

Retraction

Retracted: Mostar Index of Cycle-Related Structures

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

- (1) Discrepancies in scope
- (2) Discrepancies in the description of the research reported
- (3) Discrepancies between the availability of data and the research described
- (4) Inappropriate citations
- (5) Incoherent, meaningless and/or irrelevant content included in the article
- (6) Manipulated or compromised peer review

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

- [1] F. Asif, A. Kashif, S. Zafar, and M. O. Ojiema, "Mostar Index of Cycle-Related Structures," *Journal of Chemistry*, vol. 2022, Article ID 9411947, 11 pages, 2022.

Research Article

Mostar Index of Cycle-Related Structures

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A topological index is a numerical quantity associated with the molecular structure of a chemical compound. This number remains fixed with respect to the symmetry of a molecular graph. Diverse research studies have shown that the topological indices of symmetrical graphs are interrelated with several physicochemical properties such as boiling point, density, and heat of formation. Peripherality is also an important tool to study topological aspects of molecular graphs. Recently, a bond-additive topological index called the Mostar index that measures the peripherality of a graph is investigated which attained wide attention of researchers. In this article, we compute the Mostar index of cycle-related structures such as the Jahangir graph and the cycle graph with chord.

1. Introduction and Preliminaries

Graph theory is being extensively used in mathematical chemistry for the numerical formulation of chemical compounds by representing atoms as vertices and bonds as edges. The topological index (TI) of a molecular graph is a numerical quantity associated with the molecular structure of a chemical compound [1, 2]. These quantities are well correlated with physicochemical properties and are used as a tool to predict quantitative structure-activity relationships and quantitative structure-property relationships (QSAR/QSPR) [3, 4]. QSAR and QSPR techniques have been widely used to study the structural properties of a molecule and its biological activity [5, 6]. The TIs are majorly classified into two types, namely, degree-based and distance-based or bond-additive topological indices. The degree-based TIs focus on the role of incident bonds towards the molecular structure [7, 8] whereas the distance-based TIs emphasize on the contribution of distances between atoms towards the structure of a compound [9]. The introduction of the first distance-based TI by Wiener in [10] pointed its significance towards the physicochemical properties of the compound.

The physicochemical properties such as boiling point and melting point were shown in correlation with the Wiener index [11]. Afterwards, distance-based TIs such as the Hosoya, Shultz, and Szeged indexes were introduced [12, 13]. The bond-additive TIs have several applications; for example, they are useful in pharmaceutical sciences, in the prediction of physical properties of a molecule, and in complex network theory [14, 15]. To study further topological properties of various chemical compounds, we refer to [16, 17].

In this article, we will consider a simple and finite graph G with a vertex set $V(G)$ and an edge set $E(G)$. The degree of any vertex x , denoted by d_x , is the number of its incident vertices. The distance between any two vertices $x, y \in V(G)$, denoted by $d(x, y)$, is the length of a shortest path connecting them. For any edge $xy \in E(G)$, the collection of all vertices that are nearer to x than y will be called the resolving neighborhood of (x, y) , denoted by $\eta(x, y)$, and its cardinality is denoted by η_{xy} .

Recently, to measure the peripherality of graphs, Doslic et al. proposed a new bond-additive topological index called the Mostar index [18]. It is defined as

$$Mo(G) = \sum_{xy \in E(G)} |n_x - n_y|. \quad (1)$$

The Mostar index measures peripheral atoms and bonds to determine the physical and chemical properties of a molecular graph. Each edge (bond) is peripheral if there are an unequal number of vertices (atoms) in each neighborhood of its end vertices. In chemistry, peripherality is used to measure the nonbalancedness among the bonds of a chemical graph. Recently, many researchers took the initiative to investigate the chemical properties and mathematical perspectives of the Mostar index. Arockiaraj et al. defined the edge Mostar index and computed the Mostar index and edge Mostar index of carbon nanostructures [19]. Hayat and Zhou determined the maximum Mostar index of all n - vertex cacti and upper bound for the Mostar index of n - vertex cacti that contains k - cycles [20]. Huang et al. computed the Mostar index to find the extremal hexagonal chains [21]. The conjecture about the bicyclic graphs characterized by Doslic et al. has been proved by Tepeh [22].

The Mostar index of a graph G can be defined as

$$Mo(G) = \sum_{xy \in E(G)} Mo(xy), \quad (2)$$

where

$$Mo(xy) = |\eta_{xy} - \eta_{yx}|. \quad (3)$$

It is an NP-hard problem to derive general formulas for the Mostar index; therefore, Doslic et al. computed the Mostar index for different classes of graphs. Further, topological properties of some interesting symmetrical structures are computed in the articles [23, 24]. This motivates us to compute the Mostar index of cycle-related graphs such as the Jahangir graph and the cycle graph with chord.

