Research Article

Forgotten Coindex for the Derived Sum Graphs under Cartesian Product

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A topological index (TI) is a molecular descriptor that is applied on a chemical structure to compute the associated numerical value which measures volume, density, boiling point, melting point, surface tension, or solubility of this structure. It is an efficient mathematical method in avoiding laboratory experiments and time-consuming. The forgotten coindex of a structure or (molecular) graph H is defined as the sum of the degrees of all the possible pairs of nonadjacent vertices in H . For $D \in \{S, R, Q, T\}$ and the connected graph H , the derived graphs $D(H)$ are obtained by applying the operations S (subdivided), R (triangle parallel), Q (line superposition), and T (total graph), respectively. Moreover, a derived sum graph (D -sum graph) is obtained by the Cartesian product of the graph H_2 with the graph $D(H_1)$. In this study, we compute forgotten coindex of the D -sum graphs $H_{1+S}H_2$ (S -sum), $H_{1+R}H_2$ (R -sum), $H_{1+Q}H_2$ (Q -sum), and $H_{1+T}H_2$ (T -sum) in the form of various indices and coindices of the factor graphs H_1 and H_2 . At the end, we have analyzed our results using numerical tables and graphical behaviour for some particular D -sum graphs.

1. Introduction

Chemical graph theory being the combination of graph theory and chemistry is a branch of mathematical chemistry in which we study the various physical and chemical properties of a chemical structure or network using different graph theoretical techniques. A topological index is one of the most used graph theoretical technique that studies the different properties of the molecular structure such as volume, density, freezing point, vaporization point, boiling point, melting point, surface tension or solubility, heat of formation, and heat of evaporation numerically [1, 2].

TIs are categorized in three types such as degree, distance, and polynomial based, but according the latest survey [3], the degree-based TIs are more studied than others. Wiener was the first scientist who calculated the boiling point of paraffin by using a distance-based TI [4]. Gutman and Trinajstić [5] introduced degree-based TIs known as

Zagreb indices to calculate total π -electron energy of hydrocarbons. Rouvray [6] and Balaban [7, 8] discussed the structure-activity correlations of different chemical phenomenon using TIs. Klein et al. introduced a molecular topological index and drive a close relation with the Wiener index [9, 10]. Mendiratta et al. and Cornwell used Wiener's TI to study structure-activity study on antiviral 5-vinylpyrimidine nucleoside analogs [11]. Gutman and Estrada calculated the TIs based on the line graph of the molecular graph [12]. Biye et al. wrote a novel on TI for QSPR/QSAR study of organic compounds [13]. Qinghua et al. calculated the TI for octane number of acyclic alkane by autocorrelation [14]. Baig et al. computed the TIs for polyoxide and silicate DSL and DOX-like networks [15].

In 2015, Furtula and Gutman redefined a TI with the name forgotten index (F -index) as the sum of cubes of vertex degrees of a molecular graph [16]. Gutman et al. listed the graphs having smallest forgotten index [17]. Zhongyuan and

Zhibo computed bounds of the F -index using Cauchy-Schwarz inequality, Jensen's inequality, and Chebyshev's sum inequality [18]. Gao and Liu calculated F -index of different chemical structures [19, 20]. Ahmad et al. worked online graph of benzene ring in the 2D network and calculated the degree-based TIs for these graphs [21]. Javaid et al. calculated bound on F -index for unicyclic graphs with fixed number of pendent vertices [22]. Imran et al. [23] investigated the family of unicyclic graphs with extreme F -coindex.

Ashrafi et al. introduced the concept of coindex of graph and investigated Zagreb coindices of composite graph operations such as union, disjunction, and various product of graphs [24]. Havare et al. computed the F -index and F -coindex for carbon base nanomaterial [25]. Basavanagoud and Desai calculated the F -index and hyper-Zagreb index of generalized transformation graphs [26]. Gao et al. calculated electron energy of molecular structures F -index [27]. Yasir et al. computed F -index of dendrimers-like structure [28]. Basavanagoud and Timmanaiakar calculated first Zagreb and F -index of some dominating transformation graphs [29]. Sana et al. characterized the extremal graphs and proved the ordering among the different subfamilies of graphs with respect to F -index [30].

Various operations on a graph play the important role in the development of different new classes of graphs. Yan et al. listed five graphs line graph $L(H)$, subdivided graph $S(H)$, line superposition graph $Q(H)$, triangle parallel $R(H)$, and total $T(H)$ by performing different operations on H and computed Wiener indices of these five graphs [31]. Eliasi and Taeri defined the derived sum graphs $(H_{1+D}H_2)$ and computed their Wiener indices, where $D \in \{S, R, Q, T\}$ [32]. Later on, for these derived sum graphs, Deng et al. [33] computed first two Zagreb indices, and Akhtar and Imran calculated the F -index [34, 35]. Javaid et al. (2021) investigated the first Zagreb connection index and coindex [36], and Javaid et al. investigated forgotten index of these graphs [37]. Moreover, Pattabiraman and Peng computed F -indices and their coindices of some classes of graphs [38, 39]. Ali et al. forgotten coindex of some nontoxic dendrimers structure used in targeted drug delivery [40].

In this study, we compute forgotten coindices of D -sum graphs $H_{1+D}H_2$, where $D \in \{S, R, Q, T\}$ in the form of forgotten index, first Zagreb indices, and coincides of their basic graphs H_1 and H_2 . At the end, the obtained results are also illustrated with the assistance of the examples for some particular D -sum graphs. In Section 2, the definitions and notations are presented, Section 3 includes the main results of our work, and Section 4 presents particular examples related to the main results.

