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Research Article

The Peak Stability Analysis through Hysteresis Phenomenon on Heterogeneous Networks

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The macroscopic fundamental diagram (MFD) is a nonuniversal changing process over network traffic status which indicates different shapes in different networks. Hysteresis is observed in the MFD of some urban networks. It is a unique phenomenon when the network remains at low stability level and usually appears around the congestion period. This paper analyzed network peak stability through focusing on hysteresis. The formation mechanism of hysteresis is deduced from the mathematical method based on previous research studies. The precondition of hysteresis and the changing process of network state can be figured by mathematical deduction. It indicates that hysteresis only occurs conditionally in the period of macroscopic congestion and is not a universal phenomenon. Heterogeneity is an important factor leading to network instability. The hysteresis patterns of different peaks in MFD are different due to the variation of network flow. Real data are collected from Atlanta's urban network to verify the analysis of hysteresis. To discuss the changing process of hysteresis in different peaks, a three-stage division is proposed and time series is presented as a third dimension in MFD. It is worth noticing that the existence and form of hysteresis in morning and evening peaks are different. Although there is a higher peak flow in the morning peak, the stability of the evening peak performs better when hysteresis occurs in the network. The different fluctuations in the morning and evening peaks are exhibited through the 3D version of MFD. The otherness of hysteresis in different peaks is explained through a 3D coordinate system with cross-compared corresponding indexes.

1. Introduction

Due to the increasing number of vehicles in urban city, the requirement for accessibility and efficiency of road networks remains an essential question. However, the contradiction between the limited resources and the continuous growth of traffic demand results in the instability of the peak performances. Hence, it is important to identify the period and severity of road congestion based on corresponding indexes and to analyze the stability performances for different peaks.

Empirical and real data studies show that well-defined macroscopic relations exist in urban traffic networks rather than irregular dispersive scatter. In the urban network, instability is manifested in the inability to maintain average flow at a sustained level over the peak period. The performance level fluctuates frequently in the corresponding period and great differences in density between different road sections exist. In macroscopic traffic flow, instability usually refers to the formation of stop-and-go waves and congestions presented a subject-specific stability.

In this paper, we analyzed the network peak stability by focusing on the hysteresis phenomenon in the MFD. To focus on hysteresis, the data collected in January 2020, in Atlanta, Georgia, were used. The properties of hysteresis are discussed through real data. Through the comparison between morning peak and evening peak, the precondition of hysteresis is discussed and the network stability is analyzed. Meanwhile, traffic indexes including occupancy, flow, and speed are cross-compared to explain the difference between morning and evening peaks. Accordingly, we provide a discussion on network peak stability based on real data and present the guiding value of hysteresis to the peak stability. The main contributions include the following points:

(1) In this paper, the mechanism of hysteresis is summarized and its existence is illustrated with a real case. Through summarizing relevant studies, we mathematically deduced the process of hysteresis and recognized heterogeneity as a factor leading to network instability.

(2) A 3D MFD with time series is introduced to further observe the changing patterns of hysteresis. Combined with the 3D coordinate system of variances of traffic indexes, it brought a different perspective to comprehending the network status at different moments by integrating the characteristics of the time-varying graph with the MFD. Through the comparison of morning and evening peaks, the influencing factor to hysteresis is discussed. The differences of hysteresis in the two peaks are analyzed as well.

(3) To interpret the properties of the hysteresis loop, we divided it into three stages based on different change trends of network flow and occupancy. Each stage presents a different trend. The various degrees of spatial heterogeneity in occupancy in the onset and offset of the peak period are compared. The different performances between the morning and evening peaks are exhibited. The corresponding stability level is discussed.

2. Literature Review

Early studies focused on macroscale traffic patterns with data of lightly congested real-world networks [1] or simulations with artificial routing rules and static demand [2]. In these studies, the existence of an invariant macroscopic relation for urban networks was initially investigated. The studies of Daganzo [3] made a clear distinction between free-flow and congested network states. The empirical analysis of congestion patterns also revealed additional complexity and non-steady-state conditions of traffic states.

Based on the traditional traffic flow basic model and empirical data observed in Yokohama, a new concept of the macroscopic relationship between space-mean flow and density was proposed by Geroliminis and Daganzo [4]. This is called the macroscopic fundamental diagram (MFD), which revealed a clear relationship. Their field experiments revealed that there was MFD within a large-scale urban area [4, 5]. In several studies, the existing conditions of MFD and its corresponding influence factors including road infrastructure, traffic demand, signal control strategy, flow distribution, and the driving behaviours were investigated [6–9]. It was found that a homogeneous network will maintain a well-defined MFD [3]. In other words, the network with even and consistently distributed density will get a low-scattered MFD. Buisson and Ladier [7] showed that heterogeneity has a strong impact on the shape/scatter of MFD by analyzing the real data from a medium-sized French city. Moreover, the spatial distribution of road network density is the key component affecting the scatter of an MFD and its shape [10]. When analyzing the impacts of different transport modes on traffic performance, Loder et al. [11] extended MFD into the 3D-MFD so as to offer a novel framework at the urban scale. It gives us inspiration to observe the MFD characteristics of time-varying traffic flow and its stability.

Meanwhile, a phenomenon called hysteresis exists in some network MFD with further research. Treiterer and Myers [12] first observed a traffic loop as separation in speed-density diagrams between the acceleration and deceleration curves. Since then, hysteresis has been observed more frequently. In traffic systems, the concept of hysteresis refers to the MFD graph indicating the ratio of traffic outflow and accumulation, which forms a closed rather than a linear curve. Initially, sudden flow drop, which can also be recognized as "capacity drop" and the predecessor of traffic hysteresis, was identified disputing with the flow-density curve for a road segment. It was addressed by the discontinuity between congested and free-flow regions. Numerous researchers indicated a downstream around road capacity [13]. The hysteresis phenomenon was proposed with more studies focusing on the whole process of congestion to describe such transitions in discontinuous flow. It extended the flow drop to the evolution of network statue during the congestion period. The mechanisms of hysteresis were proposed including a mathematical theory based on acceleration, deceleration, and equilibrium flow [6, 14]. Daganzo et al. [15] mentioned that the hysteresis loop may disappear when lane-observed data were aggregated in 1–5 min intervals. The clockwise hysteresis loop was mentioned as a symbol of traffic incidents. Amini et al. [16] discussed the influence of incidents on the MFD pattern and the possibility of hysteresis loop appears around incidents. The MFD hysteresis phenomena were further explored by Geroliminis and Sun [17] in the freeway network systems. They found that hysteresis was related to the distribution of flows on urban networks. Laval [18] suggested that driver behaviour is also an influential factor of hysteresis apart from traffic flow status. The empirical implications on travel time variance yielding the hysteresis phenomenon in day-to-day travel were discussed by Yildirimoglu et al. [19, 20]. Knoop et al. [21] proposed a continuous function called GMFD relating the average flow to both the average density and the (spatial) inhomogeneity of density. It can describe hysteresis patterns in the MFD. Kieran and Connaughton [22] described and validated a data-driven method based on identifying atypical fluctuations in the relationship between density and flow, quantitatively separating atypical fluctuations from typical traffic states. The degree of fluctuations generated by network flow can be analyzed with this method. It can help to evaluate the stability level when hysteresis occurs. Raju et al. [23] developed a relationship between relative distances versus relative velocity among the leader-follower vehicles and examined the hysteresis phenomenon for vehicles under corresponding behaviours. Johari et al.

