Research Article

Comparing Zinc Oxide- and Zinc Silicate-Related Metal-Organic Networks via Connection-Based Zagreb Indices

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Metal-organic networks (MONs) are among the unique complex and porous chemical compounds. So, these chemical compounds consist of metal ions (vertices) and organic ligands (edges between vertices). These networks represent large pore volume, extreme surface area, morphology, excellent chemical stability, highly porous and crystalline materials, and octahedral clusters. MONs are mostly used in assessment of chemicals, gas and energy storage devices, sensing, separation and purification of different gases, heterogeneous catalysis, environmental hazard, toxicology, adsorption analysis, biomedical applications, and biocompatibility. Recently, drug delivery, cancer imaging, and biosensing have been investigated by biomedical applications of zinc-related MONs. The versatile applications of these MONs make them helpful tools in many fields of science in recent decade. In this paper, we discuss the two different zinc oxide and zinc silicate related MONs according to the number of increasing layers of metal and organic ligands together. We also compute the connection-based Zagreb indices such as first Zagreb connection index (ZCI), second ZCI, modified first ZCI, modified second ZCI, modified third ZCI, and modified fourth ZCI. Moreover, a comparison is also included between the zinc-related MONs by using numerical values of connection-based Zagreb indices. Finally, we conclude that zinc silicate-related MON is better than zinc oxide-related MON for all values of n .

1. Introduction

There are so many chemical compounds in the field of chemistry. One of the most popular recent chemical compounds is metal-organic network (MON) which consists of metal ions and organic ligands. A new MON with zinc as the metal ion and benzene-1,3-dicarboxylic acid as the organic ligand (linker) has been synthesized with the help of hydrothermal method. This MON is also used as a selective nano-adsorbent for the preconcentration and extraction of trace amounts of cadmium with the help of solid-phase extraction method. A category of crystalline and porous materials is porous coordination polymers which are newly known as MONs [1]. One of the most important aspects which can be considered in the matter of MONs in bioapplications is their surface modification and

control of their particle size distribution [2]. Zn-related MONs as chemical sensors could be converted into devices for luminescent characteristics [3]. The electron-rich T-conjugated fluorescent ligands are friendly to make Zn-MONs with nucleophilic properties in efficient luminescent sensors [4].

The low toxicity of zinc ions as the desirable character is considered to introduce Zn-related MONs into bio-application domains, especially drug carriers. The anti-bacterial activity has been validated by combining different antibiotic drugs and metals [5]. In reality, Zn^{2+} is an endogenous low-toxic transition metal cation which is widely used in dermatology as a cicatrizing agent and skin moisturizer with astringent, antidandruff, antibacterial, and anti-inflammatory agents [6]. In nonlinear optically active MONs, Zn^{2+} is commonly used as a connecting point

to prevail undesired d-d transitions in the visible region. Moreover, the toxicology, biomedical applications, and their biocompatibility are recently reported production procedures of zinc-related MONs. For more details, refer to [7].

Eddaoudi et al. [8] synchronized the isorecticular series (IRMOF-1 to IRMOF-16) of 16 highly crystalline materials. The free and fixed diameter of pores from IRMOF-1 to IRMOF-16 varies in the range of 3.8–19.1 and 12.8–28.8, respectively. The design of an IRMOF-10 series based on MON-5 was initiated by determining the reaction conditions necessary to produce the octahedral cluster with a ditopic linear carboxylate. Therefore, many IRMOF structures can be developed by using zinc oxide octahedral clusters ($\text{Zn}_4\text{O}(\text{CO})_2$) as the metal corners linked via diverse organic dicarboxylate linkers and different three-dimensional cubic networks are formed. For more information, see [9]. All the IRMOFs have the expected topology of CaB_6 (13) and happened through the prototype IRMOF-1 in which an oxide-centered Zn_4O tetrahedron is edge bridged. Some IRMOFs such as IRMOF-8, IRMOF-10, IRMOF-12, and IRMOF-16 have been seen in noncrystalline porous systems for SiO_2 xerogels and aerogels (16). For further investigation, see [7, 10–12].

MONs also predict the physicochemical properties such as grafting active groups [13], impregnating suitable active material [14], ion exchange [15], preparing composites with different substances [11], changing organic ligands and postsynthetic ligands [16], and biosensors enhancing sensitivity, response time, and selectivity [17]. Yap et al. [18] and Lin et al. [19] presented the recent progress in precursors on the preparation of several nanostructures and MON-related applications such as sensing, photocatalysis, electrocatalysis, supercapacitors, catalyst for production of fine chemicals, and lithium ion batteries. Graph theory provides useful tools in the field of modern chemistry which represent the chemical and physical properties of chemical compounds such as heat of formation, heat of evaporation, flash point, melting point, boiling point, temperature, pressure, density, retention in chromatography, and tension and partition coefficient [20–22]. First, distance-based topological index (TI) was discovered by Wiener to study the different properties of chemical compounds (boiling point of paraffin) in 1947 [23]. The very well-reputed first-degree-based TI was discovered by Gutman and Trinajstić to check the chemical physibility on the total π -electron energy of the chemical compounds (alternant hydrocarbons) in 1972 [24].

Recently, Zhao et al. [25] introduced two connection number (number of vertices at distance two) based TIs to compute the general results for modified Zagreb connection indices of subdivision and semitotal point operations on graphs. Nowadays, these degree and connection number-based TIs are abundantly used in the topological properties of four-layered neural networks and MONs [26, 27]. Ali et al. [28] computed connection-based indices and coindices for the product of molecular networks. Gutman and Furtula

discussed various topological properties of different molecular structures; see [29–31]. Ali and Trinajstić [32] and Javaid et al. [33] computed different connection-based TIs of graphs under different operations. Moreover, a variety of networks has been defined with the help of connection number-based TIs [34–37].

In this paper, we compute the connection-based Zagreb indices of two different zinc-related MONs such as zinc oxide (ZNOX (n) = IRMOF-10) and zinc silicate (ZNCL (n) = IRMOF-14) networks with respect to the increasing layers, $n \geq 3$, taking both metal nodes and linkers together. The rest of the paper is organized as follows. Section 2 provides the preliminaries, definitions, and some important results which can be used in the main results. Sections 3 and 4 provides the main results for zinc oxide and zinc silicate networks, and Section 5 provides comparisons and conclusions.

2. Preliminaries

The vertex and edge sets are $V(G)$ and $E(G)$ for a simple and connected network G . $|V(G)|$ and $|E(G)|$ are the cardinalities of vertex set and edge set which are equal to u and e , respectively. In a connected network, there is a path between two vertices. The distance between two vertices p and q is the shortest path between them. It is denoted by $d_G(p, q)$. In general [37], $N_G(q/m) = \{p \in V(G); d(p, q) = m\}$ is the open m -neighborhood set of q , where m represents a positive integer and $|N_G(q/m)| = d_G(q/m)$ is called m -distance degree of a vertex q . In particular,

- (i) If $m = 1$, $d_G(q/1) = |N_G(q/1)| = d_G(q) = \text{degree of } q$ (number of vertices at distance one from q)
- (ii) If $m = 2$, $d_G(q/2) = |N_G(q/2)| = \tau_G(q) = \text{connection number of } q$ (number of vertices at distance two from q)

For more terminologies and notations, see [36] and references therein.

