variations have on parking drivers.

Scholars have focused on parking reservation strategies based on the auction in light of the issues with first-come-first-served and parking permit reservation strategies in reserved parking [40–47]. Drivers can use the auction reservation strategy to select their desired parking spaces and apply for reservations to the reserved parking platform based on their willingness to pay. The highest bidder will eventually be awarded the perfect parking space. The auction reservation strategy also offers high market efficiency, transparency, and revenue for the reserved parking platform, allowing for greater awareness of the true value and scarcity of parking spaces and the realization of their maximum value. Specifically, the VCG auction strategy maximizes the overall benefits of the reserved parking platform by taking into account not only the interests of individual bidding drivers but also the final price paid by each successful reservation driver from the standpoint of the bidding drivers collectively. Despite achieving efficient reserved parking, the auction reservation strategy ignores the equity issue during the practical application.

As a result of variations in the drivers' ages, genders, economic levels, and places of residency, among other factors, it could lead to inequity issues. Assuming that the bidding price of drivers is positively correlated with their income level, the success rate of low-income residents in making reservations will continue to decline as the participation rate of high-income drivers continues to increase. This seriously undermines the interests of low-income level drivers and triggers inequity, which will also lead to poorer social welfare. At the same time, Viegas points out that equity is a crucial sign of popular acceptance [48]. Therefore, this inequity may affect the acceptance and participation rates of drivers in parking reservations and further constrain the sustainable development of parking reservations. However, when it comes to research on reserved parking, particularly research on reserved parking based on the auction, the majority of academics concentrate on the effectiveness of reserved parking, with very few researching its equity. Therefore, ensuring the equity of the reserved parking strategy is crucial for developing a sustainable transportation system and will play a significant role in future intelligent parking.

In the context of congestion charges, the question of equity has been extensively researched. Research on congestion pricing equity is mostly split into two areas. On the one hand, there are empirical studies that are based on public approval [49–51]. On the other hand, researchers mostly use the Gini coefficient and Lorenz curve to analyze the equity of congestion charge in equity evaluation research. Sumalee et al. included the Gini coefficient as a constraint in the optimization model to analyze the spatial equity of congestion charging [52]. Fridstrøm et al. analyzed the congestion charging equity problem by plotting the Lorenz curve and calculating the Gini coefficient [53]. In addition, Wang and Yu introduced the Lorenz curve and the Gini coefficient into the evaluation of traffic equity at the same time for the first time [54]. Wu and Cao further used the Gini coefficient to carry out in-depth empirical analyses and investigated the problem of traffic equity in the City of Zibo [55].

As previously stated, the auction reservation strategy may result in inequity even though it offers the benefits of a clear method, high market efficiency, and high revenue for the reserved parking platform. However, equity is a crucial determinant of public acceptance and influences drivers' acceptance and involvement in parking reservations, which limits the long-term growth of the auction reservation strategy. Therefore, this paper researches the equity of the VCG auction reservation strategy.

With reference to the definition of the Gini coefficient and the Lorenz curve, this paper proposes evaluation indicators for vertical equity of reserved parking and gives steps for analyzing vertical equity of parking reservation. It is noteworthy that this paper realizes the quantification and visualization of the equity issue of reserved parking for the first time. Subsequently, this paper analyzes the vertical equity issue of reserved parking by taking the VCG auction reservation strategy as an example. In the numerical experiments, the following questions are discussed. (1) What is the impact of rising high-income driver participation on medium to low-income drivers' reservation success rate when it comes to reserved parking? (2) What is the impact on the equity of reserved parking? (3) What kind of fluctuations may drivers with medium to low income expect in the ultimate price they pay? (4) What adjustments will be made to the reserved parking platform's overall revenue? Three categories of drivers are distinguished among those who take part in the bidding process: loss-averse, loss-neutral, and loss-seeking drivers. (5) How do different types of drivers affect the vertical equity of parking reservation? (6) Does vertical equity change with reference price?

This paper is organized as follows. In Section 2, the VCG auction reservation strategy model is explained, along with the auction reservation process and an analysis of the utility of various parking driver types. The proposed vertical equity evaluation index of reserved strategy and its calculation method combines the Lorenz curve and the Gini coefficient. In Section 3, the vertical equity index is calculated through numerical experiments. Additionally, the impact of various driver types on vertical equity is analyzed, along with parking reservation by varying percentages of high-income drivers who have access to the parking reservation platform. In Section 4, the research conclusions are compiled and further research avenues are suggested.

## 2. Vertical Equity Analysis and Modelling of Parking Reservation

Revenue from the parking reservation process is a representation of the platform's benefits. Therefore, the decline in revenue would lead to inequity and would be detrimental to the platform. The utility represents the benefits of drivers, and drivers with different income levels obtain different utilities, while a decrease in utility is equally inequity to drivers. Therefore, this paper examines the vertical equity of reserved parking under the effect of drivers with varying income levels, taking into account the reserved parking platform and drivers as a whole. It does this by utilizing the Gini coefficient. Section 2.1 introduces the parking space allocation model based on the VCG auction reservation strategy. Section 2.2 outlines the VCG auction reservation process and the driver-paid price model. Section 2.3 analyzes the parking utility of different types of drivers. Section 2.4 proposes the vertical equity index based on the Gini coefficient researching the vertical equity of parking reservation and summarizes the calculation method of the vertical equity index.

*2.1. Parking Space Allocation Model Based on the VCG Auction.* Assume that the parking reservation platform contains $M$ parks, and each park $m \in M$ provides $N$ parking spaces. Each parking space $n_i \in N$ can be divided into $T$ nonoverlapping time slots in a day. The number of drivers who need to reserve a parking space in a park $m$ is $D$. Each driver $d_m \in D$ needs to submit the reservation application to the platform at least one day in advance, which contains one or more parking spaces $n_i$ participating in a bidding process, the required consecutive time slots $t_i$, and the bidding price corresponding to each parking space $v_i(n_i^{t_i})$. Meanwhile, $P_m$ denotes the set of feasible allocation schemes for a park $m \in M$, where $P_m = \{p_{m1}, p_{m2}, \cdots, p_{mn}\}$.

In addition, it is assumed that all the parking spaces in the park are homogeneous and all drivers reserve the same space in this paper, each driver bids randomly within the bidding interval according to income level, and a driver is allowed to bid for more than one parking space at the same time.

Each driver in the auction reservation process satisfies the assumption of a limited rational actor, i.e., each driver pursues self-interested rationality or individual rationality, on the assumption of insufficient information, limited rationality, and cognitive uncertainty [56]. In other words, every driver will apply for a reservation based on how much they truly value the parking space, and every driver aims to optimize their own interest in the variety of parking spaces that are available. The model for allocating parking spaces is shown below.

$$\max \sum_{d_m \in D} \sum_{n_i \in N} v_{d_m}\left(n_i^{t_i}\right) x_{d_m}\left(n_i^{t_i}\right), \tag{1}$$

$$\text{s.t.} \sum_{n_i \in N} x_{d_m}\left(n_i^{t_i}\right) \leq 1, \quad \forall d_m \in D, \tag{2}$$

$$\sum_{p_m \in P_m} \varphi\left(p_m\right) = 1, \tag{3}$$

$$x_{d_m}\left(n_i^{t_i}\right) - \sum_{p_{n_i^{t_i}} = n_i^{t_i}} \varphi\left(p_m\right) = 0, \quad \forall d_m \in D, \tag{4}$$

$$x_{d_m}\left(n_i^{t_i}\right) \in \{0, 1\}, \tag{5}$$

$$\varphi\left(p_m\right) \in \{0, 1\}. \tag{6}$$

The objective of the parking space allocation model is to maximize the revenue of the reserved parking platform. $v_i(n_i^{t_i})$ denotes the bid price for the time slot of ti for driver $(d_m)$ to reserve a parking space $(n_i)$. $x_{d_m}(n_i^{t_i})$ as a 0, 1 variable, which when taken as 1 means that the reservation platform allocates the slot $t_i$ of the parking space $n_i$ to the driver $d_m$. Constraint (2) is a heterogeneous contingent bidding, where at most only one bid wins when each driver submits bids for more than one parking space (i.e., each driver can reserve at most one parking space per driver). Constraint (3) ensures that only one feasible solution can be matched for the parking space in park $m$. Constraint (4) indicates that the parking spaces and parking slots allocated to driver $d_m$ by the reservation platform match his reservation request. $\varphi(p_m) = 1$ indicates that the allocation scheme $P_m$ is selected as the optimal parking space allocation scheme for park $m$.

