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Retraction

Retracted: On the Edge Metric Dimension of Certain Polyphenyl Chains

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope
(2) Discrepancies in the description of the research reported
(3) Discrepancies between the availability of data and the research described
(4) Inappropriate citations
(5) Incoherent, meaningless and/or irrelevant content included in the article
(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] M. Ahsan, Z. Zahid, D. Alrowaili, A. Iampan, I. Siddique, and S. Zafar, "On the Edge Metric Dimension of Certain Polyphenyl Chains," *Journal of Chemistry*, vol. 2021, Article ID 3855172, 6 pages, 2021.

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Research Article

On the Edge Metric Dimension of Certain Polyphenyl Chains

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The most productive application of graph theory in chemistry is the representation of molecules by the graphs, where vertices and edges of graphs are the atoms and valence bonds between a pair of atoms, respectively. For a vertex w and an edge $f = c_1c_2$ of a connected graph G, the minimum number from distances of w with c_1 and c_2 is called the distance between w and f. If for every two distinct edges $f_1, f_2 \in E(G)$, there always exists $w_1 \in W_E \subseteq V(G)$ such that $d(f_1, w_1) \neq d(f_2, w_1)$, then W_E is named as an edge metric generator. The minimum number of vertices in W_E is known as the edge metric dimension of G. In this paper, we calculate the edge metric dimension of ortho-polyphenyl chain graph O_n, meta-polyphenyl chain graph M_n, and the linear [n]-tetracene graph $T[n]$ and also find the edge metric dimension of para-polyphenyl chain graph L_n. It has been proved that the edge metric dimension of O_n, M_n, and $T[n]$ is bounded, while L_n is unbounded.

1. Introduction and Preliminaries

In chemical graph theory, we use the concepts of graphs to describe the chemical structures. We can present the atomic structure of chemical compounds with the help of graphs. Atoms of molecules are expressed by the vertices of the graph and bonds of atoms are denoted by the edges. Johnson represented the new technique for graphs to show the structural changes in different chemical compounds (see [1]). The idea of metric dimension in graphs was initiated by Slater to find the location of an intruder in a network (see [2]). Harary and Melter further extended the same idea in [3]. Chartrand et al. worked on the resolvability in graphs and studied the application of drug discovery in [4]. Chartrand et al. have studied the application of chemistry by representing the distinct representations of different chemical compounds on labeled graphs (see [5]). Imran et al. discussed the application of plane graphs and calculated the metric dimension of some convex polytopes in [6]. Khuller et al. studied the application of robot navigation by using fix number of landmarks as a basis (see [7]). Caceres et al. discussed the application of games like mastermind and coin weighing and further computed the Cartesian product of graphs in [8]. Melter and Tomescu studied the application of metric dimension in digitizing an image and problems of pattern recognition (see [9]). Hallaway et al. calculated the metric dimension of graph permutations in [10]. Nadeem et al. computed the metric dimension of the ortho-polyphenyl chain, meta-polyphenyl chain, and para-polyphenyl chain graphs in [11]. Soleimani et al. computed the topological indices and polynomials for a family of linear [n]-tetracene graphs in [12]. Moreover, the resolvability of graphs was calculated by Chartrand and Zhang in [13].

Let $G = (V(G), E(G))$ be a simple, connected graph. The total number of edges adjacent to vertex $v_1 \in V(G)$ is $s \in Z^+$, and then s is called degree of v_1. $\Delta(G)$ and $\delta(G)$ denote the maximum and minimum degree of G, respectively. Let x_1 and x_2 be two distinct vertices of G, then $d(x_1, x_2)$ represents the distance between them and it is defined as the number of edges in the shortest path between x_1 and x_2. If

$d(x_1, v_1) \neq d(x_2, v_1)$, then we say that vertex $v_1 \in V(G)$ distinguishes $x_1, x_2 \in V(G)$. If any two vertices of G can be distinguished by some vertex in $W \subseteq V(G)$, then W is called metric generator of G. The cardinality of minimum W is known as the metric dimension for G, denoted by $\dim(G)$.

