Review Article

Vertex-Based Topological Indices of Double and Strong Double Graph of Dutch Windmill Graph

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A measurement of the molecular topology of graphs is known as a topological index, and several physical and chemical properties such as heat formation, boiling point, vaporization, enthalpy, and entropy are used to characterize them. Graph theory is useful in evaluating the relationship between various topological indices of some graphs derived by applying certain graph operations. Graph operations play an important role in many applications of graph theory because many big graphs can be obtained from small graphs. Here, we discuss two graph operations, i.e., double graph and strong double graph. In this article, we will compute the topological indices such as geometric arithmetic index (GA), atom bond connectivity index (ABC), forgotten index (F), inverse sum indeg index (ISI), general inverse sum indeg index $ISI_{(\alpha,\beta)}$, first multiplicative-Zagreb index (PM_1) and second multiplicative-Zagreb index (PM_2), fifth geometric arithmetic index (GA_5), fourth atom bond connectivity index (ABC_4) of double graph, and strong double graph of Dutch Windmill graph D_3^p .

1. Introduction and Preliminaries

For undetermined notations and terminologies, we recommend Robin J. Wilson book [1].

Assume that G is a simple graph that has no multiple edges and loops. $V(G)$ and $E(G)$ are the vertex and edge sets of graph G , respectively. The number of elements in $V(G)$ and $E(G)$ represents the order and size of graph G . Vertex degree is the number of edges joining to a vertex in a graph G . A vertex degree is indicated by d_r , $\{r \in V(G)\}$ and $S_r = \sum_{s \in N_G(r)} d_s$, where $N_G(r) = \{s \in V(G) \mid rs \in E(G)\}$. The following lemma is useful for computing the total number of edges in a graph G .

Lemma 1. If G is a graph of size t , then

$$\sum_{r \in V(G)} \deg(r) = 2t. \quad (1)$$

This is also called the handshake lemma and was observed by Lenford Euler in 1736. This observation is often called the first theorem of graph theory [2].

The chemical graph theory connects graph theory and chemistry to solve organic chemistry problems. Structured-property (QSPR) and structured-activity (QSAR) relationships are among the most important topics in this field. QSPR/QSAR research relies heavily on topological indices. These topological indices analyse the structure of any finite graph and are based on mathematical equations. Several different kinds of topological indices exist, i.e., degree-based topological indices [3–5], distance-based topological indices [6], and counting-related topological indices [7, 8]. The topological index concept comes from the work of Wiener, who introduced the Wiener index, and thus, topological indexing history begins.

The Wiener index is defined in [9] as follows:

$$W(G) = \frac{1}{2} \sum_{(r,s)} d(r,s), \quad (2)$$

where (r,s) is the order pair of vertices in G and $d(r,s)$ is the distance of vertex r - s in G .

The geometric-arithmetic index (GA) [10] of graph G is defined as

$$GA(G) = \sum_{rs \in E(G)} \frac{2\sqrt{d_r d_s}}{d_r + d_s}. \quad (3)$$

The atomic bond connectivity index (ABC) of graph G is defined [11] as

$$ABC(G) = \sum_{rs \in E(G)} \sqrt{\frac{d_r + d_s - 2}{d_r d_s}}. \quad (4)$$

The forgotten index (F) is defined [12] as

$$F(G) = \sum_{rs \in E(G)} (d_r^2 + d_s^2). \quad (5)$$

The inverse sum indeg index (ISI) is defined [13] as

$$ISI(G) = \sum_{rs \in E(G)} \frac{1}{1/d_r + 1/d_s}. \quad (6)$$

The general inverse sum indeg index ($ISI_{(\alpha,\beta)}$) is defined [14] as

$$ISI_{(\alpha,\beta)}(G) = \sum_{rs \in E(G)} [d_r d_s]^\alpha [d_r + d_s]^\beta, \quad (7)$$

where α and β are some real numbers.

The first multiplicative-Zagreb (PM_1) and second multiplicative-Zagreb index (PM_2) is defined [15] as

$$PM_1(G) = \prod_{rs \in E(G)} (d_r)^2, \quad (8)$$

$$PM_2(G) = \prod_{rs \in E(G)} (d_r \cdot d_s). \quad (9)$$

The first multiplicative-Zagreb index (PM_1) can also be written in the sum of the edges [16] of G :

$$PM_1(G) = \prod_{rs \in E(G)} (d_r + d_s). \quad (10)$$

The $GA_5(G)$ index is defined [17] as

$$GA_5(G) = \sum_{rs \in E(G)} \frac{2\sqrt{S_r \times S_s}}{S_r + S_s}, \quad (11)$$

where S_r is summation of degrees of all neighbor of vertex r , and the same for S_s .