*2.2. Parking Reservation Process and Driver Payment Price Model.* The VCG auction reservation process is shown below, and the specific process is shown in Figure 1.

*Step 1*. The driver ($d_m$) participates in the parking reservation of the car park ($m$) and submits a reservation application to the reserved parking platform at least one day in advance.

*Step 2*. The reserved parking platform matches drivers with parking spaces according to the parking space allocation model and solves for drivers who successfully bid for the parking space.

*Step 3*. Determine the successful bidders for parking space drivers to be included in the set $D_m^{win}$. At the same time, calculate the total revenue received by the parking reservation platform $V(D_m^{win})$.

*Step 4*. Calculate the final payment for the driver who successfully bid for a parking space according to the Vickrey payment [57]. Vickrey payment is calculated as shown in the following equation:

$$p_{d_m} = v_{d_m}\left(n_i^{t_i}\right) - \left(V\left(D_m^{win}\right) - V\left(D_m^{win}/d_m\right)\right), \tag{7}$$

where $p_{d_m}$ denotes the final amount paid by the driver. $v_{d_m}(n_i^{t_i})$ denotes the bid of the slot $t_i$ for the driver $d_m$ to reserve a parking space $n_i$. $V(D_m^{win})$ denotes the total revenue obtained by the parking reservation platform after one round of auction. $V(D_m^{win}/d_m)$ denotes the total revenue obtained by the parking reservation platform after one round of auction when driver $d_m$ is not participating.

The final price paid by a successful driver is the value of the driver's loss to other bidders. In other words, the driver's final payment equals the difference between the amount they bid and the amount that the reservation platform received in revenue. The parking driver receives a reminder to pay the parking fee on time from the reservation platform once it has calculated the total price paid by each winning bidder.

*2.3. Driver Utility Analyses.* In order to study driver behavior, Shao et al. divided drivers into three types, including the loss-averse, the gain-neutral, and the gain-seeking [23]. Set the profit and loss coefficient as $\lambda_{d_m}$; $0 \le \lambda_{d_m} < 1$ drivers are loss-averse; when facing the same amount of gains and losses, they think that losses are more intolerable; $\lambda_{d_m} = 1$ drivers are gain-neutral, which means that they do not have any obvious preference for gains or losses; $\lambda_{d_m} > 1$ drivers are gain-seeking; when facing the same amount of gains and losses, they are more inclined to pursue gains. Meanwhile, Shao et al. highlight that the auction reservation strategy can accomplish individual rationality, allocation efficiency, and incentive compatibility if there is a uniform reference price [23]. As a result, the reference price for parking space must be established by the reserved parking platform by combining the historical price paid by the driver with an analysis of the actual cost of the parking space in the parking lot. Different types of drivers will experience varying levels of profit and loss (defined by $Z_{d_m}$) in response to the final price paid by the platform feedback; these sensations are expressed as follows:

$$Z_{d_m} = \begin{cases} r_m^{n_i} - p_{d_m}, & r_m^{n_i} \ge p_{d_m}, \\ \lambda_{d_m}\left(r_m^{n_i} - p_{d_m}\right), & r_m^{n_i} < p_{d_m}, \end{cases} \tag{8}$$

where $r_m^{n_i}$ denotes the reference price of parking space $n_i$ in park $m$. $p_{d_m}$ denotes the final price paid by the successful reservation driver. $\lambda_{d_m}$ denotes the profit and loss coefficient of the driver $d_m$. The difference between the reference price and the final price paid by the driver who was successfully reserved is the profit and loss perception when the reference price is greater than or equal to the price paid by the reserved driver; when the reference price is less than the price paid by the reserved driver, the profit and loss perception is associated with the type of driver.

Then, the utility of the driver consists of two parts: one is the economic benefit, and the other is the feelings of profit and loss. The driver utility is shown in the following equation:

$$u_{d_m} = v_{d_m}\left(n_i^{t_i}\right) - p_{d_m} + Z_{d_m}, \tag{9}$$

where $u_{d_m}$ denotes the utility of the driver. $v_{d_m}(n_i^{t_i})$ denotes the bid of the slot $t_i$ for the driver $d_m$ to reserve a parking space $n_i$. $p_{d_m}$ denotes the final price paid by the successful reservation driver. $Z_{d_m}$ denotes the personal profit and loss feelings of the driver.

When the drivers participating in the auction are gain-neutral drivers, at which point the profit and loss coefficient $\lambda_{d_m} = 1$, then the driver utility is shown in the following equation:

$$u_{d_m} = 2\left(\frac{v_{d_m}\left(n_i^{t_i}\right) + r_m^{n_i}}{2} - p_{d_m}\right). \tag{10}$$

If $v'_{d_m} = v_{d_m}(n_i^{t_i}) + r_m^{n_i}/2$, then the driver can be reduced to

$$u_{d_m} = 2\left(v'_{d_m} - p_{d_m}\right). \tag{11}$$

Similarly, when the bidding driver is loss-averse ($0 \le \lambda_{d_m} < 1$) or gain-seeking ($\lambda_{d_m} > 1$), the profit and loss coefficient $\lambda_{d_m} \ne 1$; then the utility of the driver is shown in the following equation:

$$u_{d_m} = \left(1 + \lambda_{d_m}\right)\left(\frac{v_{d_m}\left(n_i^{t_i}\right) + \lambda_{d_m} r_m^{n_i}}{1 + \lambda_{d_m}} - p_{d_m}\right). \tag{12}$$

If $v''_{d_m} = (v_{d_m}(n_i^{t_i}) + \lambda_{d_m} r_m^{n_i})/(1 + \lambda_{d_m})$, then the utility of the driver can be simplified to

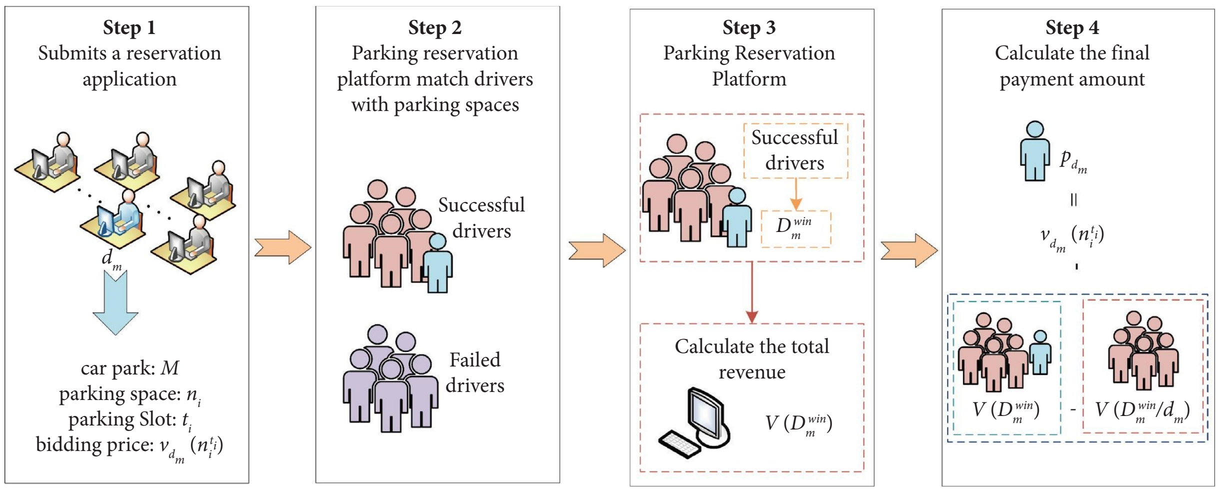


Figure 1: The VCG auction reservation flowchart.