Kelenc et al. introduced the new invariant of edge metric dimension in [14]. The distance between vertex v_1 and edge $e_1 = x_1x_2$ is given by

$$d(e_1, v_1) = \min\{d(x_1, v_1), d(x_2, v_1)\}. \tag{1}$$

If for every two distinct edges $e_1, e_2 \in E(G)$, there always exists $x \in W_E \subseteq V(G)$ such that $d(e_1, x) \neq d(e_2, x)$, then W_E is named as an edge metric generator. Minimum W_E is known as the edge basis for graph G and the minimum number of vertices in W_E is known as the edge metric dimension denoted by *edim*(G). Here we represent the edge metric dimension by *edim*.

Zubrilina showed that the ratio of *edim* to usual metric is not bounded above (see [15]). Zhang and Gao computed the *edim* of some complex convex polytopes in [16]. Peterin and Yero calculated the *edim* of corona product and lexicographic of graphs in [17]. Kratica et al. worked on the *edim* of generalized Petersen graphs in [18]. Ahsan et al. studied the *edim* of circulant graphs $C_n(1,k)$ for $k = 1$ and 2 (see [19]). Yang et al. calculated the *edim* of some families of wheel-related graphs in [20]. Wei et al. studied the *edim* of some complex convex polytopes in [21]. Deng et al. computed the *edim* of triangular, square, and hexagonal Mobius ladder networks in [22]. Ahmad et al. calculated the *edim* of the benzenoid tripod structure in [23]. Furthermore, Ahsan et al. computed the *edim* of flower graph and prism-related graphs in [24].

The following propositions are helpful throughout this article.

Proposition 1 (see [14]). *For a simple, connected graph G,*

(1) $\mathrm{edim}(G) \geq 1 + \lceil \log_2 \delta(G) \rceil$

(2) $\mathrm{edim}(G) \geq \lceil \log_2 \Delta(G) \rceil$

Ortho-polyphenyl chain graph O_n, meta-polyphenyl chain graph M_n, and para-polyphenyl chain graph L_n under topological indices have been discussed in [25]. In the present paper, we shall discuss these polyphenyl chains under the edge metric invariant.

The rest of the paper is explicit as follows. The *edim* of ortho-polyphenyl chain graph O_n, meta-polyphenyl chain graph M_n, the linear [n]-tetracene graph $T[n]$, and the para-polyphenyl chain graph L_n are calculated in Sections 2, 3, 4, and 5, respectively. In the last section, the conclusion of the article is stated.

2. Edge Metric Dimension of Ortho-Polyphenyl Chain O_n

In this section, we will find the *edim*(O_n). The graph O_n has $V(O_n) = \{w_i, v_i, u_i\colon 1 \leq i \leq 2n\}$ and $E(O_n) = \{v_{2i+1}w_{2i+1}, w_{2i+1}w_{2i+2}, w_{2i+2}v_{2i+2}, v_{2i+2}u_{2i+2}, u_{2i+1}u_{2i+2}, v_{2i+1}u_{2i+1}, u_{2j}u_{2j+1} : 0 \leq i \leq n-1, 1 \leq j \leq n-1\}$. The graph O_n for $n = 4$ is shown in Figure 1.

Now, we will find the edge dimension of ortho-polyphenyl chain O_n.

Theorem 2. *For* $n \geq 2$, $\mathrm{edim}(O_n)$ *is 2.*

Proof. Let $W_E = \{v_1, u_{2n}\} \subset V(O_n)$, we will prove that W_E is an edge basis of O_n. For this, each edge of O_n is represented in the following:

$$r(v_{2i+1}w_{2i+1}|W_E) = \begin{cases} (0, 2n), & i = 0 \\ (2+2i, 2n-2i), & \text{if } n-1 \geq i \geq 1 \end{cases}$$

$$r(w_{2i+1}w_{2i+2}|W_E) = \begin{cases} (1, 2n), & i = 0 \\ (2i+3, 2n-2i), & \text{if } n-1 \geq i \geq 1 \end{cases}$$

$$r(w_{2i+2}v_{2i+2}|W_E) = \begin{cases} (2, -1+2n), & i = 0 \\ (3+2i, -2i-1+2n), & \text{if } n-1 \geq i \geq 1 \end{cases}$$