2. Preliminaries

Let $H = (V(H), E(H))$ be a connected graph, where $V(H)$ and $E(H)$ be the set of vertices and edges of H , respectively. For any vertex $x \in V(H)$, its degree is denoted by $d(x)$ and defined as the number of edges connecting it. The joining of two vertices $x, y \in V(H)$ formed an edge denoted by $xy \in E(H)$. Gutman and Trinajstić [5] introduced Zagreb

indices $M_1(H)$ and $M_2(H)$ to calculate total π electron energy of hydrocarbons. The Zagreb indices $M_1(H)$ and $M_2(H)$ are defined as

$$\begin{aligned} M_1(H) &= \sum_{r_1 r_2 \in E(H)} [d_H(r_1) + d_H(r_2)], \\ M_2(H) &= \sum_{r_1 r_2 \in E(H)} [d_H(r_1)d_H(r_2)]. \end{aligned} \quad (1)$$

Ashrafi et al. [24] defined Zagreb coincides such as $\overline{M}_1(H)$ and $\overline{M}_2(H)$ of the Zagreb indices. The Zagreb coindices are defined as

$$\begin{aligned} \overline{M}_1(H) &= \sum_{r_1 r_2 \notin E(H)} [d_H(r_1) + d_H(r_2)], \\ \overline{M}_2(H) &= \sum_{r_1 r_2 \notin E(H)} [d_H(r_1)d_H(r_2)]. \end{aligned} \quad (2)$$

Furtula and Gutman [16] defined forgotten index (F -index), and its mathematical form is given by

$$F(H) = \sum_{r_1 r_2 \in E(H)} [d_H(r_1)^2 + d_H(r_2)^2]. \quad (3)$$

Nilanjan et al. [41] introduced the forgotten coindex (F -coindex) for F -index with mathematical formulation as

$$\overline{F}(H) = \sum_{r_1 r_2 \notin E(H)} [d_H(r_1)^2 + d_H(r_2)^2]. \quad (4)$$

Let H be a simple connected graph; then, its derived graphs are defined as follows.

- (i) $S(H)$ is a graph formed by inserting a new vertex in each edge of H
- (ii) $R(H)$ is a graph obtained from $S(H)$ by adding an edge between the adjacent vertices of H
- (iii) $Q(H)$ is a graph formed from $S(H)$ by adding an edge between the pairs of new vertices which are on the adjacent edges of H
- (iv) $T(H)$ is formed by performing both operations of $R(H)$ and $Q(H)$ on $S(H)$

Suppose that H_1 and H_2 are two connected graphs; then, their derived sum graph (D -sum graphs) is denoted by $H_{1+D}H_2$ and defined with vertex set $V(H_{1+D}H_2) = V(H_1) \cup E(H_1) \times V(H_2)$, and edge set is defined as the vertices (r_1, r_2) and (s_1, s_2) of $H_{1+D}H_2$, where $D \in \{S, R, Q, T\}$ are joined iff.

- (i) $r_1 = s_1 \in V(H_1)$ and $r_2 \sim s_2 \in H_2$
- (ii) $r_2 = s_2 \in V(H_2)$ and $r_1 \sim s_1 \in D(H_1)$

Where $r \sim s \in H_2$ presents the xy is an edge in H_2 [26]. For the D -sum graphs, refer Figures 1 and 2.

3. Main Results

In this section, we discuss main results related to F -coindex for D -sum graphs. Let H_1 be a graph of its order, and the size is denoted by n_1 and e_1 , respectively. Let $\overline{H}_1$ be the complement of H_1 , and the edge set for $\overline{H}_1$ is given by

FIGURE 1: (a) $G \cong P_4$, (b) $S(G) \cong S(P_4)$, (c) $R(G) \cong R(P_4)$, (d) $Q(G) \cong Q(P_4)$, and (e) $T(G) \cong T(P_4)$.

FIGURE 2: $G \cong P_4$, $H \cong P_2$, and $G_{+D}H \cong P_{4+D}P_2$.

$\binom{n_1}{2} - e_1$. Further assumed some sums by $\alpha, \alpha_1, \alpha_2, \alpha_3$, and α_4 are given as follows.

$$\begin{aligned} \text{(i)} \quad \alpha &= (n_2(n_2 - 1) + \bar{e}_2(\bar{n}_2 - 1)) \\ \text{(ii)} \quad \alpha_1 &= \sum_{\substack{r_1 r_2 \notin E(D(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(D(H_1)-V(H_1))}} [d_{H_1}(r_1)^2 + d_{D(H_1)}(r_2)^2] \\ \text{(iii)} \quad \alpha_2 &= \sum_{\substack{r_1 r_2 \notin E(D(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(D(H_1)-V(H_1))}} [d_{H_1}(r_1)^2 + d_{D(H_1)}(r_2)^2] \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad \alpha_3 &= \sum_{\substack{r_1 r_2 \notin E(D(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(D(H_1)-V(H_1))}} d_{H_1}(r_1) \\ \text{(v)} \quad \alpha_4 &= \sum_{\substack{r_1 r_2 \notin E(D(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(D(H_1)-V(H_1))}} d_{H_1}(r_1) \end{aligned}$$

Theorem 1. Let $H_{1+S}H_2$ be a S -sum graph; then, its F -coindex is

$$\begin{aligned} \bar{F}(H_{1+S}H_2) &= 4(n_2^2 e_1^2 - n_2 e_1) + 2\bar{e}_2 M_1(H_1) + 4e_1 \bar{M}_1(H_2) + n_1 \bar{F}(H_2) + n_2 (F(H_1) + \bar{F}(H_1)) \\ &\quad + 2(e_1 + \bar{e}_1) + (e_1(n_1 - 2))M_1(H_2) + 4e_2(M_1(H_1) + \bar{M}_1(H_1)) + 2[(e_2 + \bar{e}_2)(F(H_1) \\ &\quad + \bar{F}(H_1)) + (e_1 + \bar{e}_1)(F(H_2) + \bar{F}(H_2)) + (M_1(H_2) + \bar{M}_1(H_2))(M_1(H_1) + \bar{M}_1(H_1))] \\ &\quad + n_2 \alpha_1 + \alpha(\alpha_1 + \alpha_2) + e_1 n_1 (M_1(H_2) + \bar{M}_1(H_2)) + 4(e_2 + \bar{e}_2)(2\alpha_3 + \alpha_4). \end{aligned} \quad (5)$$