$$u_{d_m} = \left(1 + \lambda_{d_m}\right)\left(v''_{d_m} - p_{d_m}\right). \tag{13}$$

In conclusion, the utility of the driver is shown in the following equation:

$$u_{d_m} = \begin{cases} 2\left(v'_{d_m} - p_{d_m}\right), & r_m^{n_i} \geq p_{d_m}, \\ \left(1 + \lambda_{d_m}\right)\left(v''_{d_m} - p_{d_m}\right), & r_m^{n_i} < p_{d_m}. \end{cases} \tag{14}$$

### 2.4. Evaluation and Analyses of Vertical Equity in Parking Reservation Based on the Gini Coefficient

*2.4.1. The Lorenz Curve and the Gini Coefficient.* The renowned Lorenz curve was first presented in 1907 by American statistician Max Otto Lorenz (1876–1959) as a means of researching the national income distribution among citizens. The curve that results from calculating the cumulative percentage of the poorest population up to the point at which the cumulative percentage of the richest population equals the percentage of income for each percentage of the population in a sample is known as the Lorenz curve.

The cumulative percentage of the population, or the cumulative number of persons from the lowest to the highest income, is shown by the horizontal axis in Figure 2, while the cumulative percentage of income is represented by the vertical axis. The cumulative proportion of income-population is a straight line, known as the line of absolute equity when the population's income distribution is in an absolute equity scenario. The Lorenz curve gradually bends as the degree of inequality increases; a larger curvature suggests an increasingly unequal distribution of income among the population at this point. Consequently, the Gini coefficient was proposed in 1912 by Corrado Gini, which is used to represent the inequality in wealth among inhabitants of a nation or region, an Italian statistician and sociological specialist [55]. From the diagram of the Lorenz curve, the Gini coefficient is the area of graph A surrounded by the line of absolute equality and the actual Lorenz curve divided by the area of the triangle surrounded by the line of absolute equality and the horizontal and vertical coordinates, which can be calculated using the following formula:

$$Gini = \frac{S_A}{S_A + S_B} = \frac{0.5 - S_B}{0.5} = 1 - 2S_B. \tag{15}$$

The range of values for the Gini coefficient is 0 to 1. The reverse is more inequitable and the Gini coefficient is smaller in more equitable distributions. The warning line of the distribution gap is typically set at 0.382. The Gini coefficient is then divided into five degrees of equity, with 0.2, 0.3, 0.4, and 0.6 serving as the cutoff points, after being incorporated into the assessment of transport equity [58]. The caution line for the transport equity measurement is 0.4 among them. Table 1 illustrates the correlation between the Gini coefficient and the level of transport equity.

*2.4.2. The Vertical Equity Index of Parking Reservation.* Under the VCG auction strategy, due to the differences in the income levels of the drivers participating in the parking reservation, the bidding drivers with different income levels will choose parking spaces according to their economic ability and parking preferences, giving different bidding prices, which determine the probability of the bidding drivers' success in reserving the ideal parking spaces. Eventually, drivers with different income levels obtain different parking utilities.

The Lorenz curve of parking reservation can more intuitively represent the degree of vertical equity of parking reservation, whose horizontal coordinate is the cumulative percentage of the number of drivers, and the vertical coordinate is the cumulative percentage of the sum of the revenue of the platform and the driver's utility (i.e., the cumulative percentage of the revenue and utility), as shown in Figure 3.

The vertical equity index is obtained by referring to the Gini coefficient calculation. As a result, the following are the steps for determining the vertical equity index of parking reservation.

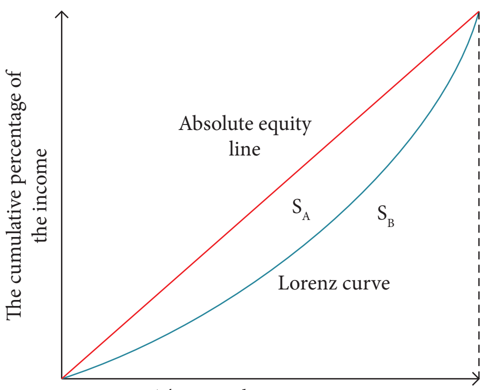


Figure 2: The Lorenz curve.

Table 1: Comparison table between transport equity and Gini coefficient.

| The Gini coefficient | Transport equity |
|---|---|
| [0, 0.2] | High degree of equity |
| [0.2, 0.3] | Relative equity |
| [0.3, 0.4] | Relatively reasonable |
| [0.4, 0.6] | Wide equity gap |
| [0.6, 1] | High inequity |

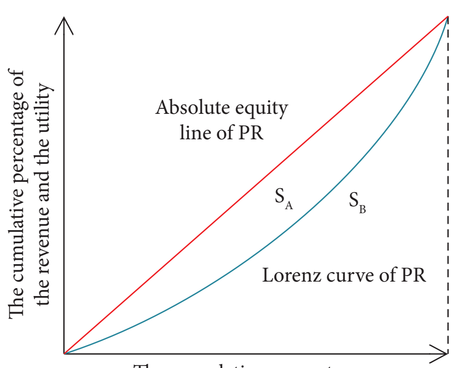


Figure 3: The Lorenz curve of parking reservation.

*Step 1*. Calculate the allocation of parking reservations to different proportions of drivers with high-income levels entering the parking reservation platform and calculate the revenue of the reserved parking platform and the utility of each driver.

*Step 2*. Calculate the sum of the number of drivers, i.e., $P = \sum_{i=1}^{n} p_i$, and the sum of the revenue of the parking reservation platform and the sum of the utility of the drivers, i.e., $\mathrm{RU} = \sum_{i=1}^{n} (r_i + u_i)$.

*Step 3*. Calculate the number of participating reserved drivers in each group of experiments as a percentage of the total number of participating reserved drivers, i.e., $N_i = p_i/P$, and calculate the sum of the benefits of the parking reservation platform and the utility of the drivers in each group of experiments as a percentage of the total, i.e., $W_i = (r_i + u_i)/(RU)$.

*Step 4*. Calculate the ratio of $N_i$ to $W_i$ (i.e., $C_i = N_i/W_i$) and sort the results of the experiment in order from smallest to largest according to the amount of $C_i$.

*Step 5*. Calculate the cumulative percentage of drivers participating in the parking reservation, i.e., $x_n = \sum_{i=1}^{n} N_i$, and the cumulative percentage of the benefit utility, i.e., $y_n = \sum_{i=1}^{n} W_i$.

*Step 6*. Calculate the vertical equity index according to the following equation:

$$G_{ev} = 1 - 2\sum_{i=1}^{n} \frac{(x_i - x_{i-1}) * (y_i + y_{i-1})}{2}. \qquad (16)$$

To make the procedure of determining the vertical equity index of parking reservation simpler to comprehend, we convert the above processes into a flowchart, as shown in Figure 4.

