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Research Article

Topological Indices of Derived Networks of Benzene Ring Embedded in P-Type Surface on $2D$

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Topological index (TI) is a numerical number assigned to the molecular structure that is used for correlation analysis in pharmacology, toxicology, and theoretical and environmental chemistry. Benzene ring embedded in the P-type surface on $2D$ network has stability similar to C_{60} and can be defined as $3D$ linkage of C_8 rings. This structure is the simplest possible tilling of the periodic minimal surface P which contains one type of carbon atom. In this paper, we compute general Randić, general Zagreb, general sum-connectivity, first Zagreb, second Zagreb, and ABC and GA indices of two operations (simple medial and stellation) of $2D$ network of benzene ring. Also, the exact expressions of ABC_4 and GA_5 indices of these structures are computed.

1. Introduction and Preliminaries

All the graphs in this work are finite and connected. Let $\mathscr{H}$ be a graph with vertex set and edge set denoted by $V(\mathscr{H})$ and $E(\mathscr{H})$, respectively. We denote the degree of a vertex $\mathfrak{u} \in V(G)$ by $d_{\mathfrak{u}}$ and it is the number of edges incident to $\mathfrak{u}$. The neighbor of a vertex $\mathfrak{v}$ is a vertex $\mathfrak{u}$ such that $\mathfrak{u}\mathfrak{v} \in E(G)$. The neighborhood of a vertex $\mathfrak{u}$ is the set of all its neighbors and is denoted by $\mathscr{N}(\mathfrak{u})$. Let $S_{\mathfrak{u}}$ be the sum of degrees of all the vertices that are adjacent to $\mathfrak{u}$. In other words,

$$S(\mathfrak{u}) = \sum_{\mathfrak{v}\in N(\mathfrak{u})} d_{\mathfrak{v}}, \quad \text{where } N(\mathfrak{u}) = \{\mathfrak{v} \in V(\mathscr{H}): \mathfrak{u}\mathfrak{v} \in E(\mathscr{H})\}. \tag{1}$$

For more insight on basic definitions and terminologies of graph theory, see [1].

In this paper, we consider two operations, stellation and simple medial of 2D network of benzene ring. The medial of a graph $\mathscr{H}$, denoted by $M(\mathscr{H})$, is defined as follows: we put a new vertex in the middle of every old edge of $\mathscr{H}$ and the new vertices have an edge if they lie on the consecutive edges. Note that the medial of a graph $\mathscr{H}$ is a 4-regular planner graph and not necessarily simple. Sjostrand [2] introduced the idea of transforming the graph with multiple edges and loops in to a simple graph by finite sequence of double edge swaps. If $M(\mathscr{H})$ is not simple, we transform the graph into simple graph and call it the simple medial of $\mathscr{H}$, denoted by $SM(\mathscr{H})$. Stellation of a graph planar $\mathscr{H}$, denoted as $St(\mathscr{H})$, is obtained by putting a vertex in every face of $\mathscr{H}$ and then we join this vertex to each vertex of respective face.

In the last couple of decades, topological and graph theoretical models have shown applications in many scientific research areas such as theoretical physics, chemistry, pharmaceutical chemistry, and toxicology. The interaction of graph theory with chemistry has enriched both the field. Topological index/descriptor is a numerical number attached to a molecular graph which is expected to predict certain physical or chemical properties of the underlying molecular structure. The simplest topological descriptors one can attach to a graph $\mathscr{H}$ is its order and size. The importance of the topological indices is because of their use in quantitative structure activity relationship (QSAR)/

quantitative structure property relationship (QSPR). The first topological index was introduced by Weiner in 1947, who showed that the index is well correlated with boiling point of alkanes. In 1975, the first degree based topological index was proposed by Milan Randić [3]. After that many degree-based topological indices were defined which were found to be useful in modeling the properties of organic molecules. Few of the important degree-based topological indices are presented in Table 1.

The Randić index was first named as branching index and is found appropriate for calculating the extent of branching of the carbon atom skeleton of saturated hydrocarbons. The first and second Zagreb indices were first introduced by Gutman and Transjistic in [8] and applied to branching problem. The Zagreb indices and their different variants are used to study chirality [16], molecular complexity [17, 18], ZE isomerism [19], and heterosystems [20]. The overall Zagreb indices are used to derive multilinear regression models. The importance of *ABC* index is due to its correlation with the thermodynamic properties of alkanes, see [21, 22]. Details on the computation of topological indices of graphs can be seen in [23–25].

2. Topological Indices of Simple Medial of $P[m,n]$

The preparation [26] of C_{60} leads to assumption about the stability of other crystalline forms of three coordinated carbons. In particular, Mackay and Terrones [27] raised the interesting prospect of creating possible tricoordinated solid carbon forms by lining the infinite periodic minimal surfaces known as *P* and *D*. These surfaces divide the space into two unconnected labyrinths. OKeeffe et al. [28] reported the results of initial calculations of molecular dynamic relaxation in the simplest treatment, which contains only one type of carbon atom. These structures contain six- and eight-membered rings in ratio of 2 : 3 and their primitive single cells have only 24 atoms. The stability of this structure is similar to C_{60} and it can be defined as a three-dimensional connection of C_8 rings. This structure is the simplest possible treatment of the periodic minimum surface *P*, which has only one type of carbon atom. From now onward, we denote the molecular structure of $2D$ network of benzene ring embedded in *P*-type surface by $P[m,n]$. Figure 1 depicts the molecular graph of $P[m,n]$.

Note that $P[m,n]$ contains $24mn$ vertices and $32mn - 2m - 2n$ edges. The medial of $P[m,n]$ is obtained as follows: we put a new vertex in the middle of every old edge of $P[m,n]$ and the new vertices have an edge if they lie on the consecutive edges. The graph of medial of $P[m,n]$ is depicted in Figure 2. Observe that the graph of medial of $P[m,n]$ contains multiple edges. It can be made simple by using the double edge swaps defined by Sjostrand [2]. Figure 3 depicts the graph of simple medial of $P[m,n]$ and we denote it by $SM(P[m,n])$. By a simple calculation, we can compute that $SM(P[m,n])$ contains $32mn - 2m - 2n$ vertices and $64mn - 20m - 20n + 12$ edges. Suppose $V_i = \{\mathfrak{u} \in V(SM(P[m,n])): d_{\mathfrak{u}} = i\}$ and $E_{i,j} = \{\mathfrak{u}\mathfrak{v} \in E(SM(P[m,n])): d_{\mathfrak{u}} = i, d_{\mathfrak{v}} = j\}$. Let n_i and $e_{i,j}$ be the cardinalities of V_i and $E_{i,j}$, respectively.