Definition 1. For a (molecular) network G , the first Zagreb index ($M_1(G)$), second Zagreb index ($M_2(G)$), and third Zagreb index ($M_3(G)$) are defined as follows:

- (a) $M_1(G) = \sum_{q \in V(G)} [d_G(q)]^2 = \sum_{pq \in E(G)} [d_G(p) + d_G(q)]$
- (b) $M_2(G) = \sum_{pq \in E(G)} [d_G(p) \times d_G(q)]$
- (c) $M_3(G) = \sum_{q \in V(G)} [d_G(q)]^3 = \sum_{pq \in E(G)} [d_G^2(p) + d_G^2(q)]$

These degree-based TIs are defined by Gutman and Trinajstić [24]. These are abundantly used to predict better findings in molecular networks such as ZE isomerism, absolute value of correlation coefficient, entropy, acentric factor, and heat capacity.

Definition 2. For a (molecular) network G , the first ZCI ($ZC_1(G)$), second ZCI ($ZC_2(G)$), and modified first ZCI ($ZC_1^+(G)$) are defined as follows:

- (a) $ZC_1(G) = \sum_{q \in V(G)} [\tau_G(q)]^2$
 (b) $ZC_2(G) = \sum_{pq \in E(G)} [\tau_G(p) \times \tau_G(q)]$
 (c) $ZC_1^*(G) = \sum_{(pq \in E(G))} [\tau_G(p) + \tau_G(q)]$

These connection-based TIs are defined by Ali and Trinajstić [32] (2018). They also reported that the modified first Zagreb connection index has better correlation coefficient value for the thirteen physicochemical properties of octane isomers than classical Zagreb indices.

Definition 3. For a (molecular) network G , the modified second Zagreb connection index ($ZC_2^*(G)$) and modified third Zagreb connection index ($ZC_3^*(G)$) are defined as follows:

- (a) $ZC_2^*(G) = \sum_{pq \in E(G)} [d_G(p)\tau_G(q) + d_G(q)\tau_G(p)]$
 (b) $ZC_3^*(G) = \sum_{pq \in E(G)} [d_G(p)\tau_G(p) + d_G(q)\tau_G(q)]$

Definition 4. For a (molecular) network G , the modified fourth Zagreb connection index ($ZC_4^*(G)$) is defined as follows:

$$ZC_4^*(G) = \sum_{pq \in E(G)} [d_G(p)\tau_G(p) \times d_G(q)\tau_G(q)]. \quad (1)$$

These connection-based TIs are defined by Javaid et al. [35] to compute the exact solutions of several wheel-related graphs.

Definition 5. Zinc oxide network ($ZNOX(n)$): a chemical compound zinc oxide (ZnO) is insoluble in water which is an inorganic compound of white powder shape with 5.61 g/cm^3 density. The zinc oxide is heated with carbon (coke) that reduces to the metal vapor to condense the liquid from which the solid metal freezes.

Zinc is a reactive metal to produce zinc ion (Zn^{2+}) and hydrogen gas. It also reduces those metal ions whose reduction potentials are higher than Zn^{2+} . Zinc oxide is mostly used in making rubber, enamels, glazes, pigment in white paint, photoconductive surfaces, and protective coating for other metals. Zinc oxide-related MON is $Zn_4O(BPDC)_3$, which is also known as IRMOF-10. IRMOF-9 is a catenated version of IRMOF-10. IRMOF-10 is three-dimensional cubic structures with a pore size diameter of $16.7/20.2 \text{ Å}^0$. In Figure 1, the zinc oxide-related MON of dimension 3 is presented. In general, the vertices and edges in $ZNOX(n)$ of dimension n are $70n + 46$ and $85n + 55$, respectively. For more understanding, see Figure 1.

Definition 6. Zinc silicate network ($ZNSL(n)$): silicate (SiO_4) is the most charming class of minerals. Silicate is the

FIGURE 1: Zinc oxide network ($ZNOX(n) \cong H$) for $n = 3$.

chemical mixture of metal oxide or metal carbonate with sand. The basic unit of silicate is tetrahedron. So, all silicates gain SiO_4 tetrahedron. In chemistry, oxygen ions and silicon ions are represented by the corner vertices and centre vertices of SiO_4 , respectively. In graph theory, we represent corner vertices and centre vertices of SiO_4 with oxygen nodes and silicon nodes. If we require a variety of silicate networks, it is easy to change the arrangement of the tetrahedron silicate. Zinc silicate-related MON is $Zn_4O(PDC)_3$, which is also known as IRMOF-14. IRMOF-14 is three-dimensional cubic structures with a pore size diameter of $14.7/20.1 \text{ Å}^0$. In Figure 2, the zinc silicate-related MON of dimension 3 is presented. In general, the vertices and edges in $ZNSL(n)$ of dimension n are $82n + 50$ and $103n + 61$, respectively. For more understanding, see Figure 2.

Now, we present some important results which are used in the main results.

Lemma 1. Let G be a connected network with u vertices and e edges. Then, $\tau_G(p) + d_G(p) \leq \sum_{q \in N_G(p)} d_G(q)$, where equality holds if and only if G is a $\{C_3, C_4\}$ -free network.

Lemma 2 (see [35]). Let G be a connected and $\{C_3, C_4\}$ -free network with u vertices and e edges. Then,

- (i) $\sum_{q \in V(G)} d_G(q) = 2e$
 (ii) $\sum_{q \in V(G)} \tau_G(q) = M_1(G) - 2e$

Lemma 3 (see [25]). Let G be a connected and $\{C_3, C_4\}$ -free network with u vertices and e edges. Also, $G \cong P_u$. Then

- (i) $ZC_2^*(G) = 8u - 22$ if $u \geq 4$
 (ii) $ZC_3^*(G) = 8u - 22$ if $u \geq 3$

FIGURE 2: Zinc silicate network (ZNSL(n) $\cong$ K) for $n = 3$.

3. Main Results Based on Zinc Oxide Network (ZNOX (n))

In this section, we compute the main results for first Zagreb connection index (ZCI), second ZCI, modified first ZCI, modified second ZCI, modified third ZCI, and modified fourth ZCI of zinc oxide-related MON (ZNOX (n)). Let $H \cong$ ZNOX(n) be the zinc oxide network of dimension n in the plane, see Figure 1. The partitions of H with respect to the vertex set and edge set are $V(H)$ and $E(H)$. We can easily see each vertex of degrees 2, 3, and 4. We have $V_1 = \{v \in V(H) | d_v = 2\}$, $V_2 = \{v \in V(H) | d_v = 3\}$, and $V_3 = \{v \in V(H) | d_v = 4\}$, where $|V_1| = 42n + 30$, $|V_2| = 26n + 14$, and $|V_3| = 2n + 2$. So, $|V(H)| = v = |V_1| + |V_2| + |V_3| = 70n + 46$. Now, the partitions of vertices according to connection number are $V_1 = \{v \in V(H) | \tau_v = 2\}$, $V_2 = \{v \in V(H) | \tau_v = 3\}$, $V_3 = \{v \in V(H) | \tau_v = 4\}$, $V_4 = \{v \in V(H) | \tau_v = 5\}$, and $V_5 = \{v \in V(H) | \tau_v = 8\}$, where $|V_1| = 2n + 6$, $|V_2| = 28n + 20$, $|V_3| = 30n + 10$, $|V_4| = 8n + 8$, and $|V_5| = 2n + 2$. So, $|V(H)| = v = |V_1| + |V_2| + |V_3| + |V_4| + |V_5| = 70n + 46$. These vertex partitions are shown in Tables 1 and 2.