In addition, it is worth noting that the larger the vertical equity index of parking reservation is, the more curved the Lorenz curve of parking reservation is, which indicates that there is a great difference in the costs paid by drivers of different income levels in this situation and that equity is poorer. In other words, low-income level drivers bear greater pressure to make reservations and have a lower level of social welfare. Conversely, the smaller the vertical equity index of parking reservations, the higher the level of social welfare.

## 3. Numerical Experiments

Drivers can be divided into two groups based on their income level when making parking reservations: high-income drivers and medium to low-income drivers. The income level of the drivers involved in parking reservations will definitely affect their valuation of the reserved parking space, which in turn will impact the bidding price. High-income drivers tend to value time more than money and consider that there is more to be gained from time-saving, while medium to low-income drivers prioritize the value of money.

Therefore, based on the income levels of the drivers participating in reserved parking and the cost of parking spaces in major cities of China (e.g., Beijing, Shenzhen, Shanghai, etc.), this section establishes the bidding price range of parking drivers in the numerical experiment. The experiment then yields the outcomes of parking allocation for drivers of varying income levels using the VCG auction process, and the experiment's findings are used to research the equity of reserved parking. In Section 3.1, the platform's revenue is examined when only drivers with medium to low income participate. Additionally, the impact of various driver types and reference prices on drivers' utility is examined. Section 3.2 discusses the vertical equity when different proportions of high-income drivers participate and

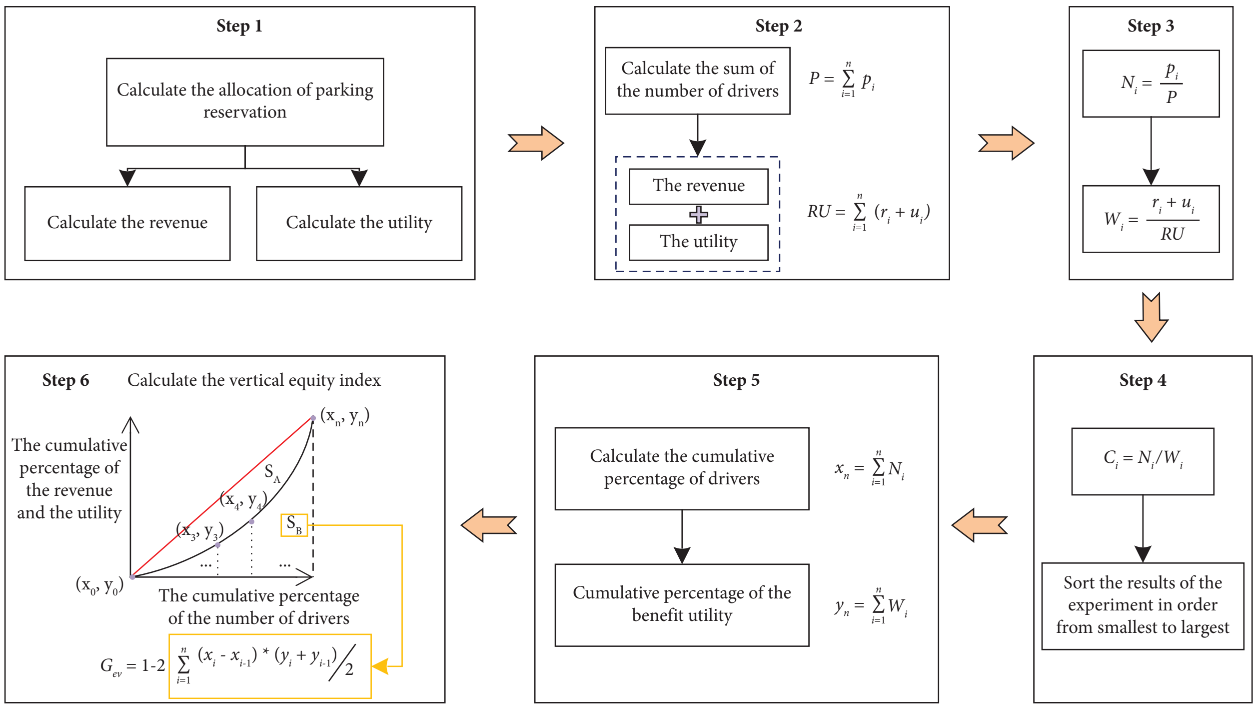


Figure 4: The flowchart for the calculation of the vertical equity index.

analyzes the changes in the vertical equity index brought about by changes in the profit and loss coefficients and reference prices for different proportions of high income.

*3.1. Experimental Reference.* In the parks of major Chinese cities, Guangzhou has the highest rate (26 RMB/h), followed by Shanghai (20 RMB/h), Shenzhen (20 RMB/h), Nanjing (20 RMB/h), Hangzhou (12 RMB/h), Beijing (10 RMB/h), Suzhou (10 RMB/h), Chengdu (10 RMB/h), Chongqing (8 RMB/h), Tianjin (8 RMB/h), and Wuhan (4 RMB/h). Consequently, the average parking fee in major cities in China is 13.45 RMB per hour. Parking drivers' bidding prices are likewise not fixed, as the income level is a range value instead of a definite amount. Assume that drivers with medium to low income will randomly bid between [10, 16] RMB/h for parking spaces. Considering the income level and time value of high-income drivers, the bidding price for parking spaces should be higher than that of the medium to low-income groups, and the random bidding interval of high-income drivers is assumed to be [17, 23] RMB/h in experiments. The two income categories' bidding discrepancies are still 6 RMB apart. In addition, RMB is the Chinese currency "Renminbi." US$1 approximates RMB7.1333 as of December 25, 2023.

Setting up an experimental reference is important to assess the change in the vertical equity index when the high-income drivers join the platform. There are 15, 25, and 35 parking spaces provided by the park, and 100, 200, and 300 drivers participating in the parking reservation, respectively, are the drivers in the experimental reference group who are all medium to low-income drivers.

As can be seen in Figure 5, the revenue of the platform rises as the number of parking spaces increases, and the revenue is basically the same when the number of parking spaces is certain. It can be seen that the VCG auction strategy ensures a stable revenue when the income level of the drivers is the same. However, the revenue of the platform only represents the benefit of the park and cannot adequately characterize the utility of the drivers. Thus, it is necessary to research the utility of drivers from the perspective of profit and loss coefficient and reference prices.

According to equation (14), the profit and loss coefficient will have an impact on the driver's utility if the platform sets a specific reference price. The change in driver utility is displayed in Tables 2–4 with the profit/loss perception values for the number of spaces (15, 25, and 35) and the reference price ($r = 13$), respectively.

In accordance with Tables 2–4, drivers who are loss-averse are more beneficial than those who are gain-neutral or gain-seeking. The utility of the driver reduces as the profit and loss coefficient rises. It is evident that drivers who are more conservative will have higher utility while gain-seeking drivers will have considerably lower utility than loss-averse and gain-neutral drivers.

Moreover, the utility of drivers is impacted by the reference price. Table 5 illustrates how the reference price affects drivers' utility when there are 15 spaces and a profit and loss coefficient of 15. Table 5 shows that when the reference price rises, driver utility rises as well. Most driver utility values are negative if the reference price is too low (e.g., when the reference price $r = 13$ in Table 5).

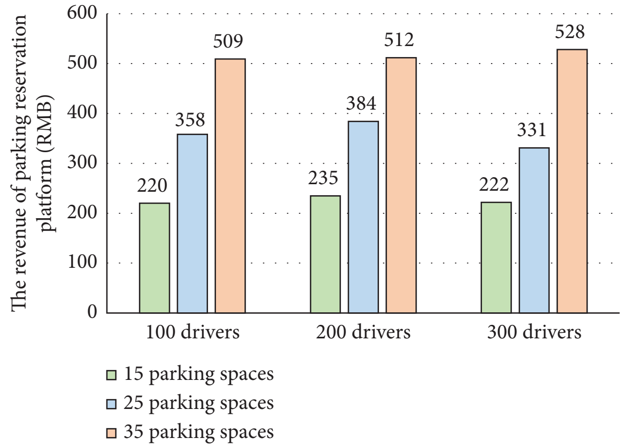


Figure 5: Parking reservation platform revenue graphs.