1.1. Main Results. The main results computed in this article are as follows.

Theorem 1. Consider the graphs $J_{n,m}$ and $C_{n,t}$; then,

(1) For $n, m \geq 5$,

$$Mo(J_{n,m}) = \begin{cases} m^2 n^2 + 2m^2 n - mn^2 + m^2 - 6mn - 5m, & \text{if } n = \text{odd}, \\ m^2 n^2 + 2m^2 n - mn^2 + m^2 - 5mn - m, & \text{if } n = \text{even}. \end{cases} \quad (4)$$

(2) For $n \geq 9$ and $t \geq 3$,

$$Mo(C_{n,t}) = \begin{cases} 2(t-2)(n-t), & \text{if } n = \text{odd}, \\ 2(n-t)(t-2), & \text{if } n = \text{even}, t = \text{odd}, \\ 2(n-t)(t-2) - 2, & \text{if } n = \text{even}, t = \text{even}. \end{cases} \quad (5)$$

consists of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm} as shown in Figure 1. The vertex set of $J_{n,m}$ is $V(J_{n,m}) = \{u, w_1, w_2, \dots, w_m\} \cup \{v_1, v_2, \dots, v_{nm}\}$. The edge set of $J_{n,m}$ is $E(J_{n,m}) = \{uw_k | 1 \leq k \leq m\} \cup \{w_1 v_1, w_1 v_{nm}, w_k v_{(k-1)n}, w_k v_{(k-1)n+1} | 2 \leq k \leq m\} \cup \{v_{ni+1} v_{ni+2}, \dots, v_{ni+(n-1)} v_{ni+n} | 0 \leq i \leq m-1\}$. The edge set of $J_{n,m}$ is partitioned as follows.

For $n = \text{even}$,

2. The Mostar Index of the Jahangir Graph

In this section, the Mostar index of the family of the Jahangir graph $J_{n,m}$ for $n, m \geq 5$ is computed. The Jahangir graph $J_{n,m}$

$$\begin{aligned} E_1(J_{n,m}) &= \{uw_k | 1 \leq k \leq m\}, \\ E_2(J_{n,m}) &= \{w_k v_{(k-1)n}, w_k v_{(k-1)n+1} | 2 \leq k \leq m\} \cup \{w_1 v_1, w_1 v_{nm}\}, \\ E_3(J_{n,m}) &= \{v_{ni+1} v_{ni+2}, \dots, v_{((2i+1)n-4)/2} v_{((2i+1)n-2)/2}, v_{((2i+1)n+4)/2} v_{((2i+1)n+6)/2}, \\ &\quad \dots, v_{ni+(n-1)} v_{ni+n} | 0 \leq i \leq m-1\}, \\ E_4(J_{n,m}) &= \{v_{((2i+1)n-2)/2} v_{((2i+1)n)/2}, v_{((2i+1)n+2)/2} v_{((2i+1)n+4)/2} | 0 \leq i \leq m-1\}, \\ E_5(J_{n,m}) &= \{v_{((2i+1)n)/2} v_{((2i+1)n+2)/2} | 0 \leq i \leq m-1\}. \end{aligned} \quad (6)$$

For $n = \text{odd}$,

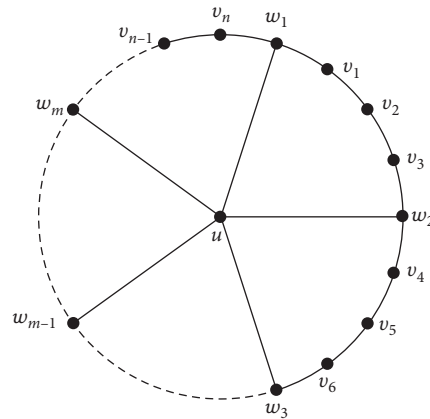


FIGURE 1: The Jahangir graph $J_{3,m}$.

$$\begin{aligned}
 E'_1(J_{n,m}) &= \{uw_k | 1 \leq k \leq m\}, \\
 E'_2(J_{n,m}) &= \{w_k v_{(k-1)n}, w_k v_{(k-1)n+1} | 2 \leq k \leq m\} \cup \{w_1 v_1, w_1 v_{mn}\}, \\
 E'_3(J_{n,m}) &= \{v_{ni+1} v_{ni+2}, \dots, v_{((2i+1)n-3)/2} v_{((2i+1)n-1)/2}, v_{((2i+1)n+3)/2} v_{((2i+1)n+5)/2} - \\
 &\quad \dots, v_{ni+(n-1)} v_{ni+n} | 0 \leq i \leq m-1\}, \\
 E'_4(J_{n,m}) &= \{v_{((2i+1)n-1)/2} v_{((2i+1)n+1)/2}, v_{((2i+1)n+1)/2} v_{((2i+1)n+3)/2} | 0 \leq i \leq m-1\}.
 \end{aligned} \tag{7}$$

In the following lemmas, the Mostar index corresponding to equations (6) and (7) of $J_{n,m}$ is computed.

Lemma 1. For $xy \in E(J_{n,m})$ where n is even and $n, m \geq 5$, then

$$Mo(xy) = \begin{cases} mn - 2n + m - 3, & \text{if } xy \in E_1(J_{n,m}), \\ mn + m - n - 1, & \text{if } xy \in E_2(J_{n,m}), \\ mn - n + m - 2, & \text{if } xy \in E_3(J_{n,m}), \\ \frac{2mn + 2m - 3n - 4}{2}, & \text{if } xy \in E_4(J_{n,m}), \\ 0, & \text{if } xy \in E_5(J_{n,m}). \end{cases} \tag{8}$$

Proof. The symmetry of graph $J_{n,m}$ allows us to discuss the Mostar index of only the following types of edges:

Case (i) (for $xy \in E_1(J_{m,n})$): let $x = u$ and $y = w_1$, the resolving neighborhoods of (u, w_1) and (w_1, u) are $\{u, w_2, \dots, w_m, v_{n+4/2}, \dots, v_{m-(n+2/2)}\}$ and $\{w_1, v_1, \dots, v_{n/2}, v_{m-(n-2/2)}, \dots, v_{mn}\}$, respectively. Hence, $\eta_{uw_1} = mn - n + m - 2$ and $\eta_{w_1u} = n + 1$. Therefore, in view of equation (3), $Mo(uw_1) = mn - 2n + m - 3$.