Proof. Using equation (4), we have

$$\begin{aligned} \bar{F}(H_{1+S}H_2) &= \sum_{(r_1, r_2) \in (S(H_1)-V(H_1)) \times (V(H_1)-V(D(H_1)-V(H_1)))} [d(r_1, s_1)^2 + d(r_2, s_2)^2], \\ \bar{F}(H_{1+S}H_2) &= \left[\sum_{r_1, r_2 \in (V(S(H_1)-V(H_1)))} + \sum_{r_1, r_2 \in V(H_1)} + \sum_{\substack{r_1, r_2 \in V(S(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(S(H_1)-V(H_1))}} \right] \sum_{s_1, s_2 \in V(H_2)} [d(r_1, s_1)^2 + d(r_2, s_2)^2], \\ &= \sum A + \sum B + \sum C, \end{aligned} \quad (6)$$

$$\begin{aligned} \sum A &= \sum_{r_1, r_2 \in V(S(H_1)-V(H_1))} \sum_{s_1, s_2 \in V(H_2)} [d_{S(H_1)}(r_1) + d_{S(H_1)}(r_2)] = \sum_{r_1, r_2 \in V(S(H_1)-V(H_1))} \sum_{s_1, s_2 \in V(H_2)} (2^2 + 2^2), \\ \sum A &= 4(n_2^2 e_1^2 - n_2 e_1), \end{aligned} \quad (7)$$

$$\begin{aligned} \sum B &= \sum B_1 + \sum B_2 + \sum B_3, \\ \sum B_1 &= \sum_{r \in V(H_1)} \sum_{s_1, s_2 \notin E(H_2)} [d(r, s_1)^2 + d(r, s_2)^2] = \sum_{r \in V(H_1)} \sum_{s_1, s_2 \notin E(H_2)} [(d(r) + d(s_1))^2 + (d(r) + d(s_2))^2] \\ &= \sum_{r \in V(H_1)} \sum_{s_1, s_2 \notin E(H_2)} [(d(r)^2 + d(s_1)^2 + 2d(r)d(s_1)) + (d(r)^2 + d(s_2)^2 + 2d(r)d(s_2))] \\ &= \sum_{r \in V(H_1)} \sum_{s_1, s_2 \notin E(H_2)} [2d(r)^2 + (d(s_1)^2 + d(s_2)^2) + 2d(r)(d(s_1) + d(s_2))] \\ &= 2\bar{e}_2 M_1(H_1) + 4e_1 \bar{M}_1(H_2) + n_1 \bar{F}(H_2), \\ \sum B_2 &= \sum_{s \in V(H_2)} \sum_{r_1, r_2 \in V(H_1)} [(d(r_1) + d(s))^2 + (d(r_2) + d(s))^2] \\ &= \sum_{s \in V(H_2)} \left(\sum_{r_1, r_2 \in E(H_1)} + \sum_{r_1, r_2 \notin E(H_1)} \right) [(d(r_1)^2 + d(s)^2 + 2d(r_1)d(s)) + (d(r_2)^2 + d(s)^2 + 2d(r_2)d(s))] \\ &= \sum_{s \in V(H_2)} \left(\sum_{r_1, r_2 \in E(H_1)} + \sum_{r_1, r_2 \notin E(H_1)} \right) [(d(r_1)^2 + d(r_2)^2) + 2d(s)^2 + 2d(s)(d(r_1) + d(r_2))] \end{aligned}$$

$$\begin{aligned}
&= n_2(F(H_1) + \bar{F}(H_1)) + 2M_1(H_2)(e_1 + \bar{e}_1) + 4e_2(M_1(H_1) + \bar{M}_1(H_1)), \\
\sum B_3 &= \left[\left(\sum_{r_1 r_2 \in E_{H_1}} + \sum_{r_1 r_2 \notin E_{H_1}} \right) \left(\sum_{s_1 s_2 \in E_{H_2}} + \sum_{s_1 s_2 \notin E_{H_2}} \right) \right] [d(r_1, s_1)^2 + d(r_2, s_2)^2] \\
&= \left[\left(\sum_{r_1 r_2 \in E_{H_1}} + \sum_{r_1 r_2 \notin E_{H_1}} \right) \left(\sum_{s_1 s_2 \in E_{H_2}} + \sum_{s_1 s_2 \notin E_{H_2}} \right) \right] [(d(r_1)^2 + d(r_2)^2) + 2(d(r_1)d(s_1) + d(r_2)d(s_2)) + (d(s_1)^2 + d(s_2)^2)] \\
&= 2[(e_2 + \bar{e}_2)(F(H_1) + \bar{F}(H_1)) + (e_1 + \bar{e}_1)(F(H_2) + \bar{F}(H_2)) + (M_1(H_2) + \bar{M}_1(H_2))(M_1(H_1) + \bar{M}_1(H_1))], \\
\sum B &= 2\bar{e}_2 M_1(H_1) + 4e_1 \bar{M}_1(H_2) + n_1 \bar{F}(H_2) + n_2(F(H_1) + \bar{F}(H_1)) + 2M_1(H_2)(e_1 + \bar{e}_1) \\
&\quad + 4e_2(M_1(H_1) + \bar{M}_1(H_1)) + 2[(e_2 + \bar{e}_2)(F(H_1) + \bar{F}(H_1)) + (e_1 + \bar{e}_1)(F(H_2) + \bar{F}(H_2))], \\
\sum C &= \sum C_1 + \sum C_2 + \sum C_3, \\
\sum C_1 &= \sum_{\substack{r_1 r_2 \notin E(S(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(S(H_1) - V(H_1))}} \sum_{s \in V_{H_2}} [d(r_1, s)^2 + d(r_2, s)^2] \\
&= \sum_{\substack{r_1 r_2 \notin E(S(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(S(H_1) - V(H_1))}} \sum_{s \in V_{H_2}} [(d(r_1)^2 + d(s)^2 + 2d(r_1)d(s)) + (d_{S(H_1)}(r_2)^2)] \\
&= n_2 \alpha_1 + e_1(n_1 - 2)M_1(H_2) + 4e_2 \alpha_3, \\
\sum C_2 &= \sum_{\substack{r_1 r_2 \notin E(S(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(S(H_1) - V(H_1))}} \sum_{s_1, s_2 \in V_{H_2}} [d(r_1, s_1)^2 + d(r_2, s_2)^2] \\
&= \sum_{\substack{r_1 r_2 \notin E(S(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(S(H_1) - V(H_1))}} \left(\sum_{s_1, s_2 \in V_{H_2}} + \sum_{s_1, s_2 \notin V_{H_2}} \right) [(d(r_1)^2 + d(s_1)^2 + 2d(r_1)d(s_1)) + (d_{S(H_1)}(r_2)^2)] \\
&= \alpha \alpha_1 + e_1(n_1 - 2)(M_1(H_2) + \bar{M}_1(H_2)) + 4(e_2 + \bar{e}_2) \alpha_3, \\
\sum C_3 &= \sum_{\substack{r_1 r_2 \in E(S(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(S(H_1) - V(H_1))}} \sum_{s_1, s_2 \in V_{H_2}} [d(r_1, s_1)^2 + d(r_2, s_2)^2] \\
&= \sum_{\substack{r_1 r_2 \in E(S(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(S(H_1) - V(H_1))}} \left(\sum_{s_1, s_2 \in V_{H_2}} + \sum_{s_1, s_2 \notin V_{H_2}} \right) [(d(r_1)^2 + d(s_1)^2 + 2d(r_1)d(s_1)) + (d_{S(H_1)}(r_2)^2)] \\
&= \alpha \alpha_2 + 2e_1(M_1(H_2) + \bar{M}_1(H_2)) + 4(e_2 + \bar{e}_2) \alpha_4.
\end{aligned}$$