The $ABC_4(G)$ index is defined [18] as

$$ABC_4(G) = \sum_{rs \in E(G)} \sqrt{\frac{S_r + S_s - 2}{S_r \times S_s}}. \quad (12)$$

We suggest that readers read the following articles for more detailed information on topological indices and molecular graphs [19–22].

FIGURE 1: Dutch windmill graph D_n^p .

Definition 1. A Dutch Windmill graph [23] is the graph which is formed by taking the p copies of the cycle graph by taking one mutual vertex. It is denoted by D_n^p , where $p \geq 1$ and $n \geq 3$. The order of Dutch Windmill graph is $(n-1)p + 1$ and size pn . Dutch Windmill graph D_n^p is depicted in Figure 1.

Definition 2. The double graph of graph G is represented by $D[G]$. Assume two copies of a graph, and join each vertex in one copy to its neighbor in the other copy in order to produce the double graph of the graph [24]. For example, the double graph of Dutch Windmill graph D_3^2 is depicted in Figure 2.

Definition 3. The strong double graph $SD[G]$ of the graph G is attained by taking two graphs and joining the closed neighbourhoods of each vertex in one graph to the adjacent vertex in the other graph [25]. For example, strong double graph of graph G is depicted in Figure 3. A new type of equienergetic and L-equienergetic graph has been found by using strong double graphs.

The following is the structure of this paper. Sections 2 and 3 will analyse some degree-based topological indices of double graphs and strong double graphs of Dutch Windmill graphs, respectively. In Section 4, we provide concluding remarks for the entire paper.

2. Degree-Based Topological Indices of Double Graph of Dutch Windmill Graph D_3^p

We will compute the vertex-based indices of the double graph of the Dutch Windmill graph D_3^p in this section.

Theorem 4. Let $D(D_3^p)$ be the double graph of Dutch Windmill graph D_3^p . Then, the geometric arithmetic index of $D(D_3^p)$ is

$$GA[D(D_3^p)] = 4p \left[1 + \frac{4\sqrt{p}}{1+p} \right]. \quad (13)$$

Proof. The total number of vertices and edges in the double graph of Dutch Windmill graph is $2[2p+1]$ and $12p$, respectively. In $[D(D_3^p)]$, we have $4p$ vertices of degree 4 and 2

FIGURE 2: (a). Dutch windmill graph D_3^2 . (b). Double graph of Dutch windmill graph D_3^2 . FIGURE 3: Strong double graph $SD[G]$ of graph G .

vertices of degree $8p$. We spilt the edges of $[D(D_3^p)]$ into those of the type $E[d_r, d_s]$ in which r, s are edges. $[D(D_3^p)]$ contains edges of type $E_{(4,4)}$ and $E_{(4,4p)}$, and Table 1 presents the edges of these types.

By applying equation (3) and Table 1, we acquire the desired results, i.e.,

$$\begin{aligned}
 GA[G] &= \sum_{rs \in E(G)} \frac{2\sqrt{d_r d_s}}{d_r + d_s}, \\
 GA[D(D_3^p)] &= |E_{(4,4)}| \sum_{rs \in E([D(D_3^p)])} \frac{2\sqrt{d_r d_s}}{d_r + d_s} \\
 &\quad + |E_{(4,4p)}| \sum_{rs \in E([D(D_3^p)])} \frac{2\sqrt{d_r d_s}}{d_r + d_s}, \quad (14) \\
 GA[D(D_3^p)] &= 4p \left[\frac{2\sqrt{4(4)}}{4+4} \right] + 8p \left[\frac{2\sqrt{4(4p)}}{4+4p} \right], \\
 GA[D(D_3^p)] &= 4p \left[1 + \frac{4\sqrt{p}}{1+p} \right].
 \end{aligned}$$

□

Theorem 5. Let $D(D_3^p)$ be the double graph of Dutch Windmill graph D_3^p . Then, the ABC index of $D(D_3^p)$ is