Theorem 1. *Let $\mathscr{K}$ be the graph of $SM(P[m,n])$ and α is a real number, then we have*

(1) $M_\alpha(\mathscr{K}) = (16m + 16n - 12)2^\alpha + (32mn - 18m - 18n + 12)4^\alpha$,

(2) $R_\alpha(\mathscr{K}) = (4m + 4n)2^{2\alpha} + (24m + 24n - 24)2^{3\alpha} + (64mn - 48m - 48n + 36)2^{4\alpha}$,

(3) $\chi_\alpha(\mathscr{K}) = (4m+4n)2^{2\alpha} + (24m+24n-24)6^\alpha + (64mn-48m-48n+36)2^{3\alpha}$,

(4) $ABC(\mathscr{K}) = (1/\sqrt{2})(28m+28n-24) + (1/2)\sqrt{(3/2)}(64mn-48m-48n+36)$,

(5) $GA(\mathscr{K}) = (4m + 4n) + (\sqrt{2}/3)(24m + 24n - 24) + (64mn - 48m - 48n + 36)$,

(6) $PM_1(\mathscr{K}) = 2^{192mn-136m-136n+108} \times 6^{24m+24n-24}$, *and*

(7) $PM_2(\mathscr{K}) = 2^{256mn-112m-112n+72}$.

Proof. We can partition $V(\mathscr{K})$ into three sets based on vertex degrees. Table 2 shows this partition. By using the values presented in Table 2, the general Zagreb index of $\mathscr{K}$ can be computed as follows:

$$M_\alpha(\mathscr{K}) = \sum_{\mathfrak{u} \in V(\mathscr{K})} (d_{\mathfrak{u}})^\alpha = (16m + 16n - 12)2^\alpha + (32mn - 18m - 18n + 12)4^\alpha. \quad (2)$$

Similarly, we can partition $E(\mathscr{K})$ into three sets based on the degree of end vertices of each edge. Table 3 shows this partition. By using the values presented in Table 3, the values of $R_\alpha, \chi_\alpha, ABC, GA, PM_1$, and PM_2 indices of $\mathscr{K}$ can be computed as

Table 1: Degree-based topological descriptors.

Topological descriptors	Mathematical forms
General Randić index [4,5]	$R_\alpha(\mathscr{H}) = \sum_{\mathfrak{uv}\in E(\mathscr{H})} (d_\mathfrak{u} d_\mathfrak{v})^\alpha$
General sum-connectivity index [6]	$\chi_\alpha(\mathscr{H}) = \sum_{\mathfrak{uv}\in E(\mathscr{H})} (d_\mathfrak{u} + d_\mathfrak{v})^\alpha$
First general Zagreb index [7]	$M_\alpha(\mathscr{H}) = \sum_{\mathfrak{u}\in V(\mathscr{H})} (d_\mathfrak{u})^\alpha$
Randić index [3]	$R_{(-1/2)}(\mathscr{H}) = \sum_{\mathfrak{uv}\in E(\mathscr{H})} (1/\sqrt{d_\mathfrak{u} d_\mathfrak{v}})$
First Zagreb index [8]	$M_1(\mathscr{H}) = \sum_{\mathfrak{uv}\in E(\mathscr{H})} (d_\mathfrak{u} + d_\mathfrak{v})$
Second Zagreb index [8]	$M_2(\mathscr{H}) = \sum_{\mathfrak{uv}\in E(\mathscr{H})} (d_\mathfrak{u} \times d_\mathfrak{v})$
First multiplicative Zagreb index [9]	$PM_1(\mathscr{H}) = \prod_{\mathfrak{uv}\in E(\mathscr{H})} (d_\mathfrak{u} + d_\mathfrak{v})$
Second multiplicative Zagreb index [10]	$PM_2(\mathscr{H}) = \prod_{\mathfrak{uv}\in E(\mathscr{H})} (d_\mathfrak{u} d_\mathfrak{v})$
Hyper-Zagreb index [11]	$HM(\mathscr{H}) = \sum_{\mathfrak{uv}\in E(\mathscr{H})} (d_\mathfrak{u} + d_\mathfrak{v})^2.$
Atom-bond connectivity index [12]	$ABC(\mathscr{H}) = \sum_{\mathfrak{uv}\in E(\mathscr{H})} \sqrt{((d_\mathfrak{u} + d_\mathfrak{v} - 2)/(d_\mathfrak{u} d_\mathfrak{v}))}$
Geometric arithmetic index [13]	$GA(\mathscr{H}) = \sum_{\mathfrak{uv}\in E(\mathscr{H})} ((2\sqrt{d_\mathfrak{u} d_\mathfrak{v}})/(d_\mathfrak{u} + d_\mathfrak{v}))$
Fourth version of atom-bond connectivity index [14]	$ABC_4(\mathscr{H}) = \sum_{\mathfrak{uv}\in E(\mathscr{H})} \sqrt{((S(\mathfrak{u}) + S(\mathfrak{v}) - 2)/(S(\mathfrak{u})S(\mathfrak{v})))}$
Fifth version of geometric arithmetic index [15]	$GA_5(\mathscr{H}) = \sum_{\mathfrak{uv}\in E(\mathscr{H})} ((2\sqrt{S(\mathfrak{u})S(\mathfrak{v})})/(S(\mathfrak{u}) + S(\mathfrak{v})))$

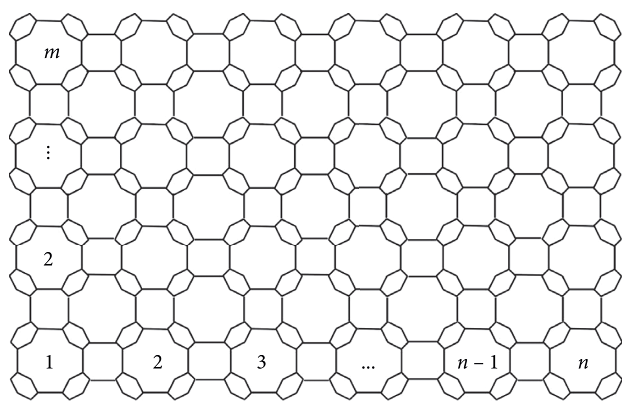

Figure 1: Graph of $P[m,n]$.

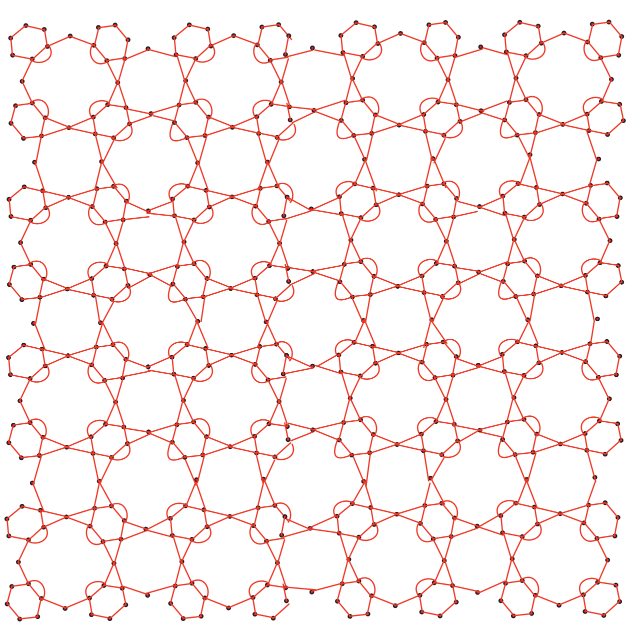

Figure 2: Molecular graph of medial of $P[m,n]$.

