

## Research Article

# An Optimization Strategy for Truncating Ultralong Bus Lines Integrated with Metro Networks

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For most ultralong bus lines, low punctuality rates and vehicle bunching are two main challenges. One potential strategy to solve these two problems is to split an ultralong line into two or three shorter lines according to historical passenger flow patterns as well as other presetting objectives and constraints. Thus, in this paper, a multiobjective optimization model is first established in consideration of both passenger flow distribution and coline metro networks. Then, the optimal truncated stop is determined by estimating the model results generated by enumerating each potential bus stop derived from related metro stops. The non-dominated sorting genetic algorithm II (NSGA-II) is applied to solve the model efficiently. A case study conducted on a real bus line associated with metro networks in Beijing, China, demonstrates the efficiency of the optimization strategy adoption in terms of operating cost and passenger travel time, even with a slight increase in passenger cost. Besides, the multivehicle type is also referred in the experiments to verify economic efficiency to reduce operational costs.

## 1. Introduction

The current public transportation system comprises bus lines of varying lengths, designed to provide travel service for passengers in residential areas or the central business district (CBD). Bus lines that surpass a specific length threshold are referred to as ultralong bus lines, with the threshold determined based on the city's development scale and the average speed of buses [1]. Through an analysis of route and passenger flow characteristics on ultralong bus lines, it was found that although these lines enhance the traffic radiation of the metropolis to the surrounding towns and reduce transfers, long-distance travel often leads to a low punctuality rate, increasing the probability of unknown security risks. In addition, it will also lead to an uneven distribution of passenger flow both in time and space, resulting in the frequent occurrence of bus bunching [2, 3]. Therefore, in order to avoid these negative effects, such ultralong bus lines in cities need to be optimized or adjusted.

Currently, the research on the operational strategy of ultralong bus lines mainly focuses on three aspects: line truncation, line shortening, and skip-stop service. For the first strategy, a bus line is split into two or more lines with unchanged stops [4, 5], while the second strategy involves creating a shorter line by selecting stops from a specific section of the original route; the last strategy is commonly employed in a public transit system that a vehicle passes through some stops that may not be continuous in a line [6, 7]. Early research on the abovementioned operation strategies mainly focuses on passenger flow distribution (PFD). Zhang et al. [8] proposed splitting strategies for long bus lines based on PFD and the average distance of the bus line by introducing the alternating coefficient. The skip-stop operation strategy is adopted when PFD is uniform, whereas for lines with nonuniform characteristics such as unimodal, symmetrical bimodal, and asymmetric bimodal PFD, the line-truncated operation strategy is commonly applied, and the truncated points are typically located near peak or valley

points [5]. In addition to relying primarily on PFD, researchers gradually focus on optimization models aimed at saving time or cost [9, 10]. Tang et al. [11] developed a bus skip-stop operation optimization model with the aim of minimizing system costs for passengers and operators while simultaneously enhancing the energy utilization rate of electric buses.

In terms of adjustment or optimization of ultralong bus lines, most scholars mainly focus on factors such as PFD and unbalance coefficient, as well as passenger or operator cost. However, the combination with the metro network is often overlooked in operating strategies, aimed at enhancing these bus lines just like the rail transit system in many megacities that have been well developed. Furthermore, since metro and bus are two integral components of urban public transport systems, it is necessary to strengthen the relationship between bus lines and metro networks to better solve public transit operation problems [12, 13]. Promoting the development and utilization of urban rail transit can also contribute in reducing greenhouse gas emissions and alleviating road congestion [14]. However, to the best of our knowledge, few studies pay special attention to ultralong bus lines with extremely long travel time and low occupancy rate, further reducing passenger satisfaction and operating benefits.

With the aim of fully improving the efficiency of urban public transport system, this paper provides a novel scheduling methodology for optimizing ultralong bus lines. We made the following threefold contributions: first, we define the ultralong line evaluation index to filter out target lines that need to be optimized. Second, we introduce a truncation optimization procedure to improve ultralong bus lines. A multiobjective optimization model is developed with the consideration of multivehicle type and cost of both passenger and operator, combining with the existing coline metro networks, and NSGA-II is employed to solve the model in order to generate the optimized truncated bus stop. Third, we perform extensive numerical experiments on a real-world instance based on IC card data, which yield valuable insights into the capability of the proposed optimization strategy.

The rest of the paper is listed as follows. Section 2 presents a review of the literature. Section 3 describes the analysis of ultralong bus lines. Section 4 introduces the multiobjective optimization model, and Section 5 represents the specific algorithm according to the model. Section 6 is case analysis. Section 7 is the conclusion, which summarizes the results of case analysis.

## 2. Literature Review

**2.1. Bus Line and Vehicle Scheduling Optimization.** The bus service planning process roughly includes the following five stages: route planning, frequency setting, schedule making, vehicle scheduling, and personnel arrangement. A high-quality bus service requires all stages to be optimized. Among them, the optimization of route planning is the basis of other stages, and it has been extensively studied by many scholars. Lin et al. [15] considered improving bus lines to

enhance the quality of passenger travel services by shortening the total length of bus routes under the premise of fully meeting the travel needs of passengers. Since an ultralong bus line may affect the punctuality rate of vehicles and cause vehicle bunching, a route optimization method referring to long-distance commuter buses is proposed to determine bus route length and each stop location and to minimize the total operating distance of the bus line under the limitation of the maximum capacity [16]. Zhang et al. [17] proposed dividing the long through-type bus lines into several independent bus lines by adjusting the stop setting and headway of bus lines, which not only effectively reduced the operational costs but also improved the unbalanced distribution of passenger flow on bus lines. The abovementioned studies mainly consider the benefits or costs of passengers and operators separately. Wu et al. [18] constructed a model with the objective of minimizing the costs of both passengers and operators using continuous functions, which saved the operating costs of enterprises and improved the travel satisfaction of passengers concurrently. Furthermore, the vehicle type is also considered in the route optimization problem [19]. For regular bus lines, previous studies mainly considered the optimization of stop location and route length, combined with the distribution of passenger flow, with the aim of reducing route length and minimizing the cost to the operators and the average travel time of passengers.

Regarding the scheduling optimization of public transport vehicles, the existing studies take the uncertainty of vehicle travel time and passenger arrival into account [20–23]. Zhang et al. [20] and Li et al. [21] considered the uncertainty of bus running time; the former optimized the existing bus schedule by fine-tuning vehicle departure time and parking vehicle at other stops, and the latter proposed a schedule optimization model containing fuzzy travel time by considering the bus running time as a fuzzy variable subject to various disturbances such as weather and traffic conditions. On the other hand, Liu et al. [22] considered the uncertainty of passenger arrival rate and built a robust optimization model of dynamic bus departure considering different passenger arrival rates under different scenarios by setting three scenarios of baseline passenger flow, high passenger flow, and low passenger flow so as to achieve higher operational efficiency. Xue et al. [23] simultaneously considered the uncertainty of bus running time and passenger arrival rate and established a bilevel programming model in order to tackle the bus scheduling problem. The abovementioned studies consider the effect of uncertain factors individually or simultaneously in determining an optimal vehicle schedule without combining associated data. Ma et al. [24] developed a data-driven bus schedule utilizing bus GPS data and IC card data to replace the current schedules based on human experience. In the abovementioned research, when combining uncertain factors or historical data to optimize vehicle scheduling, the vehicle capacity is set as fixed, while the vehicle scheduling problem referring to multiple vehicle types is not considered. Sun et al. [25] proposed a flexible scheduling optimization method considering

mixed vehicle types. The abovementioned studies mostly solve the bus route design or vehicle scheduling problem separately, but less consider jointly optimizing the above two problems.