TABLE 1: Partitioning of edges.		
$E[d_r, d_s]$	$E_{(4,4)}$	$E_{(4,4p)}$
Number of edges	$4p$	$8p$

$$ABC[D(D_3^p)] = p\sqrt{6} + 2\sqrt{2p(1+2p)}. \quad (15)$$

Proof. By applying equation (4) and Table 1, we acquire the desired results, i.e.,

$$\begin{aligned}
 ABC(G) &= \sum_{rs \in E(G)} \sqrt{\frac{d_r + d_s - 2}{d_r d_s}}, \\
 ABC[D(D_3^p)] &= |E_{(4,4)}| \sum_{rs \in E([D(D_3^p)])} \sqrt{\frac{d_r + d_s - 2}{d_r d_s}} \\
 &\quad + |E_{(4,4p)}| \sum_{rs \in E([D(D_3^p)])} \sqrt{\frac{d_r + d_s - 2}{d_r d_s}}, \\
 &= 4p \sqrt{\frac{4+4-2}{16}} + 8p \sqrt{\frac{4+4p-2}{(16p)}}, \\
 &= 4p \sqrt{\frac{6}{16}} + 8p \sqrt{\frac{2+4p}{(16p)}}, \\
 ABC[D(D_3^p)] &= p\sqrt{6} + 2\sqrt{2p(1+2p)}. \quad (16)
 \end{aligned}$$

□

Theorem: 6. Let $D(D_3^p)$ be the double graph of Dutch Windmill graph D_3^p . Then, the forgotten index of $D(D_3^p)$ is

$$F[D(D_3^p)] = 128[p + (1 + p^2)]. \quad (17)$$

Proof. By applying equation (5) and Table 1, we acquire the desired results, i.e.,

$$\begin{aligned}
F(G) &= \sum_{rs \in E(G)} (d_r^2 + d_s^2) \\
F[D(D_3^p)] &= |E_{(4,4)}| \sum_{rs \in E[D(D_3^p)]} (d_r^2 + d_s^2) \\
&\quad + |E_{(4,4p)}| \sum_{rs \in E[D(D_3^p)]} (d_r^2 + d_s^2) \quad (18) \\
&= 4p(4^2 + 4^2) + 8p(4^2 + (4p^2)) \\
F[D(D_3^p)] &= 128[p + (1 + p^2)]. \quad \square
\end{aligned}$$

Theorem 7. Let $D(D_3^p)$ be the double graph of Dutch Windmill graph D_3^p . Then, the inverse sum indeg index of $D(D_3^p)$ is

$$ISI[D(D_3^p)] = 8p \left[1 + \frac{4p}{(1+p)} \right]. \quad (19)$$

Proof. By applying equation (6) and Table 1, we acquire the desired results, i.e.,

$$\begin{aligned}
ISI[G] &= \sum_{rs \in E(G)} \frac{1}{1/d_r + 1/d_s} = \sum_{rs \in E(G)} \frac{(d_r d_s)}{(d_r + d_s)}, \\
ISI[D(D_3^p)] &= |E_{(4,4)}| \sum_{rs \in E[D(D_3^p)]} \frac{(d_r d_s)}{(d_r + d_s)} \\
&\quad + |E_{(4,4p)}| \sum_{rs \in E[D(D_3^p)]} \frac{(d_r d_s)}{(d_r + d_s)}, \\
ISI[D(D_3^p)] &= 4p \left[\frac{(4)(4)}{(4+4)} \right] + 8p \left[\frac{(4)(4p)}{(4+4p)} \right], \\
ISI[D(D_3^p)] &= 8p \left[1 + \frac{4p}{(1+p)} \right]. \quad (20)
\end{aligned}$$

□

Theorem 8. Let $D(D_3^p)$ be the double graph of Dutch Windmill graph D_3^p . Then, the general inverse sum indeg index $ISI_{(\alpha,\beta)}$ of $D(D_3^p)$ is

$$ISI_{(\alpha,\beta)}[D(D_3^p)] = 4p[16]^\alpha [8]^\beta + 8p[16p]^\alpha [4(1+p)]^\beta. \quad (21)$$

Proof. By applying equation (7) and Table 1, we acquire the desired results, i.e.,

$$\begin{aligned}
ISI_{(\alpha,\beta)}(G) &= \sum_{rs \in E(G)} [d_u d_v]^\alpha [d_u + d_v]^\beta \\
&= |E_{(4,4)}| \sum_{rs \in E[D(D_3^p)]} [d_r d_s]^\alpha [d_r + d_s]^\beta \\
&\quad + |E_{(4,4p)}| \sum_{rs \in E[D(D_3^p)]} [d_r d_s]^\alpha [d_r + d_s]^\beta, \\
ISI_{(\alpha,\beta)}[D(D_3^p)] &= 4p[(4)(4)]^\alpha [4+4]^\beta \\
&\quad + 8p[(4)(4p)]^\alpha [4+4p]^\beta, \\
ISI_{(\alpha,\beta)}[D(D_3^p)] &= 4p[16]^\alpha [8]^\beta \\
&\quad + 8p[16p]^\alpha [4(1+p)]^\beta. \quad (22)
\end{aligned}$$