$$r(v_{2i+1}u_{2i+1}|W_E) = \begin{cases} (0, 2n-1), & i = 0 \\ (2i+1, -1-2i+2n), & \text{if } n-1 \geq i \geq 1 \end{cases}$$

$$r(u_{2i+1}u_{2i+2}|W_E) = (2i+1, 2n-2i-2) \text{ for } n-1 \geq i \geq 0$$

$$r(v_{2i+2}u_{2i+2}|W_E) = (2i+2, 2n-2i-2) \text{ for } n-1 \geq i \geq 0$$

$$r(u_{2i}u_{2i+1}|W_E) = (2i, -1+2n-2i) \text{ for } n-1 \geq i \geq 0$$

We see that no two tuples have the same representations. This proves that $\mathrm{edim}(O_n) \leq 2$. Since by Proposition 1, $\mathrm{edim}(O_n) \geq 2$. Hence, $\mathrm{edim}(O_n) = 2$. □

3. Edge Metric Dimension of Meta-Polyphenyl Chain M_n

In this section, we will find the $\mathrm{edim}(M_n)$. The graph M_n has $V(M_n) = \{w_i, v_i, u_i\colon 1 \leq i \leq 2n\}$ and $E(M_n) = \{u_{2i+2}w_{2i+2}, u_{2i+1}w_{2i+2}, v_{2i+1}u_{2i+1}, v_{2i+1}w_{2i+1}, w_{2i+1}v_{2i+2}, v_{2i+2}u_{2i+2}, u_{2j+2}u_{2j+3}\colon 0 \leq i \leq n-1, 0 \leq j \leq n-2\}$. The graph M_n for $n = 5$ is shown in Figure 2.

Now, we will find the edge dimension of meta-polyphenyl chain M_n.

Theorem 3. *For* $n \geq 2$, $\mathrm{edim}(M_n)$ *is 2.*

Proof. Let $W_E = \{w_2, u_{2n}\} \subset V(M_n)$, we will prove that W_E is an edge basis of M_n. For this, each edge of M_n is represented in the following:

$$r(u_{2i+2}w_{2i+2}|W_E) = \begin{cases} (3i, 3n-3i-4), & 0 \leq i \leq n-2 \\ (3n-3, 0), & \text{if } i = n-1 \end{cases}$$

$$r(u_{2i+1}w_{2i+2}|W_E) = \begin{cases} (0, 3n-3), & i = 0 \\ (3i-1, 3n-3i-3), & \text{if } 1 \leq i \leq n-1 \end{cases}$$

$$r(v_{2i+1}u_{2i+1}|W_E) = \begin{cases} (1, n-2), & i = 0 \\ (3i-1, 3n-3i-2), & \text{if } 1 \leq i \leq n-1 \end{cases}$$

$$r(v_{2i+1}w_{2i+1}|W_E) = \begin{cases} (2, -2+3n), & \text{if } i = 0 \\ (3i, 3n-3i-3), & \text{if } 1 \leq i \leq n-1 \end{cases}$$

$$r(w_{2i+1}v_{2i+2}|W_E) = \begin{cases} (2, 3n-3), & \text{if } i = 0 \\ (3i+1, 3n-3i-3), & \text{if } 1 \leq i \leq n-2 \\ (3n-2, 2), & \text{if } i = n-1 \end{cases}$$

$$r(v_{2i+2}u_{2i+2}|W_E) = \begin{cases} (3i+1, 3n-3i-4), & 0 \leq i \leq n-2 \\ (-2+3n, 1), & \text{if } i = n-1 \end{cases}$$

