Hindawi Publishing Corporation
Journal of Chemistry
Volume 2015, Article ID 596597, 8 pages
http://dx.doi.org/10.1155/2015/596597

Research Article

A Mathematical Model for the Analysis of the Pressure Transient Response of Fluid Flow in Fractal Reservoir

Jin-Zhou Zhao,[1] Cui-Cui Sheng,[1] Yong-Ming Li,[1] and Shun-Chu Li[2]

[1] *State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation, Southwest Petroleum University, Chengdu 610500, China*
[2] *Institute of Applied Mathematics, Xihua University, Chengdu 610039, China*

Correspondence should be addressed to Shun-Chu Li; lishunchu@163.com

Received 12 August 2014; Accepted 14 October 2014

Academic Editor: Jianchao Cai

This study uses similar construction method of solution (SCMS) to solve mathematical models of fluid spherical flow in a fractal reservoir which can avoid the complicated mathematical deduction. The models are presented in three kinds of outer boundary conditions (infinite, constant pressure, and closed). The influence of wellbore storage effect, skin factor, and variable flow rate production is also involved in the inner boundary conditions. The analytical solutions are constructed in the Laplace space and presented in a pattern with one continued fraction—the similar structure of solution. The pattern can bring convenience to well test analysis programming. The mathematical beauty of fractal is that the infinite complexity is formed with relatively simple equations. So the relation of reservoir parameters (wellbore storage effect, the skin factor, fractal dimension, and conductivity index), the formation pressure, and the wellbore pressure can be learnt easily. Type curves of the wellbore pressure and pressure derivative are plotted and analyzed in real domain using the Stehfest numerical invention algorithm. The SCMS and type curves can interpret intuitively transient pressure response of fractal spherical flow reservoir. The results obtained in this study have both theoretical and practical significance in evaluating fluid flow in such a fractal reservoir and embody the convenience of the SCMS.

1. Introduction

The mechanics of oil and gas seepage is a discipline which researches the law and state of fluid flow in porous media. The practical development of oil and gas reservoirs shows that reservoir distribution and its space structure are awfully complicated. In 1982, Mandelbrot and Blumen first proposed the fractal geometry theory which used self-similar to characterize the complexity of things [1]. By combining the theories of fractal and seepage mechanics, it is able to describe fluid flow paths in porous media effectively. Acuna et al. explained the fractal characters of natural porous media with extremely complex pore structure [2]. Because flow paths of fluid in porous media can be seen as tortuous capillaries, Cai and Yu applied a fractal dimension to describe the tangle of capillary pathways and derived calculation formula of actual length of tortuous flow paths [3]. And spontaneous imbibition of wetting liquid in fractal porous media including gravity was studied [4]. The researches were regarded as a crucially important driving mechanism for enhancing oil recovery in naturally fractured reservoir. In the early 1990s, Chang and Yortsos applied the fractal theory in reservoir models and then set up new mathematical models [5, 6]. Tian and Tong established radial fluid flow models of fractal reservoirs [7]. The models have shown that the order of the fractional dimension has influence on the whole pressure behavior. Particularly, the effect on pressure behavior is larger in the early time stage. Based on fractal geometry and the semiempirical Kozeny-Carman equation which is the most famous permeability-porosity relation in the field of flow in porous media, Xu and Yu [8] developed a new form of permeability and Kozeny-Carman constant. After that, they showed that fractal dimension had significant effect on permeability, which can enhance the effective permeability [9, 10]. Santizo discussed the effect of fractal dimension and fractal conductivity index on pressure derivative in finite-conductivity fractures [11]. The papers obtained some numerical or analytical solutions of the formation pressure and the wellbore pressure in different fractal reservoirs or plotted pressure-time curves to analyze the influence of reservoir parameters [12–17].

Different versions of cylindrical flow model have been studied on the assumption that wells were completely opened and fully penetrated the productive formation. However, the assumption may seem too restrictive for practical application. Reservoirs have vertical permeability and the length of injection or extraction region was small compared to thick formations; spherical flow model can provide a good approximation in practice [18]. Schroth and Istok [19] illustrated the applicability of the derived spherical flow solution and provided a comparison with its cylindrical flow counterpart. They showed the spherical flow solution increased with increasing anisotropy in hydraulic conductivities. Joseph and Koederitz [20] presented short-time interpretation methods for radial-spherical flow in homogeneous and isotropic reservoirs inclusive of wellbore storage, wellbore phase redistribution, and damage skin effects. Liu [21] studied the transient spherical flow behavior in porous media and derived its nonlinear partial differential equation and obtained its analytical, asymptotic, and approximate solutions by using the methods of Laplace transform.

In the above proposed studies, the solution processes of their mathematical models are very complicated. Based on some study in a boundary value problem of the second-order linear homogeneous differential equation, Li and Liao recently introduced a new idea, similar construction method. The similar structure theory and its application were developed maturely [22]. Chen and Li presented similar construction method of solution (SCMS) for the boundary value problem of Bessel equation and gave the method to analyze the structure characteristics of solution [23]. After that, Sheng et al. proposed SCMS for a fractal reservoir and listed its steps [17]. They verified that the SCMS is a straightforward method to solve mathematical models in reservoir engineering, but they just did a theoretical study. SCMS of the boundary value problems of Airy equation, Weber equation, Euler hypergeometric equation, and composite first Weber system were studied [24–28]. SCMS in fractal dual-porosity reservoir and dual-permeability reservoir were presented [29–31].