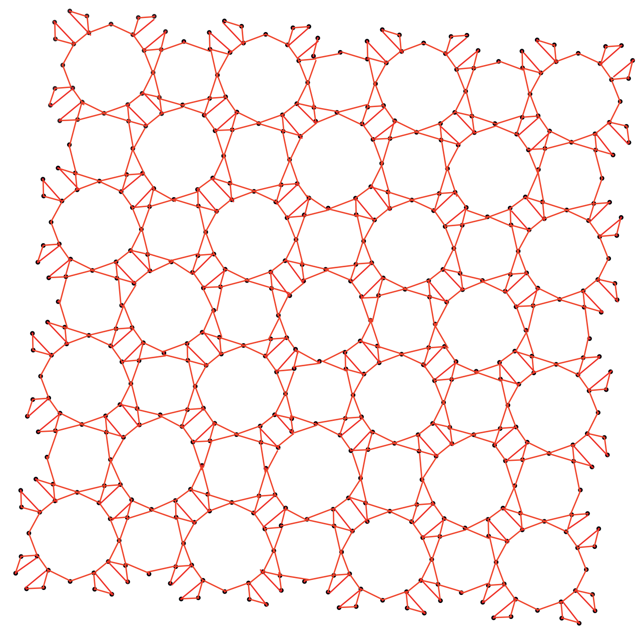

Figure 3: Molecular graph $SM(P[m,n])$ of simple medial of $P[m,n]$.

$$
\begin{aligned}
R_{\alpha}(\mathscr{K}) &= \sum_{\mathfrak{uv}\in E(\mathscr{K})} \left(d_{\mathfrak{u}} d_{\mathfrak{v}}\right)^{\alpha} \\
&= (4m+4n)(2\times 2)^{\alpha} + (24m+24n-24)(2\times 4)^{\alpha} + (64mn-48m-48n+32)(4\times 4)^{\alpha} \\
&= (4m+4n)2^{2\alpha} + (24m+24n-24)2^{3\alpha} + (64mn-48m-48n+36)2^{4\alpha}, \\
\chi_{\alpha}(\mathscr{K}) &= \sum_{\mathfrak{uv}\in E(\mathscr{K})} \left(d_{\mathfrak{u}} + d_{\mathfrak{v}}\right)^{\alpha} \\
&= (4m+4n)(2+2)^{\alpha} + (24m+24n-24)(2+4)^{\alpha} + (64mn-48m-48n+32)(4+4)^{\alpha} \\
&= (4m+4n)2^{2\alpha} + (24m+24n-24)6^{\alpha} + (64mn-48m-48n+36)2^{3\alpha}, \\
ABC(\mathscr{K}) &= \sum_{\mathfrak{uv}\in E(\mathscr{K})} \sqrt{\frac{d_{\mathfrak{u}} + d_{\mathfrak{v}} - 2}{d_{\mathfrak{u}} d_{\mathfrak{v}}}} \\
&= (4m+4n)\sqrt{\frac{2+2-2}{2\times 2}} + (24m+24n-24)\sqrt{\frac{2+4-2}{2\times 4}} \\
&\quad + (64mn-48m-48n+32)\sqrt{\frac{4+4-2}{4\times 4}} \\
&= \frac{1}{\sqrt{2}}(28m+28n-24) + \frac{1}{2}\sqrt{\frac{3}{2}}(64mn-48m-48n+36), \\
GA(\mathscr{K}) &= \sum_{\mathfrak{uv}\in E(\mathscr{K})} \frac{2\sqrt{d_{\mathfrak{u}} d_{\mathfrak{v}}}}{d_{\mathfrak{u}} + d_{\mathfrak{v}}} \\
&= (4m+4n)\frac{2\sqrt{2\times 2}}{2+2} + (24m+24n-24)\frac{2\sqrt{2\times 4}}{2+4} \\
&\quad + (64mn-48m-48n+32)\frac{2\sqrt{4\times 4}}{4+4} \\
&= (4m+4n) + \frac{\sqrt{2}}{3}(24m+24n-24) + (64mn-48m-48n+36), \\
PM_{1}(\mathscr{K}) &= \prod_{\mathfrak{uv}\in E(\mathscr{K})} \left(d_{\mathfrak{u}} + d_{\mathfrak{v}}\right) \\
&= (2+2)^{4m+4n} \times (2+4)^{24m+24n-24} \times (4+4)^{64mn-48m-48n+32} \\
&= 2^{192mn-136m-136n+108} \times 6^{24m+24n-24}, \\
PM_{2}(\mathscr{K}) &= \prod_{\mathfrak{uv}\in E(\mathscr{K})} \left(d_{\mathfrak{u}} d_{\mathfrak{v}}\right) \\
&= (2\times 2)^{4m+4n} \times (2\times 4)^{24m+24n-24} \times (4\times 4)^{64mn-48m-48n+32} \\
&= 2^{(256mn-112m-112n+72)}.
\end{aligned}
\tag{3}
$$

□

Table 2: Vertex partition of $\mathscr{K}$.

V_i	2	4
n_i	$16m+16n-12$	$32mn-18m-18n+12$

Table 3: Edge partition $E_{i,j}$ of $\mathscr{K}$.

$E_{i,j}$	$E_{2,2}$	$E_{2,4}$	$E_{4,4}$
$e_{i,j}$	$4m+4n$	$24m+24n-24$	$64mn-48m-48n+36$

From Theorem 1, we can compute the values of Randić, first Zagreb, second Zagreb, and hyper-Zagreb index of $\mathscr{K}$.

Corollary 1. *Let $\mathscr{K}$ be the graph of simple medial of $P[m,n]$, then we have*

(1) $R_{(-1/2)}(\mathscr{K}) = ((4m+4n)/2) + ((24m+24n-24)/(2\sqrt{2})) + (((64mn-48m-48n+36))/4)$,
(2) $M_1(\mathscr{K}) = 512mn - 224(m+n) + 144$,
(3) $M_2(\mathscr{K}) = 1024mn - 560(m+n) + 384$, *and*
(4) $HM(\mathscr{K}) = 4096mn - 2144(m+n) + 1440$.

Next, we will compute the ABC_4 and GA_5 indices of $\mathscr{K}$. For this, we need to find the edge partition $S_{i,j}$ of the graph $\mathscr{K}$, where $S_{i,j} = \{\mathfrak{uv} \in E(\mathscr{K})\colon S_{\mathfrak{u}} = i, S_{\mathfrak{v}} = j\}$. Let $m_{i,j}$ denote the cardinality of the set $S_{i,j}$. The edge partition $S_{i,j}$ of $\mathscr{K}$ is given in Table 4.