In the existing literature, the joint optimization for both bus routes and vehicle scheduling is mainly applied in various scenarios and also considers relative factors. In terms of the objective function, Yu et al. [26] considered the passenger cost, and with the objective of minimizing the total travel time of passengers, a vehicle scheduling model was proposed to determine vehicle frequency and bus travel route. Besides, Kenyon et al. [27] constructed an optimization model with the objective of minimizing the total vehicle travel time in view of path issues of vehicles. The abovementioned studies establish an optimization model based on a single-objective function such as passenger benefits or operator cost. However, Chen et al. [28] and Israeli et al. [29] considered both operator and passenger costs to build a model and, respectively, solved the model by using a forbidden search heuristic algorithm and an exhaustive search algorithm so as to improve the efficiency of the bus service while reducing the travel time of passengers. Li et al. [30] and Ahern et al. [31] took both operator and passenger benefits into account and established a biobjective optimization model. The former established a stochastic expected value model for bus routes in response to the random behaviors of passengers and bus vehicles and used a hybrid intelligent algorithm combining stochastic simulation and genetic algorithms to solve it. The latter proposed a model combining vehicle scheduling and the assignment of passengers to reduce the operating costs while ensuring that passengers reached their destinations without affecting vehicle travel time. Yu et al. [32] introduced headway in modeling while considering operator and passenger costs and proposed a headway optimization model for bus routes, which was solved by a parallel genetic algorithm. In the abovementioned studies, the objectives of joint optimization are divided into single objective and multiobjectives. The establishment of a multiobjective joint optimization model can take into account the benefits of both passengers and operators and the compromised proposal obtained through the solution can improve the satisfaction of both parties, which is more applicable than the single-objective optimization model. Some studies take the headway into account, and the established model referring to different types of vehicle capacity is less analyzed. In terms of model solving, it mainly consists of designing multiobjective algorithms [28–30], which is generally done by converting a multiobjective problem into a single-objective problem to be solved by setting weighting values for subobjectives [31, 32]. However, the weights of different objective values are often difficult to be determined, and if the weights are not set properly, it will have a greater impact on the optimization results. In contrast, the multiobjective optimization algorithm can effectively avoid the problem of improperly setting the weights of subobjectives; thus, it is easier to obtain a better solution. In this paper, with the objective of minimizing the cost for both operators and passengers, a model with consideration of different vehicle types is developed to

simultaneously tackle bus routes designing and vehicle scheduling problem, and it is solved by using the fast nondominant sorting genetic algorithm II (NSGA-II).

## 2.2. Bus Line Optimization Combined with Metro Networks.

Buses and metros are important components of the urban public transportation system, so it is of great significance to optimize bus lines in conjunction with metro networks. The key to the design of the integrated system of bus and metro is how to optimize the bus routes, make them adapt to metro lines, and form an organic connection in time and space, so as to shorten the transfer time of passengers, thus improving the transfer efficiency between lines, enhancing the transportation capacity of the entire integrated transportation system, and providing urban residents with more convenient and efficient travel experience. Meanwhile, it also helps in reducing traffic congestion and environmental pollution [33–35].

In recent years, research on the joint optimization of bus and metro has attracted much attention, and many researchers have conducted relevant research on the design and optimization of bus routes connecting with metro stations. Zhu et al. [36] established a route model for feeder bus service with the objective of maximizing potential demand and used genetic algorithms to generate optimal routes that start and end at rail transit stations. Wei et al. [37] proposed a feeder bus route positioning process in order to attract the potential travel and transfer demand by analyzing the attraction range of the metro station. In the process of bus route optimization, in addition to considering passenger demand, some researchers took the interests of both passengers and operators into account. Lai et al. [38] proposed a multiple-to-multiple optimization model between metro stations and feeder bus stops in the feeder service area with the objective of minimizing feeder bus operating cost and total passenger travel time; however, they overlooked transit transfers. Cao et al. [39] took into consideration the impact of passenger transfer on their travel time cost and established the optimization model in view of transfer time and the number of transfers. In recent studies, the integrated model connecting bus and metro lines is established to tackle both route optimization and vehicle scheduling problems. Xiong et al. [40, 41] studied the optimization problem of community bus routes connected to metro stations with the objective of minimizing the total cost of both operator and passenger costs. Zhang et al. [17] studied long through-type bus lines near metro stations and established a bilevel programming model considering passenger and operator cost, as well as passenger flow allocation. The experimental results show that the operating cost could be evidently reduced and the passenger flow balance coefficient of long-distance bus lines near metro stations could be improved.

The multimodal public transport network realizes the organic combination of metro, urban rail, bus, and other transportation modes, which can expand the accessibility of rapid transit lines and improve the operating efficiency of

public transportation [42]. Some researchers have studied the optimization of bus routes in the integrated system of bus and metro. Synchronization of the feeder bus network system and rail transit system was considered by Almasi et al. [43, 44], and a model with the goal of minimizing the total operational cost was established. Costescu et al. [42] considered not only the minimization of the operational cost but also the minimization of passenger travel time in the bus route planning model. Huang et al. [45] proposed a three-stage design method for the integrated bus and metro system. Hub stations and radiating nodes were selected from a clustering method, and a bilevel programming model was established with the minimum system cost to obtain the optimal passenger route. In the joint optimization of rail transit and bus networks within the transportation corridor, Chien et al. [46] gave a classification method of bus routes under the condition of rail transit and established a model to optimize the bus headway, bus stop location, and bus route distance, whereas Sun et al. [35] built a multiobjective model with the objective of maximizing rail traffic flow and minimizing total passenger travel time.

In studies on joint optimization of bus and metro networks, not all bus routes need to be optimized and only a few scholars have mentioned how to select bus lines that need to be optimized. Wei et al. [47] used the smart card data in the urban public transportation system to generate a passenger origin-destination (OD) matrix and determined the routes and stops to be adjusted by analyzing boarding and alighting numbers at each stop. Cui et al. [48] determined potential bus lines to be optimized by evaluating the utility of collocation segments of bus lines and metro networks. In addition, some scholars selected bus lines along metro lines that need to be adjusted based on passenger travel preferences. Cao et al. [49] determined the bus line to be optimized by establishing a travel choice logit model based on residents' travel preferences.

In the abovementioned studies on combined bus lines and metro networks, few studies focused on the optimization and adjustment of ultralong bus lines. To improve the transportation efficiency of ultralong bus lines and reduce the negative impact of factors such as bus bunching and low punctuality rates, a multiobjective optimization model with the aim of minimizing passenger travel cost and operator cost, respectively, is established to optimize bus lines combined with metro networks and selected based on IC card data. NSGA-II is used to solve the model to obtain the optimal truncated stop, fleet size, vehicle type, and schedule.

### 3. Analysis of Ultralong Bus Lines

*3.1. The Determination of Ultralong Bus Lines.* The first problem that needs to be addressed is how to specifically identify ultralong bus lines from numerous regular bus lines before optimizing procedures. In this study, ultralong bus lines are determined using (1) from the standard [1]. If a bus line's length exceeds the upper limit predefined in (1), it is defined as an ultralong line.

$$L_{\max} = \frac{S_n \times T_{\max}}{60}, \quad (1)$$

where  $L_{\max}$  is the theoretical maximum length of urban bus lines (km). When a city's regular bus line crosses this threshold, it is considered to be an ultralong bus line.  $S_n$  is the average running speed of buses in the line  $n$  (km/h).  $T_{\max}$  is an average maximum one-way travel time for 95% local residents using public transit. For megacities, such as Beijing and Shanghai, the value can be set as 60 minutes [1].

#### 3.2. The Determination of Ultralong Bus Line to Be Adjusted.

The ultralong bus lines that are attempted to be adjusted in this study are lines not only constrained by length limit in (1) but also with low punctuality rate and frequent bus bunching. The bus bunching phenomenon refers to the meeting of two or more vehicles on the same bus route in operation, including the meeting at one stop or between bus stops. Meanwhile, this study discusses bus bunching at stop in the following two situations: (a) the bus  $j$  has not left stop  $k$  when  $j + 1$  bus arrives at the stop  $k$  in the bus line and (b)  $j + 1$  bus leaves stop  $k$  earlier than bus  $j$ . The bus bunching rate is defined as the ratio of the number of vehicles occurring bus bunching to the total number of buses within a given time interval. A line is regarded as a high-frequent bunching line when the bunching rate is more than 5% [50]. In addition, the punctuality rate refers to the probability that a vehicle arrives at its scheduled stop within a given time window. Kho et al. [3] defined bus punctuality as the adherence to on-time performance and headway and proposed three related indicators to determine the punctuality rate. In this study, we say a vehicle is punctual when the vehicle arrives at the target stop less than 5 minutes early or late compared to the prescribed time. In this study, we define a bus route is not punctual when its punctuality rate is less than 60%.

## 4. Model Formulation

*4.1. Assumptions.* For the convenience of model development, the following assumptions are proposed:

- A1. In the optimization process, the total passenger capacity of the network is set to a fixed value
- A2. The truncation stop of the bus line is located near the metro stations
- A3. Passengers can only transfer to bus or metro
- A4. In the original bus line, 70% of the passengers passing through the truncated stop transferred to metro and 30% to the bus
- A5. Ignoring the deceleration and acceleration of the bus as it arrives and leaves the station, the average speed is taken as the travel speed
- A6. It is assumed that the time taken by the passengers in the boarding and alighting process is neglected; the time taken by the vehicle to reach the station is the boarding and alighting time of the passengers

4.2. *Notations.* The parameters and variables used in the model are defined as shown in Table 1.