By substituting the values in equation (6), we get the required result. $\square$

Theorem 2. Let $H_{1+R}H_2$ be a R -sum graph; then, its F -coindex is

$$\begin{aligned}\overline{F}(H_{1+R}H_2) = & 4(n_2^2e_1^2 - n_2e_1) + 8\overline{e}_2M_1(H_1) + 8e_1\overline{M}_1(H_2) + n_1\overline{F}(H_2) \\ & + 4n_2(\overline{F}(H_1) + (2\overline{e}_1 + (e_1(n_1 - 2))))M_1(H_2) + 8e_2\overline{M}_1(H_1) \\ & + 24(e_2 + \overline{e}_2)(F(H_1) + \overline{F}(H_1)) + (e_1 + \overline{e}_1)(F(H_2) + \overline{F}(H_2)) \\ & + 2(M_1(H_2) + \overline{M}_1(H_2))(M_1(H_1) + \overline{M}_1(H_1)) \\ & + n_2\alpha_1 + \alpha(\alpha_1 + \alpha_2) + e_1n_1(M_1(H_2) + \overline{M}_1(H_2)) + 4(e_2 + \overline{e}_2)(2\alpha_3 + \alpha_4).\end{aligned}\quad (9)$$

Proof. Using equation (4), we have

$$\begin{aligned}\overline{F}(H_{1+R}H_2) &= \sum_{(t_1, t_2) (x_1, x_2) \notin E(H_{1+R}H_2)} [d(t_1, x_1)^2 + d(t_2, x_2)^2], \\ \overline{F}(H_{1+R}H_2) &= \left[\sum_{r_1, r_2 \in (V(R(H_1)) - V(H_1))} + \sum_{r_1, r_2 \in V_{H_1}} + \sum_{\substack{r_1, r_2 \in V(R(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(R(H_1)) - V(H_1)}} \right] \sum_{s_1, s_2 \in V_{H_2}} [d(r_1, s_1)^2 + d(r_2, s_2)^2] \\ &= \sum A + \sum B + \sum C.\end{aligned}\quad (10)$$

Using equation (7),

$$\begin{aligned}\sum A &= 4(n_2^2e_1^2 - n_2e_1), \\ \sum B &= \sum B_1 + \sum B_2 + \sum B_3 \\ \sum B_1 &= \sum_{r \in V_{H_1}} \sum_{s_1, s_2 \notin E_{H_2}} [d(r, s_1)^2 + d(r, s_2)^2] = \sum_{r \in V_{H_1}} \sum_{s_1, s_2 \notin E_{H_2}} [(d(r) + d(s_1))^2 + (d(r) + d(s_2))^2] \\ &= \sum_{r \in V_{H_1}} \sum_{s_1, s_2 \notin E_{H_2}} [8d(r)^2 + (d(s_1)^2 + d(s_2)^2) + 4d(r)(d(s_1) + d(s_2))] \\ &= 8\overline{e}_2M_1(H_1) + 8e_1\overline{M}_1(H_2) + n_1\overline{F}(H_2), \\ \sum B_2 &= \sum_{s \in V_{H_2}} \sum_{r_1, r_2 \in V_{H_1}} [(d(r_1) + d(s))^2 + (d(r_2) + d(s))^2] \\ &= \sum_{s \in V_{H_2}} \left(\sum_{r_1, r_2 \in E_{H_1}} + \sum_{r_1, r_2 \notin E_{H_1}} \right) [4(d(r_1)^2 + d(r_2)^2) + 2d(s)^2 + 4d(s)(d(r_1) + d(r_2))] \\ &= 4n_2(\overline{F}(H_1) + 2\overline{e}_1M_1(H_2) + 8e_2\overline{M}_1(H_1)), \\ \sum B_3 &= \left[\left(\sum_{r_1, r_2 \in E_{H_1}} + \sum_{r_1, r_2 \notin E_{H_1}} \right) \left(\sum_{s_1, s_2 \in E_{H_2}} + \sum_{s_1, s_2 \notin E_{H_2}} \right) \right] [d(r_1, s_1)^2 + d(r_2, s_2)^2]\end{aligned}$$

