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Research Article

Pressure-Transient Behavior in a Multilayered Polymer Flooding Reservoir

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A new well-test model is presented for unsteady flow in multizone with crossflow layers in non-Newtonian polymer flooding reservoir by utilizing the supposition of semipermeable wall and combining it with the first approximation of layered stable flow rates, and the effects of wellbore storage and skin were considered in this model and proposed the analytical solutions in Laplace space for the cases of infinite-acting and bounded systems. Finally, the stable layer flow rates are provided for commingled system and crossflow system in late-time radial flow periods.

1. Introduction

Many reservoirs are formed from layers of different physical properties because of the different geological deposition rotary loops. Among them, if these layers do not communicate in terms of fluid flow through the formation but may be produced by the same wellbore, these types of reservoir are called *commingled systems*; if there exits fluid that connects between these layers, they are referred to as *crossflow systems*. The pressure-transient behavior depends on the comprehensive properties of these multilayers.

For the unstable flow of Newtonian fluids in a multilayered reservoir, Russell et al. [1, 2] studied pressure behavior of single-phase fluid in two layers with formation crossflow and derived the conclusion that it is similar to the flow behavior of the two layers without formation crossflow in early time. Raghavan et al. [3] studied the problem of well test in a multilayered reservoir. Bourdet [4] established using steady state approximation to the presentation interlayer flow model. Gao and Deans [5] studied the behavior of multilayered reservoir with formation crossflow. Ehlig-Economides [6] has systematically established the combination of commingled system and crossflow system unsteady flow models and provided the rule of the pressure and flow for each layer. Bidaux et al. [7], using layered pressure and flow data, conducted a study on the theory and practical application of multilayered reservoir.

For the unsteady flow of non-Newtonian fluids in a multilayered reservoir, van Poollen and Jargon [8] studied non-Newtonian power-law fluid unsteady flow in porous media and showed that the transient pressure response characteristics are different from that of Newtonian fluid. Ikoku and Ramey [9] studied non-Newtonian power-law fluid unsteady flow characteristics in porous medium, and the consideration of wellbore storage and skin effect is obtained in homogeneous infinite reservoir model in Laplace space solution. Lund and Ikoku [10, 11] proposed non-Newtonian power-law fluid (polymer solution) and Newtonian fluid (oil) composite model of transient well-test analysis method. Xu et al. [12] proposed the infinite reservoir Laplace space spherical transient pressure solution and discussed the characteristics of wellbore pressure at early times and later times. The above research result is to solve the problem of single layer. Escobar et al. [13] presented equations to estimate permeability, non-Newtonian bank radius, and skin factor for the well test data in reservoirs with non-Newtonian power-law fluids.

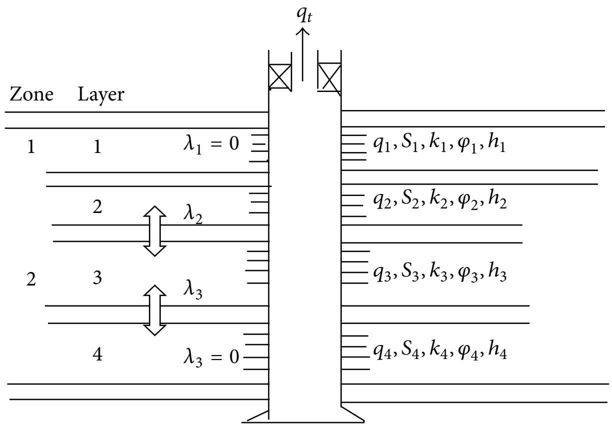

Figure 1: Sketch of a multilayer reservoir.

Martinez et al. [14] studied the transient pressure behaviors for a Bingham type fluid and the influence of the minimum pressure gradient. Escobar et al. [15, 16] studied transient pressure analysis for non-Newtonian fluids in naturally fractured formations modelled as double-porosity model. They [17] extended TDS technique to injection and fall-off tests of non-Newtonian pseudoplastic fluids. van den Hoek et al. [18] presented a simple and practical methodology to infer the in situ polymer rheology from PFO (Pressure Fall-Off) tests. To the problem of multilayered, Yu et al. [19] established a well testing model for polymer flooding and presented a numerical well testing interpretation technique to evaluate formation in crossflow double-layer reservoirs.