Case (ii) (for $xy \in E_2(J_{m,n})$): let $x = v_1$ and $y = w_1$, by means of Figure 1, $\eta(v_1, w_1) = \{v_1, \dots, v_{n+2/2}\}$ and

$\eta(w_1, v_1) = \{u, w_1, \dots, w_m, v_{n+5/2}, \dots, v_{mn}\}$. Hence, $\eta_{v_1 w_1} = (n + 2/2)$ and $\eta_{w_1 v_1} = (2mn - n + 2m - 1)/2$. Therefore, using equation (3), $Mo(v_1 w_1) = mn + m - n - 1$.

Case (iii) (for $xy \in E_3(J_{m,n})$): let $x = v_i$ and $y = v_{i+1}$ for $1 \leq i \leq (n - 3/2)$, the resolving neighborhoods of (v_i, v_{i+1}) and (v_{i+1}, v_i) are $\{u, w_1, \dots, w_m, v_1, \dots, v_i, v_{(n+2i+6)/2}, \dots, v_{mn}\}$ and $\{v_{i+1}, \dots, v_{n+2i+2/2}\}$, respectively. Hence, $\eta_{v_i v_{i+1}} = (2mn - n + 2m - 2)/2$ and $\eta_{v_{i+1} v_i} = (n + 2)/2$. Therefore, using equation (3), $Mo(v_i v_{i+1}) = mn - n + m - 2$.

Case (iv) (for $xy \in E_4(J_{m,n})$): let $x = v_{(n-2/2)}$ and $y = v_{n/2}$, by means of Figure 1, we obtain $\eta(v_{(n-2/2)}, v_{n/2}) = \{u, w_1, w_3, \dots, w_m, v_1, \dots, v_{(n-2/2)}, v_{(3n+2/2)}, \dots, v_{mn}\}$ and $\eta(v_{n/2}, v_{(n-2/2)}) = \{v_{n/2}, \dots, v_n\}$. Therefore, $\eta_{v_{(n-2/2)} v_{n/2}} = mn - n + m - 1$ and $\eta_{v_{n/2} v_{(n-2/2)}} = (n + 2/2)$. Hence, using equation (3), $Mo(v_{(n-2/2)} v_{n/2}) = (2mn + 2m - 3n - 4)/2$.

Case (v) (for $xy \in E_5(J_{m,n})$): let $x = v_{n/2}$ and $y = v_{(n+2/2)}$, the resolving neighborhoods of $(v_{n/2}, v_{(n+2/2)})$ and $(v_{(n+2/2)}, v_{n/2})$ are $\{w_1, v_1, \dots, v_{n/2}, v_{2(m-1)/2}, \dots, v_{mn}\}$ and $\{w_2, v_{(n+2/2)}, \dots, v_{(3n+2/2)}\}$, respectively. Hence, $\eta_{v_{n/2} v_{(n+2/2)}} = n + 2$ and $\eta_{v_{(n+2/2)} v_{n/2}} = n + 2$. Therefore, in view of equation (3), $Mo(v_{(n+2/2)} v_{n/2}) = 0$. \square

Lemma 2. For $xy \in E(J_{n,m})$ where n is odd and $n, m \geq 5$, then

$$Mo(xy) = \begin{cases} mn - 2n + m - 3, & \text{if } xy \in E_1'(J_{n,m}), \\ mn - n + m - 2, & \text{if } xy \in E_2'(J_{n,m}), \\ mn - n + m - 2, & \text{if } xy \in E_3'(J_{n,m}), \\ mn - 2n + m - 3, & \text{if } xy \in E_4'(J_{n,m}). \end{cases} \quad (9)$$

Proof. The symmetry of the Jahangir graph $J_{n,m}$ allows us to compute the Mostar index of the following type of edges:

Case (i) (for $xy \in E_1(J_{m,n})$): let $x = u$ and $y = w_1$, the resolving neighborhoods of (u, w_1) and (w_1, u) are $\{u, w_2, \dots, w_m, v_{(n+3)/2}, \dots, v_{mn-(n+1)/2}\}$ and $\{w_1, v_1, \dots, v_{(n+1)/2}, v_{mn-(n-1)/2}, \dots, v_{mn}\}$, respectively. Hence, $\eta_{uw_1} = mn - n + m - 1$ and $\eta_{w_1u} = n + 2$. Therefore, applying equation (3), $Mo(uw_1) = mn - 2n + m - 3$.

Case (ii) (for $xy \in E_2(J_{m,n})$): let $x = v_1$ and $y = w_1$, by means of Figure 1, $\eta(v_1, w_1) = \{v_1, \dots, v_{(n+3)/2}\}$ and $\eta(w_1, v_1) = \{u, w_1, \dots, w_m, v_{(n+5)/2}, \dots, v_{mn}\}$. Therefore, $\eta_{v_1w_1} = (n + 3)/2$ and $\eta_{w_1v_1} = (2mn - n + 2m - 1)/2$. Hence, using equation (3), $Mo(v_1w_1) = mn + m - n - 2$.

Case (iii) (for $xy \in E_3(J_{m,n})$): let $x = v_i$ and $y = v_{i+1}$ for $1 \leq i \leq (n - 4)/2$, the resolving neighborhoods of (v_i, v_{i+1}) and (v_{i+1}, v_i) are $\{u, w_1, \dots, w_m, v_1, \dots, v_i, v_{(n+2i+5)/2}, \dots, v_{mn}\}$ and $\{v_{i+1}, \dots, v_{(n+2i+1)/2}\}$, respectively. Hence, $\eta_{v_i v_{i+1}} = (2mn - n + 2m - 1)/2$ and $\eta_{v_{i+1} v_i} = (n + 3)/2$. Therefore, using equation (3), $Mo(v_i v_{i+1}) = mn - n + m - 2$.