[24] reviewed the 50-year history of macroscopic modelling of urban networks. The lack of empirical studies on the hysteresis phenomena was mentioned when it comes to the topics of network equilibrium relations. The multimodal NMFD in different traffic conditions, which contains the existence of hysteresis, might affect the NMFD shape.

Hysteresis is a unique phenomenon that most appears with congestion periods and relates to the low level of network stability. Piu and Puppo [25] investigate the mathematical modelling and the stability of multilane traffic. The stability of the steady states in the multilane system is discussed, which has a reference value for the analysis of network stability. Wang et al. [26] designed a memory feedback control signal based on the historical traffic information of the vehicle itself to improve the intelligent driver model. In the verification part, the stability condition in the heterogeneous network is discussed. It shows the distribution of network flow and the strength of memory feedback control signal affect the stability level. Control strategies that optimize the network heterogeneity can improve its stability.

However, few studies focus specifically on the network instability when hysteresis occurs. As a manifestation of the instability in a specific period, the premises and properties of hysteresis are worth concern. The stability level in different peak periods can be analyzed through hysteresis. The relative equilibrium analysis has not been explored including the differences and variances of main traffic indexes in different periods. These problems will be discussed in this paper. Based on the analysis in this paper, the correlation network indexes can be used as the measure of stability level when hysteresis occurs. This has been discussed in another prior work [27].

2.1. Mathematical Deduction. For traffic flow, by rapid changing in flow experiences, the traffic status will have an incoherent transition. Under traffic conditions of constant travel speed, the equilibrium flow is equivalent to the flow increasing linearly with the change of average density. Meanwhile, it is often observed that a rapid decrease is experienced by travel speed at a particular density level. The notion of discontinuous phase trajectories in traffic dynamics is not exotic since system theory reveals that sudden phase transitions often occur in complex nonlinear systems. This phenomenon has been extensively noticed in the macroscopic fundamental diagram (MFD) curve of urban networks relating to the onset and offset of congestion, which is regarded as a unique effect caused by discontinuous nonequilibrium traffic status.

A well-defined MFD, which can be divided into two parts by the critical accumulative vehicles and form a quadratic curve, reflects the relationship between network-weighted flow and network accumulative vehicles. For a road network, the general equation of the MFD curve can be expressed as in equations (1)–(4) for the network-weighted flow, density, speed, and occupancy, respectively. When we focus on a time period $\mathbf{t}$, we can obtain equations (5) and (6):

$$q_w = \frac{\sum_i q_i l_i}{\sum_i l_i} = \sum_i \frac{q_i}{N}, \tag{1}$$

$$k_w = \frac{\sum_i k_i l_i}{\sum_i l_i}, \tag{2}$$

$$v = \frac{q_w}{k_w}, \tag{3}$$

$$o_w = \sum_i \frac{o_i}{N}, \tag{4}$$

$$q_w(t) = \frac{\sum_i (q_i(t) \times l_i)}{\sum_i l_i}, \tag{5}$$

$$k_w(t) = \frac{\sum_i (k_i(t) \times l_i)}{\sum_i l_i}. \tag{6}$$

Since this paper discusses the network inhomogeneity in MFD, the inhomogeneity $\mathbf{h_i}$ considers all road segments at one moment and determines the standard deviation. Therefore, it can be calculated from the standard deviation of the probability density function shown as follows:

$$h_i = \sqrt{\sum_i \frac{(k_i - k_m)^2}{N}}. \tag{7}$$

From Knoop et al. [21], the network inhomogeneity $\mathbf{h_i}$ can be derived from the known standard deviation of the uniform distribution function. Combining with the function $\varnothing(\mathbf{k_i})$, we aggregate all segments in the network and obtain the inhomogeneity for time $\mathbf{t}$ from the standard deviation as follows:

$$\begin{aligned} h_i &= \sqrt{\int_0^{k_j} \left(\varnothing(k_i, t) \times \left(q_w - \int_0^{k_j} (\varnothing(k_i, t) q_w(t)) \mathrm{dk} \right)^2 \right) \mathrm{dki}} \\ &= \frac{\varnothing(k_i)}{\sqrt{3}}. \end{aligned} \tag{8}$$

It can also be simply expressed as the following function:

$$k_i = \sigma(h_i). \tag{9}$$

Densities on road segments satisfy $k_i \sim (k_m - x, k_m + x)$, with a variable $\mathbf{x}$ for the fluctuations of density. In the MFD curve, we regard it as a negative conic and have the following expression:

$$q = \begin{cases} \dfrac{\sigma(h_i)}{k_c} q_{\max}, & \text{if } k \le k_c, \\ \left(1 - \dfrac{k_i - k_c}{k_j - k_c}\right) q_{\max}, & \text{otherwise.} \end{cases} \tag{10}$$

Accordingly, when there exists $\mathbf{h_i}$, which makes MFD shape does not form a linear curve with time, network flow can be calculated as equation (11):

$$q = \frac{\int_{k_m-\sqrt{3}h_i}^{k_m+\sqrt{3}h_i} q(k_i)\mathrm{dk}}{2\sqrt{3}h_i}. \quad (11)$$

We define upper density boundary and lower density boundary as $k_u = k_m + \sqrt{3}h_i$ and $k_l = k_m - \sqrt{3}h_i$. Equation (12) can be break into

$$q = \begin{cases} \frac{k_m}{k_c} q_{\max}, & \text{if } k_u < k_c, \\ \frac{k_m - k_j}{k_c - k_j} q_{\max}, & \text{if } k_l > k_c, \\ \frac{1}{2\sqrt{3}h_i}\left(\frac{\left(k_c{}^2 - k_l{}^2\right)q_{\max}}{2k_c} + \frac{\left(k_u{}^2 - k_c{}^2\right)q_{\max}}{2\left(k_c - k_j\right)} + \frac{k_j q_{\max}\left(k_m - k_c + \sigma(h_i)\right)}{k_c - k_j}\right), & \text{otherwise.} \end{cases} \quad (12)$$

It can be observed that network flow is changing with different conditions and variables with the inhomogeneity. This process is expressed as the hysteresis in the MFD curve. It is a phenomenon presented by network status by time series, which indicates the beginning of a negative impact of congestion on the road network. When the MFD curve is chronologically described, a clip curve appears around the congestion in some networks.

Meanwhile, we can consider the tendency of different variables in hysteresis. According to Zhang [6], hysteresis owns similar changing process to "shockwave," but in the network level. Zhang uses acceleration and deceleration branches to refer to the shockwave process. As the similarity to hysteresis, we also use acceleration and deceleration phases to analyze the change trend of network flow during hysteresis. The switch position of acceleration and deceleration branches can be regarded as the max flow in hysteresis (top point in the clip shape). The traffic flow during some time period **t** is in the acceleration phase if the following expression holds:

$$\frac{\mathrm{dv}}{\mathrm{dt}} > 0, \quad (13)$$

and deceleration phase if the following expression holds:

$$\frac{\mathrm{dv}}{\mathrm{dt}} < 0. \quad (14)$$

The acceleration and deceleration phases switched at the extreme point which is also the critical density, and there should satisfy $v_a = v_e = v_d$. So we have the following equation:

$$\frac{\mathrm{dv}}{\mathrm{dt}} = -o_w v_e{}^2 \frac{\partial o_w}{\partial i}. \quad (15)$$

When it comes to the MFD curve, strong travelling/shock or rarefaction waves can be regarded as congestion. Network flow is most prominent under acceleration and deceleration phases, which should in turn highlight the fundamental structure of nonequilibrium flow-density relationships.