$$r(u_{2i+2}u_{2i+3}|W_E) = (3i+1, 3n-3i-5) \text{ for } 0 \leq i \leq n-2$$

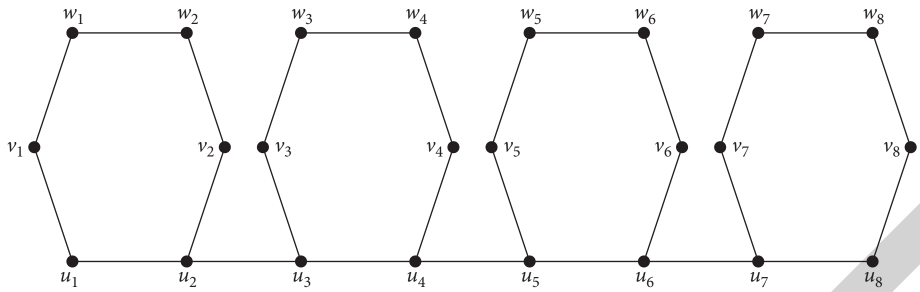

Figure 1: Graph of ortho-polyphenyl chain O_4.

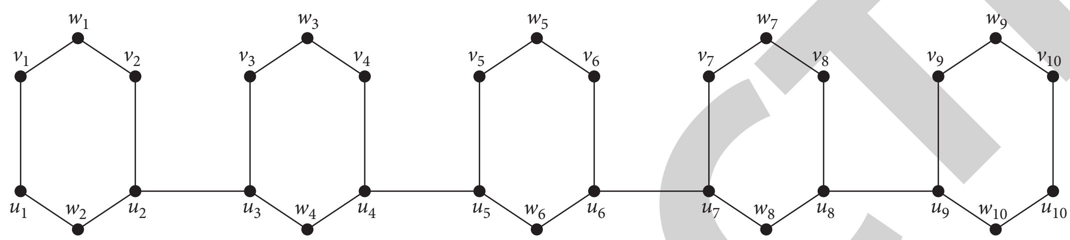

Figure 2: Graph of meta-polyphenyl chain M_5.

We see that no two tuples have the same representations. This proves that $\operatorname{edim}(M_n) \le 2$. Since by Proposition 1, $\operatorname{edim}(M_n) \ge 2$. Hence, $\operatorname{edim}(M_n) = 2$. □

4. Edge Metric Dimension of the Linear [n]-Tetracene

In this section, we will find the $e\dim(T[n])$. The graph $T[n]$ has $V(T[n]) = \{v_i, u_i, w_j\colon 1 \le i \le 5n, 1 \le j \le 8n\}$ and $E(T[n]) = \left\{\begin{array}{c} v_{5i+1}w_{8i+1}, v_{5i+2}w_{8i+3}, v_{5i+3}w_{8i+5}, v_{5i+4}w_{8i+7}, w_{8i+}1v_{5i+2}, w_{8i+3}v_{5i+3}, \\ w_{8i+5}v_{5i+4}, w_{8i+7}v_{5i+5}, v_{5i+2}u_{5i+2}, v_{5i+3}u_{5i+3}, v_{5i+4}u_{5i+4}, v_{5i+5}u_{5i+5}, u_{5i+2} \\ w8i+2, u_{5i+3}w_{8i+4}, u_{5i+4}w_{8i+6}, u_{5i+5}w_{8i+8}, u_{5i+1}w_{8i+2}, u_{5i+2}w_{8i+4}, u_{5i+3}w_{8i+6}, u_{5i+4}w_{8i+8}, v_{5i+1}u_{5i+1}, v_{5j}v_{5j+1}, u_{5j}u_{5j+1}\colon 0 \le i \le n-1, 1 \le j \le n-1 \end{array}\right\}$. The graph $T[n]$ for $n = 3$ is shown in Figure 3.

We will find the edge dimension of linear $[n]$-tetracene $T[n]$.

Theorem 4. *For $n \ge 2$, $edim(T[n])$ is 2.*

Proof. Let $W_E = \{v_1, v_{5n}\} \subset V(T[n])$, we have to prove that W_E is an edge basis of $T[n]$. For this, each edge of $T[n]$ is represented in the following:

$$r(v_{5i+1}w_{8i+1}|W_E) = (9i, 9n-9i-2) \text{ for } 0 \le i \le n-1$$
$$r(v_{5i+2}w_{8i+3}|W_E) = (9i+2, 9n-9i-4) \text{ for } 0 \le i \le n-1$$
$$r(v_{5i+3}w_{8i+5}|W_E) = (9i+4, 9n-9i-6) \text{ for } 0 \le i \le n-1$$
$$r(v_{5i+4}w_{8i+7}|W_E) = (9i+6, 9n-9i-8) \text{ for } 0 \le i \le n-1$$
$$r(w_{8i+1}v_{5i+2}|W_E) = (9i+1, 9n-9i-3) \text{ for } 0 \le i \le n-1$$
$$r(w_{8i+3}v_{5i+3}|W_E) = (9i+3, 9n-9i-5) \text{ for } 0 \le i \le n-1$$
$$r(w_{8i+5}v_{5i+4}|W_E) = (9i+5, 9n-9i-7) \text{ for } 0 \le i \le n-1$$
$$r(w_{8i+7}v_{5i+5}|W_E) = (9i+7, 9n-9i-9) \text{ for } 0 \le i \le n-1$$
$$r(v_{5i+2}u_{5i+2}|W_E) = (9i+2, 9n-9i-3) \text{ for } 0 \le i \le n-1$$
$$r(v_{5i+3}u_{5i+3}|W_E) = (9i+4, 9n-9i-5) \text{ for } 0 \le i \le n-1$$
$$r(v_{5i+4}u_{5i+4}|W_E) = (9i+6, 9n-9i-7) \text{ for } 0 \le i \le n-1$$
$$r(v_{5i+5}u_{5i+5}|W_E) = (9i+8, 9n-9i-9) \text{ for } 0 \le i \le n-1$$
$$r(u_{5i+2}w_{8i+2}|W_E) = (9i+2, 9n-9i-2) \text{ for } 0 \le i \le n-1$$
$$r(u_{5i+3}w_{8i+4}|W_E) = (9i+4, 9n-9i-4) \text{ for } 0 \le i \le n-1$$
$$r(u_{5i+4}w_{8i+6}|W_E) = (9i+6, 9n-9i-6) \text{ for } 0 \le i \le n-1$$
$$r(u_{5i+5}w_{8i+8}|W_E) = (9i+8, 9n-9i-8) \text{ for } 0 \le i \le n-1$$
$$r(u_{5i+1}w_{8i+2}|W_E) = (9i+1, 9n-9i-1) \text{ for } 0 \le i \le n-1$$
$$r(u_{5i+2}w_{8i+4}|W_E) = (9i+3, 9n-9i-3) \text{ for } 0 \le i \le n-1$$
$$r(u_{5i+3}w_{8i+6}|W_E) = (9i+5, 9n-9i-5) \text{ for } 0 \le i \le n-1$$
$$r(u_{5i+4}w_{8i+8}|W_E) = (9i+7, 9n-9i-7) \text{ for } 0 \le i \le n-1$$
$$r(v_{5i+1}u_{5i+1}|W_E) = (9i, 9n-9i-1) \text{ for } 0 \le i \le n-1$$
$$r(v_{5i}v_{5i+1}|W_E) = (9i-1, 9n-9i-1) \text{ for } 1 \le i \le n-1;$$
$$r(u_{5i}u_{5i+1}|W_E) = (9i, 9n-9i) \text{ for } 1 \le i \le n-1$$

Since every two tuples have different representations. This proves that $\operatorname{edim}(T[n]) \le 2$. Since by Proposition 1, $\operatorname{edim}(T[n]) \ge 2$. Hence, $\operatorname{edim}(T[n]) = 2$. □

5. Edge Metric Dimension of Para-Polyphenyl Chain L_n

In this section, we will find the $e\dim(L_n)$. The graph L_n has $V(L_n) = \{v_i, w_i, u_i\colon 1 \le i \le 2n\}$ and $E(L_n) = \{w_{2i+1}v_{2i+1}, v_{2i+1}v_{2i+2}, v_{2i+2}\ w_{2i+2}, w_{2i+2}u_{2i+2}, u_{2i+1}u_{2i+2}, u_{2i+1}w_{2i+1}, w_{2j}\ w_{2j+1}\colon 0 \le i \le n-1, 1 \le j \le n-1\}$. The graph L_n for $n = 6$ is shown in Figure 4.