Table 2: The effect of the profit and loss coefficient on drivers' utility ($r = 13$, parking spaces = 15).

| Profit and loss coefficient | 100 drivers | 200 drivers | 300 drivers |
|---|---|---|---|
| $\lambda = 0.2$ | 11 | −7 | 9.6 |
| $\lambda = 0.3$ | 8.5 | −11 | 6.9 |
| $\lambda = 1$ | −9 | −39 | −12 |
| $\lambda = 2$ | −34 | −79 | −39 |
| $\lambda = 3$ | −59 | −119 | −66 |

Table 3: The effect of the profit and loss coefficient on drivers' utility ($r = 13$, parking spaces = 25).

| Profit and loss coefficient | 100 drivers | 200 drivers | 300 drivers |
|---|---|---|---|
| $\lambda = 0.2$ | 25.4 | −4.8 | 61.8 |
| $\lambda = 0.3$ | 22.1 | −10.7 | 61.2 |
| $\lambda = 1$ | −1 | −52 | 57 |
| $\lambda = 2$ | −34 | −111 | 51 |
| $\lambda = 3$ | −67 | −170 | 45 |

Table 4: The effect of the profit and loss coefficient on drivers' utility ($r = 13$, parking spaces = 35).

| Profit and loss coefficient | 100 drivers | 200 drivers | 300 drivers |
|---|---|---|---|
| $\lambda = 0.2$ | 27.2 | 27.6 | 9.4 |
| $\lambda = 0.3$ | 21.8 | 21.9 | 2.1 |
| $\lambda = 1$ | −16 | −18 | −49 |
| $\lambda = 2$ | −70 | −75 | −122 |
| $\lambda = 3$ | −124 | −132 | −195 |

Table 5: The effect of reference price on drivers' utility when $\lambda = 1$ and the number of spaces is 15.

| Reference price | 100 drivers | 200 drivers | 300 drivers |
|---|---|---|---|
| $r = 13$ | −9 | −39 | −12 |
| $r = 16.5$ | 43.5 | 13.5 | 40.5 |
| $r = 20$ | 96 | 66 | 93 |
| $r = 25$ | 171 | 141 | 168 |

At the same time, the reference price also affects the utility of drivers. Table 5 shows how the utility of drivers is affected by the reference price when the profit and loss coefficient $\lambda = 1$ and the number of spaces is 15. From Table 5, it can be found that driver utility increases as the reference price increases. If the reference price is too low, most driver utility values are negative (e.g., when the reference price $r = 13$ in Table 5). This occurs when the platform pays the driver a final price that is greater than the reference price, causing the driver to feel a loss and the utility to decline. As a result, choosing a suitable reference pricing for parking reservation platforms based on past data can help safeguard drivers' utility.

*3.2. The Vertical Equity Analysis of Parking Reservation.* To research changes in the success rate of medium to low-income drivers and to calculate the vertical equity index of the overall participation under different driver types, different proportions of high-income drivers were introduced based on the experimental reference, with a certain number of drivers participating in parking reservations. These drivers' numbers accounted for 5%, 10%, and 15% of the total number of participants.

Figures 6–8 show the change in revenue with high-income levels entering the parking reservation platform when 100, 200, and 300 drivers participate in parking reservations.

The experiment demonstrates that revenue does not always increase as predicted and occasionally declines when the percentage of high-income drivers using the platform increases. When 100 drivers use parking reservations in Figure 6, the platform revenue remains relatively stable following the entry of high-income drivers, an average increase of 19.43RMB. In Figure 7, when there are 35 parking spaces and 15% of drivers are high-income, the platform revenue falls relative to the reference group. A similar situation is seen in Figure 8, where there are 15 parking spaces and 5% of drivers are high-income. The reason for these scenarios is that when a high-income driver is successful in bidding, bringing more positive value to the platform, if there is still a low-income driver who has reserved a parking space successfully, the high-income driver does not lose money to the platform in the calculation of the Vickrey payment. Therefore, high-income driver pays less than or equal to the price of their bid, and the overall revenue to the platform decreases or stays unchanged. This means that higher income level drivers will pay less to use the parking space.

The bidding price of medium to low-income drivers will also become less competitive after high-income drivers outbid them for parking spaces, which will cause a precipitous decline in the success rate of reservations. When 100, 200, and 300 drivers, respectively, participate in parking reservations, Figures 9–11 illustrate the success rate of reservations for medium to low-income drivers.

Figures 9–11 show that when high-income drivers continue to increase, medium to low-income drivers' success rate of making reservations without altering their bidding strategy drastically drops; medium to low-income drivers only have a chance to make a reservation when there are more parking spaces available than high-income drivers, and even then, their chances are slim. As a result, it influences how equitable reservation results are.

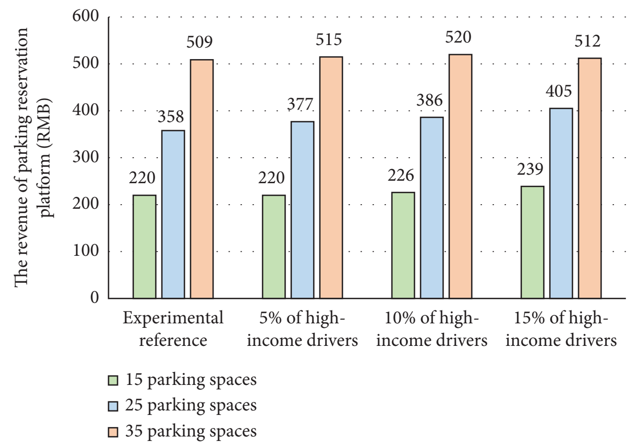


Figure 6: Change in parking reservation platform revenue graphs (100 drivers).

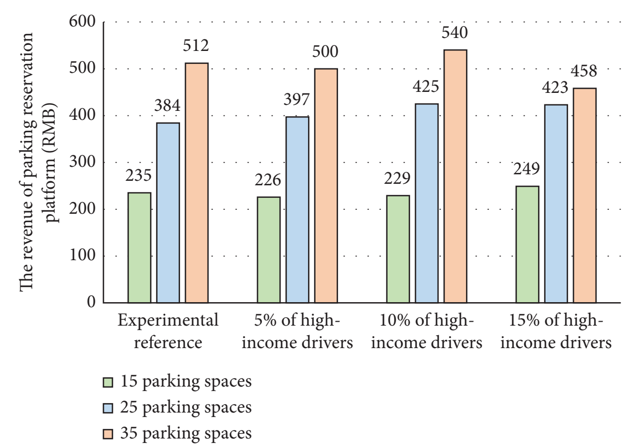


Figure 7: Change in parking reservation platform revenue graphs (200 drivers).

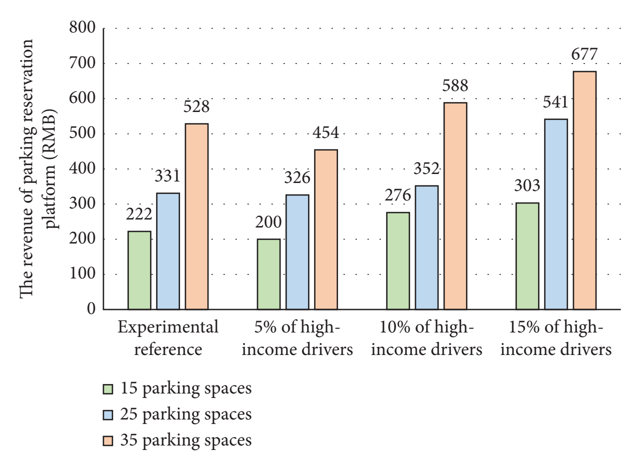


Figure 8: Change in parking reservation platform revenue graphs (300 drivers).