This paper presents a well-test model and the analytical solution for non-Newtonian polymer-flooding unsteady flow in multizone with crossflow layers and laid the foundation theory of field test data interpretation.

2. The Model Description

The reservoir model for the N-layered system is shown in Figure 1. Each layer is assumed to be homogeneous and isotropic, with injected polymer non-Newtonian power-law fluids.

A symmetrically located well penetrates all the layers, and each layer has a skin of arbitrary value S_i, wellbore storage coefficient C is assumed to be constant, and crossflow may occur in the reservoir between any two adjacent layers.

Assuming that the fluid is slightly compressible, the compression coefficient is constant, the permeability, porosity, and thickness of each layer can be different, respectively, C_{tj}, k_j, Φ_j, h_j to distinguish between layer pairs with formation crossflow with and noncommunicating layers, and the reservoir is divided into NZ ($\leq N$) zones. Between any two adjacent zones, there is no formation crossflow. Gravity and capillary forces can be neglected. Assuming weak formation crossflow, the flow is about interlayer pressure difference and has nothing to do with the shear rate, crossflow coefficient χ_j, on behalf of interlayer communicating ability. Formation crossflow is modeled as in the semipermeable-wall model of Deans and Gao [7], which assumes that all resistance to vertical flow is concentrated in the wall (layer top, bottom). Hence, the pressure difference between adjacent layers depends on only radial position and time, and flow within the layers is strictly horizontal, and assuming that each layer has the same initial pressure.

The flow in each layer j ($j = 1, 2, \ldots, N$) is governed by the following equation:

$$k_j h_j \nabla \cdot \left(\frac{1}{\mu_j} \nabla P_j \right) = \phi_j h_j C_{tj} \frac{\partial P_j}{\partial t} + \chi_{j-1} \left(P_j - P_{j-1} \right) - \chi_j \left(P_{j+1} - P_j \right), \tag{1}$$

where χ_j is given by

$$\chi_j = \frac{2}{2\left[\Delta h_j / k_{vj}\right] + x_{j+1} + x_j}, \quad j = 1, \ldots, N, \tag{2}$$

where $\chi_0 = \chi_n = 0$, Δh_j and k_{vj} are thickness and vertical permeability of a nonperforated zone between layers j and $j+1$, and $x_j = h_j / k_{zj}$, where k_{zj} is the vertical permeability for layer j that is the resistance to flow per unit length at the jth layer interface. If there is no nonperforated zone between layers j and $j+1$, then the flow resistance on behalf of a j layer interface unit length. If on the j layer and the $j+1$ layer but perforated belt, then $(\Delta h)_j$ is zero. If there is no formation crossflow between layers j and $j+1$, then χ_j is zero.

The sand surface flow of each layer as a function of time:

$$q_j(t) = \frac{-2\pi k_j h_j}{\mu_j} r_w \left. \frac{\partial P_j}{\partial r} \right|_{r_w}. \tag{3}$$

Assume that polymer solution viscosity and shear rate of power-law relations [4] are as follows:

$$\mu_i = H \nu_i^{\,n-1}, \tag{4}$$

where H is a constant and ν_i and n, respectively, represent the shear rate and power-law index; when n tends to one, fluid showed a Newtonian fluid properties.

Because of the flow in each layer j changing over time, the crossflow rate is much smaller than stable flow rate, so the flow rate q_i can be replaced by steady flow rate to calculate the shear rate:

$$\nu_i = \frac{D q_{is}}{r h_i \left(K_i \phi_i \right)^{0.5}}, \quad (i = 1, \ldots, N), \tag{5}$$

where D is a constant and layer stable flow rate q_{is} can be obtained by the late-time in unsteady flow.