This paper presents a mathematical model for the analysis of the pressure transient response of fluid spherical flow in fractal reservoir, which considers wellbore storage and skin effect in inner boundary conditions. According to a mathematical method, called SCMS, the expressions of the dimensionless formation pressure and the dimensionless wellbore pressure are constructed in the Laplace space, which avoid complicated calculations. Type curves of the wellbore pressure and pressure derivative responses are plotted and analyzed in real domain using the Stehfest numerical invention algorithm [32]. The results obtained in this study have essential significance to understand the pressure characteristics and provide theoretical basis in such a reservoir. This paper embodies the relationship between engineering and mathematics.

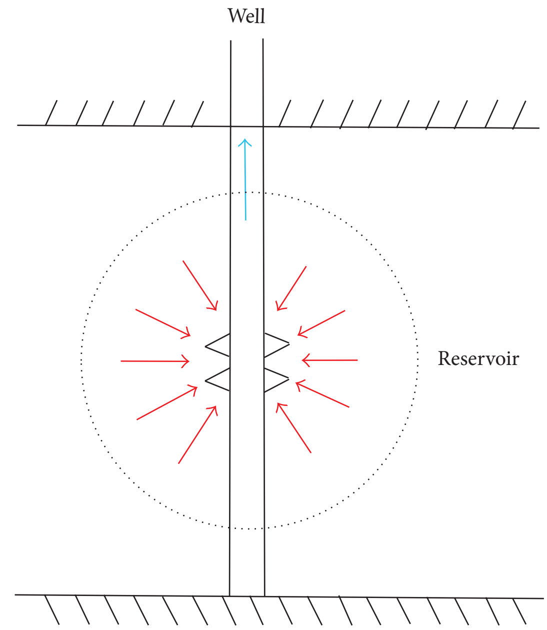

Figure 1: Schematic for spherical flow in a reservoir.

2. Materials and Methods

2.1. Reservoir Characteristics. Nonpenetrating wells that occur in a thick formation can be treated as spherical systems, as shown in Figure 1. Using fractal theory to describe the pore nature and transport properties, we assume that dimensional fractal network d_f is embedded in the dimensional Euclidean rock d. The fluid only flows in the fractal network, and the flow will obey Darcy's law from the reservoir into the wellbore. The reservoir has a uniform thickness of h and original formation pressure is p_i. The fractal reservoir is mined with a single small opening hole in the top part. The fluid flow rate is $q(t)$. The gravity and the capillary pressure are ignored.

2.2. Dimensionless Mathematical Model. The dimensionless mathematical model of the reservoir is made up of four parts, such as continuity equation, initial condition, inner boundary condition, and outer boundary condition. Appendix A presents mathematical model of fluid flow and dimensionless variables.

The continuity equation of fluid spherical flow in a fractal reservoir is

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{\beta}{r_D}\frac{\partial p_D}{\partial r_D} = r_D^{\theta}\frac{\partial p_D}{\partial t_D}. \tag{1}$$

Initial condition:

$$p_D\left(r_D, 0\right) = 0. \tag{2}$$

Inner boundary condition:

$$\left(r_D^2 \frac{\partial p_D}{\partial r_D}\right)\bigg|_{r_D=1} = -q_D(t_D) + C_D \frac{dp_{wD}}{dt_D},$$
$$p_{wD}(t_D) = \left(p_D - Sr_D \frac{\partial p_D}{\partial r_D}\right)\bigg|_{r_D=1}. \tag{3}$$

Three kinds of outer boundary conditions are the following cases.

Case 1. Infinite outer boundary condition:

$$p_D(\infty, t_D) = 0. \tag{4}$$

Case 2. Constant pressure outer boundary condition:

$$p_D(R_D, t_D) = 0. \tag{5}$$

Case 3. Closed outer boundary condition:

$$\frac{\partial p_D}{\partial r_D}\bigg|_{r_D=R_D} = 0. \tag{6}$$

2.3. A Boundary Value Problem of the Modified Bessel Equation. The dimensionless mathematical model ((1)–(6)) happens to be a boundary value problem of a second-order partial differential equation. It can be transformed into a boundary value problem of a second-order ordinary differential equation based on Laplace transform and then it can be further simplified as a boundary value problem of the modified Bessel equation:

$$r_D^2 \frac{d^2 \overline{p}_D}{dr_D^2} + \beta r_D \frac{d\overline{p}_D}{dr_D} - r_D^{\theta+2} z \overline{p}_D = 0, \tag{7}$$

$$\left[-C_D z \overline{p}_{wD}(r_D, z) + (1 + C_D z S) \frac{d\overline{p}_D(r_D, z)}{dr_D}\right]\bigg|_{r_D=1} = -\overline{q}_D(z), \tag{8}$$

$$\overline{p}_D(\infty, z) = 0 \quad \text{or} \quad \overline{p}_D(R_D, z) = 0 \quad \text{or} \quad \frac{d\overline{p}_D}{dr_D}\bigg|_{r_D=R_D} = 0. \tag{9}$$

2.4. Solution. The SCMS is based on a boundary value problem of a second-order linear differential equation and is firstly proposed by Sheng et al. [17]. Using coefficients of its right and left boundary condition and its two linearly independent solutions of the differential equation, the solution of the boundary value problem can be obtained. According to the steps of SCMS, the solution of the dimensionless mathematical model of fractal spherical flow reservoir can be constructed. The details are as follows.