$$\begin{aligned}
&= \left[\left(\sum_{r_1 r_2 \in E_{H_1}} + \sum_{r_1 r_2 \notin E_{H_1}} \right) \left(\sum_{s_1 s_2 \in E_{H_2}} + \sum_{s_1 s_2 \notin E_{H_2}} \right) \right] \left[(d(r_1)^2 + d(r_2)^2) + (d(s_1)^2 + d(s_2)^2) + 2(d(r_1)d(s_1) + d(r_2)d(s_2)) \right] \\
&= \left[\left(\sum_{r_1 r_2 \in E_{H_1}} + \sum_{r_1 r_2 \notin E_{H_1}} \right) \left(\sum_{s_1 s_2 \in E_{H_2}} + \sum_{s_1 s_2 \notin E_{H_2}} \right) \right] \left[2^2(d(r_1)^2 + d(r_2)^2) + (d(s_1)^2 + d(s_2)^2) + 4(d(r_1)d(s_1) + d(r_2)d(s_2)) \right] \\
&= 2[4(e_2 + \bar{e}_2)(F(H_1) + \bar{F}(H_1)) + (e_1 + \bar{e}_1)(F(H_2) + \bar{F}(H_2)) + 2(M_1(H_2) + \bar{M}_1(H_2))(M_1(H_1) + \bar{M}_1(H_1))] \\
\sum B &= 8\bar{e}_2 M_1(H_1) + 8e_1 \bar{M}_1(H_2) + n_1 \bar{F}(H_2) + 4n_2 (\bar{F}(H_1) + 2\bar{e}_1 M_1(H_2) + 8e_2 \bar{M}_1(H_1) \\
&\quad + 2[4(e_2 + \bar{e}_2)(F(H_1) + \bar{F}(H_1)) + (e_1 + \bar{e}_1)(F(H_2) + \bar{F}(H_2)) \\
&\quad + 2(M_1(H_2) + \bar{M}_1(H_2))(M_1(H_1) + \bar{M}_1(H_1))], \\
\sum C &= \sum C_1 + \sum C_2 + \sum C_3, \\
\sum C_1 &= \sum_{\substack{r_1 r_2 \notin E(R(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(R(H_1) - V(H_1))}} \sum_{s \in V_{H_2}} [d(r_1, s)^2 + d(r_2, s)^2] \\
&= \sum_{\substack{r_1 r_2 \notin E(R(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(R(H_1) - V(H_1))}} \sum_{s \in V_{H_2}} \left[2^2(d(r_1)^2 + d(s)^2 + 4d(r_1)d(s)) + (d_{S(H_1)}(r_2))^2 \right] \\
&= n_2 \alpha_1 + e_1 (n_1 - 2) M_1(H_2) + 4e_2 \alpha_3, \\
\sum C_2 &= \sum_{\substack{r_1 r_2 \notin E(R(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(R(H_1) - V(H_1))}} \left(\sum_{s_1, s_2 \in V_{H_2}} + \sum_{s_1, s_2 \notin V_{H_2}} \right) \left[(d(r_1)^2 + d(s_1)^2 + 2d(r_1)d(s_1)) + (d_{S(H_1)}(r_2))^2 \right] \\
&= \alpha \alpha_1 + e_1 (n_1 - 2) (M_1(H_2) + \bar{M}_1(H_2)) + 4(e_2 + \bar{e}_2) \alpha_3, \\
\sum C_3 &= \sum_{\substack{r_1 r_2 \in E(R(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(R(H_1) - V(H_1))}} \left(\sum_{s_1, s_2 \in V_{H_2}} + \sum_{s_1, s_2 \notin V_{H_2}} \right) \left[(d(r_1)^2 + d(s_1)^2 + 2d(r_1)d(s_1)) + (d_{S(H_1)}(r_2))^2 \right] \\
&= \alpha \alpha_2 + 2e_1 (M_1(H_2) + \bar{M}_1(H_2)) + 4(e_2 + \bar{e}_2) \alpha_4.
\end{aligned}$$

(11)

By substituting the values in equation (10), we get the required result. $\square$

Theorem 3. Let $H_{1+Q}H_2$ be a Q-sum graph; then, F-coindex is

$$\begin{aligned} \overline{F}(H_{1+Q}H_2) = & n_2 \overline{F}(Q'(G_1)) + 2(\overline{e}_2 + (n_2 - 1)) \left(M_1(Q'H_1) + 1 \right) F(Q'(G_1)) \\ & + \overline{F}(Q'(G_1)) + 2\overline{e}_2 M_1(H_1) + 4e_1 \overline{M}_1(H_2) + n_1 \overline{F}(H_2) + n_2 (F(H_1) + \overline{F}(H_1)) \\ & + 2M_1(H_2)(e_1 + \overline{e}_1) + 4e_2 (M_1(H_1) + \overline{M}_1(H_1)) + 2[(e_2 + \overline{e}_2)(F(H_1) + \overline{F}(H_1)) \\ & + (e_1 + \overline{e}_1)(F(H_2) + \overline{F}(H_2)) + n_2 \overline{F}(Q(H_1')) + e_1(n_1 - 2)M_1(H_2) \\ & + \alpha(\alpha_1 + \alpha_2) + e_1 n_1 (M_1(H_2) + \overline{M}_1(H_2)) + 4(e_2 + \overline{e}_2)(2\alpha_3 + \alpha_4). \end{aligned} \quad (12)$$