□

Theorem 9. Let $D(D_3^p)$ be the double graph of Dutch Windmill graph D_3^p . Then, the first multiplicative-Zagreb index of $D(D_3^p)$ is

$$PM_1[D(D_3^p)] = 1024p^2(1+p). \quad (23)$$

Proof. By applying equation (9) and Table 1, we acquire the desired results, i.e.,

$$\begin{aligned}
PM_1[G] &= \prod_{rs \in E(G)} (d_r + d_s), \\
PM_1[D(D_3^p)] &= |E_{(4,4)}| \prod_{rs \in E[D(D_3^p)]} (d_r + d_s) \\
&\quad \times |E_{(4,4p)}| \prod_{rs \in E[D(D_3^p)]} (d_r + d_s), \quad (24) \\
PM_1[D(D_3^p)] &= 4p(4+4) \times 8p(4+4p), \\
PM_1[D(D_3^p)] &= 1024p^2(1+p).
\end{aligned}$$

□

Theorem 10. Let $D(D_3^p)$ be the double graph of Dutch Windmill graph D_3^p . Then, the second multiplicative-Zagreb index of $D(D_3^p)$ is

$$PM_2[D(D_3^p)] = 8192p^3. \quad (25)$$

Proof. By applying equation (9) and Table 1, we acquire the desired results, i.e.,

$$\begin{aligned}
 \text{PM}_2[G] &= \prod_{rs \in E(G)} (d_r \cdot d_s), \\
 \text{PM}_2[D(D_3^p)] &= |E_{(4,4)}| \prod_{rs \in E[D(D_3^p)]} (d_r \cdot d_s) \\
 &\quad \times |E_{(4,4p)}| \prod_{rs \in E[D(D_3^p)]} (d_r \cdot d_s), \quad (26) \\
 \text{PM}_2[D(D_3^p)] &= 4p(4 \times 4) \times 8p(4 \times 4p), \\
 \text{PM}_2[D(D_3^p)] &= 8192p^3.
 \end{aligned}$$

Theorem 11. Let $D(D_3^p)$ be the double graph of Dutch Windmill graph D_3^p . Then, $\text{GA}_5(G)$ index of $D(D_3^p)$ is

$$\text{GA}_5[D(D_3^p)] = 4p\sqrt{(p+1)} \left[\frac{1}{(p+1)} + \frac{4p\sqrt{2}}{[3p+1]} \right]. \quad (27)$$

Proof. We split the edges of $[D(D_3^p)]$ into those of the type $E[S_r, S_s]$ in which “ rs ” is an edge. $[D(D_3^p)]$ contains edge of the type $E_{[8(p+1), 8(p+1)]}$ and $E_{[8(p+1), 16p]}$, and Table 2 presents the edges of these types.

By applying equation (11) and Table 2, we acquire the desired results, i.e.,

□

$$\begin{aligned}
 \text{GA}_5(G) &= \sum_{rs \in E(G)} \frac{2\sqrt{S_r \times S_s}}{S_r + S_s} \\
 &= |E_{[8(p+1), 8(p+1)]}| \sum_{rs \in E[D(D_3^p)]} \frac{2\sqrt{S_r \times S_s}}{S_r + S_s} + |E_{[8(p+1), 16p]}| \sum_{rs \in E[D(D_3^p)]} \frac{2\sqrt{S_r \times S_s}}{S_r + S_s}, \quad (28)
 \end{aligned}$$

$$\text{GA}_5[D(D_3^p)] = 4p \left[\frac{16\sqrt{(p+1)}}{2[8(p+1)]} \right] + 8p \left[\frac{8\sqrt{8p(p+1)}}{8[3p+1]} \right]$$

$$\text{GA}_5[D(D_3^p)] = 4p\sqrt{(p+1)} \left[\frac{1}{(p+1)} + \frac{4p\sqrt{2}}{[3p+1]} \right].$$

Theorem 12. Let $D(D_3^p)$ be the double graph of Dutch Windmill graph D_3^p . Then, the fourth atom bond connectivity index of $D(D_3^p)$ is

$$\text{ABC}_4[D(D_3^p)] = \frac{p}{2(p+1)} [\sqrt{16p-14}] + p \left[\sqrt{\frac{3(4p-1)}{p(p+1)}} \right]. \quad (29)$$

Proof. By applying equation (12) and Table 2, we acquire the desired results, i.e.,