Step 1. Solve the linearly independent solutions of (7)

$$r_D^{(1-\beta)/2} I_\nu\left(\frac{2\sqrt{z}}{2+\theta} r_D^{(2+\theta)/2}\right),$$
$$r_D^{(1-\beta)/2} K_\nu\left(\frac{2\sqrt{z}}{2+\theta} r_D^{(2+\theta)/2}\right), \tag{10}$$

where $I_\nu(\cdot)$ and $k_\nu(\cdot)$ denote the first and the second modified Bessel function of order $\nu = (1-\beta)/(2+\theta)$, respectively.

Step 2. Construct a binary function $\varphi_{0,0}(r_{D1}, r_{D2})$ employing the linearly independent solutions (10) and calculate its partial derivatives for r_{D1}, r_{D2}, respectively:

$$\varphi_{0,0}(r_{D1}, r_{D2}) = (r_{D1} r_{D2})^{(1-\beta)/2} \times \psi_{\nu,\nu}\left(r_{D1}^{(2+\theta)/2}, r_{D2}^{(2+\theta)/2}, \frac{2\sqrt{z}}{2+\theta}\right), \tag{11}$$

where

$$\psi_{m,n}(x^\varepsilon, y^\varepsilon, \xi) = K_m(\xi x^\varepsilon) I_n(\xi y^\varepsilon) + (-1)^{m-n+1} I_m(\xi x^\varepsilon) K_n(\xi y^\varepsilon), \tag{12}$$

with real numbers m, n:

$$\begin{aligned}
\varphi_{0,1}(r_{D1}, r_{D2}) &= \frac{\partial}{\partial r_{D2}} \varphi_{0,0}(r_{D1}, r_{D2}) \\
&= (1-\beta) r_{D1}^{(1-\beta)/2} r_{D2}^{-(1+\beta)/2} \\
&\quad \times \psi_{\nu,\nu}\left(r_{D1}^{(2+\theta)/2}, r_{D2}^{(2+\theta)/2}, \frac{2\sqrt{z}}{2+\theta}\right) \\
&\quad + \sqrt{z} r_{D1}^{(1-\beta)/2} r_{D2}^{(1-\beta+\theta)/2} \\
&\quad \times \psi_{\nu,\nu+1}\left(r_{D1}^{(2+\theta)/2}, r_{D2}^{(2+\theta)/2}, \frac{2\sqrt{z}}{2+\theta}\right), \\
\varphi_{1,0}(r_{D1}, r_{D2}) &= \frac{\partial}{\partial r_{D1}} \varphi_{0,0}(r_{D1}, r_{D2}) \\
&= (1-\beta) r_{D1}^{-(1-\beta)/2} r_{D2}^{(1-\beta)/2} \\
&\quad \times \psi_{\nu,\nu}\left(r_{D1}^{(2+\theta)/2}, r_{D2}^{(2+\theta)/2}, \frac{2\sqrt{z}}{2+\theta}\right) \\
&\quad - \sqrt{z} r_{D1}^{(1-\beta+\theta)/2} r_{D2}^{(1-\beta)/2} \\
&\quad \times \psi_{\nu+1,\nu}\left(r_{D1}^{(2+\theta)/2}, r_{D2}^{(2+\theta)/2}, \frac{2\sqrt{z}}{2+\theta}\right), \\
\varphi_{1,1}(r_{D1}, r_{D2}) &= \frac{\partial^2}{\partial r_{D1} \partial r_{D2}} \varphi_{0,0}(r_{D1}, r_{D2}) \\
&= (1-\beta)^2 r_{D1}^{-(1+\beta)/2} r_{D2}^{-(1+\beta)/2} \\
&\quad \times \psi_{\nu,\nu}\left(r_{D1}^{(2+\theta)/2}, r_{D2}^{(2+\theta)/2}, \frac{2\sqrt{z}}{2+\theta}\right) \\
&\quad + (1-\beta) \sqrt{z} r_{D1}^{-(1+\beta)/2} r_{D2}^{(1-\beta+\theta)/2} \\
&\quad \times \psi_{\nu,\nu+1}\left(r_{D1}^{(2+\theta)/2}, r_{D2}^{(2+\theta)/2}, \frac{2\sqrt{z}}{2+\theta}\right) \\
&\quad - (1-\beta) \sqrt{z} r_{D1}^{(1-\beta+\theta)/2} r_{D2}^{-(1+\beta)/2}
\end{aligned}$$

$$\times \psi_{\nu+1,\nu}\left(r_{D1}^{(2+\theta)/2}, r_{D2}^{(2+\theta)/2}, \frac{2\sqrt{z}}{2+\theta}\right)$$
$$- z r_{D1}^{(1-\beta+\theta)/2} r_{D2}^{(1-\beta+\theta)/2}$$
$$\times \psi_{\nu+1,\nu+1}\left(r_{D1}^{(2+\theta)/2}, r_{D2}^{(2+\theta)/2}, \frac{2\sqrt{z}}{2+\theta}\right). \tag{13}$$