Case (iv) (for $xy \in E_4(J_{m,n})$): let $x = v_{(n-1)/2}$ and $y = v_{(n+1)/2}$, the resolving neighborhoods of $(v_{(n-1)/2}, v_{(n+1)/2})$ and $(v_{(n+1)/2}, v_{(n-1)/2})$ are $\{u, w_1, w_3, \dots, w_m, v_1, \dots, v_{(n-1)/2}, v_{3(n+1)/2}, \dots, v_{mn}\}$ and $\{w_1, v_{(n+1)/2}, \dots, v_{(3n+1)/2}\}$, respectively. Therefore, $\eta_{v_{(n-1)/2} v_{(n+1)/2}} = mn - n + m - 1$ and $\eta_{v_{(n+1)/2} v_{(n-1)/2}} = n + 2$. Hence, in view of equation (3), $Mo(v_{(n-1)/2} v_{(n+1)/2}) = mn - 2n + m - 3$. \square

In the following theorem, the Mostar index of the Jahangir graph $J_{n,m}$ for $n, m \geq 5$ is computed.

Theorem 2. For $xy \in E(J_{n,m})$, then

$$Mo(J_{n,m}) = \begin{cases} m^2 n^2 + 2m^2 n - mn^2 + m^2 - 5mn - m, & \text{if } n = \text{even}, \\ m^2 n^2 + 2m^2 n - mn^2 + m^2 - 6mn - 5m, & \text{if } n = \text{odd}. \end{cases} \quad (10)$$

Proof.

Case (i) $n = \text{even}$: in view of Lemma 1 and equation (2), we have

$$\begin{aligned} Mo(J_{n,m}) &= \sum_{xy \in E_1(J_{n,m})} |n_{xy} - n_{yx}| + \sum_{xy \in E_2(J_{n,m})} |n_{xy} - n_{yx}| \\ &+ \sum_{xy \in E_3(J_{n,m})} |n_{xy} - n_{yx}| + \sum_{xy \in E_4(J_{n,m})} |n_{xy} - n_{yx}| \\ &+ \sum_{xy \in E_5(J_{n,m})} |n_{xy} - n_{yx}| \\ &= m(mn - 2n + m - 3) + 2m(mn + m - n - 1) - \\ &+ m(n - 4)(mn - n + m - 2) + m(2mn + 2m - 3n - 4) + 0, \\ &= m^2 n^2 + 2m^2 n - mn^2 + m^2 - 5mn - m. \end{aligned} \quad (11)$$

Case (ii) $n = \text{odd}$: it is easy to see from Lemma 2 and equation (2) that

$$\begin{aligned}
Mo(J_{n,m}) &= \sum_{xy \in E'_1(J_{n,m})} |n_{xy} - n_{yx}| + \sum_{xy \in E'_2(J_{n,m})} |n_{xy} - n_{yx}| \\
&+ \sum_{xy \in E'_3(J_{n,m})} |n_{xy} - n_{yx}| + \sum_{xy \in E'_4(J_{n,m})} |n_{xy} - n_{yx}|. \\
&= m(mn - 2n + m - 3) + 2m(mn + m - n - 2) \\
&+ m(n - 3)(mn - n + m - 2) + 2m(mn - 2n + m - 3), \\
&= m^2 n^2 + 2m^2 n - mn^2
\end{aligned} \tag{12}$$

Here, we present a numerical example in support of the above theorem.

Example 1. Consider the Jahangir graph $J_{5,5}$; then, the sets $E'_1(J_{5,5}) = \{uw_1, uw_2, \dots, uw_5\}$, $E'_2(J_{5,5}) = \{w_1v_{25}, w_2v_5, \dots, w_5v_{21}\}$, $E'_3(J_{5,5}) = \{v_1v_2, v_4v_5, \dots, v_{24}v_{25}\}$, and $E'_4(J_{5,5}) = \{v_2v_3, v_3v_4, \dots, v_{23}v_{24}\}$ form the edge partition of $J_{5,5}$. Now for respective edges xy , st , uv , and pq in these partitions, Lemma 1 implies that $\eta_{xy} = 24$, $\eta_{yx} = 7$, and $Mo(xy) = 17$; $\eta_{st} = 27$, $\eta_{ts} = 4$, and $Mo(st) = 23$; $\eta_{uv} = 27$, $\eta_{vu} = 4$, and $Mo(uv) = 23$; $\eta_{pq} = 24$, $\eta_{qp} = 7$, and $Mo(pq) = 17$. Therefore, using Theorem 2, $Mo(J_{5,5}) = 5 \times 17 + 2 \times 5 \times 23 + 5 \times (5 - 4) \times 23 + 2 \times 5 \times 17 = 600$.

3. The Mostar Index of Cycle with a Chord Graph

In this section, the Mostar index of a cycle with a chord graph $C_{n,t}$ is computed. The graph $C_{n,t}$ is obtained from a cycle C_n by joining its two vertices at a distance $t - 1$ as shown in Figure 2. The vertex set and edge set of $C_{n,t}$ are $V(C_{n,t}) = \{v_1, v_2, \dots, v_n\}$ and $E(C_{n,t}) = \{v_i v_{i+1} | 1 \leq i \leq n\} \cup \{v_1 v_t | 3 \leq t \leq (n + 3)/2\}$, respectively, where subscripts are taken as mod n .