Furthermore, when parking spaces are less than high-income drivers, even though the reservation of drivers with medium to low-income levels is successful, their final price

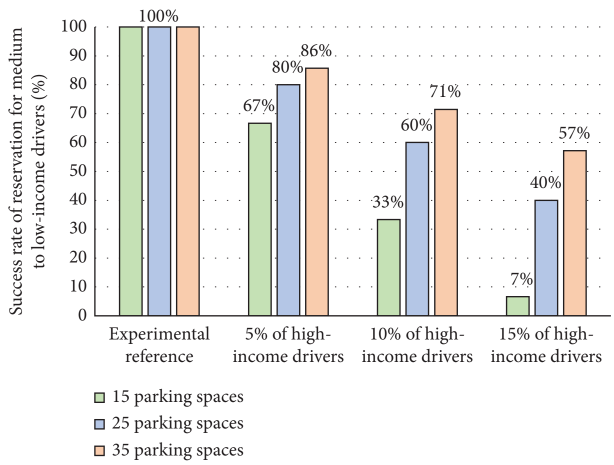


Figure 9: Success rate of reservation for medium to low-income drivers (100 drivers).

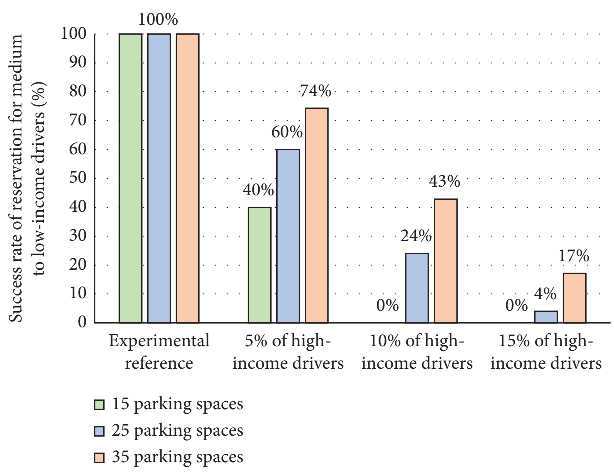


Figure 10: Success rate of reservation for medium to low-income drivers (200 drivers).

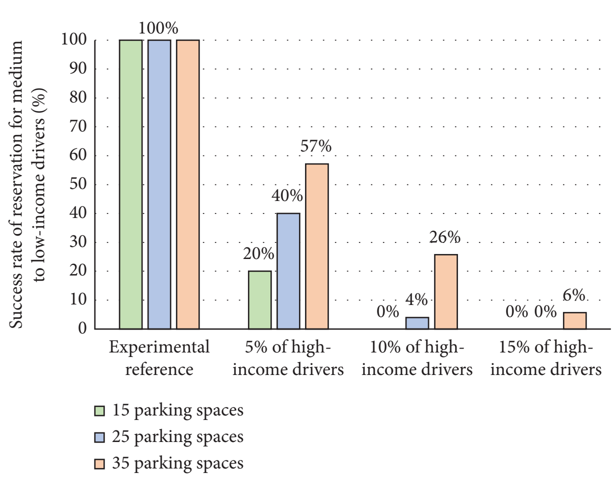


Figure 11: Success rate of reservation for medium to low-income drivers (300 drivers).

paid is higher than the bidding price. This is because, in the VCG auction strategy, drivers participating in the auction are required to give their inner truest offer, so that the final price paid by the platform back to the driver can be equal to or less than the driver's bid price. In this case, drivers with medium to low income are considered to be "dishonest," and the final price will be higher than their initial bid. The utility of medium to low-income drivers is undermined. In the long run, fewer medium to low-income drivers may choose to reserve parking, which greatly reduces the range of parking options available to medium to low-income drivers. If medium to low-income drivers could obtain the real psychological expectation of high-income drivers for parking spaces and also send the same price to the platform (i.e., to raise the original initial bid price), then the probability of their reserving a parking space will be improved, though they will bear the cost of parking in excess of the expectation of paying for it. For drivers who choose to reserve a parking space to go to work, which will undoubtedly increase their traveling burden, the introduction of the VCG auction strategy into the parking reservation in this case will lead to inequity.

The vertical equity index's variations for various reference prices are depicted in Figure 12. The reference price modifies the vertical equity index when the percentage of high-income drivers participating remains constant. Within a suitable range, the vertical equity index decreases with increasing reference price. This implies that reserved parking might be made more equal by raising the reference price.

The vertical equity index's progression with various profit and loss factors is shown in Figure 13. From Figure 13, it is found that not only does the number of drivers with high incomes affect vertical equity, but also the profit and loss coefficient will affect vertical equity. The more the high-income drivers are, the more inequity the reservation outcome is for the same profit and loss coefficient. In addition, the vertical equity index when loss-averse drivers and gain-neutral drivers participate is relatively stable and in the range of equity, while the vertical equity index of gain-seeking drivers fluctuates more. The vertical equity index under the participation of gain-seeking drivers rises quickly and even crosses the equity range as the percentage of high-income drivers rises. For instance, the vertical equity index for $\lambda = 3$ with 15% of high-income drivers participating is 0.45434, indicating a significant bias in the equity of reservation parking in this case.

Plotting the fluctuation of the Lorenz curve of reserved parking with profit and loss coefficients for the participation of 5%, 10%, and 15% of high-income drivers was done using the data derived from the computation. The Lorenz curves of high-income drivers at 5% and 10% do not vary significantly under different profit and loss coefficients, as Figures 14–16 more intuitively show. However, the difference between the Lorenz curves of different profit and loss coefficients for high-income drivers at 15% has increased and even seems to have a negative value. The degree of vertical equity is

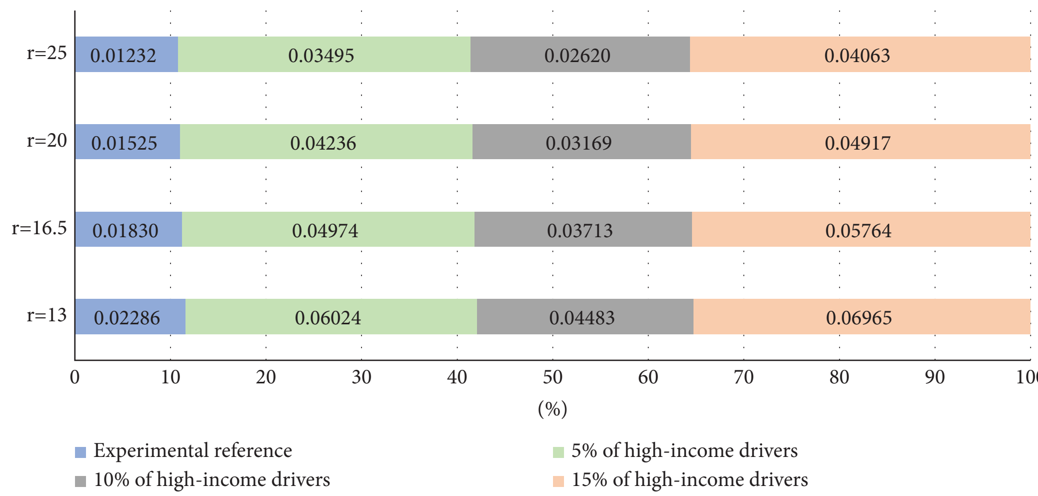


Figure 12: Comparison of vertical equity index under different reference prices.