4.3. *Problem Description.* In the study, optimization of an ultralong bus line relies on metro stops nearby. As shown in Figure 1, bus line #52 is an ultralong line, which intersects with several metro lines, and the intersection points are several stops, which can be used as potential truncation sites. The ultralong bus line is divided into several segments by these potential truncation sites. The main research in this paper is how to select the optimal truncation stop from the set of potential truncation stops, and how to obtain the optimal vehicle scheduling after the line is truncated at the site.

4.4. *Objective Function.* The objective function is a dual objective, where (2) and (3) represent the minimization of total costs for passengers and bus companies, respectively.

$$\min C_1 = C_P^1 + C_P^2 + C_P^3, \quad (2)$$

$$\min C_2 = C_F + C_V. \quad (3)$$

4.4.1. *Passenger Cost.* The total cost of passengers  $C_1$  consists of ticket purchase cost  $C_P^1$ , time cost  $C_P^2$ , and a series of penalty costs  $C_P^3$ , as shown in (2).

The sum of the ticketing costs  $C_P^1$  for each passenger's metro and bus trips can be expressed as follows:

$$C_P^1 = \sum_{n \in N} \sum_{k \in K} \sum_{y \in Y} \sum_{j \in J} \left( \sum_{i \in I} a_{ij}^k \cdot v_{jy}^n \cdot F_y + \sum_{i \in I_B^k} a_{ij}^k \cdot v_{jy}^n \cdot F_y \right) + \sum_{i \in I_M^k} F_i^D. \quad (4)$$

In (4), the first term is the ticket purchase cost for all passengers on the bus, including the first boarding and bus transfer; the second term is the ticketing cost of transferring to the metro at the truncation stop.

The set of passengers at the truncation stop  $k$  transferring to a bus or metro can be expressed as follows:

$$I_B^k = \left\{ i \mid e_i^k \cdot \lambda_i^k > 0.5, x_k = 1, i \in I \right\}, \quad k \in K, \quad (5)$$

$$I_M^k = \left\{ i \mid e_i^k \cdot (1 - \lambda_i^k) > 0.5, x_k = 1, i \in I \right\}, \quad k \in K. \quad (6)$$

Equation (5) represents the set of passengers who pass through a truncated stop and transfer to the bus at that stop with a probability greater than 50%. Equation (6) expresses the set of passengers who pass through a truncated stop and transfer to the metro at that stop with a probability greater than 50%.

The cost of passenger time including in-vehicle time and waiting time cost can be expressed as follows:

$$C_P^2 = \sum_{\substack{i \in I \\ i \notin I_B^k}} \frac{(B_i^T + B_i^W) \cdot \varepsilon}{60} + \frac{\left( \sum_{i \in I_M^k} (M_i^T + M_i^W) + \sum_{i \in I_B^k} (B_i^T + B_i^W) \right) \cdot \varepsilon}{60}. \quad (7)$$

In (7), the first term represents the bus ride time cost, which includes the waiting time cost and in-vehicle time cost before the splitting stop. The second term expresses

the waiting time cost and in-vehicle time cost after the passenger transfers to the bus or metro at the splitting stop.

TABLE 1: Notations.

Notations	Definitions
<b>Sets</b>	
$I$	Set of passengers, $i \in I$
$J$	Set of buses, $j \in J$
$K$	Set of bus stops on the bus line, $k \in K$
$Y$	Set of bus types, $y \in Y$ , $y$ representing the vehicle type, where $y = 1$ represents small bus and $y = 2$ represents a larger bus
$N$	Set of bus lines, $n \in N$
$M$	Set of metro lines, $m \in M$
$I_M^k$	Set of passengers who choose to transfer to the metro at the truncation stop $k$
$I_B^k$	Set of passengers who choose to transfer to the bus at the truncation stop $k$
<b>Parameters</b>	
$L$	Length of the original bus line (km)
$L_n$	Length of new bus line (km)
$L_A$	Average distance between bus stops (km)
$L_A^M$	Average distance between metro stops (km)
$L_{\max}$	Theoretical maximum length of urban bus lines (km)
$S_n$	The average speed of buses on a metro line $n$ (km/h)
$T_{\max}$	The average maximum travel time of 95% of the residents (minutes)
$S_m$	The average speed of trains on a metro line $m$ (km/h)
Notations	Definitions
$S_j$	The average travel speed of bus $j$ (km/h)
$A_1$	Constant, passenger waiting time penalty critical value (minutes)
$A_2$	Constant, maximum number of transfers for passengers
$A_3$	Constant, maximum waiting time for passengers, $A_3 > A_1$ (minutes)
$A_4, A_5$	Constant, maximum and minimum headway of vehicles (minutes)
$\lambda_i^k$	The probability of passenger $i$ transferring to bus at truncation stop $k$ , $0 \leq \lambda_i^k \leq 1$
$B_i^T$	Bus ride time for passenger $i$ (minutes)
$B_i^W$	Average wait time at bus stop for passenger $i$ (minutes)
$M_i^T$	Metro ride time for passenger $i$ (minutes)
$M_i^W$	Average wait time at metro stop for passenger $i$ (minutes)
$\delta$	Crowding penalty coefficient (CNY/stop)
$\alpha$	Passenger transfer penalty coefficient (CNY/person)
$\varepsilon$	Value of time (CNY/h)
$\beta$	The penalty factor when waiting time is longer than $A_1$ (CNY/person)
$\tau$	Coefficient of crowding
$H^n$	The number of buses of type $y$ at line $n$
$C_y^\phi$	Vehicle type $y$ operating cost per kilometer (CNY/km)
$F_y$	The ticket cost of riding the type $y$ bus (CNY/person)
$F_i^D$	The ticket cost of passenger $i$ when riding the metro (CNY/person)
$C_y^B$	The purchase cost of a single bus with vehicle type $y$ that is averaged to each day based on its service life (CNY/vehicle)
$C_y^P$	The salary of a single driver for bus type $y$ (CNY/person)
$I_{sum}$	The total number of passengers in the original line
$V_{\max}^y$	The maximum passenger capacity for bus type $y$ (person)
$\eta_m$	The headway of metro line $m$ (minutes)
$SK$	The total number of stops in the original bus line
$X_{\min}, X_{\max}$	Minimum and maximum truncation stop limits
$T_i^k$	Transfer time for passenger $i$ at stop $k$ (minutes)
$HM_y^n$	The maximum number of buses of type $y$ required in line $n$
<b>Auxiliary variables</b>	
$C_V$	Total operating cost of buses (CNY)
$C_P^1$	Total ticket cost for passengers (CNY)
$C_P^2$	Total time cost of passenger travel (CNY)
$C_F$	Total fixed cost of the bus company (CNY)
$C_P^3$	Penalized total cost (CNY)
$C_1$	Total travel cost of passengers (CNY)
$C_2$	Total cost of the bus company (CNY)
$V_k^j$	The number of passengers in bus $j$ at stop $k$ (person)

TABLE 1: Continued.

Notations	Definitions
$Q_n$	Number of total passengers in an hour of line $n$ (person/h)
$I_k$	The total number of passengers transferring at stop $k$ (person)
$IB_k$	The total number of passengers transferring to bus at the truncation stop $k$ (person)
$IM_k$	The total number of passengers who transferring to metro at the truncation stop $k$ (person)
$e_i^k$	Binary variable, 1 if passenger $i$ passes through stop $k$ , and 0 otherwise
$a_{ij}^k$	Binary variable, 1 if passenger $i$ chooses vehicle $j$ at the stop $k$ , and 0 otherwise
$d_{ij}^k$	Binary variable, 1 if passenger $i$ in-vehicle $j$ alights at stop $k$ , and 0 otherwise
$c_i^m$	Binary variable, 1 if passenger $i$ chooses metro line $m$ , and 0 otherwise
<b>Decision variables</b>	
$\eta_n$	The bus headway of line $n$ (minutes)
$x_k$	Binary variable, 1 if $k$ is the truncation stop, and 0 otherwise
$w_i$	Binary variable, 1 if the waiting time of passenger $i$ is greater than $A_1$ , and 0 otherwise
$r_k^j$	Binary variable, 1 if the number of passengers in-vehicle $j$ exceeds $\tau V_{\max}^y$ at stop $k$ , and 0 otherwise
$z_k$	Binary variable, 1 if existing at least a metro stop near bus stop $k$ , and 0 otherwise
$v_{jy}^n$	Binary variable, 1 if the vehicle $j$ of type $y$ is in the line $n$ , and 0 otherwise