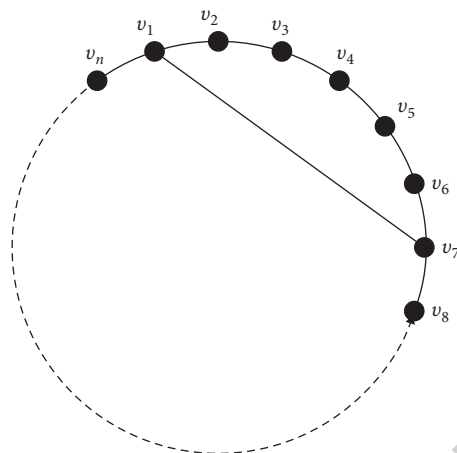
The edge set of $C_{n,t}$ is partitioned as follows.

For $n = \text{odd}$ and $t = \text{even}$,

$$\begin{aligned}
E_1^1(C_{n,t}) &= \{v_{t/2} v_{(t+2)/2}\}, \\
E_2^1(C_{n,t}) &= \{v_i v_{i+1} | 1 \leq i \leq v_{(t-2)/2}, v_{(t+2)/2} \leq i \leq t - 1\}, \\
E_3^1(C_{n,t}) &= \{v_{(n+t-1)/2} v_{(n+t+1)/2}, v_{(n+t+1)/2} v_{(n+t+3)/2}\}, \\
E_4^1(C_{n,t}) &= \{v_{(n+t-3)/2} v_{(n+t-1)/2}, v_{(n+t+3)/2} v_{(n+t+5)/2}\}, \\
E_5^1(C_{n,t}) &= \{v_i v_{i+1} | t \leq i \leq \frac{n+t-5}{2}, \frac{n+t+5}{2} \leq i \leq n\}, \\
E_6^1(C_{n,t}) &= \{v_1 v_t\}.
\end{aligned} \tag{13}$$

For $n = \text{odd}$, $t = \text{odd}$:

$$\begin{aligned}
E_1^2(C_{n,t}) &= \{v_{t/2} v_{(t+2)/2}\}, \\
E_2^2(C_{n,t}) &= \{v_i v_{i+1} | 1 \leq i \leq \frac{t-3}{2}, \frac{t+3}{2} \leq i \leq t - 1\}, \\
E_3^2(C_{n,t}) &= \{v_{(n+t)/2} v_{(n+t+2)/2}\}, \\
E_4^2(C_{n,t}) &= \{v_{(n+t-2)/2} v_{(n+t)/2}, v_{(n+t+2)/2} v_{(n+t+4)/2}\}, \\
E_5^2(C_{n,t}) &= \{v_i v_{i+1} | t \leq i \leq \frac{n+t-4}{2}, \frac{n+t+4}{2} \leq i \leq n\}, \\
E_6^2(C_{n,t}) &= \{v_1 v_t\}.
\end{aligned} \tag{14}$$

FIGURE 2: The cycle with a chord graph $C_{n,7}$.

For $n = \text{even}$ and $t = \text{even}$,

$$E_1^3(C_{n,t}) = \{v_{t/2}v_{(t+2)/2}\},$$

$$E_2^3(C_{n,t}) = \left\{v_i v_{i+1} \mid 1 \leq i \leq \frac{t-2}{2}, \frac{t+2}{2} \leq i \leq t-1\right\},$$

$$E_3^3(C_{n,t}) = \{v_{(n+t)/2}v_{(n+t+2)/2}\},$$

$$E_4^3(C_{n,t}) = \{v_{(n+t-2)/2}v_{(n+t)/2}, v_{(n+t)/2}v_{(n+t+2)/2}\},$$

$$E_5^3(C_{n,t}) = \left\{v_i v_{i+1} \mid t \leq i \leq \frac{n+t-4}{2}, \frac{n+t+2}{2} \leq i \leq n\right\},$$

$$E_6^3(C_{n,t}) = \{v_1 v_t\}.$$

For $n = \text{even}$ and $t = \text{odd}$,

$$E_1^4(C_{n,t}) = \{v_{(t-1)/2}v_{(t+1)/2}, v_{(t+1)/2}v_{(t+3)/2}\},$$

$$E_2^4(C_{n,t}) = \left\{v_i v_{i+1} \mid 1 \leq i \leq \frac{t-3}{2}, \frac{t+3}{2} \leq i \leq t-1\right\},$$

$$E_3^4(C_{n,t}) = \{v_{(n+t-1)/2}v_{(n+t+1)/2}, v_{(n+t+1)/2}v_{(n+t+3)/2}\},$$

$$E_4^4(C_{n,t}) = \{v_{(n+t-3)/2}v_{(n+t-1)/2}, v_{(n+t+3)/2}v_{(n+t+5)/2}\},$$

$$E_5^4(C_{n,t}) = \left\{v_i v_{i+1} \mid t \leq i \leq \frac{n+t-4}{2}, \frac{n+t+2}{2} \leq i \leq n\right\},$$

$$E_6^4(C_{n,t}) = \{v_1 v_t\}.$$

In the following lemmas, the Mostar index corresponding to equations (13)–(16) of $C_{n,t}$ for $n \geq 9$ and $t \geq 3$ is computed.

Lemma 3. Let $xy \in E(C_{n,t})$ where n is odd and t is even and $4 \leq t < (n+3)/2$, then

$$Mo(xy) = \begin{cases} 0, & \text{if } xy \in E_1^1(C_{n,t}), \\ n-t, & \text{if } xy \in E_2^1(C_{n,t}), \\ \frac{t-2}{2}, & \text{if } xy \in E_3^1(C_{n,t}), \\ t-2, & \text{if } xy \in E_4^1(C_{n,t}), \\ t-2, & \text{if } xy \in E_5^1(C_{n,t}), \\ 0, & \text{if } xy \in E_6^1(C_{n,t}). \end{cases} \quad (17)$$

Proof.