As the accumulative vehicles can hardly reach the fully congested number in MFD. Traffic status with irregular changes is exhibited by the uneven distribution of flows. In the urban network, with the increasing accumulated number of vehicles, the exorbitant network density will result in bottlenecks at some segments of the network and the flow on those roads will gradually reach their capacity. When link density variance is low, the average network flow is consistently higher for the same network density. The performance of the road network has not changed linearly with the traffic load. If occupancy keeps increasing until density exceeds a critical capacity, the arrival rates and network flow decrease. Hence, because of the inhomogeneous distribution of vehicular density in the network, there is a possibility for some segments of the network to become congested when the remaining parts stay noncongested or even in free flow status.

It can be regarded as, when going through a traffic congestion, there are three network stages [6]: anticipation dominant phase (stage 1); balanced anticipation and relaxation phase (stage 2); and relaxation dominant phase (stage 3). This is similar to the shockwave process. The difference is that stage 3 in hysteresis is not as stable as stages 1 and 2. There are more fluctuations and the details will be analyzed with real data in the case study.

The aforementioned conjectures lead to the following micro-macro models for vehicle **n** at a road segment $\mathbf{i_n}$:

$$\begin{cases} \dfrac{\mathrm{d}i_n(t)}{\mathrm{d}t} = q_e\left(k\left(i_n + \Delta, t\right)\right), & \text{stage 1,} \\ \dfrac{\mathrm{d}i_n(t+\gamma)}{\mathrm{d}t} = q_e\left(k\left(i_n + \Delta, t\right)\right), & \text{stage 2,} \\ \dfrac{\mathrm{d}i_n(t+\gamma)}{\mathrm{d}t} = q_e\left(k\left(i_n, t\right)\right), & \text{stage 3,} \end{cases} \tag{16}$$

where $\boldsymbol{\gamma}$ denotes the relaxation time constant and $\Delta > 0$ is the distance vehicle $\mathbf{n}$ travels in time period $\mathbf{t}$.

In this process, network in stage 1 with free flow state and going into traffic congestion. Suppose that network flows travel into a denser traffic region. The slope of the MFD curve on the left side is positive but decreasing. Together with the consistency requirement, from equation for stage 1, we can conclude that network satisfies

$$\begin{aligned} \frac{\mathrm{dq}}{\mathrm{dt}} &> 0, \\ \frac{\mathrm{dk}}{\mathrm{dt}} &> 0. \end{aligned} \tag{17}$$

Similarly, we can obtain the equations of network state for stages 2 and 3:

$$\begin{aligned} \frac{\mathrm{dq}}{\mathrm{dt}} &< 0, \\ \frac{\mathrm{dk}}{\mathrm{dt}} &> 0, \\ \frac{\mathrm{dq}}{\mathrm{dt}} &< 0, \\ \frac{\mathrm{dk}}{\mathrm{dt}} &< 0. \end{aligned} \tag{18}$$

Specific change rules and properties will be analyzed in the Case Study with real data.

Generally, the hysteresis in the MFD represents the result of macroscopic queueing and spillback and the evolution of regional congestion. It is an important property in the time dimension of the macroscopic traffic status of the road network. The properties of hysteresis are related to the onset and offset of congestion. By observing the variances of occupancy, network flow, and average speed in hysteresis, it is reasonable to analyze the change rules of regional congestion.

The conventional MFD is a 2D curve not containing the time series. It cannot be identified when hysteresis occurs beside the clip-shape. Through such a curve, the fluctuation and time-varying conditions of the network cannot be revealed. Thus, we extended it to a 3D version. There were some previous studies that used 3D MFD as well; however, the timer shaft was not introduced as the unique axial.

Furthermore, we proposed a 3-stage division for hysteresis to depict the different features and changing rules during peak hours. The basis of division between stages can correspond to the analysis above. To further describe the properties of hysteresis, a three-dimensional coordinate system on variances of traffic indexes is established. By cross-comparing, the distribution of variances, the differences between morning and evening peaks can be explained.

2.2. Case Study. In this study, we applied the data collected from 352 detectors across 9 urban roads in Atlanta, Georgia, from GDOT in January 2020. Occupancy, volume, and speed were available on average every 5 min. Considering the integrity of the data, some invalid data have been filtered. The MFD acquired by different detectors on each road is presented in Figure 1.

If we regard the fitted curve as the quadratic function of occupancy, it means $q = f(o_i)$. Then, the fitting degree of MFD on 9 roads with $\mathbf{m}$ detectors can be calculated as follows:

$$R^2 = \frac{\sum_{i=1}^{m}\left(f(o_i) - \bar{q}\right)^2}{\sum_{i=1}^{m}\left(q_i - \bar{q}\right)^2}. \tag{19}$$

It can be observed that the relation of occupancy and flow appears in different shapes on different road segments. Both the occupancy and flow have various value ranges on 9 road segments. R^2 values are different for all 9 roads. The different dispersions indicate different distributions of vehicles. Therefore, heterogeneity is clear for the traffic flow when considering these road segments as a macroscopic network. The selected network and location of detectors are shown in Figure 2(a).

Since this paper studies the network peak stability through the hysteresis phenomenon, we focus on analyzing the corresponding traffic characteristics when hysteresis occurs, including its premises, the change trend of different indexes, and the comparison of network stability in the morning and evening peaks. Therefore, in the period with complete data, we selected Jan 15$^{\text{th}}$ with relatively obvious hysteresis of the network MFD as the focus date. So, as to verify the change trend in different hysteresis stages, we proposed in the mathematical deduction above. After removing the duplicated data and the noise points, we aggregated the values of 9 road segments to describe the relation of weighted flow vs. occupancy and acquired the network MFD based on equations (1)–(6) as shown in Figure 2(b).

As the data were collected every 5 min, there were 288 scatter points distributed in the MFD curve. The fitted line conforms to a quadratic curve. When the occupancy was about 14, the average network flow reached the maximum value. When occupancy exceeded this threshold, average weighted flow decreased and network performance would relatively decline. Notice that, although we focus on the data on 15$^{\text{th}}$ in this part, there are still several other days that have occurrences of hysteresis in the period we collected (Figure 3). All these days share similar network MFD scatter pattern. The variation of weighted flow and occupancy before peak hours is higher when hysteresis occurred, which is reflected in the network MFD; that is, the scatter span of the period between free flow and peak is larger and the distribution is uneven.