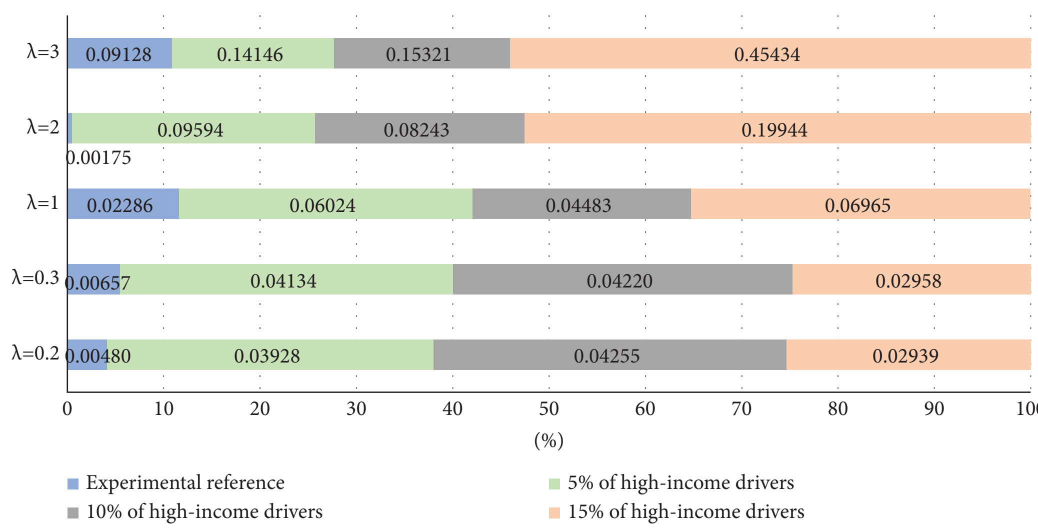


Figure 13: Comparison of vertical equity index under different profit and loss coefficients.

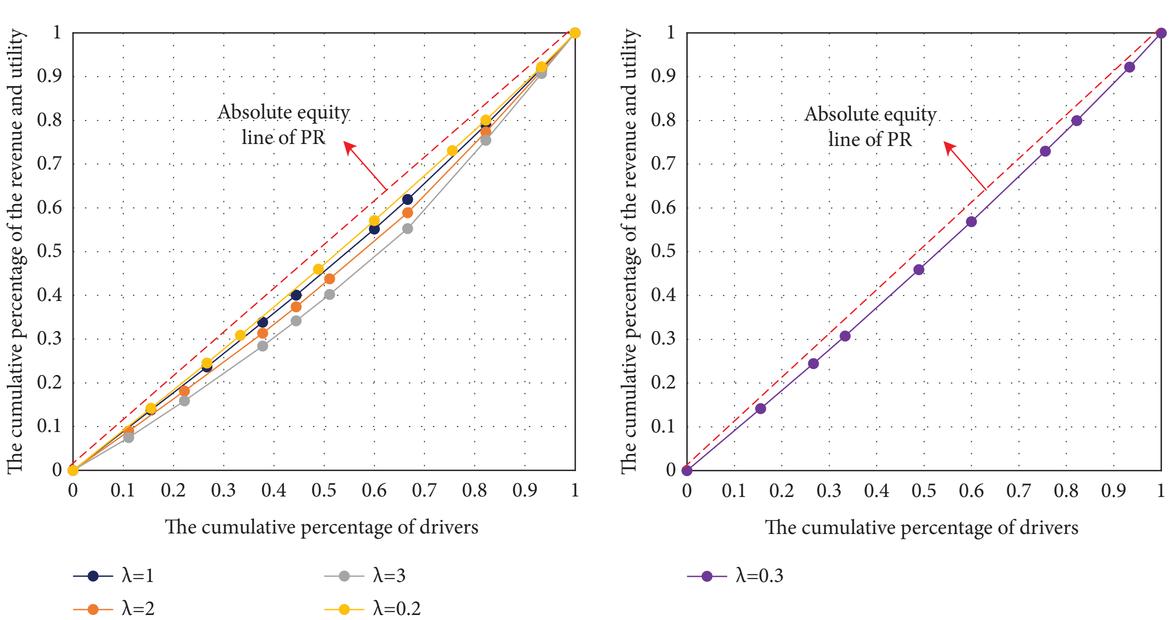


Figure 14: Lorenz curve of reserved parking with 5% high-income drivers' participation.

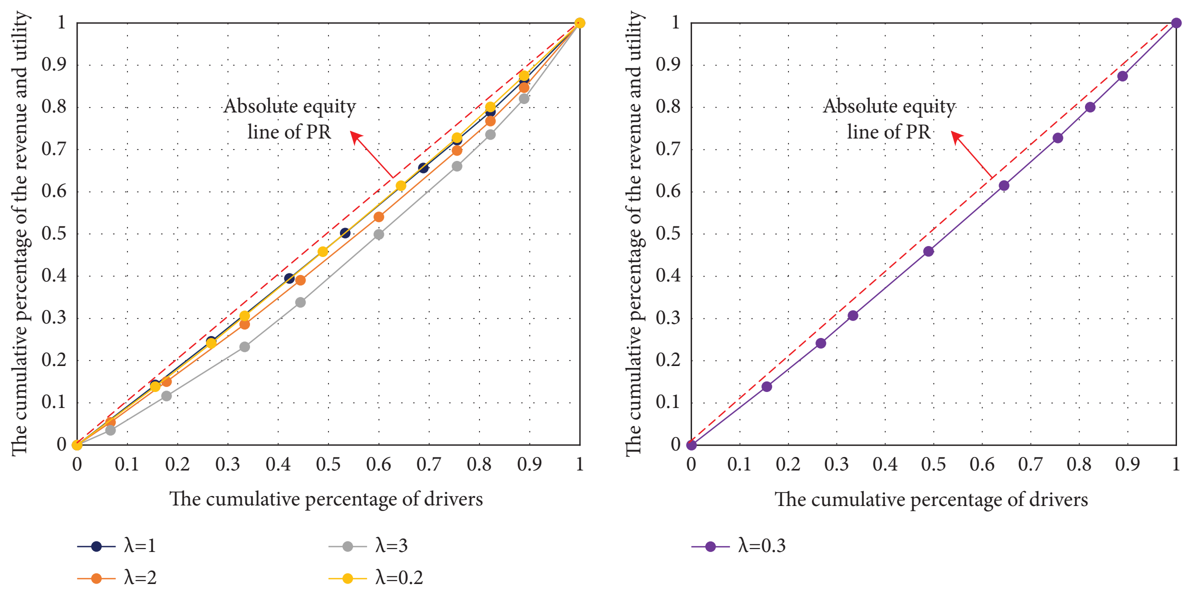


Figure 15: Lorenz curve of reserved parking with 10% high-income drivers' participation.