Case (i) (for $xy \in E_1^1(C_{n,t})$): let $x = v_{t/2}$ and $y = v_{t+2/2}$, the resolving neighborhoods of $(v_{t/2}, v_{t+2/2})$ and $(v_{t+2/2}, v_{t/2})$ are $\{v_1, \dots, v_{t/2}, v_{(n+t+3)/2}, \dots, v_n\}$ and $\{v_{t+2/2}, \dots, v_{n+t-1/2}\}$, respectively. Hence, $\eta_{v_{t/2}v_{t+2/2}} = \eta_{v_{t+2/2}v_{t/2}} = (n-1/2)$. Therefore, in view of equation (3), $Mo(v_{t/2}v_{t+2/2}) = 0$.

Case (ii) (for $xy \in E_2^1(C_{n,t})$): let $x = v_i$ and $y = v_{i+1}$ for $1 \leq i \leq (t-2)/2$, by means of Figure 2, $\eta(v_i, v_{i+1}) = \{v_1, \dots, v_i, v_{(t+2i+2)/2}, \dots, v_n\}$ and $\eta(v_{i+1}, v_i) = \{v_{i+1}, \dots, v_{(t+2i)/2}\}$. Hence, $\eta_{v_i v_{i+1}} = (2n-t)/2$ and $\eta_{v_{i+1} v_i} = (t/2)$. Therefore, equation (3) implies $Mo(v_i v_{i+1}) = n-t$.

Case (iii) (for $xy \in E_3^1(C_{n,t})$): let $x = v_{(n+t-1)/2}$ and $y = v_{(n+t+1)/2}$, then $\eta(v_{(n+t-1)/2}, v_{(n+t+1)/2}) = \{v_{(t+2)/2}, \dots, v_{(n+t-1)/2}\}$ and $\eta(v_{(n+t+1)/2}, v_{(n+t-1)/2}) = \{v_{(n+t-1)/2}, \dots, v_n\}$. Hence, $\eta_{v_{(n+t-1)/2} v_{(n+t+1)/2}} = (n-1)/2$ and

$\eta_{v_{(n+t+1)/2}v_{(n+t-1)/2}} = (n-t+1)/2$. Therefore, applying equation (3), $Mo(v_{(n+t-1)/2}v_{(n+t+1)/2}) = (t-2)/2$.

Case (iv) (for $xy \in E_4^1(C_{n,t})$): let $x = v_{(n+t-3)/2}$ and $y = v_{(n+t-1)/2}$, the resolving neighborhoods of $(v_{(n+t-3)/2}, v_{(n+t-1)/2})$ and $(v_{(n+t-1)/2}, v_{(n+t-3)/2})$ are $\{v_1, \dots, v_{(n+t-3)/2}\}$ and $\{v_{(n+t-1)/2}, \dots, v_{n-1}\}$, respectively. Therefore, $\eta_{v_{(n+t-3)/2}v_{(n+t-1)/2}} = (n+t-3)/2$ and $\eta_{v_{(n+t-1)/2}v_{(n+t-3)/2}} = (n+t-1)/2$. Hence, using equation (3), $Mo(v_{(n+t-3)/2}v_{(n+t-1)/2}) = t-2$.

Case (v) (for $xy \in E_5^1(C_{n,t})$): let $x = v_i$ and $y = v_{i+1}$ for $t \leq i \leq (n+t-5)/2$, the resolving neighborhoods of (v_i, v_{i+1}) and (v_{i+1}, v_i) are $\{v_1, \dots, v_i, v_{(n+t+2i+4)/2}, \dots, v_n\}$ and $\{v_{i+1}, \dots, v_{(n-t+2i+2)/2}\}$, respectively. Hence, $\eta_{v_i v_{i+1}} = (n+t-2)/2$ and $\eta_{v_{i+1} v_i} = (n-t+2)/2$. Therefore, in view of equation (3), $Mo(v_i v_{i+1}) = t-2$.

Case (vi) (for $xy \in E_6^1(C_{n,t})$): let $x = v_1$ and $y = v_t$, then we have $\eta(v_1, v_t) = \{v_1, \dots, v_{t/2}, v_{(n+t+3)/2}, \dots, v_n\}$ and $\eta(v_t, v_1) = \{v_{(t+2)/2}, \dots, v_{(n+t-1)/2}\}$. Hence, $\eta_{v_1 v_t} = \eta_{v_t v_1} = (n-1)/2$. Therefore, using equation (3), $Mo(v_1 v_t) = 0$. \square

Lemma 4. Let $xy \in E(C_{n,t})$ where n and t are odd and $3 \leq t \leq (n+1)/2$, then

$$Mo(xy) = \begin{cases} \frac{n-t}{2}, & \text{if } xy \in E_1^2(C_{n,t}), \\ n-t, & \text{if } xy \in E_2^2(C_{n,t}), \\ 0, & \text{if } xy \in E_3^2(C_{n,t}), \\ t-2, & \text{if } xy \in E_4^2(C_{n,t}), \\ t-2, & \text{if } xy \in E_5^2(C_{n,t}), \\ 0, & \text{if } xy \in E_6^2(C_{n,t}). \end{cases} \quad (18)$$

Proof.