With the viscosity of polymer solution into (1) for the shear rate representation, for radial flow, under cylindrical coordinate, dimensionless forms are obtained by finishing after

$$\kappa_j \left(\frac{\partial^2 P_{jD}}{\partial r_D{}^2} + \frac{n}{r_D} \frac{\partial P_{jD}}{\partial r_D} \right) = \omega_j r_D{}^{1-n} \frac{\partial P_{jD}}{\partial t_D} - r_D{}^{1-n} \lambda_{j-1} \left(P_{jD} - P_{j-1D} \right) + r_D{}^{1-n} \lambda_j \left(P_{j+1D} - P_{jD} \right). \tag{6}$$

The boundary condition at the well is given by the following equations, which account for both skin and wellbore storage:

$$P_{wD} = P_{jD}(1, t_D) - s_j \left. \frac{\partial P_{jD}}{\partial r_D} \right|_{r_D=1},$$
$$1 = C_D \frac{dP_{wD}}{dt_D} - \sum_{j=1}^{N} \kappa_j \left. \frac{\partial P_{jD}}{\partial r_D} \right|_{r_D=1}. \quad (7)$$

For infinite-outer-boundary condition,

$$P_{jD}(r_D, t_D) \longrightarrow 0 \quad (r_D \longrightarrow \infty). \quad (8)$$

For no-flow outer-boundary condition,

$$\frac{\partial P_{jD}}{\partial r_D} = 0 \quad (r_D = r_{eD}). \quad (9)$$

For initial condition,

$$P_{jD}(r_D, 0) = 0. \quad (10)$$

For dimensionless layer flow rates,

$$q_{jD}(t_D) = \frac{q_j}{q} = -\kappa_j \left. \frac{\partial P_{jD}}{\partial r_D} \right|_{r_D=1}, \quad (11)$$

where the dimensionless variables are defined by the following:

$$P_{jD} = \frac{0.02\pi}{q} \sum_{j=1}^{N} \left[\frac{k_j^{(1+n)/2} h_j^{\,n} (Dq_{js})^{1-n}}{H r_w^{\,1-n} \phi_j^{(1-n)/2}} \right] (P_j - P_0), \quad (12)$$

$$t_D = 0.01 \frac{\sum_{j=1}^{N} \left(\kappa_j^{(1+n)/2} h_j^{\,n} (Dq_{js})^{1-n} / \phi_j^{(1-n)/2} \right)}{H \sum_{j=1}^{N} \phi_j h_j C_{tj} r_w^{\,3-n}} t, \quad (13)$$

$$\kappa_j = \frac{k_j^{(1+n)/2} h_j^{\,n} \phi_j^{(n-1)/2} (Dq_{js})^{1-n}}{\sum_{j=1}^{N} k_j^{(1+n)/2} h_j^{\,n} \phi_j^{(n-1)/2} (Dq_{js})^{1-n}}, \quad (14)$$

$$\omega_j = \frac{\phi_j h_j C_{tj}}{\sum_{j=1}^{N} \phi_j h_j C_{tj}}, \quad (15)$$

$$\lambda_j = \frac{H r_w^{\,3-n} \chi_j}{\sum_{j=1}^{N} k_j^{(1+n)/2} h_j^{\,n} \phi_j^{(n-1)/2} (Dq_{js})^{1-n}}, \quad (16)$$

$$C_D = \frac{C}{2\pi r_w^{\,2} \sum_{j=1}^{N} \phi_j h_j C_{tj}}, \quad (17)$$

$$r_D = \frac{r}{r_w}, \quad (18)$$

$$r_{eD} = \frac{r_e}{r_w}. \quad (19)$$

3. The Solution of the Model

The above equation is given by Laplace transform on time.