Accordingly, it is worth concerning when network performance experienced such changes and how the hysteresis occurs. Based on the foregoing discussion and

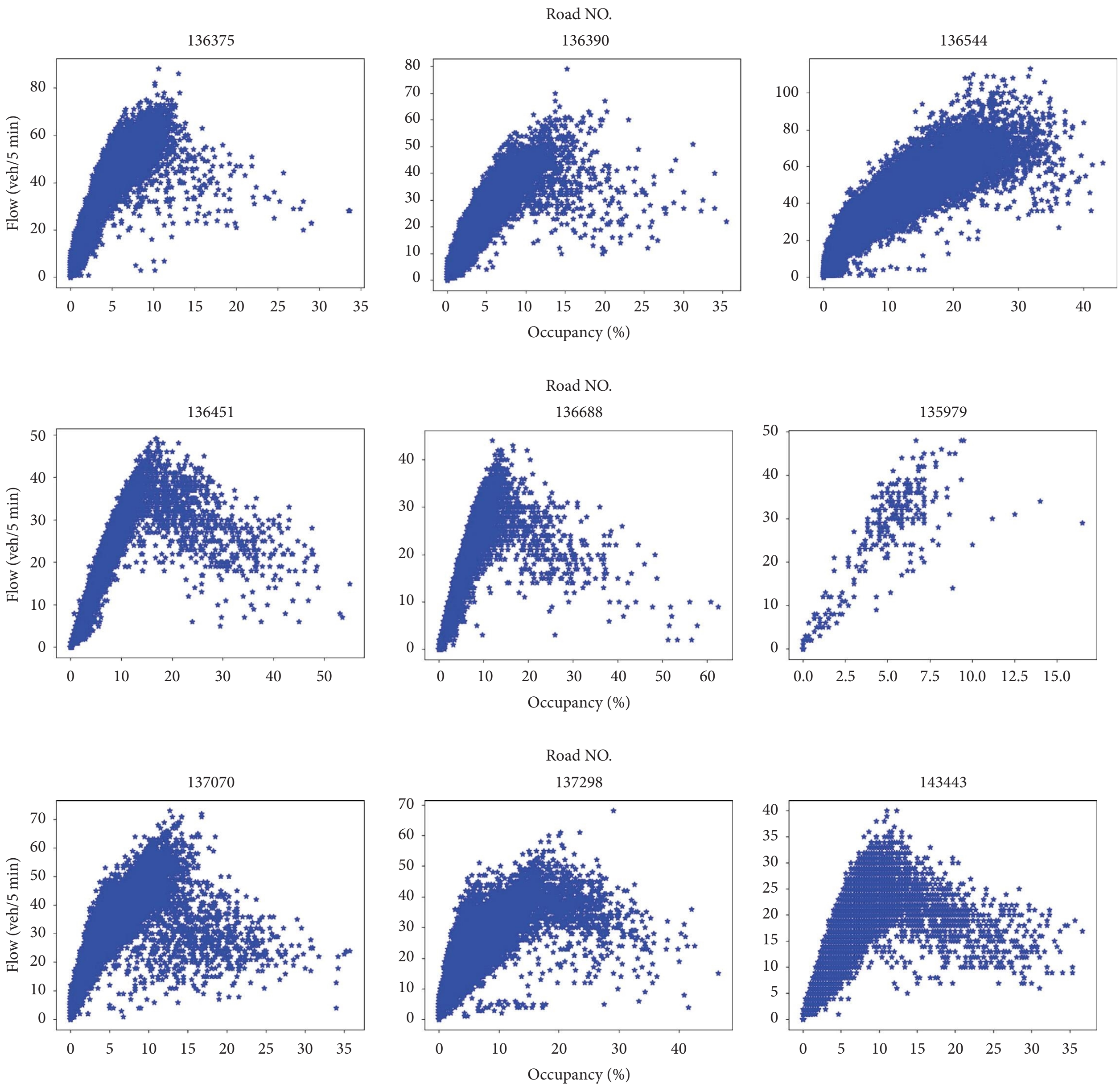

Figure 1: MFD scatter for 9 road segments.

analysis, hysteresis stands for the process of macroscopic queueing and spillback. It is an important feature in the time dimension, while the properties of time series are not completely expressed in the ordinary MFD curve like in Figure 2(b). Therefore, the hysteresis loop is not obvious.

To observe the timeline, the time-varying graph is addressed in Figure 4(a). It is observed that the morning and evening peaks occur at approximately 6:00–9:30 and 16:00–19:30, respectively. The maximum value of occupancy and network flow reaches at 17:05–17:10 and 7:25–7:30. On account of this, the MFD of the network is described by time series and the curve is fitted as shown in Figure 4(b).

There are two hysteresis loops in the MFD curve of Jan 15th. We marked 0:00–12:00 (including morning peak) with the blue line and the red line of 12:00–23:55 (including evening peak). According to Figure 4(b), the same occupancy has different corresponding values of flow representing different time points conforming to the analysis of hysteresis in the last part. For instance, when occupancy was 15, the corresponding flow was at 7:30, 8:00, and 16:30. It is also worth mentioning that there is a larger value range of weighted flow in the morning peak than in the evening peak.

We have recognized through mathematical deduction that network inhomogeneity is an important factor to the existence of hysteresis. The initial state of network in the morning and evening peaks is different. In other words, the inhomogeneity is different when different peak periods began. Corresponding to such difference, the hysteresis phenomenon of the morning peak is more obvious compared to the evening peak, and the hysteresis loop is more homogeneous in MFD fitting curve.

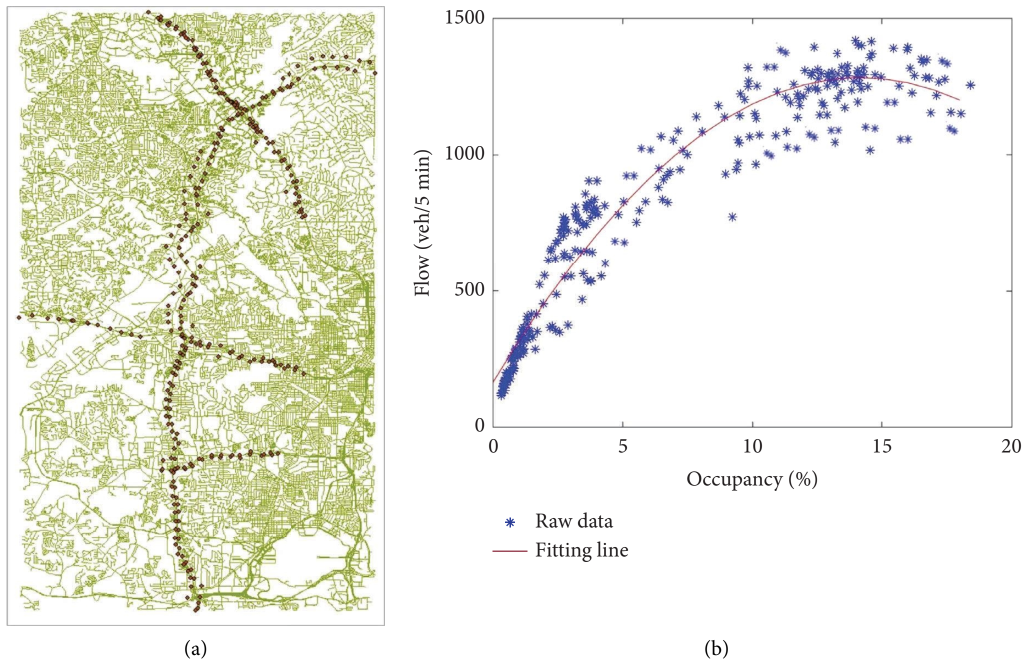

Figure 2: (a) The selected network and the detectors' distribution; (b) corresponding MFD curve.