Step 3. Construct similar kernel functions $\Phi(r_D, z)$ using the outer boundary conditions (9), the binary function equation (11), and its partial derivatives equations (13):

$$\Phi(r_D, z) = \begin{cases} \dfrac{\varphi_{0,0}(r_D, \infty)}{\varphi_{1,0}(1, \infty)}, & \overline{p}_D(\infty, z) = 0; \\ \dfrac{\varphi_{0,0}(r_D, R_D)}{\varphi_{1,0}(1, R_D)}, & \overline{p}_D(R_D, z) = 0; \\ \dfrac{\varphi_{0,1}(r_D, R_D)}{\varphi_{1,1}(1, R_D)}, & \left.\dfrac{d\overline{p}_D}{dr_D}\right|_{r_D=R_D} = 0. \end{cases} \tag{14}$$

Step 4. Construct the similar structure of solution with the similar kernel function equation (14) and coefficients $(\overline{q}_D(z), C_D z, S)$ of the inner boundary condition equation (8):

$$\overline{p}_D(r_D, z) = \overline{q}_D(z)\frac{1}{C_D z + (1/(S + \Phi(1,z)))} \cdot \frac{1}{S + \Phi(1,z)} \cdot \Phi(r_D, z). \tag{15}$$

Equation (15) is the analytical solution of the mathematical model of fluid spherical flow in a fractal reservoir. Substitute (15) into the second expression of (3); the dimensionless wellbore pressure in the Laplace space can be obtained:

$$\overline{p}_{wD}(z) = \overline{q}_D(z)\frac{1}{C_D z + \Phi(1,z)}. \tag{16}$$

3. Results and Discussions

The analytical solution of the mathematical model of fluid flow in fractal reservoir was rewritten as continued fraction, like real numbers, the structure of continued fraction is very beautiful. It is very convenient to study the influence of both wellbore storage effect and skin factor using equation (15). For different outer boundary conditions, the solution has a different kernel function, but its expression is the same so that it is convenient to program well test analysis software. Both the dimensionless formation pressure and the dimensionless wellbore pressure in the Laplace space can be inverted back into real time space using the Stehfest numerical invention algorithm (the numerical invention details are shown in Appendix B). The analysis of pressure transient response of fluid flow in fractal reservoir is studied by type curves.

3.1. Pressure Transient Response. Figure 2 shows the dimensionless wellbore pressure and derivative responses in log-log coordinate for three kinds of outer boundary conditions (infinite, constant pressure, and closed), which can be divided into the following four seepage flow stages.

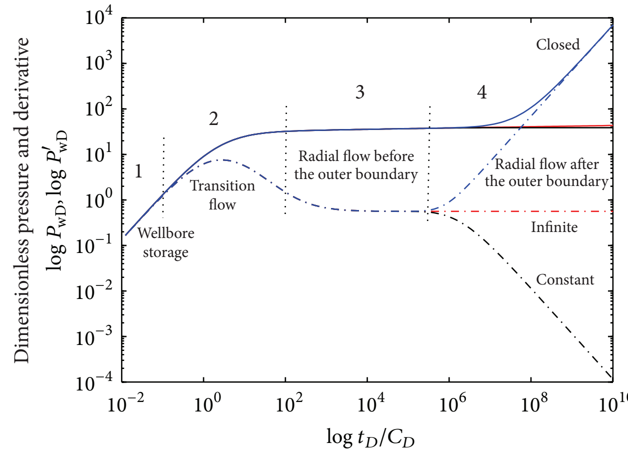

Figure 2: The type curves of the fractal reservoir with spherical flow.

Stage 1 (wellbore storage period). The fluid flow is principally affected by wellbore storage effect. In this flow period, both the dimensionless pressure change and its rate are aligned in a unite slope trend.

Stage 2 (transition flow period). The period represents the transition to early time spherical flow (Stage 3) and has a hump in the dimensionless pressure derivative curve. The fluid flow depends on both the wellbore storage effect and the skin factor.

Stage 3 (spherical flow before the outer boundary period). During this period, either the dimensionless pressure change curve or the derivative curves show a straight line with a slope of zero. The greater the radius of the outer boundary is, the longer the duration of the fluid flow period will be.

Stage 4 (spherical flow after the outer boundary period). When the outer boundary is infinite, the fluid flow still holds the same response. When pressure is constant in the outer boundary, the dimensionless pressure curve nearly remains level, but its derivative curve decreases quickly. For a closed outer boundary condition, both the dimensionless pressure and derivative curves are cocked up and overlapped. And the values of their slope curve are about 1.

As Figure 2 shows, the differences of both the dimensionless wellbore pressure and derivative responses in various outer boundary conditions only display in Stage 4. For convenience, we only analyze infinite outer boundary condition. The effect of reservoir parameters trends to be similar among the three kinds of outer boundary conditions (infinite, constant pressure, and closed), so we analyze the case of infinite outer boundary condition in the following.