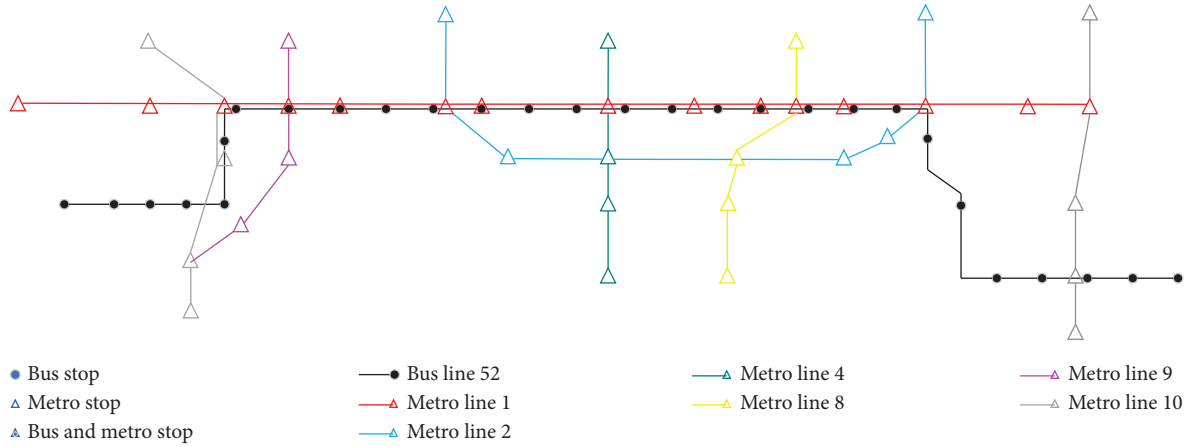


FIGURE 1: Part of #52 bus line and metro network.

$$\begin{cases}
 a_{ij}^{k_1} = 1, \\
 d_{ij}^{k_2} = 1, \\
 B_i^T = (k_2 - k_1) \cdot \frac{L_A}{S_j}, \quad i \notin I_B^k, \quad i \in I, j \in J, (k_1, k_2, k) \in K, \\
 B_i^T = (k_2 - k_1) \cdot \frac{L_A}{S_j} + T_i^k, \quad i \in I_B^k,
 \end{cases} \quad (8)$$

$$B_i^W = \sum_{k \in K} \sum_{j \in J} \sum_{n \in N} \sum_{y \in Y} \frac{1}{2} a_{ij}^k \cdot v_{jy}^n \cdot \eta_n, \quad i \in I, \quad (9)$$

$$M_i^T = \sum_{m \in M} c_i^m \cdot L_A^m \cdot \frac{\gamma_i}{S_m}, \quad i \in I, \quad (10)$$

$$M_i^W = \sum_{m \in M} c_i^m \cdot \frac{\eta_m}{2}, \quad i \in I. \quad (11)$$

The first and second items of (8) mean that passenger  $i$  traveling by bus  $j$  boarding and alighting at stops  $k_1$  and  $k_2$ , respectively; the third and fourth items calculate the travel time before and after the splitting stop. Equation (9) calculates the passenger's waiting time based on the headway of the lines. Equation (10) calculates the travel time of passengers on the metro mainly based on the average travel speed of the local metro line. The waiting time for passengers when they transfer to the subway is expressed by the authors in (11).

The total penalty cost is comprised of the following three costs: the penalty cost for an additional transfer brought on by line truncation, the penalty cost for a long wait, and the penalty cost for in-vehicle congestion.

$$C_P^3 = \sum_{k \in K} I_k \cdot \alpha + \sum_{j \in J} \sum_{k \in K} r_k^j \cdot \delta + \sum_{i \in I} w_i \cdot \beta. \quad (12)$$

In (12), the first term is the additional transfer penalty cost for passengers, which depends on the passengers passing through the truncated stop. The second term is the congestion penalty cost, when in-vehicle passengers exceed the expected load value  $\tau V_{\max}^y$ . Generally,  $\tau$  relies on how passengers actually feel. The last term is the waiting time penalty cost, when passengers' waiting time exceeds their expected value.

**4.4.2. Bus Company Cost.** The cost  $C_2$  of the bus company consists of a fixed cost  $C_F$  and an operating cost  $C_V$ , as shown in (3).

$$C_F = \sum_{n \in N} \sum_{y \in Y} (C_y^B \cdot H_y^n + H_y^n \cdot C_y^P). \quad (13)$$

The first term in (13) represents the cost of vehicle purchase, which assumes that the wage of each driver depends on the type of bus. The second term expresses the labor cost, which relies on the number of drivers deployed on the route.

$$H_y^n = \sum_{j \in J} v_{jy}^n, \quad y \in Y, n \in N. \quad (14)$$

Equation (14) calculates the number of buses of type  $y$  needed in line  $n$ , and the number of drivers equipped is equal to the number of buses. The operating cost of the bus company can be expressed as follows:

$$C_V = \sum_{n \in N} \sum_{j \in J} \sum_{y \in Y} L_n \cdot v_{jy}^n \cdot C_y^O. \quad (15)$$

In (15), the operating cost of the bus company depends on the length of the operating line  $L_n$  and the vehicle type  $y$ .

$$L_n = \begin{cases} \sum_{k \in \{k_1, 2, \dots, k\}} (1 - x_k) \cdot L_A, & n = 1, \\ \sum_{k'' \in \{k, k+1, \dots, SK\}} (1 - x_{k''}) \cdot L_A + x_k \cdot L_A, & n = 2, \end{cases} \quad k \in K. \quad (16)$$

The length of the bus line is calculated by (16), which is mostly determined by the position of the truncation stop, the number of stops, and the average stop distance. (16) defines that when only one truncated stop  $k$  exists in the line, namely,  $x_k = 1$ , the upper  $n = 1$  represents the part from the origin depot to the truncated stop and the lower  $n = 2$  is the other part from the truncated stop to the terminal stop.

#### 4.5. Constraints

$$B_i^W < A_3, \quad i \in I, \quad (17)$$

$$\sum_{k \in K} x_k < A_2, \quad (18)$$

$$v_{jy}^n \cdot V_k^j \leq V_{\max}^y, \quad y \in Y, j \in J, k \in K, n \in N, \quad (19)$$

$$V_k^j = \begin{cases} \sum_{k' \in \{k_1, 2, \dots, k\}} \left( \sum_{i \in I} a_{ij}^{k'} - \sum_{i \in I} d_{ij}^{k'} \right) up, \\ \sum_{k'' \in \{k, k+1, \dots, SK\}} \left( \sum_{i \in I} a_{ij}^{k''} - \sum_{i \in I} d_{ij}^{k''} \right) down, \end{cases} \quad k \in K, j \in J, \quad (20)$$



$$\begin{cases} A_4 \geq \eta_n \geq A_5, A_4 \leq \frac{60V_{\max}^y}{Q_n}, \\ 60V_{\max}^y/Q_n \geq \eta_n \geq A_5, A_4 > \frac{60V_{\max}^y}{Q_n}, \end{cases} \quad n \in N, y \in Y, \quad (21)$$

$$\begin{cases} \sum_{k \in K} x_k \cdot z_k \geq 1, \\ X_{\min} \leq k \leq X_{\max}, \quad x_k = 1, k \in K, \end{cases} \quad (22)$$

$$X_{\min} = \frac{(L - L_{\max})}{L_A}, \quad (23)$$

$$X_{\max} = SK - X_{\min}, \quad (24)$$

$$\sum_{j \in J} v_{jy}^n \leq HM_y^n, \quad y \in Y, n \in N, \quad (25)$$

$$HM_y^n = \frac{2L_n}{(S_n \cdot \eta_n)}, \quad y \in Y, n \in N, \quad (26)$$

$$w_i \geq 0, r_k^j \geq 0, c_i^m \geq 0, \quad i \in I, j \in J, k \in K, m \in M. \quad (27)$$

The waiting time limit for each passenger is represented by formula (17). The limit number of truncated stops is expressed by formula (18). Formula (19) represents the bus capacity constraints, where  $V_k^j$  is determined by the number of passengers alighting and boarding at each stop. Equation (20) calculates the number of passengers in the bus  $j$  when it arrives at stop  $k$  in the ascending or descending direction, which is mainly determined by the number of passengers boarding and alighting at each stop. Formula (21) reflects the headway limit, which is primarily determined by the maximum passenger flow in an hour and bus capacity, as well as applicable restrictions such as minimum and maximum headway limits. Formula (22) is the constraint for the number of truncated stops derived from the metro stop set. The upper and lower limits of truncated stops can be represented by equations (23) and (24). Formula (25) expresses the quantity constraint of each vehicle type in each line. Equation (26) calculates the maximum number of buses allocated to the bus line determined by the length of the line, the average speed, and the headway. Formula (27) represents a constraint on the 0-1 variable.

## 5. Algorithm Solution

The solution of the model is primarily divided into two steps. First, alternative truncation stops that satisfy relevant constraints are identified as potential truncation stops. Second, an algorithm is employed to determine the optimal headway, fleet size, and schedule for each alternative truncation stop. The target line optimization process is shown in Figure 2.