Case (i) (for $xy \in E_1^2(C_{n,t})$): let $x = v_{(t-1)/2}$ and $y = v_{(t-1)/2}$, the resolving neighborhoods of $(v_{(t-1)/2}, v_{(t+1)/2})$ and $(v_{(t+1)/2}, v_{(t-1)/2})$ are $\{v_1, \dots, v_{(t-1)/2}, v_{(n+t+2)/2}, \dots, v_n\}$ and $\{v_{(t-1)/2}, \dots, v_{t-1}\}$, respectively. Hence, $\eta_{v_{(t-1)/2}v_{(t+1)/2}} = (n-1)/2$ and $\eta_{v_{(t+1)/2}v_{(t-1)/2}} = (t-1)/2$. Therefore, using equation (3), $Mo(v_{(t-1)/2}v_{(t+1)/2}) = (n-t)/2$.

Case (ii) (for $xy \in E_2^2(C_{n,t})$): let $x = v_i$ and $y = v_{i+1}$ for $1 \leq i \leq (t-3)/2$, by means of Figure 2, $\eta(v_i, v_{i+1}) = \{v_1, \dots, v_i, v_{(t+2i+3)/2}, \dots, v_n\}$ and $\eta(v_{i+1}, v_i) = \{v_{i+1}, \dots, v_{(t+2i-1)/2}\}$. Hence, $\eta_{v_i v_{i+1}} = (2n -$

$t-1)/2$ and $\eta_{v_{i+1} v_i} = (t-1)/2$. Therefore, in view of equation (3), $Mo(v_i v_{i+1}) = n-t$.

Case (iii) (for $xy \in E_3^2(C_{n,t})$): let $x = (n+t)/2$ and $y = v_{(n+t+2)/2}$, then $\eta(v_{(n+t)/2}, v_{(n+t+2)/2}) = \{v_1, \dots, v_{(t-1)/2}, v_{(n+t+2)/2}, \dots, v_n\}$ and $\eta(v_{(n+t+2)/2}, v_{(n+t)/2}) = \{v_{(t+3)/2}, \dots, v_{(n+t)/2}\}$. Hence, $\eta_{v_{(n+t)/2}v_{(n+t+2)/2}} = (n-1)/2$ and $\eta_{v_{(n+t+2)/2}v_{(n+t)/2}} = (n-1)/2$. Hence, equation (3) implies $Mo(v_{(n+t)/2}v_{(n+t+2)/2}) = 0$.

Case (iv) (for $xy \in E_4^2(C_{n,t})$): let $x = v_{(n+t-2)/2}$ and $y = v_{(n+t)/2}$, the resolving neighborhoods of $(v_{(n+t-2)/2}, v_{(n+t)/2})$ and $(v_{(n+t)/2}, v_{(n+t-2)/2})$ are $\{v_1, \dots, v_{(n+t-2)/2}\}$ and $\{v_{(n+t)/2}, \dots, v_n\}$, respectively. Therefore, $\eta_{v_{(n+t-2)/2}v_{(n+t)/2}} = (n+t-2)/2$ and $\eta_{v_{(n+t)/2}v_{(n+t-2)/2}} = (n-t+2)/2$. Henceforth, applying equation (3), $Mo(v_{(n+t-2)/2}v_{(n+t)/2}) = t-2$.

Case (v) (for $xy \in E_5^2(C_{n,t})$): let $x = v_i$ and $y = v_{i+1}$ where $t \leq i \leq (n+t-4)/2$, the resolving neighborhoods of $v_i v_{i+1}$ and $v_{i+1} v_i$ are $\{v_1, \dots, v_i, v_{(n-t+2i+4)/2}, \dots, v_n\}$ and $\{v_{i+1}, \dots, v_{(n-t+2i+2)/2}\}$, respectively. Hence, $\eta_{v_i v_{i+1}} = (n+t-2)/2$ and $\eta_{v_{i+1} v_i} = (n-t+2)/2$. Therefore, using equation (3), $Mo(v_i v_{i+1}) = t-2$.

Case (vi) (for $xy \in E_6^2(C_{n,t})$): let $x = v_1$, $y = v_t$, the resolving neighborhoods of (v_1, v_t) and (v_t, v_1) are $\{v_1, \dots, v_{(t-1)/2}, v_{(n+t+2)/2}, \dots, v_n\}$ and $\{v_{(t+3)/2}, \dots, v_{(n+t)/2}\}$ respectively. Hence, $\eta_{v_1 v_t} = (n-1)/2$ and $\eta_{v_t v_1} = (n-1)/2$. Therefore, equation (3) implies $Mo(v_1 v_t) = 0$. \square

Lemma 5. Let $xy \in E(C_{n,t})$ where n and t are even and $4 \leq t \leq (n+2)/2$, then

$$Mo(xy) = \begin{cases} 0, & \text{if } xy \in E_1^3(C_{n,t}), \\ n-t, & \text{if } xy \in E_2^3(C_{n,t}), \\ 0, & \text{if } xy \in E_3^3(C_{n,t}), \\ t-2, & \text{if } xy \in E_4^3(C_{n,t}), \\ t-2, & \text{if } xy \in E_5^3(C_{n,t}), \\ 0, & \text{if } xy \in E_6^3(C_{n,t}). \end{cases} \quad (19)$$

Proof.