Proof. Using equation (4), we have

$$\begin{aligned} \overline{F}(H_{1+Q}H_2) &= \sum_{(t_1, t_2)(x_1, x_2) \notin E(H_{1+Q}H_2)} [d(t_1, x_1)^2 + d(t_2, x_2)^2], \\ \overline{F}(H_{1+Q}H_2) &= \left[\sum_{r_1, r_2 \in (V(Q(H_1)) - V(H_1))} \sum_{s_1, s_2 \in V_{H_2}} + \sum_{r_1, r_2 \in V_{H_1}} + \sum_{\substack{r_1, r_2 \in V(Q(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(Q(H_1)) - V(H_1)}} \right] \\ &\quad \cdot \sum_{s_1, s_2 \in V_{H_2}} [d(r_1, s_1)^2 + d(r_2, s_2)^2] \\ &= \sum A + \sum B + \sum C, \\ \sum A &= \sum A_1 + \sum A_2 + \sum A_3 + \sum A_4, \\ \sum A_1 &= \sum_{\substack{r_1, r_2 \notin E(Q(H_1)) \\ r_1, r_2 \in V(Q(H_1)) - H_1}} \sum_{s \in V_{H_2}} [d_{Q(H_1)}(r_1)^2 + d_{Q(H_1)}(r_2)^2] = n_2, \\ &\quad \cdot \sum_{\substack{r_1, r_2 \notin E(Q(H_1)) \\ r_1, r_2 \in V(Q(H_1)) - H_1}} [d_{Q(H_1)}(r_1)^2 + d_{Q(H_1)}(r_2)^2] \\ \text{if } \sum_{\substack{r_1, r_2 \notin E(Q(H_1)) \\ r_1, r_2 \in V(Q(H_1)) - H_1}} [d_{Q(H_1)}(r_1)^2 + d_{Q(H_1)}(r_2)^2] &= \overline{F}(Q'(H_1)), \text{ then } \sum A_1 = n_2 \overline{F}(Q'(H_1)), \end{aligned}$$

$$\begin{aligned} \sum A_2 &= \sum_{r \in V(Q(H_1)) - (H_1)} \sum_{s_1, s_2 \in E_{H_2}} [d_{Q(H_1)}(r_1)^2 + d_{Q(H_1)}(r_2)^2] \\ &= \sum_{r \in V(Q(H_1)) - V(H_1)} \left(\sum_{s_1, s_2 \notin E_{H_2}} + \sum_{s_1, s_2 \in E_{H_2}} \right) [d_{Q(H_1)}(r_1)^2 + d_{Q(H_1)}(r_2)^2] \\ &= 2(\overline{e}_2 + (n_2 - 1)) \sum_{r \in V(Q(H_1)) - V(H_1)} [d_{Q(H_1)}(r)^2] \end{aligned}$$

$$\begin{aligned}
& \text{if } \sum_{r \in V(Q(H_1) - (H_1))} \left[d_{Q(H_1)}(r)^2 \right] = M_1(Q'H_1), \text{ then } = 2(\bar{e}_2 + (n_2 - 1))M_1(Q'H_1), \\
& \sum A_3 = \sum_{\substack{r_1 r_2 \notin E(Q(H_1)) \\ r_1, r_2 \in V(Q(H_1) - H_1)}} \sum_{s_1, s_2 \in V_{H_2}} \left[d_{Q(H_1)}(r_1)^2 + d_{Q(H_1)}(r_2)^2 \right] \\
& = \sum_{\substack{r_1 r_2 \notin E(Q(H_1)) \\ r_1, r_2 \in V(Q(H_1) - V(H_1))}} \left[\sum_{s_1, s_2 \in E_{H_2}} \left(d_{Q(H_1)}(r_1)^2 + d_{Q(H_1)}(r_2)^2 \right) \right. \\
& \quad \left. + \sum_{s_1, s_2 \notin E_{H_2}} \left(d_{Q(H_1)}(r_1)^2 + d_{Q(H_1)}(r_2)^2 \right) \right] \\
& = 2(\bar{e}_2 + (n_2 - 1)) \sum_{\substack{r_1 r_2 \notin E(Q(H_1)) \\ r_1, r_2 \in V(Q(H_1) - V(H_1))}} \left[d_{Q(H_1)}(r_1)^2 + d_{Q(H_1)}(r_2)^2 \right]
\end{aligned}$$

$$\text{if } \sum_{\substack{r_1 r_2 \notin E(Q(H_1)) \\ r_1, r_2 \in V(Q(H_1) - V(H_1))}} \left[d_{Q(H_1)}(r_1)^2 + d_{Q(H_1)}(r_2)^2 \right] = F(Q'(H_1)) = 2(\bar{e}_2 + (n_2 - 1))F(Q'(H_1)),$$

$$\begin{aligned}
& \sum A_4 = \sum_{\substack{r_1 r_2 \notin E(Q(H_1)) \\ r_1, r_2 \in V(Q(H_1) - V(H_1))}} \sum_{s_1, s_2 \in V_{H_2}} \left[d_{Q(H_1)}(r_1)^2 + d_{Q(H_1)}(r_2)^2 \right] \\
& = \sum_{\substack{r_1 r_2 \notin E(Q(H_1)) \\ r_1, r_2 \in V(Q(H_1) - V(H_1))}} \left[\sum_{s_1, s_2 \in E_{H_2}} \left(d_{Q(H_1)}(r_1)^2 + d_{Q(H_1)}(r_2)^2 \right) \right. \\
& \quad \left. + \sum_{s_1, s_2 \notin E_{H_2}} \left(d_{Q(H_1)}(r_1)^2 + d_{Q(H_1)}(r_2)^2 \right) \right] \\
& = 2(\bar{e}_2 + (n_2 - 1)) \sum_{\substack{r_1 r_2 \notin E(Q(H_1)) \\ r_1, r_2 \in V(Q(H_1) - V(H_1))}} \left[d_{Q(H_1)}(r_1)^2 + d_{Q(H_1)}(r_2)^2 \right].
\end{aligned}$$

$$\text{Take } \sum_{\substack{r_1 r_2 \notin E(Q(H_1)) \\ r_1, r_2 \in V(Q(H_1) - V(H_1))}} \left[d_{Q(H_1)}(r_1)^2 + d_{Q(H_1)}(r_2)^2 \right] = \bar{F}(Q'(H_1)), \text{ then } = 2(\bar{e}_2 + (n_2 - 1))\bar{F}(Q'(H_1)).$$

(13)