□

$$\begin{aligned}
 \text{ABC}_4(G) &= \sum_{rs \in E(G)} \sqrt{\frac{S_r + S_s - 2}{S_r \times S_s}} \\
 &= |E_{[8(p+1), 8(p+1)]}| \sum_{rs \in E[D(D_3^p)]} \sqrt{\frac{S_r + S_s - 2}{S_r \times S_s}} + |E_{[8(p+1), 16p]}| \sum_{rs \in E[D(D_3^p)]} \sqrt{\frac{S_r + S_s - 2}{S_r \times S_s}}, \quad (30)
 \end{aligned}$$

$$\text{ABC}_4[D(D_3^p)] = 4p \left[\sqrt{\frac{16p-14}{[8(p+1)]^2}} \right] + 8p \left[\sqrt{\frac{6(4p-1)}{128p(p+1)}} \right],$$

$$\text{ABC}_4[D(D_3^p)] = \frac{p}{2(p+1)} [\sqrt{16p-14}] + p \left[\sqrt{\frac{3(4p-1)}{p(p+1)}} \right].$$

□

TABLE 2: Partitioning of edges.

$E[S_u, S_v]$	$E_{[8(p+1), 8(p+1)]}$	$E_{[8(p+1), 16p]}$
Number of edges	$4p$	$8p$

3. Degree-Based Topological Indices of Strong Double Graphs of Dutch Windmill Graph D_3^p

We will compute the vertex-based indices of the strong double graph of the Dutch Windmill graph D_3^p in this section. The strong double graph of D_3^p is depicted in Figure 4.

Theorem 13. *Let $SD(D_3^p)$ be the strong double graph of Dutch Windmill graph D_3^p . Then, the geometric arithmetic index of $SD(D_3^p)$ is*

$$\begin{aligned}
 GA[G] &= \sum_{rs \in E(G)} \frac{2\sqrt{d_r d_s}}{d_r + d_s} \\
 &= |E_{(5,5)}| \sum_{rs \in E([D(D_3^p)])} \frac{2\sqrt{d_r d_s}}{d_r + d_s} + |E_{(5,4p+1)}| \sum_{rs \in E([D(D_3^p)])} \frac{2\sqrt{d_r d_s}}{d_r + d_s} \\
 &\quad + |E_{(4p+1,4p+1)}| \sum_{rs \in E([D(D_3^p)])} \frac{2\sqrt{d_r d_s}}{d_r + d_s} \\
 &= (6p) \frac{\sqrt{25}}{5} + (8p) \frac{2\sqrt{5(4p+1)}}{5 + (4p+1)} + (1) \frac{2\sqrt{(4p+1)(4p+1)}}{(4p+1) + (4p+1)}, \\
 GA[SD(D_3^p)] &= 1 + 6p + \frac{8p\sqrt{20p+5}}{(2p+3)}.
 \end{aligned} \tag{32}$$

Theorem 14. *Let $SD(D_3^p)$ be the strong double graph of Dutch Windmill graph D_3^p . Then, ABC index for*

$$GA[SD(D_3^p)] = 1 + 6p + \frac{8p\sqrt{20p+5}}{(2p+3)}. \tag{31}$$

Proof. The total number of vertices and edges in $[SD(D_3^p)]$ are $2[2p+1]$ and $14p+1$, respectively. In $[SD(D_3^p)]$, we have $4p$ vertices of degree 5 and 2 vertices of degree $4p+1$. We split the edges of $[SD(D_3^p)]$ into those of the type $E[d_r, d_s]$ in which “ rs ” is an edge. $[SD(D_3^p)]$ contains edges of the type $E_{(5,5)}$, $E_{(5,4p+1)}$, and $E_{(4p+1,4p+1)}$, and Table 3 presents the edges of these types.

By applying Equation (3) and Table 3, we acquire the desired results, i.e.,

$SD(D_3^p)$ is □

$$ABC[SD(D_3^p)] = \frac{12p\sqrt{2}}{5} + 16p\sqrt{\frac{(p+1)}{(20p+5)}} + \frac{2\sqrt{2p}}{(4p+1)}. \tag{33}$$

FIGURE 4: (a) Dutch Windmill graph D_3^2 . (b). Strong double graph of Dutch Windmill graph D_3^p .

TABLE 3: Partitioning of edges.