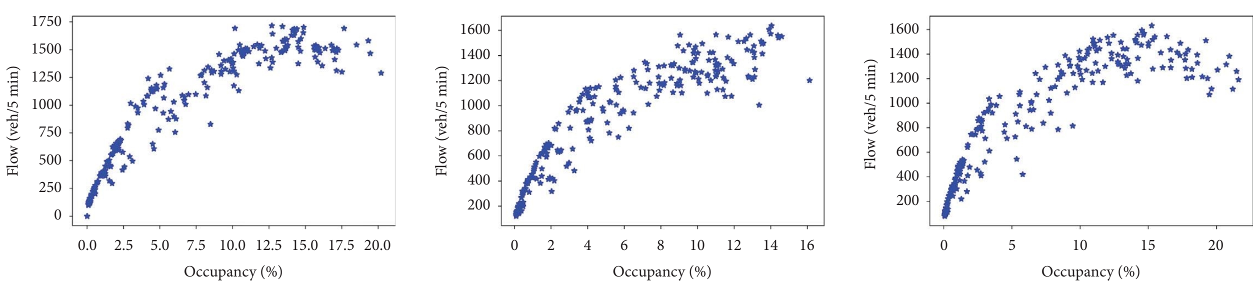

Figure 3: Network MFD scatter for several other dates.

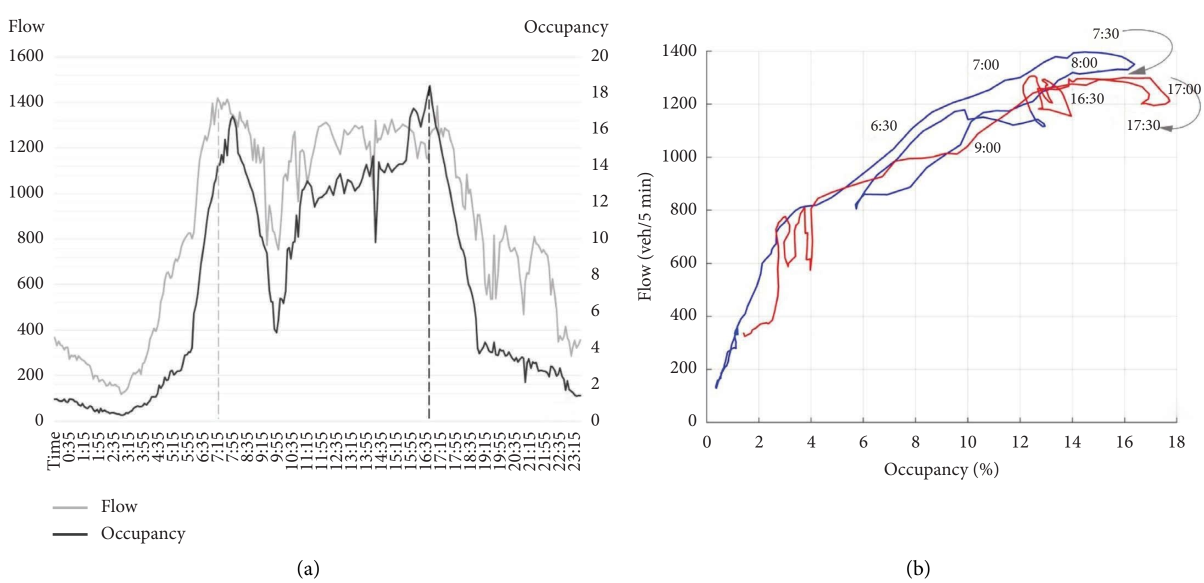

Figure 4: (a) Time series of network flow and occupancy; (b) MFD as time series in 2-dimensional vision.

It is reasonable that, on one hand, the initial distribution of vehicles on the network has a higher degree of heterogeneity in the morning peak than that in the evening peak due to the different travel demands on different road segments. On the other hand, the network initial state in the morning peak is closer to the empty load, which means there is more space for loading the critical flow. The variation rate of network flow has a greater difference in each segment. It will be compared with the evening peak through data analysis in the following section. Furthermore, the performance of the network in different traffic indexes in both the morning and evening peaks should be considered along with the distinction between them. However, we divide hysteresis into stages according to the aforementioned mathematical analysis so as to better observe its features.

2.3. Three Stages for Hysteresis. A clear hysteresis loop can be observed by focusing on the blue half. However, the red half is more concentrated indicating the irregular distribution of MFD points and the stronger fluctuation of network flow in the afternoon. Therefore, we first focus on the blue half. Comparing a common simulated MFD curve (Figure 5(a)) to this real data based on the MFD curve with time series, the difference of the peak can be seen. Meanwhile, the span of the hysteresis loop is not limited to peak hours. To better understand the evolution of hysteresis, we divided it into three stages as shown in Figure 5(b). The division is based on the relative change rate of network-weighted flow and occupancy in the MFD curve.

Stage 1: 6:00–7:25. From the beginning of morning peak to the maximum weighted flow. Both flow and occupancy change positively over time in this stage. Generally, it can be considered as the evolution of free flow to the capacity of the network. Due to the network heterogeneity brought by different change rates of flows on different road segments, this stage possesses a larger growth span of occupancy than that of flow compared to the ideal state of the network (e.g., in simulation), which comes as an important premise for the form of the hysteresis loop. In this stage, the network satisfies the following expression:

$$\begin{aligned} \frac{\mathrm{dq}}{\mathrm{dt}} &> 0, \\ \frac{\mathrm{dk}}{\mathrm{dt}} &> 0. \end{aligned} \tag{20}$$

Stage 2: 7:25–8:10. From the max network-weighted flow to the highest occupancy. The slope of the MFD curve gradually declines to 0 at the beginning of this stage. The network-weighted flow starts to experience negative change after reaching the capacity while network occupancy is still increasing. Compared to the duration of this stage, its span on MFD is smaller than other stages. Despite that, the network becomes congested at this stage. The MFD evolves from the maximum value of flow to the maximum value of occupancy in a single peak period. In this stage, the network satisfies the following expression:

$$\begin{aligned} \frac{\mathrm{dq}}{\mathrm{dt}} &< 0, \\ \frac{\mathrm{dk}}{\mathrm{dt}} &> 0. \end{aligned} \tag{21}$$

Stage 3: 8:10–9:30. After the occupancy climaxes at about 8:10, hysteresis enters stage 3, and both flow and occupancy of the network decrease. There is the main difference between the real data MFD and the well-defined MFD without hysteresis. The curve did not go back linearly and rather formed the half bottom of the clip shape of hysteresis instead. As the flow starts to decline earlier in stage 2 than occupancy, there is a larger reduction span of occupancy compared to the flow in stage 3. The offset of congestion mainly lies in this period. Similarly, the network satisfies the following expression:

$$\begin{aligned} \frac{\mathrm{dq}}{\mathrm{dt}} &< 0, \\ \frac{\mathrm{dk}}{\mathrm{dt}} &< 0. \end{aligned} \tag{22}$$

It is worth mentioning that when hysteresis enters Stage 3, network returns to a low load status gradually and cumulative vehicles gradually get back to a conventional level. A wavelike rise is experienced after a period of decline by network flow and occupancy. The regain of flow comes faster than occupancy, which is not similar to the variation trend in stage 1. As a result, the hysteresis loop does not appear to be a well-formed closed loop in the MFD curve. It forms a clip shape with a regular upper part and a fluctuating lower part.