There are four types of partitions of edge sets of H according to the degree as $|E(H)| = |E_{2,2}^d| + |E_{2,3}^d| + |E_{3,3}^d| + |E_{3,4}^d| = 85n + 55$, and there are seven types of partitions of edge sets of H according to the connection number of vertices as $|E(H)| = |E_{2,3}^c| + |E_{3,3}^c| + |E_{3,4}^c| + |E_{3,5}^c| + |E_{4,4}^c| + |E_{4,5}^c| + |E_{5,8}^c| = 85n + 55$. These edge partitions are shown in Tables 3 and 4.

TABLE 1: Partitions of H 's vertices according to the degree.

d_v	2	3	4
$ d_v $	$42n + 30$	$26n + 14$	$2n + 2$

TABLE 2: Partitions of H 's vertices according to the connection number.

τ_v	2	3	4	5	8
$ \tau_v $	$2n + 6$	$28n + 20$	$30n + 10$	$8n + 8$	$2n + 2$

TABLE 3: Partitions of H 's edges according to the degree.

$E_{d(p),d(q)}^d$	$E_{2,2}^d$	$E_{2,3}^d$	$E_{3,3}^d$	$E_{3,4}^d$
$ E_{d(p),d(q)}^d $	$6n + 16$	$52n + 28$	$9n + 3$	$8n + 8$

Theorem 1. Let $H \cong$ ZNOX(n) be a zinc oxide network of dimensions $n \geq 3$. Then, the first Zagreb connection index is

$$ZC_1(H) = 1068n + 692. \quad (3)$$

Proof. By definition,

$$\begin{aligned}
 ZC_1(G) &= \sum_{q \in V(G)} [\tau_G(q)]^2 \\
 &= (2n + 6)(2)^2 + (28n + 20)(3)^2 + (30n + 10)(4)^2 + (8n + 8)(5)^2 + (2n + 2)(4)^2 \\
 &= 8n + 24 + 252n + 180 + 480n + 160 + 200n + 200 + 128n + 128 \\
 &= 1068n + 692.
 \end{aligned} \quad (4)$$

Theorem 2. Let $H \cong$ ZNOX(n) be a zinc oxide network of dimensions $n \geq 3$. Then, the second Zagreb connection index is

$$ZC_2(H) = 1376n + 896. \quad (5)$$

□

TABLE 4: Partition of H 's edges according to the connection number.

$E_{\tau(p),\tau(q)}^c$	$E_{2,3}^c$	$E_{3,3}^c$	$E_{3,4}^c$	$E_{3,5}^c$	$E_{4,4}^c$	$E_{4,5}^c$	$E_{5,8}^c$
$ E_{\tau(p),\tau(q)}^c $	$4n + 12$	$4n + 12$	$24n + 12$	$4n + 12$	$21n + 7$	$12n + 4$	$8n + 8$

Proof. By definition,

$$\begin{aligned}
ZC_2(G) &= \sum_{pq \in E(G)} [\tau_G(p) \times \tau_G(q)] \\
&= \sum_{pq \in E_{2,3}^c} [\tau_H(p) \times \tau_H(q)] + \sum_{pq \in E_{3,3}^c} [\tau_H(p) \times \tau_H(q)] + \sum_{pq \in E_{3,5}^c} [\tau_H(p) \times \tau_H(q)] \\
&\quad + \sum_{pq \in E_{4,5}^c} [\tau_H(p) \times \tau_H(q)] + \sum_{pq \in E_{4,4}^c} [\tau_H(p) \times \tau_H(q)] + \sum_{pq \in E_{3,4}^c} [\tau_H(p) \times \tau_H(q)] \\
&\quad + \sum_{pq \in E_{4,4}^c} [\tau_H(p) \times \tau_H(q)] + \sum_{pq \in E_{5,8}^c} [\tau_H(p) \times \tau_H(q)] \\
&= |E_{(2,3)(H)}| (2 \times 3) + |E_{(3,3)(H)}| (3 \times 3) + |E_{(3,5)(H)}| (3 \times 5) + |E_{(4,5)(H)}| (4 \times 5) \\
&\quad + |E_{(4,4)(H)}| (4 \times 4) + |E_{(3,4)(H)}| (3 \times 4) + |E_{(4,4)(H)}| (4 \times 4) + |E_{(5,8)(H)}| (5 \times 8) \\
&= (4n + 12)(2 \times 3) + (12n + 4)(3 \times 3) + (4n + 12)(3 \times 5) + (12n + 4)(4 \times 5) \\
&\quad + (12n + 4)(4 \times 4) + (24n + 8)(3 \times 4) + (9n + 3)(4 \times 4) + (8n + 8)(5 \times 8) \\
&= 24n + 72 + 108n + 36 + 60n + 180 + 240n + 80 + 192n + 64 \\
&\quad + 288n + 96 + 144n + 48 + 320n + 320 \\
&= 1376n + 896.
\end{aligned} \tag{6}$$

□

$$ZC_1^*(H) = 672n + 432. \tag{7}$$

Theorem 3. Let $H \cong ZNOX(n)$ be a zinc oxide network of dimensions $n \geq 3$. Then, the modified first Zagreb connection index is

Proof. By definition,

$$\begin{aligned}
ZC_1^*(G) &= \sum_{pq \in E(G)} [\tau_G(p) + \tau_G(q)] \\
&= \sum_{pq \in E_{2,3}^c} [\tau_H(p) + \tau_H(q)] + \sum_{pq \in E_{3,3}^c} [\tau_H(p) + \tau_H(q)] + \sum_{pq \in E_{3,5}^c} [\tau_H(p) + \tau_H(q)] \\
&\quad + \sum_{pq \in E_{4,5}^c} [\tau_H(p) + \tau_H(q)] + \sum_{pq \in E_{4,4}^c} [\tau_H(p) + \tau_H(q)] + \sum_{pq \in E_{3,4}^c} [\tau_H(p) + \tau_H(q)] \\
&\quad + \sum_{pq \in E_{4,4}^c} [\tau_H(p) + \tau_H(q)] + \sum_{pq \in E_{5,8}^c} [\tau_H(p) + \tau_H(q)] \\
&= |E_{(2,3)(H)}| (2 + 3) + |E_{(3,3)(H)}| (3 + 3) + |E_{(3,5)(H)}| (3 + 5) + |E_{(4,5)(H)}| (4 + 5) \\
&\quad + |E_{(4,4)(H)}| (4 + 4) + |E_{(3,4)(H)}| (3 + 4) + |E_{(4,4)(H)}| (4 + 4) + |E_{(5,8)(H)}| (5 + 8) \\
&= (4n + 12)(5) + (12n + 4)(6) + (4n + 12)(8) + (12n + 4)(9) + (12n + 4)(8) \\
&\quad + (24n + 8)(7) + (9n + 3)(8) + (8n + 8)(13) \\
&= 20n + 60 + 72n + 24 + 32n + 96 + 108n + 36 + 96n + 32 + 168n + 56 \\
&\quad + 72n + 24 + 104n + 104 \\
&= 672n + 432.
\end{aligned} \tag{8}$$