The data utilized in this research encompass the arrival time information of each passenger. The algorithm's

variables, including passengers' waiting time, maximum passenger count, and vehicle type, are determined by the vehicle's headway as depicted in Figure 3. Consequently, headway serves as the fundamental variable to be resolved, influencing both other variables' outcomes and optimization performance.

Bus network design and frequency setting are non-deterministic polynomial (NP-hard) problems, and heuristic algorithms have obvious advantages in solving these problems [51]. The improved NSGA-II based on the genetic algorithm can effectively solve the difficult problem of weight setting in multiobjective problems [52]. Among them, fast nondominated sorting, crowding distance assignment, and building partial ordering sets, which are all influenced by the quantity of targets and individuals, are the key factors that affect the computing efficiency of NSGA-II. In the algorithm, it is imperative to continuously calculate how many people board and alight at each stop with the constraints of a problem, the program's space complexity is large, as well as constraints on its computing efficiency [53]. In summary, NSGA-II is used to solve a multiobjective mathematical programming model for the optimization of ultralong bus line in this study, which yields a Pareto optimal solution through the process of fast nondominated sorting.

**5.1. Chromosome Representation.** In this study, each chromosome represents the headway of two subroutes after being truncated. An 8-bit binary encoding is used to keep the headway of each line within the range of 8 to 15 minutes. Since the first and fifth positions are set to fixed values, these two genes in the chromosome are ignored, as shown in Figure 4.

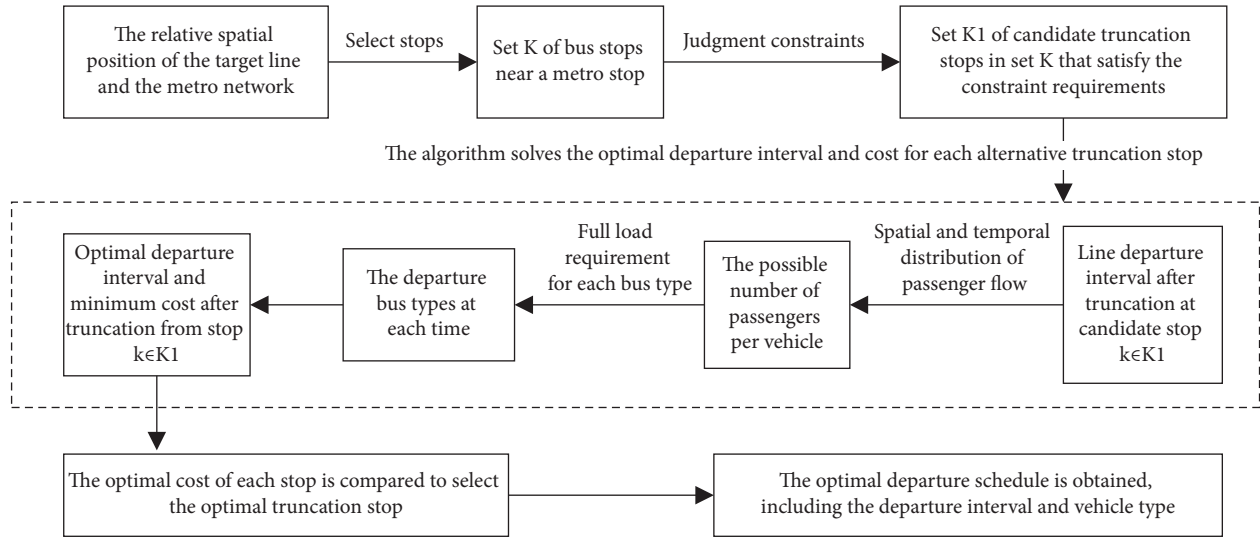


FIGURE 2: Optimization process of ultralong bus lines.

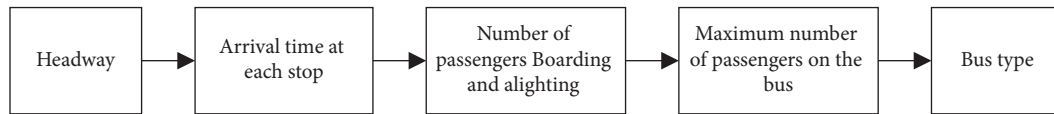


FIGURE 3: Relationships between variables.

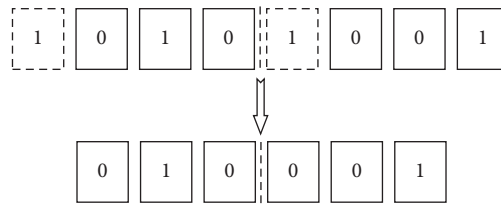


FIGURE 4: Chromosome diagram.

The crossover operation is shown in Figure 5. A partial segment crossover operation is utilized involving exchanging the two segments from two parent chromosomes to create new individuals according to a predetermined probability.

The process of mutation is shown in Figure 6.

The selection method is tournament selection, as shown in Figure 7.

5.2. Procedures. The algorithm flow is as follows:

- Step 1: set the algorithm parameter values, including the number of individuals  $N$  in the population, the maximum number of iterations  $Q$ , the probability of crossover and mutation, and the iteration number  $t = 0$
- Step 2: initialize, encode the individuals, and generate populations
- Step 3: calculate the fitness values of the individuals in the initial population

- Step 4: check each individual whether it satisfies the constraints; if not, its fitness value is set to infinity
- Step 5: the individuals in the population are sorted by fast nondominated sorting according to their fitness values
- Step 6: the crowding distance between individuals is calculated to measure the quality of each solution in the same Pareto front
- Step 7: using elitism, according to the results of fast nondominated sorting and crowding distance calculation, the individuals in the population are sorted and the better ones are selected
- Step 8: generate a new population through crossover and mutation
- Step 9: calculate the fitness values of all individuals covering the parent population and the offspring population, and then set,  $t = t + 1$
- Step 10: if  $t < Q$ , then return to Step 4; otherwise, end.

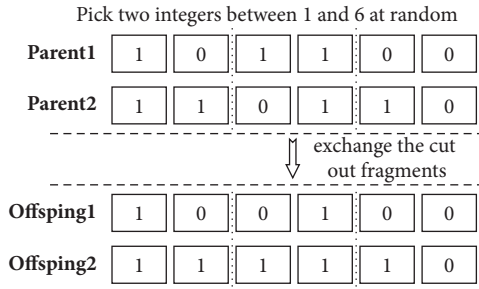


FIGURE 5: Fragment crossover.

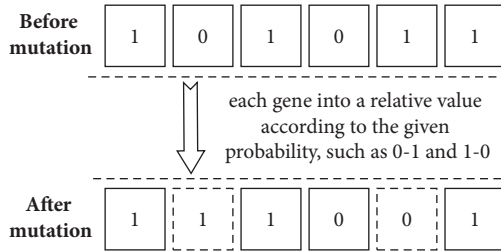


FIGURE 6: Gene mutation.

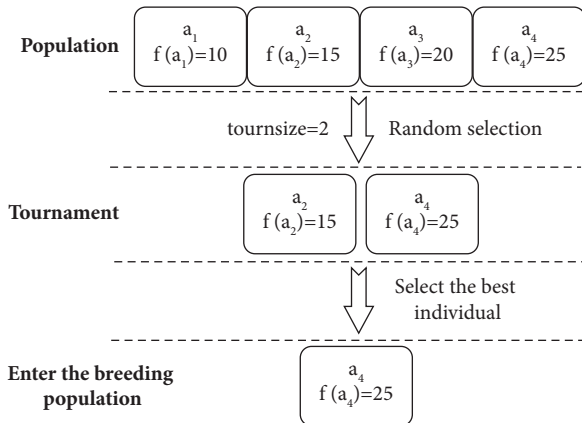


FIGURE 7: Tournament selection.

5.3. *Algorithm Results.* The initial population of the algorithm is set to 16 individuals, and it is iterated for 30 generations with a crossover probability of 0.7 and a mutation probability of 0.05. The algorithm is run on a personal computer with an Intel(R) Core (TM) i7-7700HQ (2.80 GHz) central processing unit, and the average running time is approximately 5 hours.

The best individual is selected from the Pareto front solutions as a representative in each iteration and it plots the iteration curve as shown in Figure 8 (only S15 is shown as an example). The horizontal axis represents the number of iterations, and the vertical axis represents the minimum fitness value obtained by the weighted summation of the two objective function values of each individual on the current Pareto frontier. The current optimal solution is determined

by normalizing the objective values of all options in the Pareto frontier and weighting them. Therefore, the solution with the lowest value is then considered to be optimal.

Figure 8 demonstrates that the method reaches the convergence stage after 17 iterations when the weighted sum value of the fitness of the best individual no longer varies.