3.2. Effect of Wellbore Storage. Figure 3 shows the effect of the coefficient of wellbore storage, C_D, on pressure transient responses of a fractal reservoir with spherical flow. It can be seen that, with the other parameters kept constant, the curves have obvious impact on the wellbore storage period and the

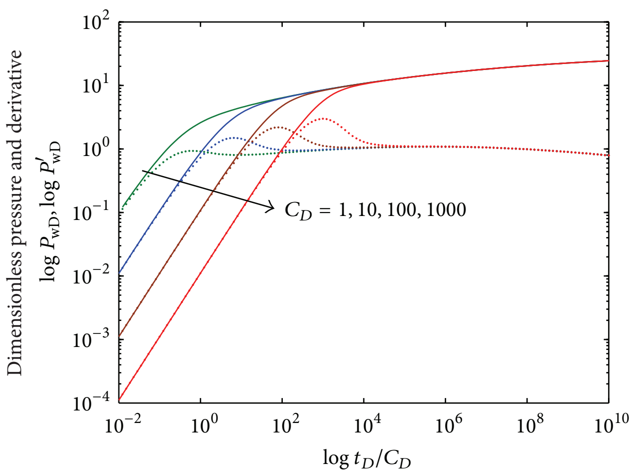

FIGURE 3: Effect of C_D for the type curves of the fractal reservoir with spherical flow.

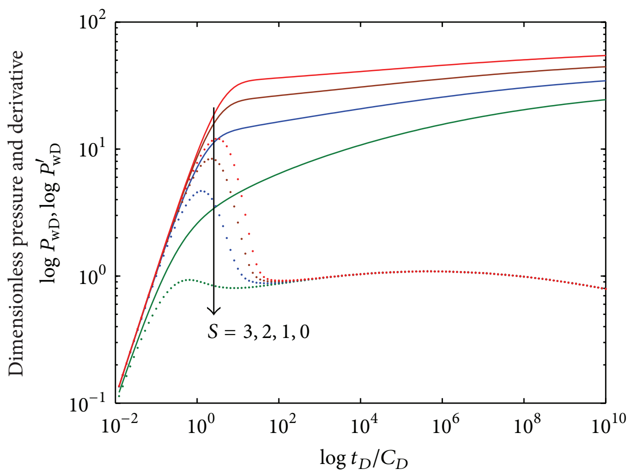

FIGURE 4: Effect of S for the type curves of the fractal reservoir with spherical flow.

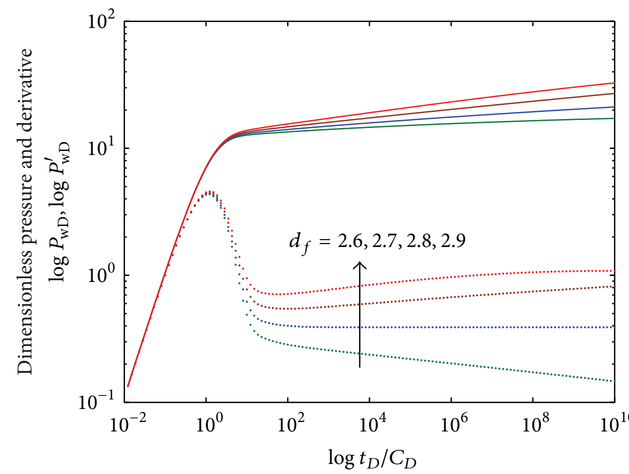

FIGURE 5: Effect of d_f for the type curves of the fractal reservoir with spherical flow.

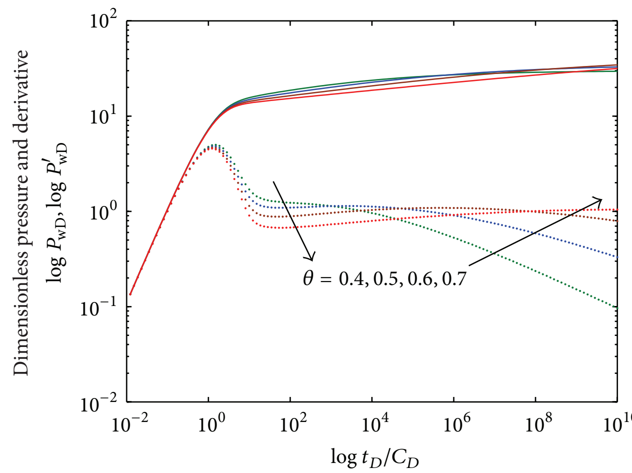

FIGURE 6: Effect of θ for the type curves of the fractal reservoir with spherical flow.

transition flow period. The curves of dimensionless pressure derivative indicate that the larger coefficient of wellbore storage increases the duration of the wellbore storage period.

3.3. Effect of Skin Factor. Figure 4 shows the effect of the skin factor, S, on pressure transient responses of a fractal reservoir with spherical flow. If the other parameters keep constant, the larger the skin factor is, the higher the hump of the wellbore storage period in the derivative becomes.

3.4. Effect of Fractal Dimension. The effect of the fractal dimension, d_f, is shown in Figure 5. We can find that the fluid flow behaviors are not affected in the wellbore storage period. The bigger the d_f is, the higher the derivative curve will become. As time goes on, the difference could be observed obviously. That is because of the fact that, with fractal dimension increasing, the fractal porosity and permeability of a reservoir will become bigger, which also can be understood from (A.1).