Set

$$l = \frac{2}{3-n}, \qquad m = \frac{1-n}{3-n}. \quad (20)$$

The basic control equation solution is as follows:

$$\overline{P}_{jD} = \sum_{k=1}^{N} r_D^{(1-n)/2} \left[A_j^{\,k} K_m \left(l\sigma_k r_D^{(3-n)/2} \right) + B_j^{\,k} I_m \left(l\sigma_k r_D^{(3-n)/2} \right) \right], \quad (21)$$

where the subscript on A indexes the layer and the superscript indexes σ, A, and B are, respectively, the first and the two class of m order modified Bessel function. $k_j\sigma^2$ is eigenvalue of real symmetric three diagonal positive definite matrices $[a'_{jk}]$, where

$$a'_{jk} = \{ -\lambda_{j-1}, k = j-1, j > 1;\ \omega_j z - \lambda_{j-1} - \lambda_j, k = j;\ -\lambda_j, k = j+1, j < N;\ 0, k \neq j-1, j \text{ or } j+1 \}. \quad (22)$$

According to the boundary conditions, a function relationship between $A_j^{\,k}$ and $B_j^{\,k}$ is as follows:

$$A_2^{\,k} = \frac{-a_{11}}{a_{12}} A_1^{\,k} = \alpha_2^{\,k} A_1^{\,k},$$
$$A_3^{\,k} = \frac{-\left(a_{21} A_1^{\,k} + a_{22} A_2^{\,k}\right)}{a_{23}} = \alpha_3^{\,k} A_1^{\,k},$$
$$\vdots$$
$$A_N^{\,k} = \frac{-\left(a_{N-1,N-2} A_{N-2}^{\,k} + a_{N-1,N-1} A_{N-1}^{\,k}\right)}{a_{N-1,N}} = \alpha_N^{\,k} A_1^{\,k},$$
$$B_j^{\,k} = \alpha_j^{\,k} B_1^{\,k}, \quad (j = 1, \ldots, N). \quad (23)$$

Then, N^2 values for $\alpha_j^{\,k}$ can be computed directly from the above recursion formula. Let zone i ($i = 1, \ldots, NZ$) contain m_i layer, the zone i with a specific layer j, dimensionless pressure:

$$\overline{P}_{jD} = \sum_{k_i=1}^{m_i} r_D^{(1-n)/2} \left[A_j^{\,k_i} K_m \left(l\sigma_{k_i} r_D^{(3-n)/2} \right) + B_j^{\,k_i} I_m \left(l\sigma_{k_i} r_D^{(3-n)/2} \right) \right], \quad (24)$$

where each layer index, k_i, in the sum refers to the same zone i as Layer j.

Finally, the coefficients for each zone can be expressed as multiples of the coefficients, ${A_1}^{k_i}, {B_1}^{k_i}$, of the uppermost layer in zone i with (12):

$$\overline{P}_{jD} = \sum_{k_i=1}^{m_i} {r_D}^{(1-n)/2} \left[{A_1}^{k_i} {\alpha_j}^{k_i} K_m \left(l\sigma_{k_i} {r_D}^{(3-n)/2} \right) + {B_1}^{k_i} {\alpha_j}^{k_i} I_m \left(l\sigma_{k_i} {r_D}^{(3-n)/2} \right) \right]. \quad (25)$$

External boundary condition implies the relationship between ${A_1}^{k_i}$ and ${B_1}^{k_i}$ is as follows:

$${B_1}^{k_i} = b^{k_i} {A_1}^{k_i}. \quad (26)$$

For infinite-outer-boundary condition,

$$b^{k_i} = 0. \quad (27)$$

For no-flow outer-boundary conditions,

$$b^{k_i} = \frac{K_l \left(l\sigma_{k_i} {r_{eD}}^{(3-n)/2} \right)}{I_{-l} \left(l\sigma_{k_i} {r_{eD}}^{(3-n)/2} \right)}. \quad (28)$$