## 6. Case Study

In this case study, an ultralong bus line is determined based on public transport IC card data from Beijing in 2018, and then the selected bus line is utilized to show the optimization results and applicability of the proposed strategy on the ultralong bus line.

6.1. *Determination of the Ultralong Bus Line.* The IC card data mainly include bus number, line number, passenger boarding, and alighting time and stop. The specific fields are shown in Table 2.

In this study, Beijing Bus #52 is selected as the target line. The total length of this line is 22 km, and the average bus speed is 16.9 km/h. The population of Beijing exceeds 20 million, so  $T = 60$  minutes, and the maximum length limit of the line can be calculated as 16.9 kilometers according to formula (1). After processing and analyzing the passenger flow data, it is found that numerous bunching phenomena exist on this line, and its punctuality rate is relatively low. The partial bus bunching phenomenon will affect the overall service efficiency of the whole public transport network, resulting in a serious imbalance of in-vehicle passengers and waiting time on several bus stops, as well as a low punctuality rate.

In Figures 9(a) and 9(b), 8 vehicle trips are listed both in the up and down directions between 7:00 a.m. and 8:00 a.m. at each stop in #52. The bunching rate in the up direction is up to 44.1% and the punctuality rate is 50.0%, while in the contrary direction, the bunching rate is up to 58.5% and the punctuality rate is 43.9% during the peak travel hours from 7:00 a.m. to 9:00 a.m. and 5:00 p.m. to 7:00 p.m. Consequently, urgent adjustments to the bus line are necessary to address the frequent bunching and low punctuality rate during this time period.

6.2. *Results' Analysis.* In this paper, #52 is taken as a specific example, and the main parameters are presented in Table 3. The distribution of metro lines and stops along #52 is shown in Figure 10.

In Table 3, the vehicle costs are calculated on a daily basis and determined by the purchase price and service life of each vehicle. The fare of #52 is 2 CNY for 10 kilometers and adds 1 CNY for an additional 5 kilometers. In 2018, the per capita GDP in Beijing was approximately 140,000 CNY, so the travel time cost for residents during that period is about 34.5 CNY/h [54]. The selection of truncation stops in this study is primarily based on the analysis of passenger flow patterns and the presence of subway stops under specific constraints.

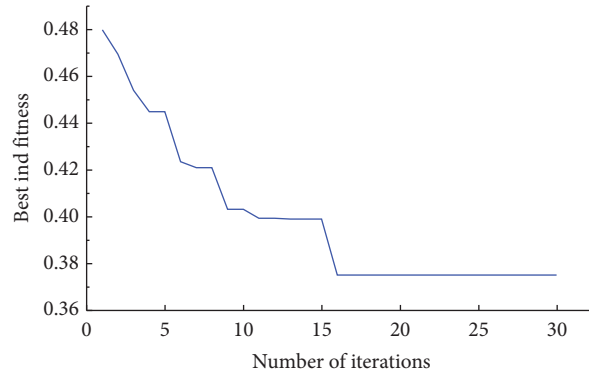
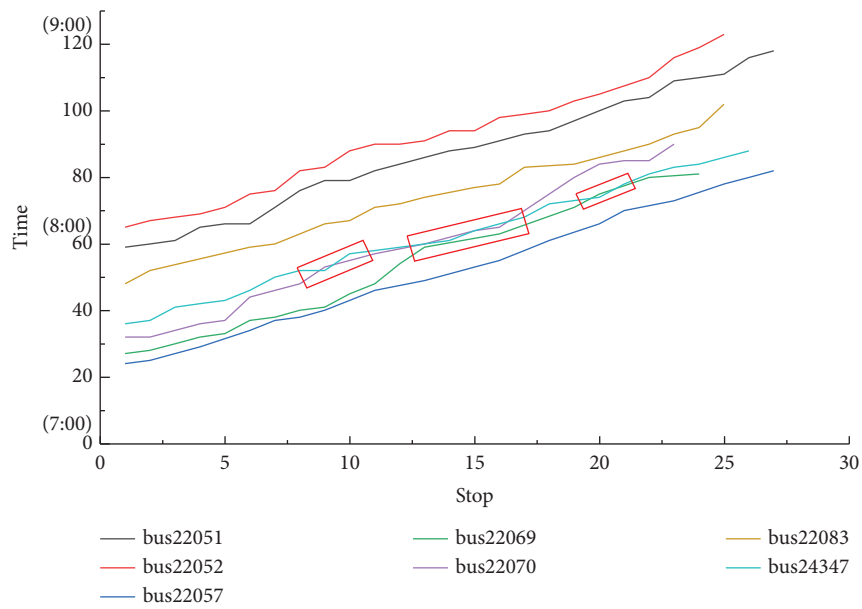


FIGURE 8: Iteration curve: the horizontal axis represents the number of iterations, and the vertical axis represents the minimum fitness value obtained by the weighted summation of the two objective function values of each individual on the current Pareto frontier.

TABLE 2: Partial fields of passenger travel IC card data.

Line number	Bus number	Boarding stop	Alighting stop	Boarding time	Alighting time
52	22049	2	7	20181201182500	20181201183853
52	22049	2	3	20181201182600	20181201182804
52	22049	2	3	20181201182600	20181201182812
...	...	...	...	...	...



(a)

FIGURE 9: Continued.

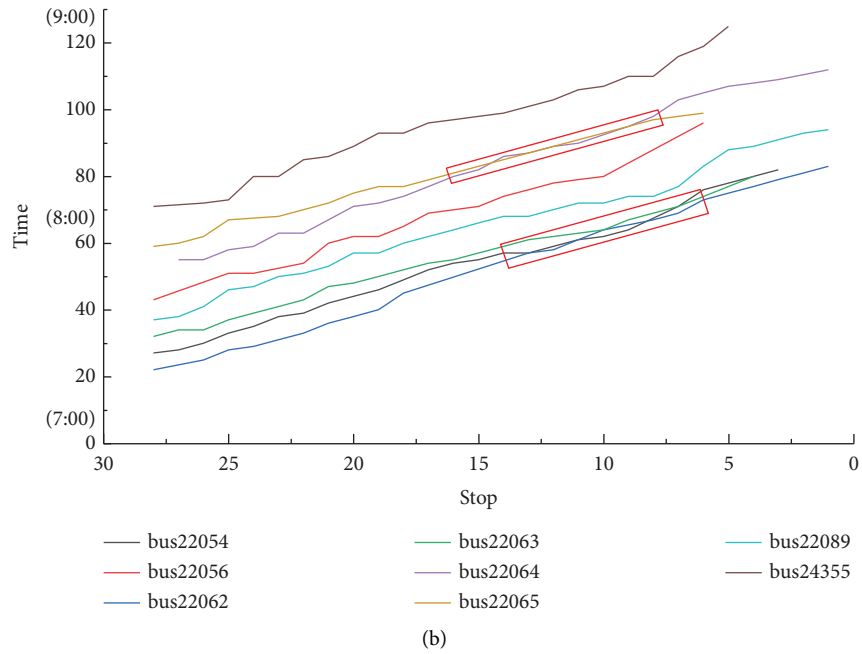


FIGURE 9: Bunching in the up and down direction. The arrival time of vehicles at each stop: the intersection is the phenomenon of bunching. (a) Up direction. (b) Down direction.

TABLE 3: Parameters of #52.

Parameters	Data
Maximum section passenger flow per hour	412 person/h
Total passenger flow	21011 persons
Line length	22 km
Number of stops	Up: 27 stops, down: 28 stops
Value of time	34.5 CNY/h
The vehicle cost of a small bus	205.48 CNY/(vehicle-day)
The operating cost of a small bus	3 CNY/km
Maximum capacity of a small bus	60 people
The vehicle cost of a large bus	410.96 CNY/(vehicle-day)
The operating cost of a large bus	5 CNY/km
Maximum capacity of a large bus	120 people

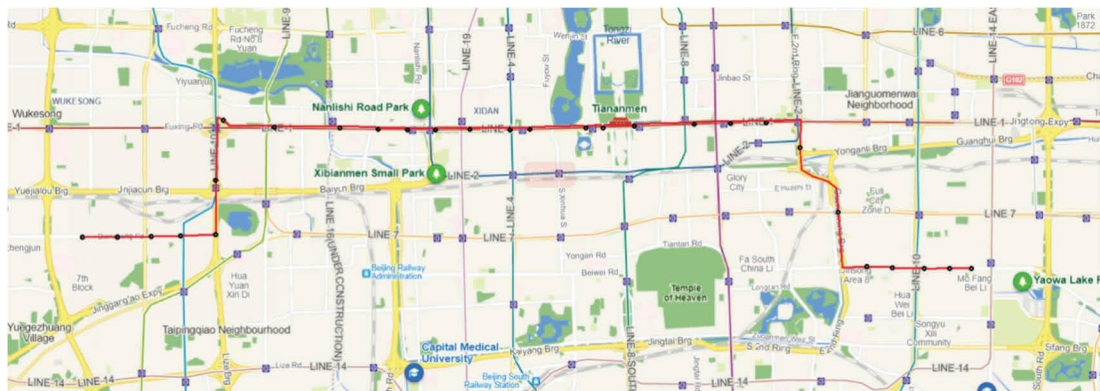


FIGURE 10: Distribution of metro lines and stops along #52. Red lines and black dots represent bus lines and stops, and other colored lines and purple dots represent metro lines and stops.