3.5. Effect of Conductivity Index. Figure 6 shows the influence of conductivity index, θ, on pressure transient responses of a fractal reservoir with spherical flow. The conductivity index reflects the curvature of the pore system. It holds that the more circuitous pore system is, the worse connectivity and conductivity will be. When the fractal dimension is small enough and makes $d + \theta - d_f < 1$, the derivative curve decreases at the late time such as the blue curve and the green curve because the reservoir is similar to the situation of the constant pressure outer boundary. When the fractal dimension makes $d+\theta-d_f \geq 1$, the derivative curve (the red curve) inclines upward at the late time because the reservoir is similar to the situation of the closed outer boundary.

4. Conclusions

Based on the reasoning mentioned above, the following conclusions can be drawn.

(1) The mathematical models of fluid spherical flow in fractal reservoirs with three kinds of outer boundary conditions (infinite, constant pressure, and closed) were presented, which comprehensively took into consideration the effect of wellbore storage effect, the skin factor, fractal dimension, and conductivity index.

(2) The dimensionless formation pressure in the Laplace space was constructed by SCMS and was written as a continued fraction. The analytical solution presented in this paper has afforded theoretical basis to understand the effect of reservoir parameters and practical significance to make the well test analysis software.

(3) The dimensionless pressure and derivative type curves were plotted and their characteristics were analyzed. For three kinds of outer boundary conditions, the pressure transient responses make no difference when the fluid does not reach to the outer boundary condition.

(4) The time and the high of the hump of the dimensionless wellbore pressure derivative have connection with the coefficient of wellbore storage C_D and the skin factor S, respectively. The paper has studied the effects of the conductivity index θ and the fractal dimension d_f. The fractal dimension d_f is beneficial to the fluid flow. However, the trend is reversing with the conductivity index θ. In addition, the behaviors in wellbore storage period are not affected for the fractal dimension d_f and the conductivity index θ.

Appendices

A. Mathematical Model of Fractal Spherical Flow Reservoir

In Acuna et al.'s research [2], the fractal porosity and permeability of a reservoir as a function of distance are

$$\phi(r) = \phi_w\left(\frac{r}{r_w}\right)^{d_f-d}, \qquad k(r) = k_w\left(\frac{r}{r_w}\right)^{d_f-d-\theta}, \tag{A.1}$$

where d, d_f, θ, and r represent Euclid dimension, fractal dimension, conductivity index, and radial distance, respectively.

The fluid flow from reservoir into the wellbore follows Darcy's law. The fluid flow in the reservoir is isothermal and single-phase with constant fluid viscosity. Based on the principle of mass and energy conservation, the mathematical equations of single fluid phase in fractal reservoir can be written in the form as

$$\frac{\partial^2 p}{\partial r^2} + \frac{\beta}{r}\frac{\partial p}{\partial r} = \left(\frac{r}{r_w}\right)^{\theta}\frac{\mu\phi_w C_t}{k_w}\frac{\partial p}{\partial t}, \tag{A.2}$$

where

$$\beta = d_f - d - \theta + 2, \qquad C_t = C_l + C_f, \qquad C_l = \frac{1}{\rho}\frac{\partial\rho}{\partial p}, \qquad C_f = \frac{1}{\phi_f}\frac{\partial\phi_f}{\partial p}. \tag{A.3}$$

Initial condition:

$$p(r,0) = 0. \tag{A.4}$$

Inner boundary conditions:

$$\left(r^2\frac{\partial p}{\partial r}\right)\bigg|_{r=r_w} = \frac{1.842\times10^{-3}\mu}{k_w}\left[Bq(t) + C\frac{dp_w}{dt}\right], \qquad p_w(t) = \left(p(r_w,t) - Sr\frac{\partial p}{\partial r}\right)\bigg|_{r=r_w}. \tag{A.5}$$

The three kinds of outer boundary condition are as follows.

Case 1. Infinite outer boundary condition:

$$p(\infty,t) = p_0. \tag{A.6}$$

Case 2. Constant pressure outer boundary condition:

$$p(R,t) = p_0. \tag{A.7}$$

Case 3. Closed outer boundary condition:

$$\frac{\partial p}{\partial r}\bigg|_{r=R} = 0. \tag{A.8}$$

For the convenience of calculation, the dimensionless variables (formation pressure, wellbore pressure, production rate, radial distance, outer boundary radius, time, and wellbore storage coefficient) are defined as follows:

$$p_D = \frac{k_w r_w}{1.842\times10^{-3}B\mu q_e}(p_i - p), \qquad p_{wD} = \frac{k_w r_w}{1.842\times10^{-3}B\mu q_e}(p_i - p_w),$$
$$q_D(t) = \frac{q(t)}{q_e}, \qquad r_D = \frac{r}{r_w}, \qquad R_D = \frac{R}{r_w},$$
$$t_D = \frac{3.65\times10^{-3}k_w t}{\mu\phi_w C_t r_w^2}, \qquad C_D = \frac{7.96\times10^{-2}C}{\phi_w C_t r_w^3}. \tag{A.9}$$

Taking the above dimensionless variables into (A.2)–(A.8), the dimensionless mathematical model of fractal reservoir ((1)–(6)) can be obtained.