According to the boundary conditions, the layer $j-1$ and layer j which in the same zone have the following relations:

$$\begin{aligned} \sum_{k_i=1}^{m_i} {A_1}^{k_i} \Big(& {\alpha_{j-1}}^{k_i} \left\{ K_m \left(l\sigma_{k_i} \right) + b^{k_i} I_m \left(l\sigma_{k_i} \right) + s_{j-1} \sigma_{k_i} \left[\left(l\sigma_{k_i} \right) - b^{k_i} I_{-l} \left(l\sigma_{k_i} \right) \right] \right\} \\ & - {\alpha_j}^{k_i} \left\{ K_m \left(l\sigma_{k_i} \right) + b^{\kappa_i} I_m \left(l\sigma_{k_i} \right) + s_j \sigma_{k_i} \left[K_l \left(l\sigma_{k_i} \right) - b^{\kappa_i} I_{-l} \left(l\sigma_{k_i} \right) \right] \right\} \Big) = 0. \end{aligned} \quad (29)$$

The layer $j-1$ of zone $i-1$ and the layer j of zone $i-1$ have the following relations:

$$\begin{aligned} & \sum_{k_{i-1}=1}^{m_{i-1}} \left({A_1}^{k_{i-1}} {\alpha_{j-1}}^{k_{i-1}} \left\{ K_m \left(l\sigma_{k_{i-1}} \right) + b^{k_{i-1}} I_m \left(l\sigma_{k_{i-1}} \right) + s_{j-1} \sigma_{k_{i-1}} \left[K_l \left(l\sigma_{k_{i-1}} \right) - b^{k_{i-1}} I_{-l} \left(l\sigma_{k_{i-1}} \right) \right] \right\} \right) \\ & - \sum_{k_i=1}^{m_i} \left({A_1}^{k_i} {\alpha_j}^{k_i} \left\{ K_m \left(l\sigma_{k_i} \right) + b^{\kappa_i} I_m \left(l\sigma_{k_i} \right) + s_j \sigma_{k_i} \left[K_l \left(l\sigma_{k_i} \right) - b^{\kappa_i} I_{-l} \left(l\sigma_{k_i} \right) \right] \right\} \right) = 0. \end{aligned} \quad (30)$$

The wellbore pressure is as follows:

$$\overline{P}_{wD} = \frac{1}{\left(C_D z^2 + 1/\overline{P}_{wDC_D=0} \right)}. \quad (31)$$

Equation (31) is a general solution, and it is by no wellbore storage pressure solution conversion into the wellbore storage pressure solutions. Therefore, we will solve the $C_D = 0$ cases of equations.

Equations (29)-(30) are linear equations with coefficient ${A_1}^{k_i}$ which can be solved by numerical method. The wellbore pressure without wellbore storage is given as

$$\overline{P}_{wDC_D=0} = \sum_{k_i=1}^{m_i} \left({A_1}^{k_i} {\alpha_1}^{k_i} \left\{ K_m \left(l\sigma_{k_i} \right) + b^{k_i} I_m \left(l\sigma_{k_i} \right) + s_1 \sigma_{k_i} \left[K_l \left(l\sigma_{k_i} \right) - b^{k_i} I_{-l} \left(l\sigma_{k_i} \right) \right] \right\} \right). \quad (32)$$

In addition, layer flow rate is given as follows:

$$\overline{q}_{jD} = \left(1 - C_D \overline{P}_{wD} z^2 \right) \cdot \kappa_j \sum_{k_i=1}^{m_i} {A_1}^{k_i} {\alpha_j}^{k_i} \sigma_{k_i} \left\{ K_l \left(l\sigma_{k_i} \right) - b^{k_i} I_{-l} \left(l\sigma_{k_i} \right) \right\}. \quad (33)$$

Distribution of radial pressure on each layer becomes

$$\overline{P}_{jD} = \left(1 - C_D \overline{P}_{wD} z^2 \right) \overline{P}_{jDC_D=0} = \left(1 - C_D \overline{P}_{wD} z^2 \right) \cdot \sum_{k_i=1}^{m_i} \left({A_1}^{k_i} {\alpha_j}^{k_i} \left\{ K_m \left(l\sigma_{k_i} {r_D}^{(3-n)/2} \right) + b^{k_i} I_m \left(l\sigma_{k_i} {r_D}^{(3-n)/2} \right) \right\} \right). \quad (34)$$

For the case of double-layer reservoirs with formation cross-flow, assume that choosing parameters is as follows.