Because of the limitations of the 2D perspective, the fluctuation of indexes over time in each stage is not always clear. It will be clearer in the 3D version with individual time coordinates. Especially for the evening peak, hysteresis is not as obvious as that in the morning. Therefore, we extend it to a 3D MFD to better understand each stage and analyze the changing process.

3. Results

3.1. Peak Stability Analysis. Although there are some fluctuations of flow or occupancy in the morning peak, the hysteresis loop and its properties in different stages can still be recognized through the curve. On the other hand, it is difficult to identify the hysteresis in the evening peak when the fluctuations become more frequent and irregular as the red line presents for the afternoon. It represents the stability level in different period changes with time. During hysteresis, the change interval is not as large as the blue half. However, there are always points of different periods distributed in the same area on this kind of 2-dimensional MFD. To observe the existence of hysteresis in the evening

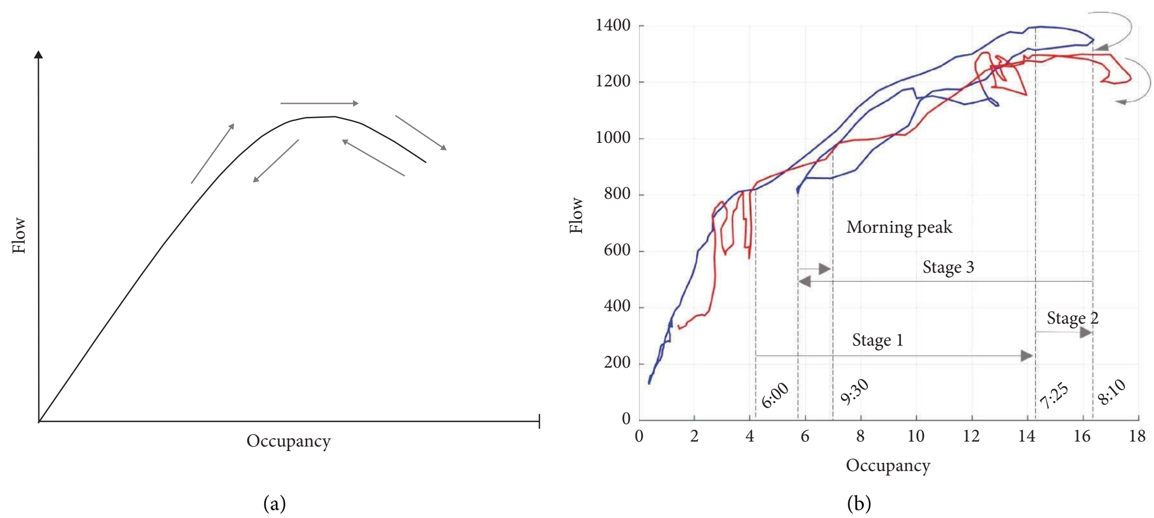

FIGURE 5: (a) Conventional simulated MFD; (b) 3-stage division in case study.

peak and identify the variance of the MFD curve in time series, we extended the MFD curve to three dimensions. The time series was considered as the Z-axis. Network MFD was developed from Figures 4(b) to 6, which combines the characterization indexes of Figures 4(a) and 4(b).

In such a 3-dimensional MFD, the semitransparent surface represents the originally fitted curve under 2D. The blue and red half-lines correspond to the time half of the day in Figure 4(b). Specifically, the timeline of hysteresis in the morning peak in Figure 6(a) conforms to the three stages divided in Figure 5. Relevant transformation rules are revealed by both flow and occupancy in different stages. When it comes to the evening peak presented in Figure 6(b), a different changing process is observed. The change interval of occupancy remains considerable. By contrast, the value interval of flow is limited to a small range. There are similar stages 1 and 2 to morning peak for 16:00–17:05 and 17:05–17:30, respectively. However, stage 3 is not present at the evening peak. To better understand the reason, the difference between the morning and evening peaks should be considered. The network is under low load status before the morning peak starts. The accumulative vehicles are low in the early morning. However, for the evening peak, it comes to the opposite situation. The network accumulative vehicles remain at a medium level in the afternoon leading to the smaller interval change than in the morning. In addition, medium-level initial vehicles also bring trip diversity to the network. Therefore, the fluctuation and irregularity become more notable in the evening peak.

By extending MFD to a three-dimensional form, it can conveniently observe the relevant indexes and traffic status of the road network at any time of the day. For instance, when the maximum flow of the day appears at 7:25 in the morning peak for about 1400 veh/5 min, the corresponding occupancy is around 14.5%. Moreover, when the maximum occupancy appears at 17:05 in the evening peak for 18.5%, the corresponding flow is about 1280 veh/5 min. Meanwhile, based on the data, it is verified that not only exorbitant vehicles in the network in total restrict the performance but they are also assembled at some shorter jams at parts of the road networks. It can be explained that the probability of spillover is increased by the inhomogeneity in spatial distribution, which continuously decreases the network flow. Combined with the analysis above, the change trend of the network in the morning peak is more fixed than that in the evening peak, with a higher network max flow. However, though the flow fluctuated more frequently in the evening peak, the network maintains average flow at a sustained level of around 1280 veh/5 min. Therefore, the stability level in the evening peak comes higher than that in the morning peak.

The peak stability is different from the network capacity. It indicates the ability to maintain average flow at a sustained level. In the following section, we confirmed the division of three stages with the variances of different traffic indicators in each period and observed the variation trend of network traffic status through the comparison between morning and evening peaks.

3.2. Comparison of Morning and Evening Peaks. Three-dimensional MFD provides the traffic status of the road network at any time point and also locates congestion on the corresponding period when the hysteresis appears. Furthermore, it brings a kind of angle to comprehend the change rule of the network in different periods through the comparison of traffic index variances.

To be specific, we selected the morning and evening peaks as the study objects and found a corresponding three-dimensional coordinate system with three network indicators, including occupancy variance, flow variance, and speed variance every 5 min. In order to show the completeness of the change rules at the peak, 1 hour before and after the peak is also considered in this case. That means the period of 5:00–10:30 and 15:00–20:30 with 66 counted points. Similarly, blue and red points are marked as morning and evening peaks in Figures 7 and 8, respectively.