Consequently,

$$\sum A = n_2 \bar{F}(Q'(G_1)) + 2(\bar{e}_2 + (n_2 - 1))M_1(Q'H_1) + 2(\bar{e}_2 + (n_2 - 1))F(Q'(G_1)) + \bar{F}(Q'(G_1)). \quad (14)$$

TABLE 1: For two graphs, $H_1 \cong P_n$ and $H_2 \cong P_m$ values of forgotten coindex for their D -sum graphs by taking different values for n and m .

n, m	$\overline{F}(P_{n+S}P_m)$	$\overline{F}(P_{n+R}P_m)$	$\overline{F}(P_{n+Q}P_m)$	$\overline{F}(P_{n+T}P_m)$
3, 2	332	528	410	632
4, 2	840	1268	1760	1820
5, 2	1476	2552	3214	4002
6, 2	2276	4585	4160	6552
7, 2	3332	6448	7034	9830
8, 2	4840	9512	9514	14362

FIGURE 3: Graphical presentations of F -coincides for $P_{n+D}P_m$ graphs such as $H_{1+S}H_2$, $H_{1+R}H_2$, $H_{1+Q}H_2$, and $H_{1+T}H_2$ are represented by blue, orange, gray, and yellow, respectively.

Using equation (8), we directly have

$$\begin{aligned}
\sum B &= 2\overline{e}_2 M_1(H_1) + 4e_1 \overline{M}_1(H_2) + n_1 \overline{F}(H_2) + n_2 (F(H_1) + \overline{F}(H_1) + 2M_1(H_2)(e_1 + \overline{e}_1) \\
&\quad + 4e_2 (M_1(H_1) + \overline{M}_1(H_1)) + 2[(e_2 + \overline{e}_2)(F(H_1) + \overline{F}(H_1)) + (e_1 + \overline{e}_1)(F(H_2) + \overline{F}(H_2))], \\
\sum C &= \sum C_1 + \sum C_2 + \sum C_3, \\
\sum C_1 &= \sum_{\substack{r_1 r_2 \notin E(S(H_1)) \\ r_1 \in V(H_1) r_2 \in V(Q(H_1) - V(H_1))}} \sum_{s \in V_{H_2}} [d(r_1, s)^2 + d(r_2, s)^2] \\
&= \sum_{\substack{r_1 r_2 \notin E(Q(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(S(H_1) - V(H_1))}} \sum_{s \in V_{H_2}} [(d(r_1)^2 + d(r_2)^2) + d(s)^2 + 2d(r_1)d(s)] = n_2 \alpha_1 + e_1 (n_1 - 2) M_1(H_2) + 4e_2 \alpha_3, \\
\sum C_2 &= \sum_{\substack{r_1 r_2 \notin E(Q(H_1)) \\ r_1 \in V(H_1) \\ r_2 \in V(S(H_1) - V(H_1))}} \left(\sum_{s_1, s_2 \in V_{H_2}} + \sum_{s_1, s_2 \notin V_{H_2}} \right) [(d(r_1, s_1)^2 + d(r_2, s_2)^2)] \\
&= \alpha \alpha_1 + e_1 (n_1 - 2) (M_1(H_2) + \overline{M}_1(H_2)) + 4(e_2 + \overline{e}_2) \alpha_3, \\
\sum C_3 &= \sum_{\substack{r_1 r_2 \in E(Q(H_1)) \\ r_1 \in V(H_1) r_2 \in V(S(H_1) - V(H_1))}} \left(\sum_{s_1, s_2 \in V_{H_2}} + \sum_{s_1, s_2 \notin V_{H_2}} \right) [(d(r_1, s_1)^2 + d(r_2, s_2)^2)] \\
&= \alpha \alpha_2 + 2e_1 (M_1(H_2) + \overline{M}_1(H_2)) + 4(e_2 + \overline{e}_2) \alpha_4.
\end{aligned} \tag{15}$$

By substituting the values in equation (13), we get the required result. $\square$

Theorem 4. Let $H_{1+T}H_2$ be a T -sum graph; then, its F -coindex is

$$\begin{aligned}\overline{F}(H_{1+T}H_2) = & n_2\overline{F}(Q'(G_1)) + 2(\overline{e}_2 + (n_2 - 1))(M_1(Q'H_1) + 1)F(Q'(G_1)) \\ & + 2(\overline{e}_2 + (n_2 - 1))\overline{F}(Q'(G_1)) + \overline{e}_2M_1(H_1) + 8e_1\overline{M}_1(H_2) + n_1\overline{F}(H_2) \\ & + 4n_2(\overline{F}(H_1) + 2\overline{e}_1M_1(H_2) + 8e_2\overline{M}_1(H_1) + 2[4(e_2 + \overline{e}_2)(F(H_1) \\ & + \overline{F}(H_1)) + (e_1 + \overline{e}_1)(F(H_2) + \overline{F}(H_2)) + 2(M_1(H_2) + \overline{M}_1(H_2))(M_1(H_1) \\ & + \overline{M}_1(H_1))] + n_2\overline{F}(T(H_1')) + e_1(n_1 - 2)M_1(H_2) + \alpha(\alpha_1 + \alpha_2) \\ & + e_1n_1(M_1(H_2) + \overline{M}_1(H_2)) + 4(e_2 + \overline{e}_2)(2\alpha_3 + \alpha_4).\end{aligned}\quad (16)$$

Proof. It follows from Theorems 2 and 3. $\square$

4. Conclusion

In this study, we have computed forgotten coindex of D -sum graphs such as $\overline{F}(H_{1+S}H_2)$, $\overline{F}(H_{1+R}H_2)$, $\overline{F}(H_{1+Q}H_2)$, and $\overline{F}(H_{1+T}H_2)$ in their general forms. If $H_1 = P_n$ and $H_2 = P_m$ of order $n, m \geq 2$, then

- (i) In this study, we computed the exact values of forgotten coindex of D -sum graphs such as $(H_{1+D}H_2)$, where $D \in \{S, R, Q, T\}$ in the form of

forgotten index, first Zagreb indices, and coincides their basic graphs H_1 and H_2 .