□

Theorem 4. Let $H \cong ZNOX(n)$ be a zinc oxide network of dimensions $n \geq 3$. Then, the modified second Zagreb connection index is

$$ZC_2^*(H) = 1740n + 1124. \quad (9)$$

Proof. By definition,

$$\begin{aligned} ZC_2^*(G) &= \sum_{pq \in E(G)} [d_G(p)\tau_G(q) + d_G(q)\tau_G(p)] \\ &= \sum_{pq \in E_{2,3}^c} [d_H(p)\tau_H(q) + d_H(q)\tau_H(p)] + \sum_{pq \in E_{3,3}^c} [d_H(p)\tau_H(q) + d_H(q)\tau_H(p)] \\ &\quad + \sum_{pq \in E_{3,5}^c} [d_H(p)\tau_H(q) + d_H(q)\tau_H(p)] + \sum_{pq \in E_{4,5}^c} [d_H(p)\tau_H(q) + d_H(q)\tau_H(p)] \\ &\quad + \sum_{pq \in E_{3,4}^c} [d_H(p)\tau_H(q) + d_H(q)\tau_H(p)] + \sum_{pq \in E_{3,4}^c} [d_H(p)\tau_H(q) + d_H(q)\tau_H(p)] \\ &\quad + \sum_{pq \in E_{4,4}^c} [d_H(p)\tau_H(q) + d_H(q)\tau_H(p)] + \sum_{pq \in E_{5,8}^c} [d_H(p)\tau_H(q) + d_H(q)\tau_H(p)] \\ &= |E_{2,3}^c| (2 \times 3 + 2 \times 2) + |E_{3,3}^c| (2 \times 3 + 2 \times 3) + |E_{3,5}^c| (2 \times 5 + 3 \times 3) + |E_{4,5}^c| (2 \times 5 + 3 \times 4) \\ &\quad + |E_{4,4}^c| (2 \times 4 + 3 \times 4) + |E_{3,4}^c| (2 \times 4 + 3 \times 3) + |E_{4,4}^c| (3 \times 4 + 3 \times 4) + |E_{5,8}^c| (3 \times 8 + 4 \times 5) \\ &= (4n + 12)(10) + (12n + 4)(12) + (4n + 12)(19) + (12n + 4)(22) + (12n + 4)(20) \\ &\quad + (24n + 8)(17) + (9n + 3)(24) + (8n + 8)(44) \\ &= 40n + 120 + 144n + 48 + 76n + 228 + 264n + 88 + 240n + 80 \\ &\quad + 408n + 136 + 216n + 72 + 352n + 352 \\ &= 1740n + 1124. \end{aligned} \quad (10)$$

□

Theorem 5. Let $H \cong ZNOX(n)$ be a zinc oxide network of dimensions $n \geq 3$. Then, the modified third Zagreb connection index is

$$ZC_3^*(H) = 1808n + 1184. \quad (11)$$

Proof. By definition,

$$\begin{aligned} ZC_3^*(G) &= \sum_{pq \in E(G)} [d_G(p)\tau_G(p) + d_G(q)\tau_G(q)] \\ &= \sum_{pq \in E_{2,3}^c} [d_H(p)\tau_H(p) + d_H(q)\tau_H(q)] + \sum_{pq \in E_{3,3}^c} [d_H(p)\tau_H(p) + d_H(q)\tau_H(q)] \\ &\quad + \sum_{pq \in E_{3,5}^c} [d_H(p)\tau_H(p) + d_H(q)\tau_H(q)] + \sum_{pq \in E_{4,5}^c} [d_H(p)\tau_H(p) + d_H(q)\tau_H(q)] \\ &\quad + \sum_{pq \in E_{3,4}^c} [d_H(p)\tau_H(p) + d_H(q)\tau_H(q)] + \sum_{pq \in E_{3,4}^c} [d_H(p)\tau_H(p) + d_H(q)\tau_H(q)] \\ &\quad + \sum_{pq \in E_{4,4}^c} [d_H(p)\tau_H(p) + d_H(q)\tau_H(q)] + \sum_{pq \in E_{5,8}^c} [d_H(p)\tau_H(p) + d_H(q)\tau_H(q)] \\ &= |E_{2,3}^c| (2 \times 2 + 2 \times 3) + |E_{3,3}^c| (2 \times 3 + 2 \times 3) + |E_{3,5}^c| (2 \times 3 + 3 \times 5) + |E_{4,5}^c| (2 \times 4 + 3 \times 5) \\ &\quad + |E_{4,4}^c| (2 \times 4 + 3 \times 4) + |E_{3,4}^c| (2 \times 3 + 3 \times 4) + |E_{4,4}^c| (3 \times 4 + 3 \times 4) + |E_{5,8}^c| (3 \times 5 + 4 \times 8) \\ &= (4n + 12)(10) + (12n + 4)(12) + (4n + 12)(21) + (12n + 4)(23) + (12n + 4)(20) \\ &\quad + (24n + 8)(18) + (9n + 3)(24) + (8n + 8)(47) \\ &= 40n + 120 + 144n + 48 + 84n + 252 + 276n + 92 + 240n + 80 \\ &\quad + 432n + 144 + 216n + 72 + 376n + 376 \\ &= 1808n + 1184. \end{aligned} \quad (12)$$