Now, we will find the edge dimension of para-polyphenyl chain L_n.

Lemma 5. *Let $Y = \{v_1, v_2, v_3, \ldots, v_{2n}\} \subseteq V(L_n)$. Then any edge metric generator W_E of L_n has at least n vertices of Y.*

Proof. Suppose on contrarily that W_E has at most $n-1$ vertices of Y. Without loss of generality for $1 \le k \le n$, we assume that $v_{2k-1}, v_{2k} \notin W_E$, and then we have $(v_{2k-1}v_{2k}|W_E) = (u_{2k-1}u_{2k}|W_E)$,

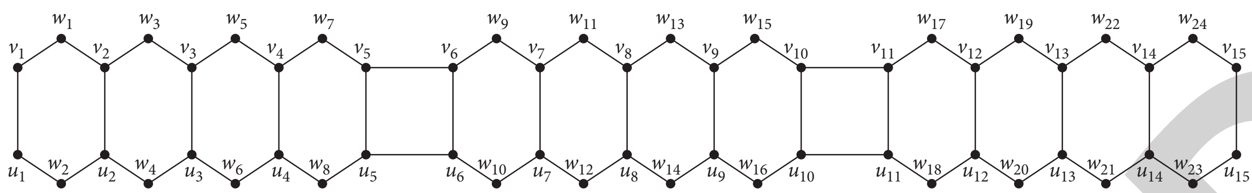

Figure 3: Graph of the linear [3]-tetracene.

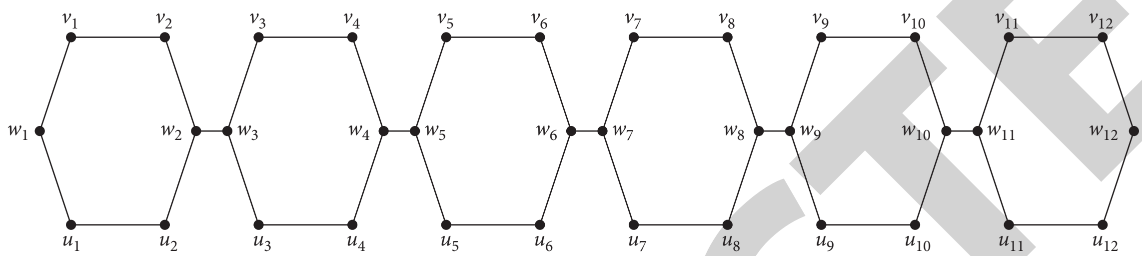

Figure 4: Graph of para-polyphenyl chain L_6.

$(v_{2k-1}w_{2k-1}|W_E) = (u_{2k-1}w_{2k-1}|W_E)$, and $(v_{2k}w_{2k}|W_E) = (u_{2k}w_{2k}|W_E)$, so we get a contradiction. □

Remark 5. Let W_E is an edge basis of L_n. For all n, W_E contains all vertices of Y having vertex indices odd.

Theorem 5. *For $n \geq 2$, we have*

$$edim(L_n) = n. \tag{2}$$

Proof. Let $W_E = \{v_1, v_3, v_5, \ldots, v_{2n-1}\}$. We will prove that W_E is an edge basis of L_n.

For $3 \leq k \leq n$, let $W_1 = \{v_1, v_3, v_{2k-1}\}$. Now, each edge representation of L_n with respect to W_1 is given in the following:

$$r(w_{2i+1}v_{2i+1}|W_1) = \begin{cases} (0, 4, 4k-4) & i = 0, \\ (3, 0, 4k-8) & i = 1, \\ (4i-1, 4i-5, 4k-4i-4) & 2 \leq i \leq k-1, \\ (4k-1, 4k-5, 3) & i = k, \\ (4i-1, 4i-5, 4i-4k+3) & k+1 \leq i \leq n-1, \end{cases}$$

$$r(v_{2i+1}v_{2i+2}|W_1) = \begin{cases} (0, 3, 4k-5) & i = 0, \\ (4i, 4i-4, 4k-4i-5) & 1 \leq i \leq k-2, \\ (4k-4, 4k-8, 0) & i = k-1, \\ (4i, 4i-4, 4i-4k+4) & k \leq i \leq n-1. \end{cases}$$