In Figure 10, S13, S15, S16, S17, and S18 have been identified as potential candidates for truncation stops. A detailed analysis of these candidates will be presented in the subsequent section.

**6.2.1. Total Cost Analysis.** The paper calculates the total cost for passengers and bus companies before and after splitting at bus stops S13, S14, S15, S16, S17, and S18. This information is presented in Table 4. Among these stops, S13, S14, S15, S17, and S18 are close to metro stops, while S16 is a nonmetro stop, meaning it is a bus stop without a metro station nearby.

In Table 4, compared with the average cost value at splitting metro stops, the total travel cost for passengers after the truncation operation is not significantly changed, but the total cost of bus companies is reduced by 65.1%. This indicates that the splitting strategy operation is efficient for the ultralong bus line. Compared with nonmetro stops, the average travel cost is decreased by 12.0% at metro stops, but no significant change in the bus company cost occurs. Moreover, the costs of both the bus company and passengers at S14 and S15 are relatively lower than those at S13, S17, and S18. Therefore, the optimized truncation stop is chosen from S14 and S15 in order to obtain better performance.

**6.2.2. Passenger Cost Analysis.** Various types of passenger travel costs before and after truncation are shown in Figure 11. After truncation, compared with the average value of splitting metro stops, the average passenger ticket purchase cost increases by 39.2% due to more expensive metro purchases, and the time cost decreases by 5.3%. Also, the reduction in travel time costs means less travel time is taken for passengers with slightly higher ticket costs. Since the average ticket purchase fee per passenger increased by 0.88 CNY, the maximum savings for passengers is about 20 minutes in a trip. The results will be much better if the travel fee has a discount or the transfer fee is free, as seen in the analysis in the next section. However, for the nonmetro truncated stops, the ticketing cost is reduced by 8.6%, but the time cost and penalty cost are increased by 14.4% and 74.5%, respectively, compared with the average value. It is indicated that if the truncation stop is a nonmetro stop, passengers still have to transfer to bus travel, which may increase extra transfer time without much travel time savings.

**6.2.3. Bus Company Cost Analysis.** The same conclusion can be drawn from Figures 12 and 13 that after splitting both the operation cost and fixed cost reduce to relatively low values, 41.0% and 70.0%, compared to average values, 40.0% and 67.6%, compared to nonmetro stops. It may be because the length of the bus line becomes shorter after the truncation operation, and the required fleet size on the line is reduced. At the same time, with the adoption of different bus types, both the fixed cost and the operating cost are reduced.

Compared to the nonmetro stops, the average fixed cost and operating cost of splitting stops near metro stops decrease by 1.7% and 7.3%, respectively. Figure 14 shows that

the passenger flow at metro stops is significantly lower than the flow at nonmetro stops. It may indicate that part of the passengers will transfer to the metro, thus reducing the number of passengers on the new split line. However, when the truncated stop is a nonmetro stop, passengers can only transfer to the split bus line, resulting in a relatively large amount of passenger flow on the latter line. Therefore, it is necessary to reduce the headway and increase the number of vehicles in order to deliver more transfer passengers.

From the perspective of total cost, the optimized truncation stop is attempted to be generated from S14 and S15. The costs for both passengers and bus company at S15 are relatively lower than those at other stops. Therefore, S15 is determined to be the optimal truncation stop, and the relative line information of the two parts is shown in Table 5. Part of the timetable after truncation is shown in Table 6, and the complete information is represented in the attachment.

### 6.3. Sensitivity Analysis

**6.3.1. Impact of the Vehicle Type.** In this paper, two types of buses with different capacities are utilized. Among them, the results of the bus company cost splitting at S15 stop with different bus type combinations are detailed in Table 7. In Table 7, the bus company cost for the combination of large- and small-capacity vehicle types is relatively lower than the other two single types. Although the vehicle purchase cost of the small capacity type is lower than the cost of the large-capacity type, more vehicles and drivers with a much higher purchase cost and driver salary are needed for this type. However, the load rate of vehicles with larger capacity is generally lower, especially during off-peak hours. In Figure 15, during the period from 9:00 to 10:00, the minimum load rate of the large-capacity bus type is 21.7%, and the average load rate is about 36.4%, which will lead to a waste of resources and a decrease in the transportation efficiency of the whole bus system, violating the concepts of energy saving, emission reduction, and low-carbon travel. Therefore, the combination of large and small bus types can not only reduce the operating cost of the bus company associated with meeting the travel needs of passengers but also contribute to alleviating road traffic congestion and improving the transportation efficiency of public transportation to a certain extent.

**6.3.2. Results of Different Travel Mode Expectations.** This paper starts by assuming that 70% of the passengers choose to transfer to the metro and 30% choose to transfer to the bus at the truncated stop. Due to the difference in passengers' preferences for public transport, the proportion of transfers to the subway or bus may change. Therefore, three values of proportion are attempted to be utilized to verify the impact on passenger travel costs, as shown in Table 8. The total passenger flow of #52 passing through S15 is 8021. Based on the results, as the number of bus-transferring passengers continues to increase, the total ticket purchase cost is gradually decreasing, caused by the fact that metro tickets are generally much more expensive than bus tickets. For

TABLE 4: Cost results at different truncation stops.

Truncation stop	Cost of passengers (CNY)	Bus company cost (CNY)
S13	343,483	25,929
S14	337,347	25,762
S15	336,540	25,817
S17	343,117	25,551
S18	342,905	25,278
Metro stop average	340,678	25,667
S16 (nonmetro stop)	387,048	27,239
Before truncation	347,238	73,583

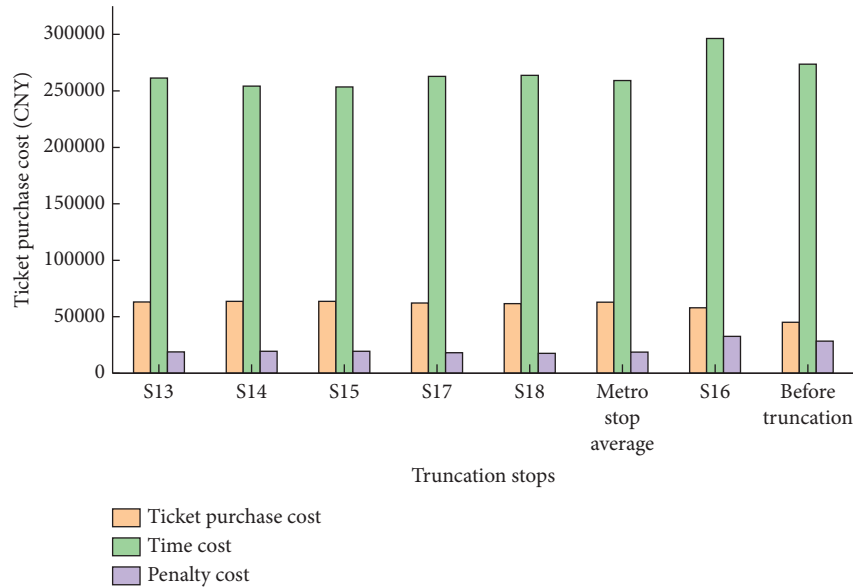


FIGURE 11: Various travel costs for passengers include passenger ticketing costs, travel time costs, and penalty costs at each truncated stop, respectively.