Figure 7 illustrates the scatter of the variances of three indexes from 5:00 to 10:30, which contains the morning peak. As mentioned in Figure 4(a), the maximum value of

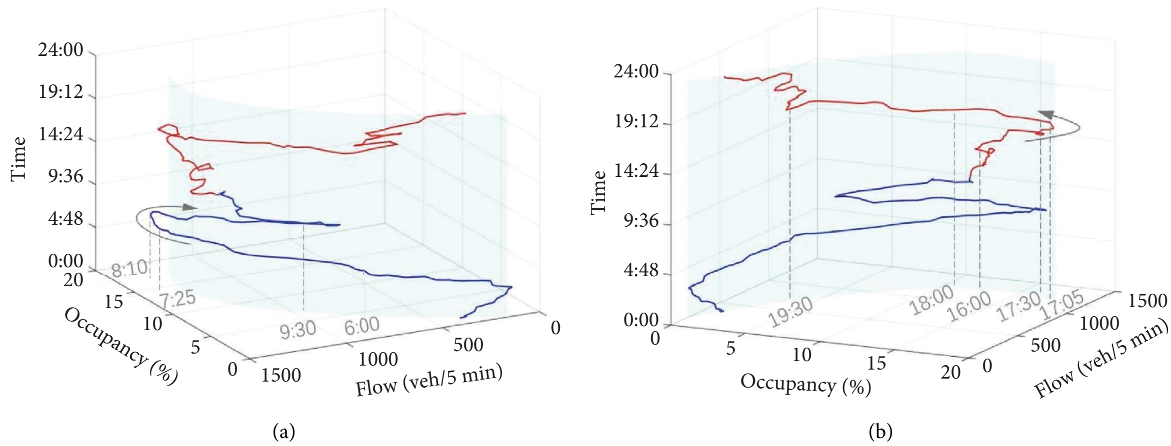

Figure 6: 3-dimensional MFD with time series: (a) morning peak; (b) evening peak.

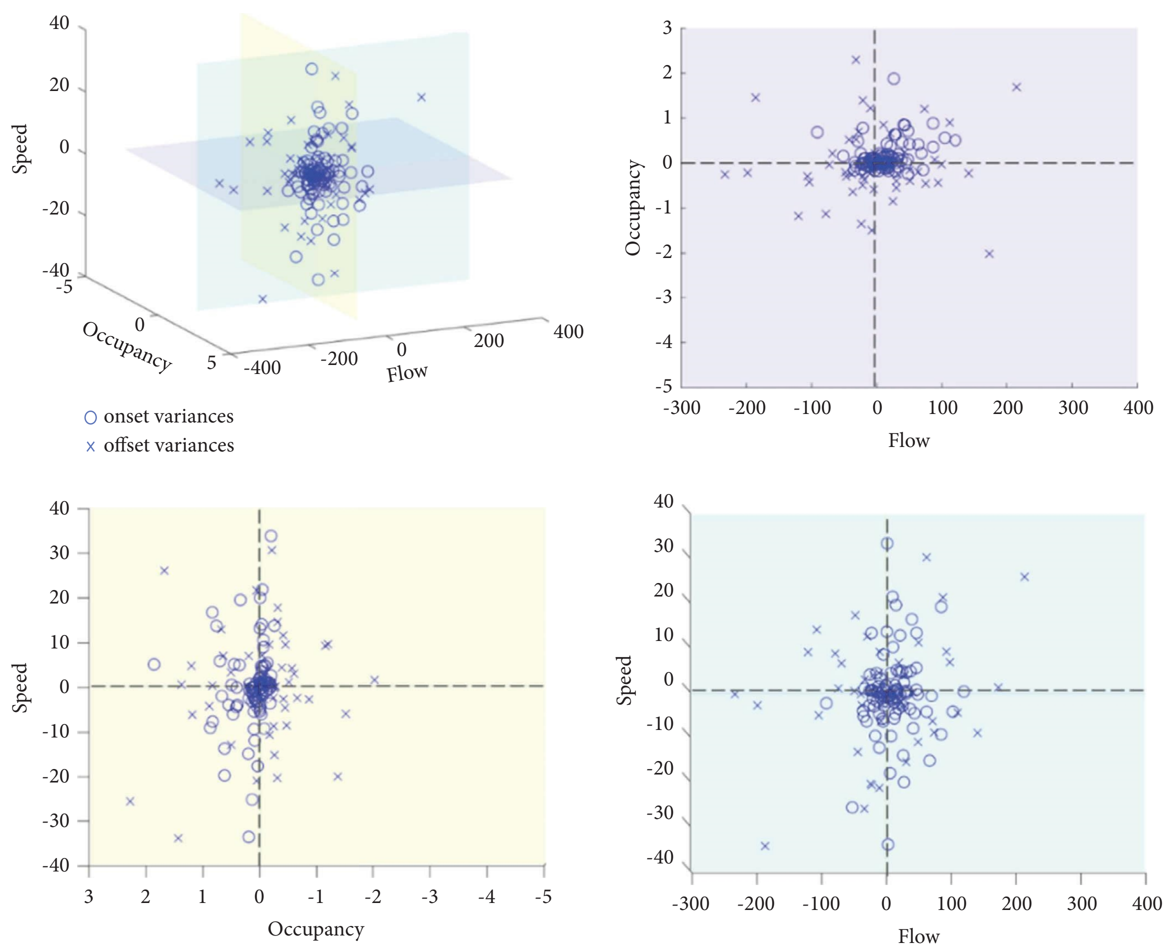

Figure 7: The variance of occupancy, flow, and speed every 5 min for 5:00–10:30.

network flow reached at 7:25–7:30; therefore, we regard 7:30 as the demarcation point of the morning peak. In this case, "o" and "×" represent two different halves, which can be also considered as the onset and offset of congestion, respectively. Generally, most points approach the origin point and the dispersion is low in this three-dimensional coordinate. It can be observed that the distribution of speed variances is more uneven compared to the flow and occupancy. The dispersion of speed variances is higher both in speed-flow and speed-occupancy subgraphs.

Meanwhile, we corresponded this three-dimensional coordinate to the 3 stages for hysteresis introduced in the last section and further verified those properties in the pairwise comparison of the three indicators. Compared to