Theorem 2. *Let $\mathscr{K}$ be the graph of simple medial of $P[m,n]$, then we have*

$$\begin{aligned}
&(1)\, ABC_4(\mathscr{K}) = \frac{1}{3}\sqrt{\frac{5}{2}}(4m+4n) + \sqrt{\frac{7}{30}}(8m+8n) + \sqrt{\frac{1}{5}}(4m+4n) + \frac{1}{2}\sqrt{\frac{5}{7}}(4m+4n-8)\\
&\quad + \frac{12}{5}\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{7}}(4m+4n-8) + \frac{1}{2}\sqrt{\frac{13}{21}}(4m+4n-8) + \frac{1}{2}\sqrt{\frac{1}{2}}(4m+4n-8)\\
&\quad + \sqrt{\frac{11}{70}}(4m+4n-8) + \frac{1}{8}\sqrt{\frac{15}{2}}(64mn-60m-60n+56),\\
&(2)\, GA_5(\mathscr{K}) = (4m+4n) + \frac{1}{4}\sqrt{15}(8m+8n) + \frac{4}{9}\sqrt{5}(4m+4n) + \frac{4}{11}\sqrt{7}(4m+4n-8) + (4)\\
&\quad + \frac{1}{3}\sqrt{35}(4m+4n-8) + \frac{2}{13}\sqrt{42}(4m+4n-8) + \frac{4}{7}\sqrt{3}(4m+4n-8)\\
&\quad + \frac{4}{15}\sqrt{14}(4m+4n-8) + 64mn-60m-60n+56.
\end{aligned} \tag{4}$$

Proof. The edge partition of $\mathscr{K}$ depending on the sum of degree of end vertices is presented in Table 4. The result follows by using the values from Table 4 in the definition of $ABC_4(\mathscr{K})$ and $GA_5(\mathscr{K})$. □

3. Topological Indices of Stellation of $P[m,n]$

Let $\mathscr{L}$ be the molecular graph of stellation of $P[m,n]$. It is obtained adding a vertex in each face of $P[m,n]$ and then joining this vertex to each vertex of the respective face. The graph of $\mathscr{L}$ is shown in Figure 4. In $\mathscr{L}$, there are $32mn - 2n + 1$ vertices and $96mn - 22m - 22n + 12$ edges. Suppose $V_i = \{\mathfrak{u} \in V(\mathscr{L})\colon d_{\mathfrak{u}} = i\}$ and $E_{i,j} = \{\mathfrak{uv} \in E(\mathscr{L})\colon d_{\mathfrak{u}} = i, d_{\mathfrak{v}} = j\}$. Let n_i and $e_{i,j}$ be the cardinalities of the vertex set V_i and edge set $E_{i,j}$, respectively.

Theorem 3. *Let $\mathscr{L}$ be the graph of stellation of $P[m,n]$ and α is a real number, then we have*

(1) $M_\alpha(\mathscr{L}) = (8m+8n-4)3^\alpha + (8mn-4m-4n+4)4^\alpha + (8m+8n-8)5^\alpha + (20mn-)12m-12n+8)6^\alpha + (2mn-6m-6n)8^\alpha + (2mn-m-n+1)12^\alpha$,
(2) $R_\alpha(\mathscr{L}) = (4m+4n)3^{2\alpha} + (8m+8n-8)15^\alpha + (8m+8n-4)18^\alpha + (4m+4n)20^\alpha + (24mn-16m-16n+12)24^\alpha + (24mn-16m-16n+12)48^\alpha + (4m+4n-4)5^{2\alpha} + (12m+12n-16)30^\alpha + (4m+4n-8)40^\alpha + (4m+4n)60^\alpha + (32mn-26m-26n+20)6^{2\alpha} + (16mn-12m-12n+8)72^\alpha$,
(3) $\chi_\alpha(\mathscr{L}) = (4m+4n)6^\alpha + (8m+8n-8)8^\alpha + (12m+12n-4)3^{2\alpha} + (24mn-16m-16n+12)10^\alpha + (8mn-4m-4n+4)16^\alpha + (4m+4n-4)10^\alpha + (12m+12n-16)11^\alpha + (4m+4n-8)13^\alpha + (4m+4n)17^\alpha + (32mn-26m-26n+20)12^\alpha + (16mn-12m-12n+8)14^\alpha + (16mn-12m-12n+8)18^\alpha$,
(4) $ABC(\mathscr{L}) = (((24\sqrt{3}+2\sqrt{42}+16\sqrt{10}+16\sqrt{2})/3)+8)mn + (((4\sqrt{14}-16\sqrt{3}-\sqrt{42}-13\sqrt{10}-4)/3) + ((8\sqrt{10}+2\sqrt{35}+6\sqrt{30}+\sqrt{110}-12\sqrt{2})/5))(m+n) + (((-2\sqrt{14}+12\sqrt{3}+10\sqrt{10}+\sqrt{42}+8\sqrt{2})/3) + ((-8\sqrt{2}-8\sqrt{30}-2\sqrt{110}-8\sqrt{10})/5)+4)$,
(5) $GA(\mathscr{L}) = (32 + ((48\sqrt{6})/5) + ((32\sqrt{2})/3) + ((92\sqrt{3})/7))mn + (((16\sqrt{5})/9) - ((32\sqrt{6})/5) + ((24\sqrt{30})/11) + ((16\sqrt{10})/13) - ((8\sqrt{2})/3) + ((50\sqrt{15})/17) - ((62\sqrt{3})/7))(m+n) + (((24\sqrt{6})/5) - ((32\sqrt{30})/11) - (32\sqrt{10}/13) + ((8\sqrt{2})/3) + ((46\sqrt{3})/7) - 2\sqrt{15})$,
(6) $PM_1(\mathscr{L}) = 2^{(152mn-76m-76n+56)} \times 3^{(64mn-22m-22n+28)} \times 5^{(24mn-12m-12n+8)} \times 7^{(16mn-12m-12n+8)} \times 11^{(12m+12n-16)} \times 13^{(4m+4n-8)} \times 17^{(4m+4n)}$, *and*

Table 4: Edge partition $S_{i,j}$ of $\mathscr{K}$.

$S_{i,j}$	$S_{6,6}$	$S_{6,10}$	$S_{8,10}$	$S_{8,12}$	$S_{8,14}$	$S_{10,10}$
$m_{i,j}$	$4m+4n$	$8m+8n$	$4m+4n$	$4m+4n-8$	$4m+4n-8$	4
$S_{i,j}$	$S_{10,14}$	$S_{12,14}$	$S_{12,16}$	$S_{14,16}$	$S_{16,16}$	
$m_{i,j}$	$4m+4n-8$	$4m+4n-8$	$4m+4n-8$	$4m+4n-8$	$64mn-60m-60n+56$	

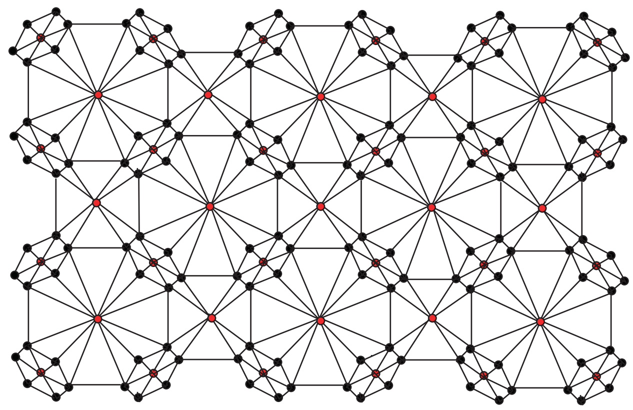

Figure 4: Molecular graph of $st(P[m,n])$.