□

Theorem 6. Let $H \cong ZNOX(n)$ be a zinc oxide network of dimensions $n \geq 3$. Then, the modified fourth Zagreb connection index is

$$ZC_4^*(H) = 10344n + 7224. \quad (13)$$

Proof. By definition,

$$\begin{aligned} ZC_4^*(G) &= \sum_{pq \in E(G)} [d_G(p)\tau_G(q) \times d_G(q)\tau_G(p)] \\ &= \sum_{pq \in E_{2,3}^c} [d_H(p)\tau_H(q) \times d_H(q)\tau_H(p)] + \sum_{pq \in E_{3,3}^c} [d_H(p)\tau_H(q) \times d_H(q)\tau_H(p)] \\ &\quad + \sum_{pq \in E_{3,5}^c} [d_H(p)\tau_H(q) \times d_H(q)\tau_H(p)] + \sum_{pq \in E_{4,5}^c} [d_H(p)\tau_H(q) \times d_H(q)\tau_H(p)] \\ &\quad + \sum_{pq \in E_{4,4}^c} [d_H(p)\tau_H(q) \times d_H(q)\tau_H(p)] + \sum_{pq \in E_{3,4}^c} [d_H(p)\tau_H(q) \times d_H(q)\tau_H(p)] \\ &\quad + \sum_{pq \in E_{4,4}^c} [d_H(p)\tau_H(q) \times d_H(q)\tau_H(p)] + \sum_{pq \in E_{5,8}^c} [d_H(p)\tau_H(q) \times d_H(q)\tau_H(p)] \\ &= |E_{2,3}^c| (2 \times 3 \times 2 \times 2) + |E_{3,3}^c| (2 \times 3 \times 2 \times 3) + |E_{3,5}^c| (2 \times 5 \times 3 \times 3) + |E_{4,5}^c| (2 \times 5 \times 3 \times 4) \\ &\quad + |E_{4,4}^c| (2 \times 4 \times 3 \times 4) + |E_{3,4}^c| (2 \times 4 \times 3 \times 3) + |E_{4,4}^{C*}| (3 \times 4 \times 3 \times 4) + |E_{5,8}^c| (3 \times 8 \times 4 \times 5) \\ &= (4n + 12)(24) + (12n + 4)(36) + (4n + 12)(90) + (12n + 4)(120) + (12n + 4)(96) \\ &\quad + (24n + 8)(72) + (9n + 3)(144) + (8n + 8)(480) \\ &= 96n + 288 + 432n + 144 + 360n + 1080 + 1440n + 480 + 1152n + 384 \\ &\quad + 1728n + 576 + 1296n + 432 + 3840n + 3840 \\ &= 10344n + 7224. \end{aligned} \quad (14)$$

4. Main Results Based on Zinc Silicate Network (ZNSL (n))

In this section, we compute the main results for first Zagreb connection index (ZCI), second ZCI, modified first ZCI, modified second ZCI, modified third ZCI, and modified fourth ZCI of zinc silicate-related MON (ZNSL (n)). Let $K \cong ZNSL(n)$ be the zinc silicate network of dimension n in the plane, see Figure 2. The partitions of K with respect to the vertex set and edge set are $V(K)$ and $E(K)$. We can easily see each vertex of degrees 2, 3, and 4. We have $V_1 = \{v \in V(K) | d_v = 2\}$, $V_2 = \{v \in V(K) | d_v = 3\}$, and $V_3 = \{v \in V(K) | d_v = 4\}$, where $|V_1| = 42n + 30$, $|V_2| = 38n + 18$, and $|V_3| = 2n + 2$. So, $|V(K)| = v = |V_1| + |V_2| + |V_3| = 82n + 50$. Now, the partitions of vertices according to connection number are $V_1 = \{v \in V(K) | \tau_v = 2\}$, $V_2 = \{v \in V(K) | \tau_v = 3\}$, $V_3 = \{v \in V(K) | \tau_v = 4\}$, $V_4 = \{v \in V(K) | \tau_v = 5\}$, $V_5 = \{v \in V(K) | \tau_v = 6\}$, and $V_6 = \{v \in V(K) | \tau_v = 8\}$, where $|V_1| = 2n + 6$, $|V_2| = 16n + 16$, $|V_3| = 48n + 16$, $|V_4| = 8n + 8$, $|V_5| = 6n + 2$, and $|V_6| = 2n + 2$. So, $|V(K)| = v = |V_1| + |V_2| + |V_3| + |V_4| + |V_5| + |V_6| = 82n + 50$. These vertex partitions are shown in Tables 5 and 6.

There are four types of partitions of edge sets of K according to the degree as $|E(K)| = |E_{2,2}^d| + |E_{2,3}^d| + |E_{3,3}^d| + |E_{3,4}^d| = 103n + 61$, and there are seven types of partitions of edge sets of K according to the connection number of vertices as $|E(K)| = |E_{2,3}^c| + |E_{3,3}^c| + |E_{3,4}^c| + |E_{3,5}^c| + |E_{4,4}^c| + |E_{4,5}^c| + |E_{4,6}^c| + |E_{5,8}^c| + |E_{6,6}^c| = 103n + 61$. These edge partitions are shown in Tables 7 and 8.

Theorem 7. Let $K \cong ZNSL(n)$ be a zinc silicate network of dimensions $n \geq 3$. Then, the first Zagreb connection index is

$$ZC_1(K) = 1464n + 824. \quad (15)$$

Proof. By definition,

$$\begin{aligned} ZC_1(G) &= \sum_{q \in V(G)} [\tau_G(q)]^2 \\ &= (2n + 6)(2)^2 + (16n + 16)(3)^2 + (48n + 16)(4)^2 + (8n + 8)(5)^2 + (6n + 2)(6)^2 + (2n + 2)(8)^2 \\ &= 8n + 24 + 144n + 144 + 768n + 256 + 200n + 200 + 216n + 72 + 128n + 128 \\ &= 1464n + 824. \end{aligned} \quad (16)$$

TABLE 5: Partitions of K 's vertices according to the degree.

d_v	2	3	4
$ d_v $	$42n + 30$	$38n + 18$	$2n + 2$

TABLE 6: Partitions of K 's vertices according to the connection number.

τ_v	2	3	4	5	6	8
$ \tau_v $	$2n + 6$	$16n + 16$	$48n + 16$	$8n + 8$	$6n + 2$	$2n + 2$

TABLE 7: Partitions of K 's edges according to the degree.

$E_{d(p),d(q)}^d$	$E_{2,2}^d$	$E_{2,3}^d$	$E_{3,3}^d$	$E_{3,4}^d$
$ E_{d(p),d(q)}^d $	$10n + 14$	$64n + 32$	$21n + 7$	$8n + 8$

TABLE 8: Partition of K 's edges according to the connection number.

$E_{\tau(p),\tau(q)}^c$	$E_{2,3}^c$	$E_{3,3}^c$	$E_{3,4}^c$	$E_{3,5}^c$	$E_{4,4}^c$	$E_{4,5}^c$	$E_{4,6}^c$	$E_{5,8}^c$	$E_{6,6}^c$
$ E_{\tau(p),\tau(q)}^c $	$4n + 12$	$6n + 2$	$12n + 4$	$4n + 12$	$42n + 14$	$12n + 4$	$12n + 4$	$8n + 8$	$3n + 1$