$$r(v_{2i+2}w_{2i+2}|W_1) = \begin{cases} (1, 2, -6+4k) & i = 0, \\ (4i+1, -3+4i, -6+4k-4i) & 1 \leq i \leq k-2, \\ (1+4i, 4i-3, 4i+5-4k) & k-1 \leq i \leq n-1. \end{cases}$$

$$r(w_{2i+2}u_{2i+2}|W_1) = \begin{cases} (2, 2, -6+4k) & i = 0, \\ (2+4i, 4i-2, 4k-4i-6) & 1 \leq i \leq k-2, \\ (4i+2, 4i-2, 4i-4k+6) & k-1 \leq i \leq n-1. \end{cases}$$

$$
r(u_{2i+1}u_{2i+2}|W_1) = \begin{cases} (2, 3, 4k-5) & i = 0, \\ (4, 2, 4k-9) & i = 1, \\ (4i, 4i-4, 4k-4i-5) & 2 \le i \le k-2, \\ (-4+4k, 4k-8, 2) & i = k-1, \\ (4i, 4i-4, 4i+4-4k) & k \le i \le n-1. \end{cases}
$$

$$
r(u_{2i+1}w_{2i+1}|W_1) = \begin{cases} (1, 4, 4k-4) & i = 0, \\ (3, 1, -8+4k) & i = 1, \\ (-1+4i, 4i-5, 4k-4i-4) & 2 \le i \le k-2, \\ (4k-5, 4k-9, 1) & i = k-1, \\ (-1+4i, 4i-5, 4i-4k+3) & k \le i \le n-1. \end{cases} \tag{3}
$$

$$
r(w_{2i}w_{2i+1}|W_1) = \begin{cases} (2, 1, -7+4k) & i = 1, \\ (4i-2, -8+4i, -3+4k-4i) & 2 \le i \le k-1, \\ (-2+4i, 4i-6, 4i+2-4k) & k \le i \le n-1. \end{cases}
$$

From above representation we see that $(v_{2i-1}v_{2i}|W_1) = (u_{2i-1}u_{2i}|W_1)$, $(v_{2i-1}w_{2i-1}|W_1) = (u_{2i-1}w_{2i-1}|W_1)$ and $(v_{2i}w_{2i}|W_1) = (u_{2i}w_{2i}|W_1)$ when $1 \le i \le n$ and $i \ne 1, 2, k$ and no other edges have same representation. If we take $1 \le i \le n$ and $i \ne 1, 2, k$ such that $(v_{2i-1}v_{2i}|W_1 \cup \{v_{2i-1}\}) \ne (u_{2i-1}u_{2i}|W_{11} \cup \{v_{2i-1}\})$, $(v_{2i-1}w_{2i-1}|W_1 \cup \{v_{2i-1}\}) \ne (u_{2i-1}w_{2i-1}|W_1 \cup \{v_{2i-1}\})$ and $(v_{2i}w_{2i}|W_1 \cup \{v_{2i-1}\}) \ne (u_{2i}w_{2i}|W_1 \cup \{v_{2i-1}\})$. It follows that $(v_{2i-1}v_{2i}|W_E) \ne (u_{2i-1}u_{2i}|W_E)$, $(v_{2i-1}w_{2i-1}|W_E) \ne (u_{2i-1}w_{2i-1}|W_E)$, and $(v_{2i}w_{2i}|W_E) \ne (u_{2i}w_{2i}|W_E)$ for $1 \le i \le n$. So from Lemma 5 and Remark 5, W_E is an edge basis for L_n and $e\dim(L_n) = n$. □

6. Conclusion

In this paper, we have calculated the *edim* of ortho-polyphenyl chain graph O_n, meta-polyphenyl chain graph M_n, linear [n]-tetracene graph $T[n]$, and the para-polyphenyl chain graph L_n. It has been proved that the *edim* of these polyphenyl chain graphs is constant while the para-polyphenyl chain graph L_n has unbounded.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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