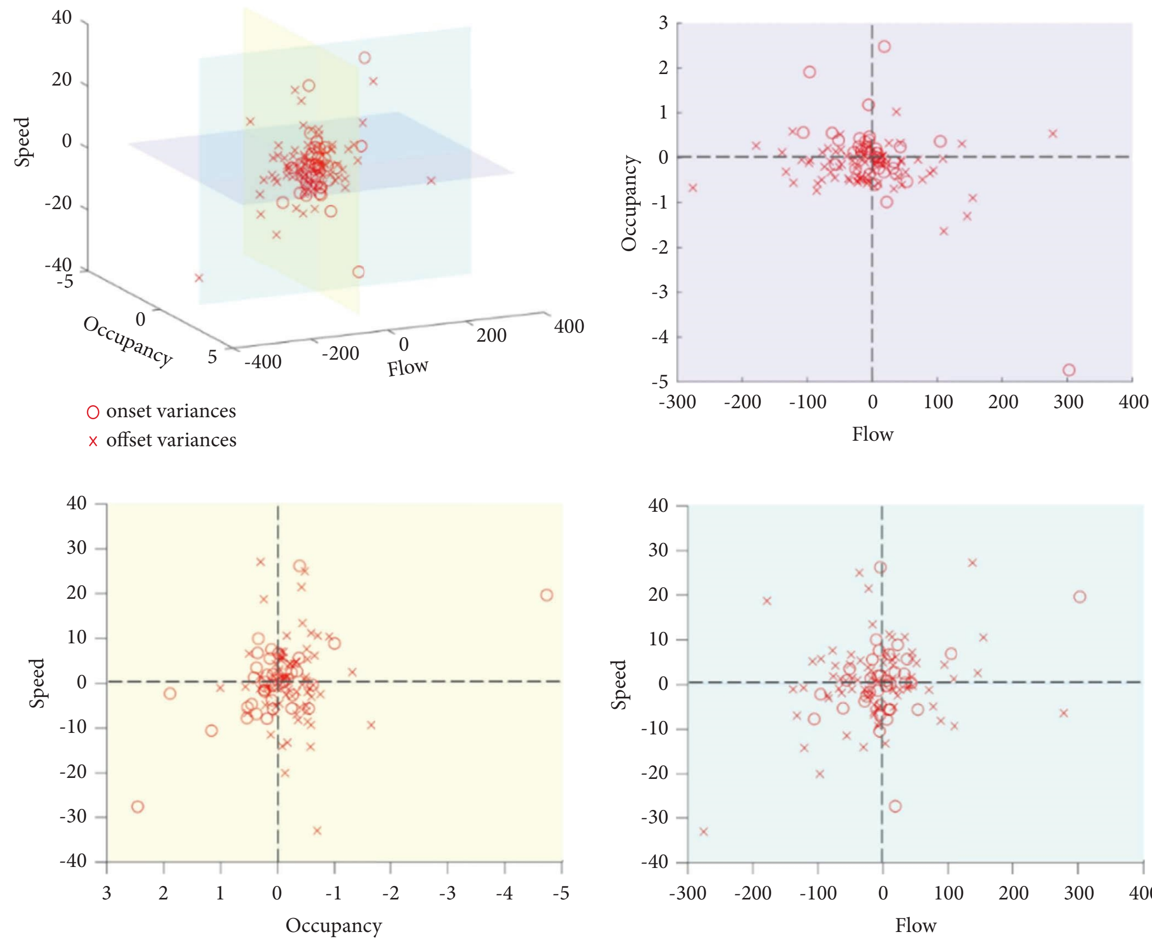

Figure 8: The variance of occupancy, flow, and speed every 5 min for 15:00–20:30.

the flow, the scatter of occupancy variance is more concentrated through the occupancy-flow subgraph. This is consistent with the causes of hysteresis analyzed in stage 1. In the occupancy-flow subgraph, numerous scatter points are located in the first and third quadrants, a small number in the fourth quadrant. The second quadrant has the fewest points. It corresponds to the changing trend in stages 1 and 2. With the onset of congestion in the network, the average flow starts to decline before occupancy, while the variances of occupancy are relatively small.

Moreover, the distribution of "×" points is more irregular in all three subgraphs, which aligned with the properties of stage 3. The points in the third quadrant are mostly related to stage 3, representing negative changes in both occupancy and flow. However, there are both "×" points distributed in the second and third quadrants for stage 3. It is in the period that the network is negatively affected but not blocked by reaching maximum capacity. Few points were scattered around other quadrants, for each stage, the volatility of different traffic indexes was proved during the whole hysteresis. Through speed-flow and speed-occupancy subgraph, the distribution of speed variances is more uneven. It makes sense that more "o" points are distributed in the second and third quadrants for the speed-occupancy subgraph, especially in the third quadrant. Furthermore, "o" points were distributed in the first and fourth quadrants for the speed-flow subgraph, especially in the fourth quadrant. It means that the fluctuation of the speed of vehicles in the network is more obvious compared to the occupancy and flow in the morning peak, while such fluctuation has less impact on network efficiency within a certain range.

Figure 8 shows the variances of occupancy, flow, and speed from 15:00 to 20:30, including the evening peak with red "o" and "×," respectively. Similarly, we regarded 17:10 as the demarcation point. Compared to Figure 7, the regularities of distribution of each index variance are not exactly the same. Through the occupancy-flow subgraph, most points were distributed in the second, third, and fourth quadrants. There were only 8 points located in the first quadrant opposite the morning peak, because of the difference of the initial accumulative vehicles in the network analyzed in the last section. Both "o" and "×" points in the second and fourth quadrants represent persistent volatility around the evening peak. It is consistent with the previous discussion on the properties of network performance and confirms the reason for no hysteresis loop existing in the evening peak. In addition, the dispersion of speed variances in speed-flow and speed-occupancy subgraphs is smaller than that in the morning peak. It represents a gentle change during this whole period. The heterogeneity of the network is an important condition for the hysteresis phenomena with higher initial accumulative vehicles.

In general, there is a more obvious hysteresis in the morning peak and higher stability in the evening peak. The trend of network is constant in the morning peak, but the cumulative flow change is more significant. Although more frequent fluctuations exist in the evening peak, the network maintains average flow at a sustained level. Combining the 3 stages analyzed above, the initial network status and heterogeneous process of change are both premises of hysteresis in MFD. Precisely, comparing the evening peak with the morning peak, the heterogeneity in the growth process of network flow is the key factor. When hysteresis exists in the MFD curve, the upper part of the loop is more regular than that of the lower part. There are more fluctuations after the max network occupancy was reached.

4. Conclusions

In this paper, hysteresis is studied as a representative phenomenon of network in low stability level. The mechanism of hysteresis is summarized and its existence was illustrated with real cases while discussing its basis in heterogeneity. Through the mathematical deduction, the network inhomogeneity is recognized as a precondition of hysteresis. The way in which the formation of hysteresis causes changes in network stability is discussed. Real data are collected from a road network in Atlanta to exemplify the analysis. We also extended the MFD to a 3D version with time series to further observe the change rules of hysteresis. Through the comparison of morning and evening peaks, we revealed that the different initial network status is also an influencing factor of hysteresis. Therefore, the heterogeneity in the growth process of network flow is the key factor to the network peak stability. To interpret the properties of the hysteresis loop, we divided it into three stages based on different change trends of network flow and occupancy. Each stage presents a different trend. The various degrees of spatial heterogeneity in occupancy in the onset and offset of the peak period are compared.

Furthermore, to verify the properties of hysteresis and compare the stability level in different peaks, a 3D coordinate system of variances of traffic indexes is introduced. These 3D graphs can contribute to intuitively comprehend the network status at different moments through combining the characteristics of the time-varying graph with the MFD curve. Through the cross-comparison of occupancy, average flow, and speed variances, we explained the different performances between the morning and evening peaks. Notice that hysteresis does not always occur in a network. The morning peak has a higher possibility to display significant hysteresis than the evening peak caused by the different heterogeneity and initial status, which indicates a higher peak flow does not represent a high stability level. The factors that cause the possibility of hysteresis occurring in different networks (network structure, flow environment, control strategy, etc.) deserve further comparative study. How to establish a mathematical expression of network peak stability to reflect the equilibrium status through the proposed coordinate system of variances will be investigated in the following study.

Notations

Variables and Functions

q_i: The flow on the road segment **i**
k_i: The density on the road segment **i**
o_i: The occupancy on the road segment **i**
q_w: Network-weighted flow
k_w: Network-weighted density
o_w: Network-weighted occupancy
v: Vehicle speed on road segment and network
l_i: The length of each road segment
N: The number of detectors in the network
h_i: The network inhomogeneity
k_m: Mean value of all road segments density
$q_{\max}$: Maximum network flow
k_c: Network critical density
k_j: Network jam density
$\varnothing(k_i)$: The probability density function on road segment **i**
v_e: Network equilibrium speed

Abbreviations

MFD: Macroscopic fundamental diagram.

Data Availability

The data used to support the findings of the study are collected in Atlanta, Georgia, from Transportation Department. Raw data can be obtained from the corresponding author upon request.

Disclosure

I would like to declare on behalf of my co-authors that the work described was original research that has not been published previously and not under consideration for publication elsewhere, in whole or in part.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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