$E[d_r, d_s]$	$E_{(5,5)}$	$E_{(5,4p+1)}$	$E_{(4p+1,4p+1)}$
Number of edges	$6p$	$8p$	1

Proof. By applying equation (4) and Table 3, we acquire the desired results, i.e.,

$$\begin{aligned}
 \text{ABC}[G] &= \sum_{rs \in E(G)} \sqrt{\frac{d_r + d_s - 2}{d_r d_s}}, \\
 \text{ABC}[\text{SD}(D_3^p)] &= |E_{(5,5)}| \sum_{rs \in E([D(D_3^p)])} \sqrt{\frac{d_r + d_s - 2}{d_r d_s}} \\
 &\quad + |E_{(5,4p+1)}| \sum_{rs \in E([D(D_3^p)])} \sqrt{\frac{d_r + d_s - 2}{d_r d_s}} \\
 &\quad + |E_{(4p+1,4p+1)}| \sum_{rs \in E([D(D_3^p)])} \sqrt{\frac{d_r + d_s - 2}{d_r d_s}}, \\
 \text{ABC}[\text{SD}(D_3^p)] &= 6p\sqrt{\frac{8}{25}} + 8p\sqrt{\frac{5 + (4p+1) - 2}{(5)(4p+1)}} + \sqrt{\frac{(8p+2) - 2}{(4p+1)^2}}, \\
 \text{ABC}[\text{SD}(D_3^p)] &= \frac{12p\sqrt{2}}{5} + 16p\sqrt{\frac{(p+1)}{(20p+5)}} + \frac{2\sqrt{2p}}{(4p+1)}.
 \end{aligned} \tag{34}$$

□

Theorem 15. Let $SD(D_3^p)$ be the strong double graph of Dutch Windmill graph D_3^p . Then, the forgotten index of $SD(D_3^p)$ is

$$F[SD(D_3^p)] = 128p^3 + 96p^2 + 524p + 2. \quad (35)$$

Proof. By applying equation (5) and Table 3, we acquire the desired results, i.e.,

$$\begin{aligned} F(G) &= \sum_{rs \in E(G)} (d_r^2 + d_s^2) \\ F[SD(D_3^p)] &= |E_{(5,5)}| \sum_{rs \in E[D(D_3^p)]} (d_r^2 + d_s^2) \\ &\quad + |E_{(5,4p+1)}| \sum_{rs \in E[D(D_3^p)]} (d_r^2 + d_s^2) \\ &\quad + |E_{(4p+1),(4p+1)}| \sum_{rs \in E[D(D_3^p)]} (d_r^2 + d_s^2) \\ &= 6p(50) + 8p(25 + 16p^2 + 8p + 1) \\ &\quad + 1[(16p^2 + 8p + 1) + (16p^2 + 8p + 1)], \\ F[SD(D_3^p)] &= 128p^3 + 96p^2 + 524p + 2. \end{aligned} \quad (36)$$

□

Theorem 16. Let $SD(D_3^p)$ be the strong double graph of Dutch Windmill graph D_3^p . Then, the inverse sum indeg index of $SD(D_3^p)$ is

$$ISI[SD(D_3^p)] = 15p + 4p \left[\frac{20p + 5}{(2p + 3)} \right] + \left[\frac{(4p + 1)^2}{2(4p + 1)} \right]. \quad (37)$$

Proof. By applying equation (6) and Table 3, we acquire the desired results, i.e.,

$$\begin{aligned} ISI[G] &= \sum_{rs \in E(G)} \frac{1}{1/d_r + 1/d_s} = \sum_{rs \in E(G)} \frac{(d_r d_s)}{(d_r + d_s)}, \\ ISI[SD(D_3^p)] &= |E_{(5,5)}| \sum_{rs \in E[D(D_3^p)]} \frac{(d_r d_s)}{(d_r + d_s)} + |E_{(5,(4p+1))}| \sum_{rs \in E[D(D_3^p)]} \frac{(d_r d_s)}{(d_r + d_s)} \\ &\quad + |E_{(4p+1),(4p+1)}| \sum_{rs \in E[D(D_3^p)]} \frac{(d_r d_s)}{(d_r + d_s)}, \\ ISI[SD(D_3^p)] &= 6p \left[\frac{25}{10} \right] + 8p \left[\frac{(5)(4p + 1)}{(5 + 4p + 1)} \right] + \left[\frac{(4p + 1)(4p + 1)}{(8p + 2)} \right], \\ ISI[SD(D_3^p)] &= 15p + 4p \left[\frac{20p + 5}{(2p + 3)} \right] + \left[\frac{(4p + 1)^2}{2(4p + 1)} \right]. \end{aligned} \quad (38)$$

Theorem 17. Let $SD(D_3^p)$ be the strong double graph of Dutch Windmill graph D_3^p . Then, general inverse sum indeg index $ISI_{(\alpha,\beta)}$ of $SD(D_3^p)$ is

$$\begin{aligned} ISI_{(\alpha,\beta)}[SD(D_3^p)] &= 6p[25]^\alpha [10]^\beta \\ &\quad + 8p[20p + 5]^\alpha [2(2p + 3)]^\beta \\ &\quad + [(4p + 1)^2]^\alpha [2(4p + 1)]^\beta. \end{aligned} \quad (39)$$