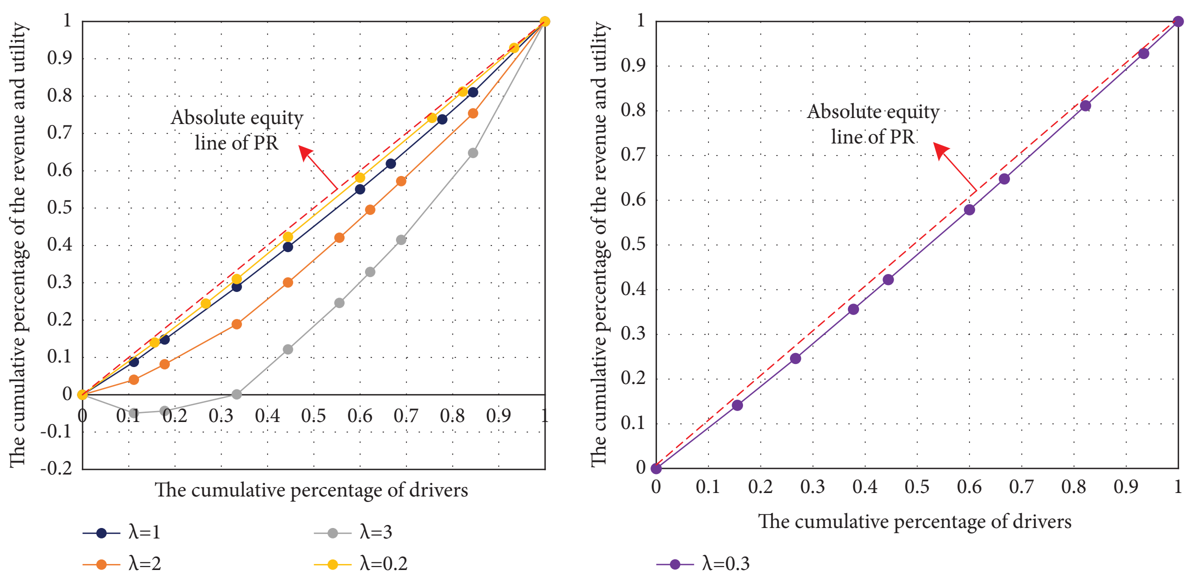


Figure 16: Lorenz curve of reserved parking with 15% high-income drivers' participation.

represented by the degree of bending of the Lorenz curve for parking reservations; the more bent the curve is, the more unfair the situation is at the moment. This suggests that the larger the profit and loss coefficient, the more inequitable the parking reservation outcome will be, given a consistent proportion of high-income drivers. In other words, inequity increases when all drivers are more profit-seeking.

In conclusion, through numerical experiments, we can find that the driver's income level, the type of profit and loss, and the platform's reference price all impact the equity of the auction reservation strategy. Therefore, the auction reservation strategy can be implemented in the process of bidding for part of the parking spaces to provide high-quality reservation parking services for drivers who have urgent needs and are willing to pay higher parking fees. This can avoid large fluctuations in the price paid by drivers with low-income levels, thus guaranteeing equity and improving the level of parking space services. Meanwhile, the reservation parking platform can also regulate equity by setting a reasonable reference price. In addition, this paper suggests integrating technologies such as ChatGPT into the auction reservation system, which can constrain drivers to bid in accordance with real psychological expectations and reduce the probability of gain-seeking drivers to ensure the equity of the auction reservation strategy.

## 4. Conclusions and Prospects

Using the VCG auction strategy as an example, this paper addresses the parking utility of various driver types in the reserved parking process and investigates the vertical equity problem of reserved parking from this perspective. To quantitatively research the vertical equity problem of reserved parking, this paper introduces the Gini coefficient and the Lorenz curve into the analysis of the equity problem of reserved parking and proposes a vertical equity evaluation index of parking reservation that is easier to understand and implement.

Python simulation is used to determine the outcomes of the parking space allocation process as well as the total amount that winning bidders under the VCG auction strategy paid. Also, a calculation was made of drivers' parking utilities. The experimental data were calculated via the steps for determining the parking reservation vertical equity index, and the analysis's findings were then displayed. Furthermore, by varying the profit and loss coefficients, the variations in the parking reservation platform's income and the vertical equity index under various driver involvement scenarios were examined in more detail. Additionally, the effect of reference price on vertical equity is explored.

The following four conclusions are drawn from this study.

Firstly, the reference price and the profit and loss coefficient have an impact on drivers' utility when all drivers involved in a parking reservation have the same income. According to the experiment, drivers who are loss-averse are more than those who are gain-neutral or gain-seeking when the reference price rises.

Secondly, as the percentage of high-income drivers using the parking reservation platform rises, the success rate of reservations for medium to low-income drivers declines sharply when high-income drivers exceed parking spaces; conversely, the parking reservation platform's revenue also declines in comparison to the reference group.

Thirdly, the growing proportion of high-income drivers presents a concern with reservation outcome equity when driver types remain constant. When gain-neutral and loss-averse drivers participate, the vertical equity index remains relatively steady, whereas the vertical equity index of gain-seeking drivers is more volatile.

Lastly, the reference price modifies the vertical equity index when the percentage of drivers who participate at high-income levels remains constant. The vertical equity index decreases as the reference price increases.

In conclusion, the reference price, the profit and loss coefficients, and the drivers' income levels all have an impact on the vertical equity of reserved parking. While the auction strategy satisfies market efficiency standards and increases parking reservation platform revenue, it is unfair to drivers with lower incomes. The lack of consideration for social welfare issues by the auction strategy restricts low-income drivers' travel alternatives in places with a dearth of parking spaces and is detrimental to the steady and long-term growth of society. In light of this, the parking reservation strategy can be optimized based on the vertical equity problem, so that the parking reservation strategy becomes more equitable in the process of application. Thus, it is possible to strike a compromise between efficiency and equity. Simultaneously, the results of this paper clearly show that residents' income levels have a significant influence on the vertical equity of parking reservation allocation results. In order to provide efficient reserved parking services for residents who have higher demand and are willing to pay higher parking fees, this paper advises adopting the auction reservation strategy into particular parking spaces during practical application.

Additionally, the reserved parking vertical equity analysis methodology that this study presents is universal, meaning that in addition to the auction reservation strategy, it can be used to evaluate the vertical equity of other reserved parking strategies. The analysis will help the reservation parking platform choose a more equitable reservation parking strategy that will raise the participation rate of resident parking reservations as well as the acceptability of the driver's parking reservation strategy. This will accomplish the goal of guiding most citizens to utilize the city's parking resources sensibly and regulate parking, which will increase the degree of harmony and happiness index in the city.

While the vertical equity of reserved parking has been quantified in this research, there are still three shortcomings as the discussion of the reserved parking equity problem is still in the theoretical research stage.

(1) This paper has done numerical experiments and examined the subject of vertical equity of reserved parking because the auction reservation strategy has not yet been implemented. During the process of practical application as a result of the ongoing implementation of various reservation parking strategies, managers of the reservation parking platform will be able to integrate more real data and take advantage of the vertical equity evaluation index suggested in this paper to further discuss the vertical equity of the reservation parking strategy in the future. When combined with the features of the real application area, the reservation parking strategy is promptly adjusted to enhance the parking utility of various driver categories while effectively and fairly allocating parking spaces, offering drivers improved reservation parking services and ensuring the parking reservation platform develops steadily in long term.

(2) The individual socioeconomic characteristics of drivers include a wide range of information, including the driver's age, gender, family structure, degree of education, employment status, and residential location. The driver's psychological assessment of a parking space is influenced by all of these elements, and this in turn influences how a parking space is allotted and how much is ultimately paid. In turn, the reserved parking outcome will counteract the driver's acceptance, which will affect the issue of vertical equity of reserved parking. In view of this, in

the future, the research in this paper can be enriched from other perspectives of drivers' personal socioeconomic factors by combining them with practical applications through questionnaires and other methods.

(3) Parking reservation strategies have a crucial impact on the equity of parking reservations. Therefore, the parking reservation strategies can be optimized with the goal of social welfare to ensure equity.

## Data Availability

The data used to support the findings of this research are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Authors' Contributions

Rong Chen was responsible for conceptualization, formal analysis, original draft preparation, data curation, and methodology. Ge Gao was responsible for original draft preparation, methodology, supervision, and funding acquisition. Fahui Pan, Shuo Liu, and Xinbo Mao were responsible for software and grammar check. All authors reviewed the results and approved the final version of the manuscript.

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