$C_D = 1$, $s_1 = s_2 = 1$, $\lambda = 1e-5$, $\kappa = 0.95$, $\omega = 0.1$, and $n = 0.1, 0.3, 0.5, 0.7, 0.9$, through the numerical inversion of the Laplace transform of Stehfest [20]. Pressure and flow in the real space are obtained. The wellbore pressure curve is shown in Figure 1. The wellbore pressure curve is shown in Figure 2. It can be seen that the pressure derivative curve is unit slope straight line in the early-time; namely, linear unit slope represents pure effect of wellbore storage; in the medium term, the curve is concave interporosity flow transition characteristics; in the late stage, the pressure derivative curve is approximately straight line up, and the slope is related to the power law index. The slope is $m_L = (1-n)/(3-n)$. And it can be seen that the pressure derivative rises and increases with the reduction of power-law index n steepened.

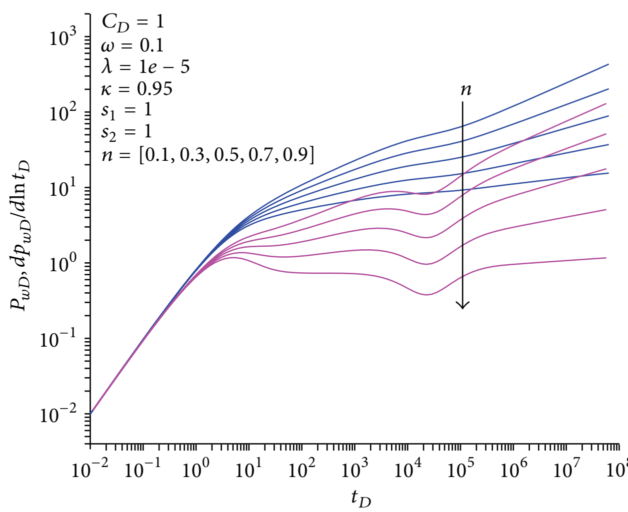

Figure 2: Dimensionless pressure for two-layer system.

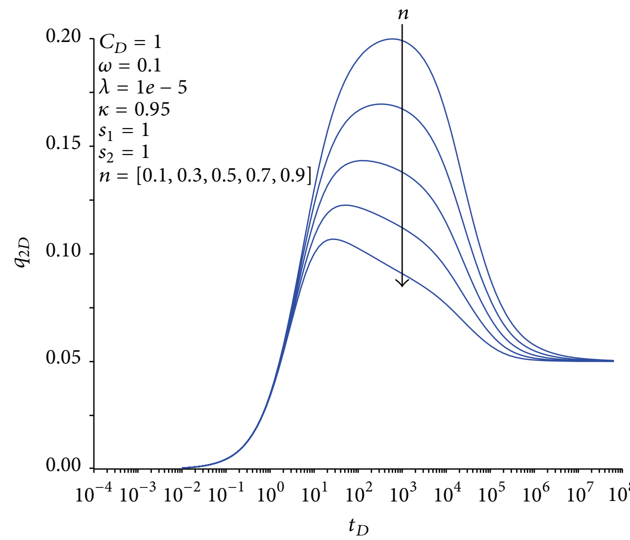

Figure 4: Dimensionless flow-rate for the second layer.