□

Proof. By applying equation (7) and Table 3, we acquire the desired results, i.e.,

$$\begin{aligned}
 \text{ISI}_{(\alpha,\beta)}(G) &= \sum_{rs \in E(G)} [d_r d_s]^\alpha [d_r + d_s]^\beta \\
 &= |E_{(5,5)}| \sum_{rs \in E(G)} [d_r d_s]^\alpha [d_r + d_s]^\beta + |E_{(5,4p+1)}| \sum_{rs \in E(G)} [d_r d_s]^\alpha [d_r + d_s]^\beta + |E_{(4p+1),(4p+1)}| \sum_{rs \in E(G)} [d_r d_s]^\alpha [d_r + d_s]^\beta \\
 &= 6p[25]^\alpha [10]^\beta + 8p[(5)(4p+1)]^\alpha [4p+6]^\beta + [(4p+1)^2]^\alpha [(4p+1) + (4p+1)]^\beta \\
 \text{ISI}_{(\alpha,\beta)}[\text{SD}(D_3^p)] &= 6p[25]^\alpha [10]^\beta + 8p[20p+5]^\alpha [2(2p+3)]^\beta + [(4p+1)^2]^\alpha [2(4p+1)]^\beta.
 \end{aligned} \tag{40}$$

Theorem 18. Let $\text{SD}(D_3^p)$ be the strong double graph of Dutch Windmill graph D_3^p . Then, the first multiplicative-Zagreb index of $\text{SD}(D_3^p)$ is

$$\text{PM}_1[\text{SD}(D_3^p)] = 15360p^4 + 26880p^3 + 5760p^2. \tag{41}$$

Proof. By applying equation (10) and Table 3, we acquire the desired results, i.e.,

$$\begin{aligned}
 \text{PM}_1[G] &= \prod_{rs \in E(G)} (d_r + d_s), \\
 \text{PM}_1[\text{SD}(D_3^p)] &= |E_{(5,5)}| \prod_{rs \in E[D(D_3^p)]} (d_r + d_s) \\
 &\quad \times |E_{(5,4p+1)}| \prod_{rs \in E[D(D_3^p)]} (d_r + d_s) \\
 &\quad \times |E_{(4p+1,4p+1)}| \prod_{rs \in E[D(D_3^p)]} (d_r + d_s), \\
 \text{PM}_1[\text{SD}(D_3^p)] &= 6p(10) \times 16p(2p+3) \times (8p+2), \\
 \text{PM}_1[\text{SD}(D_3^p)] &= 15360p^4 + 26880p^3 + 5760p^2.
 \end{aligned} \tag{42}$$

Theorem 19. Let $\text{SD}(D_3^p)$ be the strong double graph of Dutch Windmill graph D_3^p . Then, the second multiplicative-Zagreb index of $\text{SD}(D_3^p)$ is

$$\text{PM}_2[\text{SD}(D_3^p)] = (4p+1)^2(24000p^3 + 6000p^2). \tag{43}$$

Proof. By applying equation (9) and Table 3, we acquire the desired results, i.e.,

$$\begin{aligned}
 \text{PM}_2[G] &= \prod_{rs \in E(G)} (d_r \cdot d_s), \\
 \text{PM}_2[\text{SD}(D_3^p)] &= |E_{(5,5)}| \prod_{rs \in E[D(D_3^p)]} (d_r \cdot d_s) \\
 &\quad \times |E_{(5,4p+1)}| \prod_{rs \in E[D(D_3^p)]} (d_r \cdot d_s) \\
 &\quad \times |E_{(4p+1),(4p+1)}| \prod_{rs \in E[D(D_3^p)]} (d_r \cdot d_s), \\
 \text{PM}_2[D(D_3^p)] &= 6p(25) \times 40p(4p+1) \times (4p+1)^2, \\
 \text{PM}_2[\text{SD}(D_3^p)] &= (4p+1)^2(24000p^3 + 6000p^2).
 \end{aligned} \tag{44}$$

Theorem 20. Let $\text{SD}(D_3^p)$ be the strong double graph of Dutch Windmill graph D_3^p . Then, the GA_5 index of $\text{SD}(D_3^p)$ is

$$\text{GA}_5[\text{SD}(D_3^p)] = \frac{2p(48p+27+4\sqrt{8p(24p+52)+17})}{16p+9}. \tag{45}$$

Proof. We split the edges of $[\text{SD}(D_3^p)]$ into those of the type $E[S_r, S_s]$ in which “ rs ” is an edge. $[\text{SD}(D_3^p)]$ contains edges of the type $E_{(8p+17,8+17)}$, $E_{(8p+17,24p+1)}$, and $E_{(24p+1,24p+1)}$, and Table 4 presents the edges of these types.