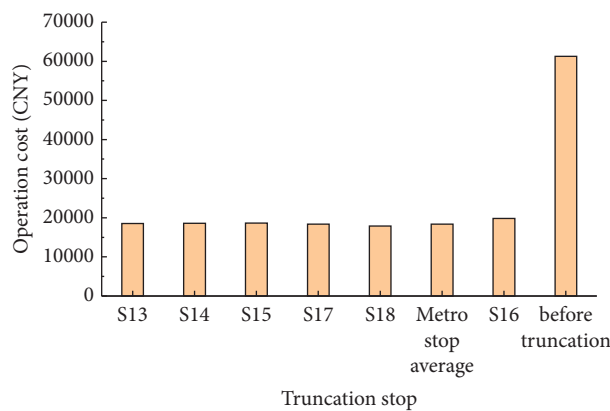


FIGURE 12: Operating costs at each truncation stop.

instance, in Beijing, the average cost per ticket for a metro ride is 3 CNY/person, whereas the average cost for a bus ticket is 2 CNY/person. However, the ticket cost will fall sharply if the transfer fee has a discount or is free for a period so as to encourage passengers to choose transit trips, which is

now implemented in several cities in China such as Chengdu and Wuhan. In addition, the travel cost of passengers is decreasing, especially the travel time cost. It is obvious that transferring to the metro can significantly save passengers' travel time and improve the efficiency of the public

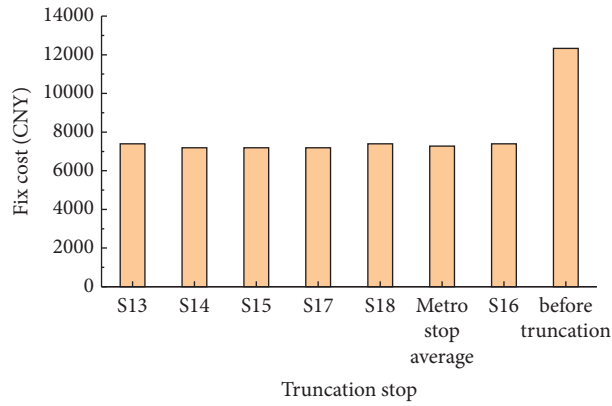


FIGURE 13: Fixed costs at each truncation stops.

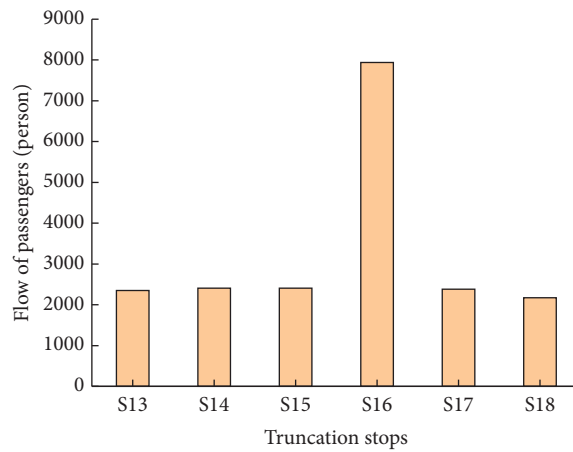


FIGURE 14: The passenger flow shared by buses at each truncation stop.

TABLE 5: New line information.

Line	Length (km)	Headway (min)	Fleet size
First part line (S1–S15)	11.5	11	8 large and 9 small buses
Second part line (S15–S28)	10.5	11	1 large and 8 small buses

TABLE 6: Partial timetable after truncation (up-direction).

First part line		Second part line	
Time of departure	Bus type	Time of departure	Bus type
5: 00	Small bus	5: 00	Small bus
11	Small bus	11	Small bus
22	Small bus	22	Small bus
33	Small bus	33	Small bus
44	Small bus	44	Small bus
55	Small bus	55	Small bus
6: 06	Large bus	6: 06	Small bus
17	Large bus	17	Small bus
28	Large bus	28	Small bus
39	Small bus	39	Small bus
50	Small bus	50	Small bus
7: 01	Large bus	7: 01	Small bus
.....	.....	.....	.....



TABLE 7: Cost of bus company when using a single model.

Bus types	Quantity	Bus company cost (CNY)	Fix cost (CNY)	Operation cost (CNY)
Large bus	17	33,836	6986	26850
Small bus	30	34,230	6,164	28,066
Large and small bus	9 + 17	25.817	7,152	18,625

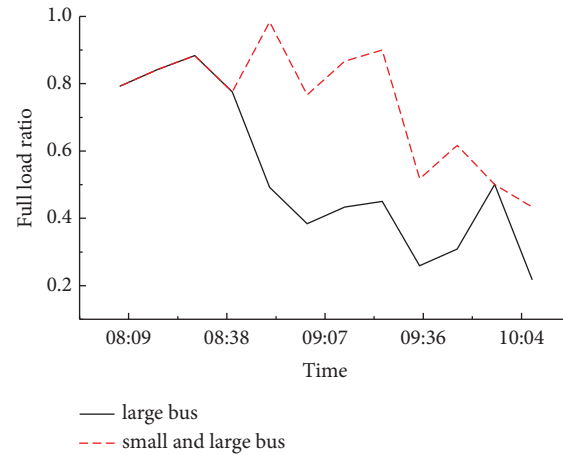


FIGURE 15: Maximum load ratio at each time points for two different fleet types.

TABLE 8: Passenger travel costs under different transfer proportion.

Proportion of transfer (metro : bus)	Total passengers' cost (CNY)	Total time cost (CNY)	Total ticket cost (CNY)
7 : 3	336,540	253,451	63,678
5 : 5	349,677	268,191	62,074
3 : 7	362,812	282,931	60,470

TABLE 9: Passenger travel costs under different transfer preferential measures (#52 at S15).

Transfer preferential measures	Passenger cost (CNY)	Ticket cost (CNY)
No discount	336,540	63,678
50% discount for transfer tickets	325,712	52,850
Free transfer	314,884	42,022

transportation system. This also confirms the necessity of choosing the splitting stop near the metro in an ultralong bus line.

**6.3.3. Transfer Ticket Preferential Measure.** The truncated operation of the ultralong bus line will result in extra transfer time cost and purchase costs for passengers passing through the truncated stop. In order to reduce the loss of time and money for the passengers when facing additional transfers, several preferential measures, such as transfer discounts or free in a period, are attempted to be adopted. With these measures in place, the cost of passenger travel is shown in Table 9. As can be seen from Table 9, compared to not taking any preferential measures, when transfer fares are discounted by half, the cost of passenger tickets is reduced by 17% and the total cost of travel is reduced by 6.2%; when

transfer fares are free, the cost of passenger tickets is decreased by 34% and the total cost of travel is decreased by 9.3%. Therefore, in the truncation and optimization of ultralong bus lines, the adoption of transfer concessions at the truncated stops can effectively reduce passenger cost and the resistance generated by transfer.

## 7. Conclusion

This study develops a multiobjective model for optimizing the operation of long bus routes by truncating them. The objectives are to minimize passenger travel costs and minimize bus company costs. The model incorporates the influence of subway networks and adopts a coordinated optimization approach between bus routes and subway networks. Furthermore, it considers the operational mode of coordinating multiple types of buses in order to enhance the

operational efficiency of the bus company. Based on an analysis and verification using Beijing's #52 bus route, several key conclusions can be drawn.

- (1) By implementing a truncated operation method for ultralong bus lines, significant travel time savings can be achieved at the expense of a slight increase in purchase ticket cost. In this scenario, the average ticket cost after truncation is raised by 0.88 CNY/person, while passengers experience a decrease of 5.8% in overall travel time costs and the bus company achieves up to a 64.9% reduction in total costs. However, it should be noted that the maximum possible reduction in travel time after optimization is 20 minutes. Therefore, it is recommended to focus on improving ultralong lines with minimal operational benefits.
- (2) The selection of a splitting stop depends on its ability to significantly reduce costs for both bus companies and passengers. Based on this analysis, locating splitting stops near metro stations can effectively decrease the aforementioned expenditures. Therefore, in order to maximize the synergistic benefits between these two modes of transportation, it is essential to consider the placement of splitting stops along ultralong bus routes in relation to nearby metro stations.
- (3) Combining large and small capacity bus types in an optimization model's operation can reduce the bus company's operating costs while simultaneously increasing the full load rate and lowering certain carbon emissions and traffic congestion.
- (4) Considering the additional transfers following the route truncation, it is advisable to implement preferential measures such as fare discounts or complementary transfers in order to mitigate ticket purchasing costs. These measures can effectively address the operational deficit resulting from truncation and enhance the attractiveness of public transportation.

Although this work makes a fresh attempt to enhance the optimization of ultralong bus lines in both theory and reality, there are still a number of issues that need to be addressed. (1) Regardless of a person's preferred mode of transportation, a predetermined and fixed percentage of passengers will take a bus or metro. In the future study, a model of individual travel mode selection will be taken into account; (2) after truncation, the effect on passenger choice or the overall passenger flow of the bus trip is not taken into account. In order to gather pertinent data for the ensuing study, an SP survey based on the operation tactics on the extremely long bus route will be conducted; (3) on the ultralong bus line, only one line-truncated optimization technique is investigated. More tactics, including line-truncated, skip-stop, and line-shortened ones as well as combination strategies, will be used in the future study to

optimize ultralong lines; (4) the implementation scope is intended to be broadened to include all relevant transit networks, not only the one bus or metro line at the splitting stop.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Acknowledgments

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