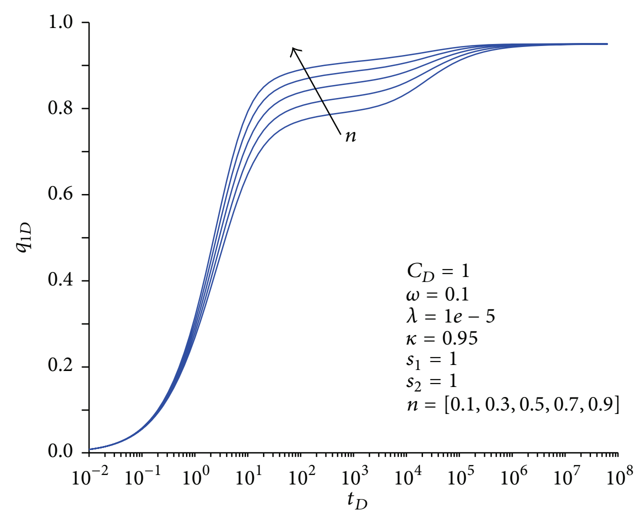

Figure 3: Dimensionless flow-rate for the first layer.

In Figures 3 and 4 flow curve can be seen; for the first layer with high quasi capacity coefficient, the dimensionless flow rate increases with the time increasing and finally tends to be stable flow rate κ; for the second layer with low quasi capacity coefficient, the dimensionless flow rate increases with the time increasing. In a certain period, due to high permeability layer crossflow, pressure tends to be balanced, and the crossflow that is more and more weak, finally, tends to be stable flow rate $1-\kappa$.

4. The Late Stable Flow Model

For both cases stable layer flow-rates were discussed, including (1) without formation crossflow and (2) with formation crossflow. We only discussed the behavior in late time, so ignore the effect of wellbore storage effect.

(1) Without Formation Crossflow.

for the infinite-out-boundary, the stable flow rate, for layer j, is as follows:

$$q_{js} = \frac{q\kappa_j{}^l\omega_j{}^m}{\sum_{k=1}^{N}\kappa_k{}^l\omega_k{}^m}. \tag{35}$$

For the no-flow-boundary, the stable flow rate, for layer j, is as follows:

$$q_{js} = \frac{q\phi_j h_j C_{tj}}{\sum_{j=1}^{N}\phi_j h_j C_{tj}}. \tag{36}$$

(2) With Formation Crossflow.

Regarding the eigenvalues of matrix $\{a'_{jk}\}$, there is a value to meet $\sigma^2 = z$, and the rest is independent of z and ω_j and depends only on $\kappa_1, \kappa_2, \ldots, \kappa_n$ and $\lambda_1, \lambda_2, \ldots, \lambda_n$. Assume that $\sigma_1{}^2 = z$. Stable layer flow rate through the determinant value can be expressed as

$$\begin{aligned} q_{js} = {} & q\kappa_j \lim_{z\to 0}\left[\frac{\det\{C_{jk}\}_{1,1}}{\det\{C_{jk}\}}\alpha_j{}^1\sigma_1\left\{K_l(l\sigma_1)\right.\right. \\ & \left.\left.-b^1 I_{-l}(l\sigma_1)\right\}\right] + q\kappa_j\sum_{k_i=2}^{N}\lim_{z\to 0}\left[\frac{\det\{C_{ik}\}_{k_i,k_i}}{\det\{C_{ik}\}}\right. \\ & \left.\cdot\alpha_j{}^{k_i}\sigma_{k_i}\left\{K_l(l\sigma_k) - b^{k_i}I_{-l}(l\sigma_k)\right\}\right], \end{aligned} \tag{37}$$

where $\det\{C_{jk}\}$ is determinant of matrix $\{C_{jk}\}$.

The matrix element is as follows:

$$\begin{aligned} C_{jk} = {} & \alpha_j{}^k \left\{K_m\left(l\sigma_k\right) + b^k I_m\left(l\sigma_k\right)\right. \\ & \left. + s_k\sigma_k\left[K_l\left(l\sigma_k\right) - b^k I_{-l}\left(l\sigma_k\right)\right]\right\} \\ & - \alpha_{j+1}{}^k \left\{K_m\left(l\sigma_{k+1}\right) + b^k I_m\left(l\sigma_{k+1}\right)\right. \\ & \left. + s_{k+1}\sigma_{k+1}\left[K_l\left(l\sigma_{k+1}\right) - b^k I_{-l}\left(l\sigma_{k+1}\right)\right]\right\}. \end{aligned} \tag{38}$$

The matrix $\{C_{jk}\}_{i,i}$ is used in matrix in $\{C_{jk}\}$ column i replacement for $(0, 0, \ldots, A, 1/z)^T$, where superscript T is the vector transpose.