- (ii) We illustrated Theorems 1–4 with the help of some particular path graphs such as $P_{n+S}P_m$, $P_{n+R}P_m$, $P_{n+Q}P_m$, and $P_{n+T}P_m$ for $n, m \geq 2$. Forgotten coindex of D -sum graphs from the paths is also computed as applications of the obtained results.

- (iii) Table 1 and Figure 3 present that forgotten coindex of $(P_{n+T}P_m)$ is dominant than forgotten coindex of $P_{n+S}P_m$, $P_{n+R}P_m$, and $P_{n+Q}P_m$.

$$\begin{aligned}\overline{F}(P_{m+S}P_n) = & 4m(n-1)(m(n-1)-1) + (m-1)(m-2)M(P_n) + 4(n-1)\overline{M}P_m + n\overline{F}(P_m) \\ & + m(F(P_n) + \overline{F}(P_n) + n(n-1)M_1(P_m) + 4(m-1)(M_1(P_n) + \overline{M}_1(P_n)) \\ & + m(m-1)(F(P_n) + \overline{F}(P_n)) + n(n-1)(F(P_m) + \overline{F}(P_m)) + 2(M_1(P_m) + \overline{M}_1(P_m))(M_1(P_n) \\ & + \overline{M}_1(P_n)) + m\overline{F}(S(P'_n)) + (n-1)(n-2)M_1(P_m) + 4(m-1)\overline{B}(S(P'_n)) + \alpha\overline{F}(S(P'_n)) \\ & + (n-1)(n-2)(M_1(P_m) + \overline{M}_1(P_m)) + 2m(m-1)\overline{\beta}(S(P'_n)) + \alpha F(S(P'_n)) \\ & + 2(n-1)(M_1(P_m) + \overline{M}_1(P_m)) + 2m(m-1)\beta(S(P'_n)), \\ \overline{F}(P_{m+R}P_n) = & 4m(n-1)(m(n-1)-1) + 4(m-1)(m-2)M(P_n) + 8(n-1)\overline{M}P_m + n\overline{F}(P_m) \\ & + 4m\overline{F}(P_n + n(n-1)M_1(P_m) + 8(m-1)\overline{M}_1(P_n) + 4m(m-1)(F(P_n) + \overline{F}(P_n)) + n(n-1)(F(P_m) \\ & + \overline{F}(P_m)) + 4(M_1(P_m) + \overline{M}_1(P_m))(M_1(P_n) + \overline{M}_1(P_n)) + m\overline{F}(R(P'_n)) + (n-1)(n-2)M_1(P_m) \\ & + 4(m-1)\overline{\beta}(R(P'_n)) + \alpha\overline{F}(R(P'_n)) + (n-1)(n-2)(M_1(P_m) + \overline{M}_1(P_m)) + 2m(m-1)\overline{\beta}(R(P'_n)) \\ & + \alpha F(R(P'_n)) + 2(n-1)(M_1(P_m) + \overline{M}_1(P_m)) + 2m(m-1)\beta(R(P'_n)), \\ \overline{F}(P_{m+Q}P_n) = & n\overline{F}(Q'P_n) + 2(m+n-2)(M_1(Q'H_1) + \overline{F}(Q'P_n) + F(Q'P_n)) \\ & + (m-1)(m-2)M_{P_n} + 4(n-1)\overline{M}_{P_m} + n\overline{F}(P_m) + m(F(P_n) + \overline{F}(P_n) + n(n-1)M_1(P_m) \\ & + 4(m-1)(M_1(P_n) + \overline{M}_1(P_n)) + m(m-1)(F(P_n) + \overline{F}(P_n)) + n(n-1)(F(P_m) + \overline{F}(P_m))\end{aligned}$$

$$\begin{aligned}
& + 2(M_1(P_m) + \overline{M}_1(P_m))(M_1(P_n) + \overline{M}_1(P_n)) + m\overline{F}(Q(P'_n)) + (n-1)(n-2)M_1(P_m) \\
& + 4(m-1)\overline{\beta}(S(P'_n)) + \alpha\overline{F}(S(Q'_n)) + 2m(m-1)\overline{\beta}(S(Q'_n)) \\
& + (n-1)(n-2)(M_1(P_m) + \overline{M}_1(P_m)) + \alpha F(Q(P'_n)) + 2(n-1)(M_1(P_m) + \overline{M}_1(P_m)) \\
& + 2m(m-1)\beta(Q(P'_n)), \\
\overline{F}(P_{m+T}P_n) = & n\overline{F}(Q'_nP_n) + 2(m+n-2)(M_1(Q'_nH_1) + \overline{F}(Q'_nP_n) + F(Q'_nP_n)) \\
& + 4(m-1)(m-2)M_{P_n} + 8(n-1)\overline{M}_{P_m} + n\overline{F}(P_m) + 4m\overline{F}(P_n + n(n-1)M_1(P_m) + 8(m-1)\overline{M}_1(P_n) \\
& + 4m(m-1)(F(P_n) + \overline{F}(P_n)) + n(n-1)(F(P_m) + \overline{F}(P_m)) + 4(M_1(P_m) + \overline{M}_1(P_m))(M_1(P_n) \\
& + \overline{M}_1(P_n)) + m\overline{F}(T(P'_n)) + (n-1)(n-2)M_1(P_m) + 4(m-1)\overline{\beta}(\beta(P'_n)) + \alpha\overline{F}(T(P'_n)) \\
& + 2m(m-1)\beta(S(P'_n)) + (n-1)(n-2)(M_1(P_m) + \overline{M}_1(P_m)) + 2m(m-1)\overline{\beta}(T(P'_n)) \\
& + \alpha F(T(P'_n)) + 2(n-1)(M_1(P_m) + \overline{M}_1(P_m)).
\end{aligned} \tag{17}$$

Open problem 1. Investigate the existence of the distance-based coindices for the D -sum graphs.

Data Availability

The data used to support the findings of this study are included within the article and are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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