B. Stehfest Numerical Invention Algorithm

The solution in real space can be obtained by the method of the Stehfest numerical invention algorithm [32]. $p_{wD}(t_D)$ is the primary function of $\overline{p}_{wD}(z)$ using the Laplace transform; then $p_{wD}(t_D)$ is given by Wooden et al. [33]. Consider

$$p_{wD}(t_D) = \frac{\ln 2}{t_D}\sum_{i=1}^{N} V_i \overline{p}_{wD}\left(\frac{\ln 2}{t_D} i\right), \quad \text{(B.1)}$$

where

$$\begin{aligned} V_i = {} & (-1)^{(N/2)+i} \\ & \times \sum_{k=[(i+1)/2]}^{\min(i,N/2)} \left(k^{N/2}(2k+1)!\right) \\ & \times \left((k+1)!k!\left(\left(\frac{N}{2}\right)-k+1\right)! \right. \\ & \left. \times (i-k+1)!(2k-i+1)!\right)^{-1}. \end{aligned} \quad \text{(B.2)}$$

Here N is an even number which is usually in the range $10 < N < 30$, and k is computed using integer arithmetic.

Nomenclature

B: Formation volume factor, RB/STB
C: Coefficient of wellbore storage, m^3/Pa
C_t: Total compressibility, MPa^{-1}
d: Euclid dimension
d_f: Fractal dimension
k: Permeability, mD
p: Reservoir pressure, MPa
q: Production rate or injection rate, m^3/d
R: Radial distance of the outer boundary, m
r: Radial distance in spherical coordinate, m
S: Skin factor
t: Time, h
z: Laplace transform variable.

Greek Symbols

θ: Conductivity index
μ: Viscosity, mPa·s
ϕ: Porosity.

Subscripts

D: Dimensionless
i: Initial
w: Wellbore parameter.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The authors would like to thank the anonymous referee for his/her helpful suggestions and comments. The research is supported by the National Natural Science Foundation of China (no. 51274169), the National Basic Research Program of China (973 Program) (no. 2013CB228004), the New Century Excellent Talents in University (no. NCET-11-1062), and the Scientific Research Fund of Sichuan Provincial Education Department of China (no. 12ZA164).

References

[1] B. B. Mandelbrot, "Fractal geometry: what is it, and what does it do?" *Proceedings of the Royal Society of London, Series A: Mathematical, Physical and Engineering Sciences*, vol. 423, no. 1864, pp. 3–16, 1989.

[2] J. A. Acuna, I. Ershagh, and Y. C. Yortsos, *Fractal Analysis of Pressure Transient in the Geysers Geothermal Field*, Petroleum Engineering Program, University of Southern California, Los Angeles, Calif, USA, 1992.

[3] J. Cai and B. Yu, "A discussion of the effect of tortuosity on the capillary imbibition in porous media," *Transport in Porous Media*, vol. 89, no. 2, pp. 251–263, 2011.

[4] J. C. Cai, X. Y. Hu, D. C. Standnes, and L. J. You, "An analytical model for spontaneous imbibition in fractal porous media including gravity," *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, vol. 414, pp. 228–233, 2012.

[5] J. Chang and Y. C. Yortsos, "Pressure-transient analysis of fractal reservoirs," *SPE Formation Evaluation*, vol. 5, no. 1, pp. 31–38, 1990.

[6] J. Chang and Y. C. Yortsos, "Note on pressure-transient analysis of fractal reservoirs," *SPE Advanced Technology Series*, vol. 1, no. 2, pp. 170–171, 1993.

[7] J. Tian and D. K. Tong, "The flow analysis of fluids in fractal reservoir with the fractional derivative," *Journal of Hydrodynamics B*, vol. 18, no. 3, pp. 287–293, 2006.

[8] P. Xu and B. Yu, "Developing a new form of permeability and Kozeny-Carman constant for homogeneous porous media by means of fractal geometry," *Advances in Water Resources*, vol. 31, no. 1, pp. 74–81, 2008.

[9] X. Peng, B. Yu, X. Qiao, S. Qiu, and Z. Jiang, "Radial permeability of fractured porous media by Monte Carlo simulations," *International Journal of Heat and Mass Transfer*, vol. 57, no. 1, pp. 369–374, 2013.

[10] P. Xu, S. Qiu, B. Yu, and Z. Jiang, "Prediction of relative permeability in unsaturated porous media with a fractal approach," *International Journal of Heat and Mass Transfer*, vol. 64, pp. 829–837, 2013.

[11] M. C. Santizo, "A semi-analytic solution for flow in finite-conductivity vertical fractures using fractal theory," in *SPE Annual Technical Conference and Exhibition*, pp. 1–18, 2012, 163057-STU.

[12] M. A. Prott and W. C. Chin, "New exact spherical flow solution with storage and skin for early-time interpretation with applications to wireline formation and early-evaluation drillstem testing," in *Proceedings of the SPE Annual Technical Conference and Exhibition*, SPE-49140-MS, pp. 463–478, New Orleans, La, USA, September 1998.

[13] B. Q. He and K. L. Xiang, "Analytical solutions of effective well radius model of unsteady flow in fractal reservoirs," *Journal of Southwest Petroleum Institute*, vol. 22, no. 3, pp. 37–40, 2000 (Chinese).

[14] F. Flamenco-López and R. Camacho-Velázquez, "Determination of fractal parameters of fracture networks using pressure-transient data," *SPE Reservoir Evaluation and Engineering*, vol. 6, no. 1, pp. 39–47, 2003.