Theorem 8. Let $K \cong \text{ZNSL}(n)$ be a zinc silicate network of dimensions $n \geq 3$. Then, the second Zagreb connection index is

$$ZC_2(K) = 1910n + 1074. \quad (17)$$

Proof. By definition,

$$\begin{aligned}
 ZC_2(G) &= \sum_{pq \in E(G)} [\tau_G(p) \times \tau_G(q)] \\
 &= \sum_{pq \in E_{2,3}^c} [\tau_K(p) \times \tau_K(q)] + \sum_{pq \in E_{3,3}^c} [\tau_K(p) \times \tau_K(q)] + \sum_{pq \in E_{3,5}^c} [\tau_K(p) \times \tau_K(q)] \\
 &\quad + \sum_{pq \in E_{4,5}^c} [\tau_K(p) \times \tau_K(q)] + \sum_{pq \in E_{4,4}^c} [\tau_K(p) \times \tau_K(q)] + \sum_{pq \in E_{3,4}^c} [\tau_K(p) \times \tau_K(q)] \\
 &\quad + \sum_{pq \in E_{4,4}^c} [\tau_K(p) \times \tau_K(q)] + \sum_{pq \in E_{4,6}^c} [\tau_K(p) \times \tau_K(q)] + \sum_{pq \in E_{6,6}^c} [\tau_K(p) \times \tau_K(q)] \\
 &\quad + \sum_{pq \in E_{5,8}^c} [\tau_K(p) \times \tau_K(q)] \\
 &= |E_{(2,3)(K)}| (2 \times 3) + |E_{(3,3)(K)}| (3 \times 3) + |E_{(3,5)(K)}| (3 \times 5) + |E_{(4,5)(K)}| (4 \times 5) \\
 &\quad + |E_{(4,4)(K)}| (4 \times 4) + |E_{(3,4)(K)}| (3 \times 4) + |E_{(4,4)(K)}| (4 \times 4) + |E_{(4,6)(K)}| (4 \times 6) \\
 &\quad + |E_{(6,6)(K)}| (6 \times 6) + |E_{(5,8)(K)}| (5 \times 8) \\
 &= (4n + 12)(2 \times 3) + (6n + 2)(3 \times 3) + (4n + 12)(3 \times 5) + (12n + 4)(4 \times 5) \\
 &\quad + (36n + 12)(4 \times 4) + (12n + 4)(3 \times 4) + (6n + 2)(4 \times 4) + (12n + 4)(4 \times 6) \\
 &\quad + (3n + 1)(6 \times 6) + (8n + 8)(5 \times 8) \\
 &= 24n + 72 + 54n + 18 + 60n + 180 + 240n + 80 + 576n + 192 \\
 &\quad + 144n + 48 + 96n + 32 + 288n + 96 + 108n + 36 + 320n + 320 \\
 &= 1910n + 1074.
 \end{aligned} \quad (18)$$

□

Theorem 9. Let $K \cong \text{ZNSL}(n)$ be a zinc silicate network of dimensions $n \geq 3$. Then, the modified first Zagreb connection index is

$$ZC_1^*(K) = 876n + 500. \quad (19)$$

Proof. By definition,

$$\begin{aligned} ZC_1^*(G) &= \sum_{pq \in E(G)} [\tau_G(p) + \tau_G(q)] \\ &= \sum_{pq \in E_{2,3}^c} [\tau_K(p) + \tau_K(q)] + \sum_{pq \in E_{3,3}^c} [\tau_K(p) + \tau_K(q)] + \sum_{pq \in E_{3,5}^c} [\tau_K(p) + \tau_K(q)] \\ &\quad + \sum_{pq \in E_{4,5}^c} [\tau_K(p) + \tau_K(q)] + \sum_{pq \in E_{4,4}^c} [\tau_K(p) + \tau_K(q)] + \sum_{pq \in E_{3,4}^c} [\tau_K(p) + \tau_K(q)] \\ &\quad + \sum_{pq \in E_{4,4}^c} [\tau_K(p) + \tau_K(q)] + \sum_{pq \in E_{4,6}^c} [\tau_K(p) + \tau_K(q)] + \sum_{pq \in E_{6,6}^c} [\tau_K(p) + \tau_K(q)] \\ &\quad + \sum_{pq \in E_{5,8}^c} [\tau_K(p) + \tau_K(q)] \\ &= |E_{(2,3)(K)}|(2+3) + |E_{(3,3)(K)}|(3+3) + |E_{(3,5)(K)}|(3+5) + |E_{(4,5)(K)}|(4+5) \\ &\quad + |E_{(4,4)(K)}|(4+4) + |E_{(3,4)(K)}|(3+4) + |E_{(4,4)(K)}|(4+4) + |E_{(4,6)(K)}|(4+6) \\ &\quad + |E_{(6,6)(K)}|(6+6) + |E_{(5,8)(K)}|(5+8) \\ &= (4n+12)(2+3) + (6n+2)(3+3) + (4n+12)(3+5) + (12n+4)(4+5) \\ &\quad + (36n+12)(4+4) + (12n+4)(3+4) + (6n+2)(4+4) + (12n+4)(4+6) \\ &\quad + (3n+1)(6+6) + (8n+8)(5+8) \\ &= 20n+60+36n+12+32n+96+108n+36+288n+96+84n+28+48n+16 \\ &\quad + 120n+40+36n+12+104n+104 \\ &= 876n+500. \end{aligned} \quad (20)$$

Theorem 10. Let $K \cong \text{ZNSL}(n)$ be a zinc silicate network of dimensions $n \geq 3$. Then, the modified second Zagreb connection index is

$$ZC_2^*(K) = 2340n + 1324. \quad (21)$$

Proof. By definition,

$$\begin{aligned}
ZC_2^*(G) &= \sum_{pq \in E(G)} [d_G(p)\tau_G(q) + d_G(q)\tau_G(p)] \\
&= \sum_{pq \in E_{2,3}^c} [d_K(p)\tau_K(q) + d_K(q)\tau_K(p)] + \sum_{pq \in E_{3,3}^c} [d_K(p)\tau_K(q) + d_K(q)\tau_K(p)] \\
&\quad + \sum_{pq \in E_{3,5}^c} [d_K(p)\tau_K(q) + d_K(q)\tau_K(p)] + \sum_{pq \in E_{4,5}^c} [d_K(p)\tau_K(q) + d_K(q)\tau_K(p)] \\
&\quad + \sum_{pq \in E_{4,4}^c} [d_K(p)\tau_K(q) + d_K(q)\tau_K(p)] + \sum_{pq \in E_{3,4}^c} [d_K(p)\tau_K(q) + d_K(q)\tau_K(p)] \\
&\quad + \sum_{pq \in E_{4,4}^{c*}} [d_K(p)\tau_K(q) + d_K(q)\tau_K(p)] + \sum_{pq \in E_{4,6}^c} [d_K(p)\tau_K(q) + d_K(q)\tau_K(p)] \\
&\quad + \sum_{pq \in E_{6,6}^c} [d_K(p)\tau_K(q) + d_K(q)\tau_K(p)] + \sum_{pq \in E_{5,8}^c} [d_K(p)\tau_K(q) + d_K(q)\tau_K(p)] \\
&= |E_{2,3}^c| (2 \times 3 + 2 \times 2) + |E_{3,3}^c| (2 \times 3 + 2 \times 3) + |E_{3,5}^c| (2 \times 5 + 3 \times 3) + |E_{4,5}^c| (2 \times 5 + 3 \times 4) + |E_{4,4}^c| (2 \times 4 + 3 \times 4) \\
&\quad + |E_{3,4}^c| (2 \times 4 + 3 \times 3) + |E_{4,4}^{c*}| (3 \times 4 + 3 \times 4) + |E_{4,6}^c| (3 \times 6 + 3 \times 4) + |E_{6,6}^c| (3 \times 6 + 3 \times 6) + |E_{5,8}^c| (3 \times 8 + 4 \times 5) \\
&= (4n + 12)(10) + (6n + 2)(12) + (4n + 12)(19) + (12n + 4)(22) + (36n + 12)(20) + (12n + 4)(17) \\
&\quad + (6n + 2)(24) + (12n + 4)(30) + (3n + 1)(36) + (8n + 8)(44) \\
&= 40n + 120 + 72n + 24 + 76n + 228 + 264n + 88 + 720n + 240 + 204n + 68 + 144n + 48 + 360n + 120 \\
&\quad + 108n + 36 + 352n + 352 \\
&= 2340n + 1324.
\end{aligned}$$