By applying equation (11) and Table 4, we acquire the desired results, i.e.,

$$\begin{aligned}
 GA_5(G) &= \sum_{rs \in E(G)} \frac{2\sqrt{S_r \times S_s}}{S_r + S_s}, \\
 GA_5[SD(D_3^p)] &= |E_{(8p+17, 8p+17)}| \sum_{rs \in E[D(D_3^p)]} \frac{2\sqrt{S_r \times S_s}}{S_r + S_s} \\
 &\quad + |E_{(8p+17, 24p+1)}| \sum_{rs \in E[D(D_3^p)]} \frac{2\sqrt{S_r \times S_s}}{S_r + S_s} + |E_{(24p+1, 24p+1)}| \sum_{rs \in E[D(D_3^p)]} \frac{2\sqrt{S_r \times S_s}}{S_r + S_s}, \\
 &= 6p \frac{2\sqrt{(8p+17)^2}}{16p+34} + 8p \frac{2\sqrt{(8p+17)(24p+1)}}{32p+18} + (1) \frac{2\sqrt{(24p+1)^2}}{48p+2} \\
 GA_5[SD(D_3^p)] &= \frac{2p(48p+27+4\sqrt{8p(24p+52)+17})}{16p+9}.
 \end{aligned} \tag{46}$$

Theorem 21. Let $SD(D_3^p)$ be the strong double graph of Dutch Windmill graph D_3^p . Then, the fourth atom bond connectivity index of $SD(D_3^p)$ is

$$\begin{aligned}
 ABC_4[SD(D_3^p)] &= \frac{12p\sqrt{4p+8}}{8p+17} + 32p \left[\sqrt{\frac{2p+1}{8p(24p+52)+17}} \right] \\
 &\quad + \frac{4\sqrt{3p}}{24p+1}.
 \end{aligned} \tag{47}$$

Proof. By applying equation (11) and Table 4, we acquire the desired results, i.e.,

$$\begin{aligned}
 ABC_4(G) &= \sum_{rs \in E(G)} \sqrt{\frac{S_r + S_s - 2}{S_r \times S_s}} \\
 &= |E_{(8p+17, 8p+17)}| \sum_{rs \in E[D(D_3^p)]} \sqrt{\frac{S_r + S_s - 2}{S_r \times S_s}} \\
 &\quad + |E_{(8p+17, 24p+1)}| \sum_{rs \in E[D(D_3^p)]} \sqrt{\frac{S_r + S_s - 2}{S_r \times S_s}} + |E_{(24p+1, 24p+1)}| \sum_{rs \in E[D(D_3^p)]} \sqrt{\frac{S_r + S_s - 2}{S_r \times S_s}}, \\
 ABC_4[SD(D_3^p)] &= 6p \left[\sqrt{\frac{16p+32}{(8p+17)^2}} \right] + 8p \left[\sqrt{\frac{32p+16}{(8p+17)(24p+1)}} \right] + \left[\sqrt{\frac{48p}{(24p+1)^2}} \right], \\
 ABC_4[SD(D_3^p)] &= \frac{12p\sqrt{4p+8}}{8p+17} + 32p \left[\sqrt{\frac{2p+1}{8p(24p+52)+17}} \right] + \frac{4\sqrt{3p}}{24p+1}.
 \end{aligned} \tag{48}$$

TABLE 4: Partitioning of edges.

$E[S_r, S_s]$	$E_{(8p+17, 8p+17)}$	$E_{(8p+17, 24p+1)}$	$E_{(24p+1, 24p+1)}$
Number of edges	$6p$	$8p$	1

4. Conclusion

The topological indices provide key information about a molecule's chemical structure and chemical activity, for example, in order to derive quantitative structure-activity relationships (QSARs). These models are derived by applying statistical measures of the molecular structure or properties with descriptors representative of the biological activity (including undesirable side effects) of chemicals (drugs, toxicants, and environmental pollutants). Drug discovery, lead optimization, and toxicity prediction are just a few of the areas in which QSAR is being used. Our purpose in this article was to construct two new graphs from the Dutch Windmill graph using two graph operations, namely, double graph and strong double graph. After that, we calculated some vertex-based topological indices of double graph and strong double graph of the Dutch Windmill graph. Many graph theory applications rely on graph operations. We recommend the readers to compute the topological indices for double and strong double graphs of some other classes of graphs or networks.

Data Availability

No data were used in this manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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