(7) $PM_2(\mathscr{L}) = 2^{(216mn-100m-100n+64)} \times 3^{(144mn-60m-60n+48)} \times 5^{(40m+40n-40)}$.

Proof. We can partition $V(\mathscr{L})$ into six sets based on vertex degrees. Table 5 shows this partition. By using the values presented in Table 5, the general Zagreb index of $\mathscr{L}$ can be computed as

$$\begin{aligned} M_\alpha(\mathscr{L}) &= \sum_{\mathfrak{u}\in V(\mathscr{L})} (d_\mathfrak{u})^\alpha \\ &= (8m+8n-4)3^\alpha + (8mn-4m-4n+4)4^\alpha + (8m+8n-4)5^\alpha \\ &\quad + (20mn-12m-12n+8)6^\alpha + (2mn-m-n)8^\alpha + (2mn-m-n+1)12^\alpha. \end{aligned} \tag{5}$$

Similarly, we can partition $E(\mathscr{L})$ into three sets based on the degree of end vertices of each edge. Table 6 shows this partition. By using the values presented in Table 6, the values of R_α, χ_α, ABC, GA, PM_1, and PM_2 indices of $\mathscr{L}$ can be computed as

$$\begin{aligned} R_\alpha(\mathscr{L}) &= \sum_{\mathfrak{u}\mathfrak{v}\in E(\mathscr{L})} (d_\mathfrak{u} d_\mathfrak{v})^\alpha \\ &= (4m+4n)(3\times 3)^\alpha + (24mn-16m-16n+12)(3\times 5)^\alpha + (8m+8n-8)(3\times 6)^\alpha \\ &\quad + (4m+4n)(4\times 5)^\alpha + (8m+8n-8)(4\times 6)^\alpha + (8mn-4m-4n+4)(4\times 12)^\alpha \\ &\quad + (4m+4n-4)(5\times 5)^\alpha + (12m+12n-16)(5\times 6)^\alpha + (4m+4n-8)(5\times 8)^\alpha \\ &\quad + (4m+4n)(5\times 12)^\alpha + (32mn-26m-26n+20)(6\times 6)^\alpha \\ &\quad + (16mn-12m-12n+8)(6\times 8)^\alpha + (16mn-12m-12n+8)(6\times 12)^\alpha \\ &= (4m+4n)3^{2\alpha} + (8m+8n-8)15^\alpha + (8m+8n-4)18^\alpha + (4m+4n)20^\alpha \\ &\quad + (24mn-16m-16n+12)24^\alpha + (24mn-16m-16n+12)48^\alpha + (4m+4n-4)5^{2\alpha} \\ &\quad + (12m+12n-16)30^\alpha + (4m+4n-8)40^\alpha + (4m+4n)60^\alpha \\ &\quad + (32mn-26m-26n+20)6^{2\alpha} + (16mn-12m-12n+8)72^\alpha. \end{aligned}$$

$$
\begin{aligned}
\chi_{\alpha}(\mathscr{L}) &= \sum_{\mathfrak{uv}\in E(\mathscr{L})} (d_{\mathfrak{u}}+d_{\mathfrak{v}})^{\alpha} \\
&= (4m+4n)(3+3)^{\alpha}(24mn-16m-16n+12)(3+5)^{\alpha}+(8m+8n-8)(3+6)^{\alpha} \\
&\quad +(4m+4n)(4+5)^{\alpha}+(8m+8n-8)(4+6)^{\alpha}+(8mn-4m-4n+4)(4+12)^{\alpha} \\
&\quad +(4m+4n-4)(5+5)^{\alpha}+(12m+12n-16)(5+6)^{\alpha}+(4m+4n-8)(5+8)^{\alpha} \\
&\quad +(4m+4n)(5+12)^{\alpha}+(32mn-26m-26n+20)(6+6)^{\alpha} \\
&\quad +(16mn-12m-12n+8)(6+8)^{\alpha}+(16mn-12m-12n+8)(6+12)^{\alpha} \\
&= (4m+4n)6^{\alpha}+(8m+8n-8)8^{\alpha}+(12m+12n-4)3^{2\alpha} \\
&\quad +(24mn-16m-16n+12)10^{\alpha}+(8mn-4m-4n+4)16^{\alpha}+(4m+4n-4)10^{\alpha} \\
&\quad +(12m+12n-16)11^{\alpha}+(4m+4n-8)13^{\alpha}+(4m+4n)17^{\alpha} \\
&\quad +(32mn-26m-26n+20)12^{\alpha}+(16mn-12m-12n+8)14^{\alpha} \\
&\quad +(16mn-12m-12n+8)18^{\alpha}. \\
ABC(\mathscr{L}) &= \sum_{\mathfrak{uv}\in E(\mathscr{L})} \sqrt{\frac{d_{\mathfrak{u}}+d_{\mathfrak{v}}-2}{d_{\mathfrak{u}}d_{\mathfrak{v}}}} \\
&= (4m+4n)\sqrt{\frac{3+3-2}{3\times 3}}+(24mn-16m-16n+12)\sqrt{\frac{3+5-2}{3\times 5}} \\
&\quad +(8m+8n-8)\sqrt{\frac{3+6-2}{3\times 6}}+(4m+4n)\sqrt{\frac{4+5-2}{4\times 5}} \\
&\quad +(8m+8n-8)\sqrt{\frac{4+6-2}{4\times 6}}+(8mn-4m-4n+4)\sqrt{\frac{4+12-2}{4\times 12}} \\
&\quad +(4m+4n-4)\sqrt{\frac{5+5-2}{5\times 5}}+(12m+12n-16)\sqrt{\frac{5+6-2}{5\times 6}} \\
&\quad +(4m+4n-8)\sqrt{\frac{5+8-2}{5\times 8}}+(4m+4n)\sqrt{\frac{5+12-2}{5\times 12}} \\
&\quad +(32mn-26m-26n+20)\sqrt{\frac{6+6-2}{6\times 6}}+(16mn-12m-12n+8)\sqrt{\frac{6+8-2}{6\times 8}} \\
&\quad +(16mn-12m-12n+8)\sqrt{\frac{6+12-2}{6\times 12}} \\
&= \left(\frac{24\sqrt{3}+2\sqrt{42}+16\sqrt{10}+16\sqrt{2}}{3}+8\right)mn \\
&\quad +\left(\frac{4\sqrt{14}-16\sqrt{3}-\sqrt{42}-13\sqrt{10}-4}{3}+\frac{8\sqrt{10}+2\sqrt{35}+6\sqrt{30}+\sqrt{110}-12\sqrt{2}}{5}\right)(m+n) \\
&\quad +\left(\frac{-2\sqrt{14}+12\sqrt{3}+10\sqrt{10}+\sqrt{42}+8\sqrt{2}}{3}+\frac{-8\sqrt{2}-8\sqrt{30}-2\sqrt{110}-8\sqrt{10}}{5}+4\right) \\
GA(\mathscr{L}) &= \sum_{\mathfrak{uv}\in E(\mathscr{L})} \frac{2\sqrt{d_{\mathfrak{u}}d_{\mathfrak{v}}}}{d_{\mathfrak{u}}+d_{\mathfrak{v}}} \\
&= (4m+4n)\frac{2\sqrt{3\times 3}}{3+3}+(24mn-16m-16n+12)\frac{2\sqrt{3\times 5}}{3+5} \\
&\quad +(8m+8n-8)\frac{2\sqrt{3\times 6}}{3+6}+(4m+4n)\frac{2\sqrt{4\times 5}}{4+5} \\
&\quad +(8m+8n-8)\frac{2\sqrt{4\times 6}}{4+6}+(8mn-4m-4n+4)\frac{2\sqrt{4\times 12}}{4+12} \\
&\quad +(4m+4n-4)\frac{2\sqrt{5\times 5}}{5+5}+(12m+12n-16)\frac{2\sqrt{5\times 6}}{5+6} \\
&\quad +(4m+4n-8)\frac{2\sqrt{5\times 8}}{5+8}+(4m+4n)\frac{2\sqrt{5\times 12}}{5+12} \\
&\quad +(32mn-26m-26n+20)\frac{2\sqrt{6\times 6}}{6+6}+(16mn-12m-12n+8)\frac{2\sqrt{6\times 8}}{6+8} \\
&\quad +(16mn-12m-12n+8)\frac{2\sqrt{6\times 12}}{6+12} \\
&= \left(32+\frac{48\sqrt{6}}{5}+\frac{32\sqrt{2}}{3}+\frac{92\sqrt{3}}{7}\right)mn \\
&\quad +\left(\frac{16\sqrt{5}}{9}-\frac{32\sqrt{6}}{5}+\frac{24\sqrt{30}}{11}+\frac{16\sqrt{10}}{13}-\frac{8\sqrt{2}}{3}+\frac{50\sqrt{15}}{17}-\frac{62\sqrt{3}}{7}\right)(m+n) \\
&\quad +\left(\frac{24\sqrt{6}}{5}-\frac{32\sqrt{30}}{11}-\frac{32\sqrt{10}}{13}+\frac{8\sqrt{2}}{3}+\frac{46\sqrt{3}}{7}-2\sqrt{15}\right),
\end{aligned}
$$