(22)

□

$$ZC_3^*(K) = 2396n + 1380. \quad (23)$$

Theorem 11. Let $K \cong \text{ZNSL}(n)$ be a zinc silicate network of dimensions $n \geq 3$. Then, the modified third Zagreb connection index is

Proof. By definition,

$$\begin{aligned}
ZC_3^*(G) &= \sum_{pq \in E(G)} [d_G(p)\tau_G(p) + d_G(q)\tau_G(q)] \\
&= \sum_{pq \in E_{2,3}^c} [d_K(p)\tau_K(p) + d_K(q)\tau_K(q)] + \sum_{pq \in E_{3,3}^c} [d_K(p)\tau_K(p) + d_K(q)\tau_K(q)] \\
&\quad + \sum_{pq \in E_{3,5}^c} [d_K(p)\tau_K(p) + d_K(q)\tau_K(q)] + \sum_{pq \in E_{4,5}^c} [d_K(p)\tau_K(p) + d_K(q)\tau_K(q)] \\
&\quad + \sum_{pq \in E_{4,4}^c} [d_K(p)\tau_K(p) + d_K(q)\tau_K(q)] + \sum_{pq \in E_{3,4}^c} [d_K(p)\tau_K(p) + d_K(q)\tau_K(q)] \\
&\quad + \sum_{pq \in E_{4,4}^{c*}} [d_K(p)\tau_K(p) + d_K(q)\tau_K(q)] + \sum_{pq \in E_{4,6}^c} [d_K(p)\tau_K(p) + d_K(q)\tau_K(q)] \\
&\quad + \sum_{pq \in E_{6,6}^c} [d_K(p)\tau_K(p) + d_K(q)\tau_K(q)] + \sum_{pq \in E_{5,8}^c} [d_K(p)\tau_K(p) + d_K(q)\tau_K(q)] \\
&= |E_{2,3}^c| (2 \times 2 + 2 \times 3) + |E_{3,3}^c| (2 \times 3 + 2 \times 3) + |E_{3,5}^c| (2 \times 3 + 3 \times 5) + |E_{4,5}^c| (2 \times 4 + 3 \times 5) + |E_{4,4}^c| (2 \times 4 + 3 \times 4) \\
&\quad + |E_{3,4}^c| (2 \times 3 + 3 \times 4) + |E_{4,4}^{c*}| (3 \times 4 + 3 \times 4) + |E_{4,6}^c| (3 \times 4 + 3 \times 6) + |E_{6,6}^c| (3 \times 6 + 3 \times 6) + |E_{5,8}^c| (3 \times 5 + 4 \times 8) \\
&= (4n + 12)(10) + (6n + 2)(12) + (4n + 12)(21) + (12n + 4)(23) + (36n + 12)(20) + (12n + 4)(18) \\
&\quad + (6n + 2)(24) + (12n + 4)(30) + (3n + 1)(36) + (8n + 8)(47) \\
&= 40n + 120 + 72n + 24 + 84n + 252 + 276n + 92 + 720n + 240 + 216n + 72 + 144n + 48 + 360n \\
&\quad + 120 + 108n + 36 + 376n + 376 \\
&= 2396n + 1380.
\end{aligned}$$

(24)

□

Theorem 12. Let $K \cong \text{ZNSL}(n)$ be a zinc silicate network of dimensions $n \geq 3$. Then, the modified fourth Zagreb connection index is

$$ZC_4^*(K) = 14700n + 8676. \quad (25)$$

Proof. By definition,

$$\begin{aligned} ZC_4^*(G) &= \sum_{pq \in E(G)} [d_G(p)\tau_G(q) \times d_G(q)\tau_G(p)] \\ &= \sum_{pq \in E_{2,3}^c} [d_K(p)\tau_K(q) \times d_K(q)\tau_K(p)] + \sum_{pq \in E_{3,3}^c} [d_K(p)\tau_K(q) \times d_K(q)\tau_K(p)] \\ &\quad + \sum_{pq \in E_{3,5}^c} [d_K(p)\tau_K(q) \times d_K(q)\tau_K(p)] + \sum_{pq \in E_{4,5}^c} [d_K(p)\tau_K(q) \times d_K(q)\tau_K(p)] \\ &\quad + \sum_{pq \in E_{4,4}^c} [d_K(p)\tau_K(q) \times d_K(q)\tau_K(p)] + \sum_{pq \in E_{3,4}^c} [d_K(p)\tau_K(q) \times d_K(q)\tau_K(p)] \\ &\quad + \sum_{pq \in E_{4,4}^{c*}} [d_K(p)\tau_K(q) \times d_K(q)\tau_K(p)] + \sum_{pq \in E_{4,6}^c} [d_K(p)\tau_K(q) \times d_K(q)\tau_K(p)] \\ &\quad + \sum_{pq \in E_{6,6}^c} [d_K(p)\tau_K(q) \times d_K(q)\tau_K(p)] + \sum_{pq \in E_{5,8}^c} [d_K(p)\tau_K(q) \times d_K(q)\tau_K(p)] \\ &= |E_{2,3}^c| (2 \times 3 \times 2 \times 2) + |E_{3,3}^c| (2 \times 3 \times 2 \times 3) + |E_{3,5}^c| (2 \times 5 \times 3 \times 3) + |E_{4,5}^c| (2 \times 5 \times 3 \times 4) + |E_{4,4}^c| (2 \times 4 \times 3 \times 4) \\ &\quad + |E_{3,4}^c| (2 \times 4 \times 3 \times 3) + |E_{4,4}^{c*}| (3 \times 4 \times 3 \times 4) + |E_{4,6}^c| (3 \times 6 \times 3 \times 4) + |E_{6,6}^c| (3 \times 6 \times 3 \times 6) + |E_{5,8}^c| (3 \times 8 \times 4 \times 5) \\ &= (4n + 12)(24) + (6n + 2)(36) + (4n + 12)(90) + (12n + 4)(120) + (36n + 12)(96) + (12n + 4)(72) \\ &\quad + (6n + 2)(144) + (12n + 4)(216) + (3n + 1)(324) + (8n + 8)(480) \\ &= 96n + 288 + 216n + 72 + 360n + 1080 + 1440n + 480 + 3456n + 1152 \\ &\quad + 864n + 288 + 864n + 288 + 2592n + 864 + 972n + 324 + 3840n + 3840 \\ &= 14700n + 8676. \end{aligned} \quad (26)$$

5. Comparisons and Conclusions

In this section, we compare zinc oxide (H) and zinc silicate (K) related MONs via some Zagreb connection indices (ZCIs) such as first ZCI, second ZCI, modified first ZCI, modified second ZCI, modified third ZCI, and modified fourth ZCI with the help of Tables 9–14 that have been constructed by using numerical values of the aforementioned ZCIs. The graphical presentations for ZCIs of MONs are presented in Figures 3–10.