[15] X. Kong, D. Li, and D. Lu, "Transient pressure analysis in porous and fractured fractal reservoirs," *Science in China, Series E: Technological Sciences*, vol. 52, no. 9, pp. 2700–2708, 2009.

[16] C. C. Sheng, S. C. Li, and D. X. Xu, "Fractal sphere seepage flow oil-gas reservoir model and its solution," *Oil and Gas Field Development*, vol. 29, no. 6, pp. 55–58, 2011 (Chinese).

[17] C. C. Sheng, J. Z. Zhao, Y. M. Li, S. C. Li, and H. Jia, "Similar construction method of solution for solving the mathematical model of fractal reservoir with spherical flow," *Journal of Applied Mathematics*, vol. 2013, Article ID 219218, 8 pages, 2013.

[18] R. W. Tang, "Model of limited-entry completions undergoing spherical flow," *SPE Formation Evaluation*, vol. 3, no. 4, pp. 761–770, 1988.

[19] M. H. Schroth and J. D. Istok, "Approximate solution for solute transport during spherical-flow push-pull tests," *Ground Water*, vol. 43, no. 2, pp. 280–284, 2005.

[20] J. A. Joseph and L. F. Koederitz, "Unsteady-state spherical flow with storage and skin," *Society of Petroleum Engineers journal*, vol. 25, no. 6, pp. 804–822, 1985.

[21] C. Q. Liu, "Transient spherical flow of non-newtonian power-law fluids in porous media," *Applied Mathematics and Mechanics*, vol. 9, no. 6, pp. 521–525, 1988.

[22] S. C. Li and Z. J. Liao, "Constructing the solution to a boundary value problem for a differential equation with its arbitrary non-trivial solution," *Journal of Sichuan University*, vol. 49, no. 6, pp. 1209–1213, 2012 (Chinese).

[23] Z. R. Chen and S. C. Li, "The similar structure method solving the boundary value problem of Bessel equation," *Journal of Sichuan Normal University*, vol. 34, no. 6, pp. 850–853, 2011 (Chinese).

[24] F. R. Wang, S. C. Li, and D. X. Xu, "The similarity construction method of a kind of boundary-value problem of Airy equation," *Journal of Hubei Normal University*, vol. 33, no. 1, pp. 79–85, 2013 (Chinese).

[25] R. J. Huang, S. C. Li, and D. X. Xu, "Similar structured method of solution to boundary value of first Weber equation," *Journal of Mianyang Normal University*, vol. 31, no. 11, pp. 1–5, 2012 (Chinese).

[26] X. X. Xiao and S. C. Li, "Similar structure of solutions to a boundary value problem for Euler's hypergeometric differential equation," *Journal of Inner Mongolia Normal University*, vol. 41, no. 6, pp. 597–600, 2012 (Chinese).

[27] J. H. Shi, S. C. Li, X. L. Wang, and D. D. Gui, "A new method for solving boundary value problem of composite Hermit Equations," *Advanced Materials Research*, vol. 753–755, pp. 2851–2854, 2013.

[28] X. X. Dong, S. C. Li, D. D. Gui, J. Pu, and H. C. Li, "Similar constructing method for solving the boundary value problem of the composite first Weber system," *The American Journal of Applied Mathematics and Statistics*, vol. 1, no. 4, pp. 76–82, 2013.

[29] X. T. Bao, S. C. Li, and D. D. Gui, "Similar constructive method for solving the nonlinear spherical percolation model in dual-porosity media," *Advanced Materials Research*, vol. 631-632, pp. 265–271, 2013.

[30] L. Xu, X. Liu, L. Liang, S. Li, and L. Zhou, "The similar structure method for solving the model of fractal dual-porosity reservoir," *Mathematical Problems in Engineering*, vol. 2013, Article ID 954106, 9 pages, 2013.

[31] C. Y. Fan, S. C. Li, D. D. Gui, M. Hu, and H. C. Li, "Similar constructing method for solving the model of the plane radial flow of dual permeability reservoir," *Applied Mechanics and Materials*, vol. 419, pp. 43–50, 2013.

[32] H. Stehfest, "Numerical inversion of Laplace transforms," *Communications of the ACM*, vol. 13, no. 1, pp. 47–49, 1970.

[33] B. Wooden, M. Azari, and M. Soliman, "Well test analysis benefits from new method of Laplace space inversion," *Journal of Oil & Gas*, vol. 90, no. 29, pp. 108–110, 1992.


International Journal of Medicinal Chemistry
Organic Chemistry International
International Journal of Photoenergy
International Journal of Analytical Chemistry
Advances in Physical Chemistry
International Journal of Carbohydrate Chemistry
Hindawi
Submit your manuscripts at
http://www.hindawi.com
Journal of Quantum Chemistry
The Scientific World Journal
Journal of Analytical Methods in Chemistry
Journal of Spectroscopy
International Journal of Inorganic Chemistry
International Journal of Electrochemistry
Journal of Applied Chemistry
Bioinorganic Chemistry and Applications
Journal of Theoretical Chemistry
Chromatography Research International
Journal of Catalysts
Journal of Chemistry
International Journal of Spectroscopy