Table 5: Vertex partition of $\mathscr{L}$.

V_i	3	4	5
n_i	$8m+8n-4$	$8mn-4m-4n+4$	$8m+8n-8$
V_i	6	8	12
n_i	$20mn-12m-12n+8$	$2mn-m-n$	$2mn-m-n+1$

Table 6: Edge partition of $E_{i,j}$ of $\mathscr{L}$.

$E_{i,j}$	$E_{3,3}$	$E_{3,5}$	$E_{3,6}$
$e_{i,j}$	$4m+4n$	$8m+8n-8$	$8m+8n-4$
$E_{i,j}$	$E_{4,5}$	$E_{4,6}$	$E_{4,12}$
$e_{i,j}$	$4m+4n$	$24mn-16m-16n+12$	$8mn-4m-4n+4$
$E_{i,j}$	$E_{5,5}$	$E_{5,6}$	$E_{5,8}$
$e_{i,j}$	$4m+4n-4$	$12m+12n-16$	$4m+4n-8$
$E_{i,j}$	$E_{5,12}$	$E_{6,6}$	$E_{6,8}$
$e_{i,j}$	$4m+4n$	$32mn-26m-26n+20$	$16mn-12m-12n+8$
$E_{i,j}$	$E_{6,12}$		
$e_{i,j}$	$16mn-12m-12n+8$		

$$\begin{aligned}
PM_1(\mathscr{L}) &= \prod_{\mathfrak{uv}\in E(\mathscr{L})} (d_{\mathfrak{u}} + d_{\mathfrak{v}}) \\
&= (3+3)^{4m+4n} \times (3+5)^{24mn-16m-16n+12} \times (3+6)^{(8m+8n-8)} \\
&\quad \times (4+5)^{4m+4n} \times (4+6)^{8m+8n-8} \times (4+12)^{8mn-4m-4n+4} \\
&\quad \times (5+5)^{4m+4n-4} \times (5+6)^{12m+12n-16} \times (5+8)^{4m+4n-8} \\
&\quad \times (5+12)^{4m+4n} \times (6+6)^{32mn-26m-26n+20} \times (6+8)^{16mn-12m-12n+8} \\
&\quad \times (6+12)^{16mn-12m-12n+8} \\
&= 2^{(152mn-76m-76n+56)} \times 3^{(64mn-22m-22n+28)} \times 5^{(24mn-12m-12n+8)} \times 7^{(16mn-12m-12n+8)} \\
&\quad \times 11^{(12m+12n-16)} \times 13^{(4m+4n-8)} \times 17^{(4m+4n)}, \\
PM_2(\mathscr{L}) &= \prod_{\mathfrak{uv}\in E(\mathscr{L})} (d_{\mathfrak{u}} d_{\mathfrak{v}}) \\
&= (3\times 3)^{4m+4n} \times (3\times 5)^{24mn-16m-16n+12} \times (3\times 6)^{(8m+8n-8)} \\
&\quad \times (4\times 5)^{4m+4n} \times (4\times 6)^{8m+8n-8} \times (4\times 12)^{8mn-4m-4n+4} \\
&\quad \times (5\times 5)^{4m+4n-4} \times (5\times 6)^{12m+12n-16} \times (5\times 8)^{4m+4n-8} \\
&\quad \times (5\times 12)^{4m+4n} \times (6\times 6)^{32mn-26m-26n+20} \times (6\times 8)^{16mn-12m-12n+8} \\
&\quad \times (6\times 12)^{16mn-12m-12n+8} \\
&= 2^{(216mn-100m-100n+64)} \times 3^{(144mn-60m-60n+48)} \times 5^{(40m+40n-40)}.
\end{aligned} \tag{6}$$

□

From Theorem 3, we can compute the values of Randić, first Zagreb, second Zagreb, and hyper-Zagreb index of $\mathscr{L}$.