The comparative study of zinc-related MONs is highlighted by the following conclusions:

- (i) From Tables 9–14 and Figures 3–8, we see that the behaviors for all the ZCIs of zinc silicate MONs have more values and upper lines than zinc oxide MONs with the following order:

$$ZC_1(K) \geq ZC_1(H), ZC_2(K) \geq ZC_2(H), ZC_1^*(K) \geq ZC_1^*(H), ZC_2^*(K) \geq ZC_2^*(H), ZC_3^*(K) \geq ZC_3^*(H), \text{ and } ZC_4^*(K) \geq ZC_4^*(H).$$

- (ii) From Tables 15 and 16 and Figures 9 and 10, we see that modified fourth ZCI (ZC_4^*) attains more values

and upper lines than other ZCIs for both zinc-related MONs. □

- (iii) The modified first Zagreb connection index (ZC_1^*) attained better values of correlation coefficient for the thirteen physicochemical properties of octane isomers than other classical Zagreb indices. In this paper, novel connection-based Zagreb index ZC_4^* attains better values of correlation coefficient for increasing order in both the cases of zinc-related MONs.
- (iv) Table 17 shows that zinc silicate-related MON of dimension n for the aforesaid ZCIs has attained upward position than zinc oxide-related MON.
- (v) Moreover, these general relations (Tables 15–17) indicate that the chemical capability of zinc silicate-related MON is better than zinc oxide-related MON for all values of n .

Now, the problem is still open for prism, product, subdivision, and their compliment networks with the help of connection-based Zagreb indices.

TABLE 9: Numerical values of ZC_1 for H and K networks on dimensions $3 \leq n \leq 10$.

ZCIs	3	4	5	6	7	8	9	10
$ZC_1(H)$	3896	4964	6032	7100	8168	9236	10304	11372
$ZC_1(K)$	5216	6680	8144	9608	11072	12536	14000	15464

TABLE 10: Numerical values of ZC_2 for H and K networks on dimensions $3 \leq n \leq 10$.

ZCIs	3	4	5	6	7	8	9	10
$ZC_2(H)$	5024	6400	7776	9152	10528	11904	13280	14656
$ZC_2(K)$	6804	8714	10624	12534	14444	16354	18264	20174

TABLE 11: Numerical values of ZC_1^* for H and K networks on dimensions $3 \leq n \leq 10$.

ZCIs	3	4	5	6	7	8	9	10
$ZC_1^*(H)$	2448	3120	3792	4464	5136	5808	6480	7152
$ZC_1^*(K)$	3128	4004	4880	5756	6632	7508	8384	9260

TABLE 12: Numerical values of ZC_2^* for H and K networks on dimensions $3 \leq n \leq 10$.

ZCIs	3	4	5	6	7	8	9	10
$ZC_2^*(H)$	6344	8084	9824	11564	13304	15044	16784	18524
$ZC_2^*(K)$	8344	10684	13024	15364	17704	20044	22384	24724

TABLE 13: Numerical values of ZC_3^* for H and K networks on dimensions $3 \leq n \leq 10$.

ZCIs	3	4	5	6	7	8	9	10
$ZC_3^*(H)$	6608	8416	10224	12032	13840	15648	17456	19264
$ZC_3^*(K)$	8568	10964	13360	15756	18152	20548	22944	25340

TABLE 14: Numerical values of ZC_4^* for H and K networks on dimensions $3 \leq n \leq 10$.

ZCIs	3	4	5	6	7	8	9	10
$ZC_4^*(H)$	38256	48600	58944	69288	79632	89976	100320	110664
$ZC_4^*(K)$	52776	67476	82176	96876	111576	126276	140976	155676

FIGURE 3: Comparison of ZC_1 index for H and K networks on dimensions $3 \leq n \leq 10$.

FIGURE 4: Comparison of ZC_2 index for H and K networks on dimensions $3 \leq n \leq 10$.

FIGURE 5: Comparison of ZC_1^* index for H and K networks on dimensions $3 \leq n \leq 10$.

FIGURE 6: Comparison of ZC_2^* index for H and K networks on dimensions $3 \leq n \leq 10$.

FIGURE 7: Comparison of ZC_3^* index for H and K networks on dimensions $3 \leq n \leq 10$.

FIGURE 8: Comparison of ZC_4^* index for H and K networks on dimensions $3 \leq n \leq 10$.

FIGURE 9: Comparison of ZCIs of network H on dimensions $3 \leq n \leq 10$.

FIGURE 10: Comparison of ZCIs of network K on dimensions $3 \leq n \leq 10$.

TABLE 15: Numerical table for indicated ZCIs of network H on dimensions $3 \leq n \leq 10$.

ZCIs	$ZC_1(H)$	$ZC_2(H)$	$ZC_1^*(H)$	$ZC_2^*(H)$	$ZC_3^*(H)$	$ZC_4^*(H)$
3	3896	5024	2448	6344	6608	38256
4	4964	6400	3120	8084	8416	48600
5	6032	7776	3792	9824	10224	58944
6	7100	9152	4464	11564	12032	69288
7	8168	10528	5136	13304	13840	79632
8	9236	11904	5808	15044	15648	89976
9	10304	13280	6480	16784	17456	100320
10	11372	14656	7152	18524	19264	110664

TABLE 16: Numerical table for indicated ZCIs of network K on dimensions $3 \leq n \leq 10$.

ZCIs	$ZC_1(K)$	$ZC_2(K)$	$ZC_1^*(K)$	$ZC_2^*(K)$	$ZC_3^*(K)$	$ZC_4^*(K)$
3	5216	6804	3128	8344	8568	52776
4	6680	8714	4004	10684	10964	67476
5	8144	10624	4880	13024	13360	82176
6	9608	12534	5756	15364	15756	96876
7	11072	14444	6632	17704	18152	111576
8	12536	16354	7508	20044	20548	126276
9	14000	18264	8384	22384	22944	140976
10	15464	20174	9260	24724	25340	155676

TABLE 17: Comparison of indicated ZCIs for all n .

ZCIs	$K - H = \text{ZNSL}(n) - \text{ZNOX}(n)$	Results
First ZCI	$396n + 132$	$K > H$
Second ZCI	$534n + 178$	$K > H$
Modified first ZCI	$204n + 68$	$K > H$
Modified second ZCI	$600n + 200$	$K > H$
Modified third ZCI	$588n + 196$	$K > H$
Modified fourth ZCI	$4356n + 1452$	$K > H$

Data Availability

All data are included within this article. However, the reader may contact the corresponding author for more details of the data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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