5. Conclusions

Establishing the multilayer reservoir well-test model for polymer flooding gives the formation and wellbore transient pressure and the layer flow-rate finally. The expressions of late time stable flow rate in each layer are given.

The unsteady well-test model provides theoretical method for polymer flooding well test analysis of multilayer reservoir; the experimental data and the theoretical curve fitting can be used to determine the permeability, skin factor and effective interlayer vertical permeability, and other important parameters.

Nomenclature

a'_{jk}: Matrix elements defined in (22)
$A_j{}^k, B_j{}^k$: Coefficient for jth layer, kth root defined in (23)
$A_1{}^{k_i}, B_1{}^{k_i}$: Coefficient for jth layer, kth root, in zone I, defined in (25)
b^{k_i}: Coefficient for outer boundary condition defined in (27) or (28)
C: Wellbore-storage coefficient (cm^3/kPa)
C_{tj}: Total compressibility in layer I (kPa^{-1})
C_{jk}: Matrix elements defined in (38)
D: Constant defined in (5)
h_j: Formation thickness in layer j (cm)
H : Constant defined in (4)
$I_m(\cdot), K_m(\cdot)$: Modified m order Bessel function of the first and second kind
Δh_j: Thickness of a nonperforated zone between layers j and $j+1$ (cm)
K_j: Horizontal permeability in layer j, $\mu\mathrm{m}^2$
K_{vj}: Vertical permeability of a tight zone between Layers j and $j-1$ ($\mu\mathrm{m}^2$)
K_{zj}: Vertical permeability in Layer j ($\mu\mathrm{m}^2$)
l, m: Constant defined in (20)
m_i: Number of layers in Zone i
n: Power-law index
N: Number of layers in reservoir system
NZ: Number of Zones in reservoir system
P_i: Reservoir pressure in layer I (kPa)
$P_j{}^i$: Reservoir pressure in layer j in Zone i (kPa)
P_{jD}: Dimensionless reservoir pressure in layer j defined in (12)
P_0: Initial reservoir pressure (kPa)
$\overline{P}_{wD}$: Dimensionless bottomhole pressure in Laplace space
$\overline{P}_{wDC_D=0}$: Dimensionless bottomhole pressure without wellbore storage in Laplace space
Q: Surface production rate (cm^3/s)
q_j: Flow rate for layer j (cm^3/s)
q_{js}: Stable flow rate for layer j (cm^3/s)
r: Radial distance (cm)
r_D: Dimensionless radius, defined in (18)
r_e: Reservoir outer radius (cm)
r_{eD}: Dimensionless outer reservoir radius, defined in (19)
r_w: Wellbore radius (cm)
S_j: Wellbore skin factor for layer j
t: Time (s)
Δt: Elapsed time after rate change (s)
t_D: Dimensionless time referenced to producing layer
ν_i: Shear rate in layer I (s^{-1})
χ_j: Crossflow coefficient between layers j and $j+1$ defined in (2)
x_j: Resistance to flow per unit length at the jth layer interface
Z: Laplace space variable i
$\alpha_j{}^k$: Coefficient for layer j, Root k, defined in (23)
$\alpha_j{}^{k_i}$: Coefficient for layer j, Root k, defined in Zone I, defined in (25)
κ_j: Coefficient for layer j, defined in (14)
λ_j: Dimensionless semipermeability between layers j and $j+1$, defined in (16)
M: Dynamic viscosity (mpa·s)
Φ_j: Porosity fraction for layer j
ω_j: Coefficient for layer j, defined in (15)
ν_j: Shear rate for layer j defined in (5)
$\det\{C_{jk}\}$: Determinant of matrix $\{C_{jk}\}$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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