Corollary 2. *Let $\mathscr{L}$ be the graph of stellation of $P[m,n]$, then we have*

(1) $R_\alpha(\mathscr{L}) = (\sqrt{24} + (24/\sqrt{48}) + (16/\sqrt{72}) + (16/3))mn + ((8/\sqrt{15}) - (8/\sqrt{18}) + (4/\sqrt{20}) - (16/\sqrt{24}) - (16/\sqrt{48}) + (12/\sqrt{30}) + (4/\sqrt{40}) - (4/\sqrt{60}) - (12/\sqrt{72}) - (11/5))(m+n) - (8/\sqrt{15}) - (4/\sqrt{18}) - (12/\sqrt{24}) + (12/\sqrt{48}) - (16/\sqrt{30}) - (8/\sqrt{40}) + (8/\sqrt{72}) + (12/5)$,

(2) $M_1(\mathscr{L}) = 1372mn - 432(m+n) + 260$,

(3) $M_2(\mathscr{L}) = 4032mn - 1712(m+n) + 1068$, *and*

(4) $HM(\mathscr{L}) = 17376mn - 7296(m+n) + 4740$.

Next, we will compute the ABC_4 and GA_5 indices of $\mathscr{L}$. For this, we need to find the edge partition $S_{i,j}$ of the graph $\mathscr{L}$, where $S_{i,j} = \{\mathfrak{uv} \in E(\mathscr{L}): S_{\mathfrak{u}} = i, S_{\mathfrak{v}} = j\}$. Let $m_{i,j}$ denote the cardinality of the set $S_{i,j}$. The edge partition $S_{i,j}$ of $\mathscr{L}$ is given in Table 7.

Theorem 4. *Let $\mathscr{L}$ be the graph of stellation of $P[m,n]$, then we have*

$$\begin{aligned}
(1)\ ABC_4(\mathscr{L}) = {} & \left(\frac{8\sqrt{82}}{21}+\frac{8\sqrt{2}}{3}+\frac{4\sqrt{42}}{7}+\frac{4\sqrt{273}}{21}+\frac{\sqrt{690}}{15}+\frac{8\sqrt{77}}{21}+2\right)mn+\left(\frac{2\sqrt{10}}{5}+\frac{\sqrt{58}}{15}+\frac{3\sqrt{6}}{14}+\right.\\
& \frac{4\sqrt{3458}}{91}+\frac{2\sqrt{41538}}{249}+\frac{4\sqrt{5}}{7}+\frac{2\sqrt{26}}{7}+\frac{2\sqrt{14}}{7}+\frac{2\sqrt{39962}}{377}+\frac{2\sqrt{5510}}{145}+\frac{4\sqrt{80852}}{1189}+\frac{2\sqrt{19229}}{287}+\frac{6\sqrt{130}}{65}+\\
& \frac{2\sqrt{410}}{41}+\frac{2\sqrt{160310}}{943}+\frac{12\sqrt{161}}{161}+\frac{6\sqrt{1722}}{287}-\frac{3\sqrt{82}}{7}-\frac{8\sqrt{2}}{3}-\frac{4\sqrt{42}}{7}-\frac{2\sqrt{273}}{7}-\frac{\sqrt{690}}{10}-\frac{4\sqrt{77}}{7}+\frac{2\sqrt{160022}}{899}+\\
& \left.\frac{2\sqrt{256742}}{1271}+\frac{4\sqrt{186}}{31}+\frac{4\sqrt{7378}}{217}-2\right)(m+n)+\left(-\frac{3\sqrt{6}}{7}-\frac{8\sqrt{3458}}{91}-\frac{8\sqrt{41538}}{483}-\frac{8\sqrt{5}}{7}-\frac{4\sqrt{26}}{7}\right.\\
& \frac{4\sqrt{14}}{7}-\frac{4\sqrt{39962}}{377}-\frac{4\sqrt{5510}}{145}-\frac{8\sqrt{80852}}{1189}-\frac{4\sqrt{19229}}{287}-\frac{12\sqrt{130}}{65}-\frac{4\sqrt{410}}{41}-\frac{4\sqrt{160310}}{943}-\frac{24\sqrt{161}}{161}-\\
& \frac{12\sqrt{1722}}{287}+\frac{10\sqrt{82}}{21}+\frac{8\sqrt{2}}{3}+\frac{4\sqrt{42}}{7}+\frac{10\sqrt{273}}{21}+\frac{\sqrt{690}}{6}+\frac{16\sqrt{77}}{21}-\frac{8\sqrt{160022}}{899}-\frac{8\sqrt{256742}}{1271}-\frac{16\sqrt{186}}{31}-\\
& \left.\frac{16\sqrt{7378}}{217}+\frac{8\sqrt{7}}{7}+\frac{4\sqrt{230}}{23}+\frac{8\sqrt{15}}{15}+\frac{4\sqrt{5}}{5}+\frac{4\sqrt{6765}}{205}+\frac{4\sqrt{70}}{21}+\frac{2\sqrt{253}}{23}+\frac{2\sqrt{161}}{23}+\frac{\sqrt{9030}}{105}+\frac{20\sqrt{11}}{15}+2\right),
\end{aligned}$$

$$\begin{aligned}
(2)\ GA_5(\mathscr{L}) = {} & \left(8\frac{\sqrt{35}}{3}+\frac{128\sqrt{21}}{37}+\frac{64\sqrt{15}}{31}+\frac{128\sqrt{42}}{53}+\frac{64\sqrt{30}}{47}+\frac{64\sqrt{14}}{15}+16\right)mn+\left(\frac{4\sqrt{105}}{11}+\frac{4\sqrt{91}}{5}+\right.\\
& \frac{4\sqrt{483}}{11}+\frac{8\sqrt{2}}{3}+\frac{8\sqrt{182}}{27}+\frac{8\sqrt{754}}{55}+\frac{8\sqrt{870}}{59}+\frac{4\sqrt{1189}}{35}+\frac{16\sqrt{287}}{69}+\frac{2\sqrt{195}}{7}+\frac{8\sqrt{1066}}{67}+\frac{8\sqrt{1886}}{87}+\\
& \frac{8\sqrt{322}}{37}+\frac{8\sqrt{1722}}{83}-\frac{8\sqrt{35}}{3}-\frac{128\sqrt{21}}{37}-\frac{64\sqrt{15}}{31}-\frac{192\sqrt{42}}{53}-\frac{96\sqrt{30}}{47}-\frac{96\sqrt{14}}{15}+\frac{8\sqrt{1798}}{91}+\frac{8\sqrt{2542}}{103}+\\
& \left.\frac{8\sqrt{465}}{23}+\frac{8\sqrt{651}}{26}-10\right)(m+n)+\left(-\frac{8\sqrt{91}}{5}-\frac{8\sqrt{483}}{11}-\frac{16\sqrt{182}}{27}-\frac{16\sqrt{754}}{55}-\frac{16\sqrt{870}}{59}-\frac{8\sqrt{1189}}{35}-\right.\\
& \frac{32\sqrt{287}}{69}-\frac{4\sqrt{195}}{7}-\frac{16\sqrt{1066}}{67}-\frac{16\sqrt{1886}}{87}-\frac{16\sqrt{322}}{37}-\frac{16\sqrt{1722}}{83}+\frac{8\sqrt{35}}{3}+\frac{128\sqrt{21}}{37}+\frac{64\sqrt{15}}{31}+\frac{320\sqrt{42}}{53}+\\
& \frac{160\sqrt{30}}{47}+\frac{128\sqrt{14}}{15}-\frac{32\sqrt{1798}}{91}-\frac{32\sqrt{2542}}{103}-\frac{32\sqrt{465}}{23}-\frac{16\sqrt{651}}{13}+\frac{16\sqrt{42}}{13}+\frac{16\sqrt{322}}{37}+\frac{16\sqrt{690}}{53}+\\
& \left.\frac{16\sqrt{210}}{29}+\frac{32\sqrt{435}}{89}+\frac{32\sqrt{615}}{101}+\frac{16\sqrt{70}}{17}+\frac{16\sqrt{69}}{35}+\frac{61\sqrt{161}}{51}+\frac{4\sqrt{105}}{11}+8\sqrt{2}+16\right).
\end{aligned}$$

$$\begin{aligned}
& \frac{4\sqrt{3458}}{91}+\frac{2\sqrt{41538}}{249}+\frac{4\sqrt{5}}{7}+\frac{2\sqrt{26}}{7}+\frac{2\sqrt{14}}{7}+\frac{2\sqrt{39962}}{377}+\frac{2\sqrt{5510}}{145}+\frac{4\sqrt{80852}}{1189}+\frac{2\sqrt{19229}}{287}+\frac{6\sqrt{130}}{65}+\\
& \frac{2\sqrt{410}}{41}+\frac{2\sqrt{160310}}{943}+\frac{12\sqrt{161}}{161}+\frac{6\sqrt{1722}}{287}-\frac{3\sqrt{82}}{7}-\frac{8\sqrt{2}}{3}-\frac{4\sqrt{42}}{7}-\frac{2\sqrt{273}}{7}-\frac{\sqrt{690}}{10}-\frac{4\sqrt{77}}{7}+\frac{2\sqrt{160022}}{899}+\\
& \frac{2\sqrt{256742}}{1271}+\frac{4\sqrt{186}}{31}+\frac{4\sqrt{7378}}{217}-2)(m+n)+(-\frac{3\sqrt{6}}{7}-\frac{8\sqrt{3458}}{91}-\frac{8\sqrt{41538}}{483}-\frac{8\sqrt{5}}{7}-\frac{4\sqrt{26}}{7}-\\
& \frac{4\sqrt{14}}{7}-\frac{4\sqrt{39962}}{377}-\frac{4\sqrt{5510}}{145}-\frac{8\sqrt{80852}}{1189}-\frac{4\sqrt{19229}}{287}-\frac{12\sqrt{130}}{65}-\frac{4\sqrt{410}}{41}-\frac{4\sqrt{160310}}{943}-\frac{24\sqrt{161}}{161}-\\
& \frac{12\sqrt{1722}}{287}+\frac{10\sqrt{82}}{21}+\frac{8\sqrt{2}}{3}+\frac{4\sqrt{42}}{7}+\frac{10\sqrt{273}}{21}+\frac{\sqrt{690}}{6}+\frac{16\sqrt{77}}{21}-\frac{8\sqrt{160022}}{899}-\frac{8\sqrt{256742}}{1271}-\frac{16\sqrt{186}}{31}-\\
& \frac{16\sqrt{7378}}{217}+\frac{8\sqrt{7}}{7}+\frac{4\sqrt{230}}{23}+\frac{8\sqrt{15}}{15}+\frac{4\sqrt{5}}{5}+\frac{4\sqrt{6765}}{205}+\frac{4\sqrt{70}}{21}+\frac{2\sqrt{253}}{23}+\frac{2\sqrt{161}}{23}+\frac{\sqrt{9030}}{105}+\frac{20\sqrt{11}}{15}+2).
\end{aligned}$$

(7)

Table 7: Edge partition $S_{i,j}$ of $\mathscr{L}$.

$S_{i,j}$	$S_{12,14}$	$S_{12,23}$	$S_{14,14}$	$S_{14,23}$	$S_{14,26}$
$m_{i,j}$	8	4	$4m+4n-8$	8	$8m+8n-16$
$S_{i,j}$	$S_{14,28}$	$S_{14,30}$	$S_{23,28}$	$S_{23,30}$	$S_{26,28}$
$m_{i,j}$	$4m+4n-8$	$4m+4n$	4	8	$4m+4n-8$
$S_{i,j}$	$S_{26,29}$	$S_{26,30}$	$S_{26,41}$	$S_{28,28}$	$S_{28,30}$
$m_{i,j}$	$2m+2n-4$	$4m+4n-8$	$4m+4n-8$	$2m+2n-4$	8
$S_{i,j}$	$S_{28,41}$	$S_{28,46}$	$S_{28,60}$	$S_{29,30}$	$S_{29,41}$
$m_{i,j}$	$4m+4n-8$	$4m+4n-8$	4	$4m+4n-8$	$4m+4n-8$
$S_{i,j}$	$S_{29,60}$	$S_{29,62}$	$S_{30,30}$	$S_{30,32}$	$S_{30,42}$
$m_{i,j}$	8	$4m+4n-16$	$2m+2n$	$8mn-8m-8n+8$	$16mn-16m-16n+16$
$S_{i,j}$	$S_{30,60}$	$S_{30,62}$	$S_{30,64}$	$S_{32,42}$	$S_{41,42}$
$m_{i,j}$	20	$8m+8n-32$	$8mn-12m-12n+20$	$16mn-16m-16n+16$	$4m+4n-8$
$S_{i,j}$	$S_{41,46}$	$S_{41,60}$	$S_{41,62}$	$S_{42,42}$	$S_{42,46}$
$m_{i,j}$	$4m+4n-8$	8	$4m+4n-16$	$16mn-18m-18n+20$	$8m+8n-16$
$S_{i,j}$	$S_{42,48}$	$S_{42,60}$	$S_{42,62}$	$S_{42,64}$	
$m_{i,j}$	$16mn-24m-24n+32$	8	$4m+4n-16$	$16mn-24m-24n+40$	

Proof. The edge partition of $\mathscr{L}$ depending on the sum of degree of end vertices is presented in Table 7. The result follows by using the values from Table 7 in the definition of $ABC_4(\mathscr{L})$ and $GA_5(\mathscr{L})$. □

4. Conclusion

In this work, we have considered two transformations (medial and stellation) on benzene ring embedded in P-type surface on 2 D network. We have computed general Randić, general Zagreb, general sum-connectivity, first Zagreb, second Zagreb, first multiple Zagreb, second multiple Zagreb, ABC, GA, ABC_4, and GA_5 indices of these transformation graphs.

Data Availability

No data were used in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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