Case (i) (for $xy \in E_1^3(C_{n,t})$): let $x = v_{t/2}$ and $y = v_{(t+2)/2}$, the resolving neighborhoods of $(v_{t/2}, v_{(t+1)/2})$ and $(v_{(t+1)/2}, v_{t/2})$ are $\{v_1, \dots, v_{t/2}, v_{(n+t+2)/2}, \dots, v_n\}$ and $\{v_{(t+2)/2}, \dots, v_{(n+t)/2}\}$, respectively. Hence, $\eta_{v_{t/2}v_{(t+1)/2}} = n/2$ and $\eta_{v_{(t+1)/2}v_{t/2}} = n/2$. Therefore, using equation (3), $Mo(v_{t/2}v_{(t+2)/2}) = 0$.

Case (ii) (for $xy \in E_2^3(C_{n,t})$): let $x = v_i$ and $y = v_{i+1}$ for $1 \leq i \leq (t-2)/2$, the resolving neighborhoods of (v_i, v_{i+1}) and v_{i+1}, v_i are $\{v_1, \dots, v_i, v_{(t+2i+2)/2}, \dots, v_n\}$ and $\{v_{i+1}, \dots, v_{(t+2i)/2}\}$, respectively. Therefore, $\eta_{v_i v_{i+1}} =$

$(2n - t)/2$ and $\eta_{v_{i+1}, v_i} = t/2$. Hence, equation (3) implies $Mo(v_i, v_{i+1}) = n - t$.

Case (iii) (for $xy \in E_3^3(C_{n,t})$): let $x = v_{(n+t)/2}$ and $y = v_{(n+t+2)/2}$, by means of Figure 2, $\eta(v_{(n+t)/2}, v_{(n+t+2)/2}) = \{v_{(t+2)/2}, \dots, v_{(n+t)/2}\}$ and $\eta(v_{(n+t+2)/2}, v_{(n+t)/2}) = \{v_1, \dots, v_{t/2}, v_{(n+t+2)/2}, \dots, v_n\}$. Therefore, $\eta_{v_{(n+t)/2}, v_{(n+t+2)/2}} = n/2$ and $\eta_{v_{(n+t+2)/2}, v_{(n+t)/2}} = n/2$. Hence, applying equation (3), $Mo(v_{(n+t)/2}, v_{(n+t+2)/2}) = 0$.

Case (iv) (for $xy \in E_4^3(C_{n,t})$): let $x = v_{(n+t-2)/2}$ and $y = v_{(n+t)/2}$, then $\eta(v_{(n+t-2)/2}, v_{(n+t)/2}) = \{v_1, \dots, v_{(n+t-2)/2}\}$ and $\eta(v_{(n+t)/2}, v_{(n+t-2)/2}) = \{v_{(n+t)/2}, \dots, v_n\}$. Hence, $\eta_{v_{(n+t-2)/2}, v_{(n+t)/2}} = (n+t-2)/2$ and $\eta_{v_{(n+t)/2}, v_{(n+t-2)/2}} = (n-t+2)/2$. Therefore, in view of equation (3), $Mo(v_{(n+t-2)/2}, v_{(n+t)/2}) = t - 2$.

Case (v) (for $xy \in E_5^3(C_{n,t})$): let $x = v_i$ and $y = v_{i+1}$ for $t \leq i \leq (n+t-4)/2$, the resolving neighborhoods of (v_i, v_{i+1}) and (v_{i+1}, v_i) are $\{v_1, \dots, v_i, v_{(n-t+2i+4)/2}, \dots, v_n\}$ and $\{v_{i+1}, \dots, v_{(n-t+2i+2)/2}\}$, respectively. Hence, $\eta_{v_i, v_{i+1}} = (n+t-2)/2$ and $\eta_{v_{i+1}, v_i} = (n-t+2)/2$. Therefore, $Mo(v_i, v_{i+1}) = t - 2$.

Case (vi) (for $xy \in E_6^3(C_{n,t})$): let $x = v_1$ and $y = v_t$, then resolving neighborhoods are $\eta(v_1, v_t) = \{v_1, \dots, v_{t/2}, v_{(n+t+2)/2}, \dots, v_n\}$ and $\eta(v_t, v_1) = \{v_{(t+2)/2}, \dots, v_{(n+t)/2}\}$. Hence, $\eta_{v_1, v_t} = n/2$ and $\eta_{v_t, v_1} = n/2$. Therefore, using equation (3), $Mo(v_1, v_t) = 0$. \square

Lemma 6. Let $xy \in E(C_{n,t})$ where n is even and t is odd and $3 \leq t \leq (n+2)/2$, then

$$Mo(xy) = \begin{cases} \frac{n-t+1}{2}, & \text{if } xy \in E_1^4(C_{n,t}), \\ n-t, & \text{if } xy \in E_2^4(C_{n,t}), \\ \frac{t-1}{2}, & \text{if } xy \in E_3^4(C_{n,t}), \\ t-2, & \text{if } xy \in E_4^4(C_{n,t}), \\ n-t-3, & \text{if } xy \in E_5^4(C_{n,t}), \\ 0, & \text{if } xy \in E_6^4(C_{n,t}). \end{cases} \quad (20)$$

Proof.

Case (i) (for $xy \in E_1^4(C_{n,t})$): let $x = v_{(t-1)/2}$ and $y = v_{(t+1)/2}$, the resolving neighborhoods of $(v_{(t-1)/2}, v_{(t+1)/2})$ and $(v_{(t+1)/2}, v_{(t-1)/2})$ are

$\{v_1, \dots, v_{(t-1)/2}, v_{(n+t+1)/2}, \dots, v_n\}$ and $\{v_{(t+1)/2}, \dots, v_{t-1}\}$, respectively. Hence, $\eta_{v_{(t-1)/2}, v_{(t+1)/2}} = n/2$ and $\eta_{v_{(t+1)/2}, v_{(t-1)/2}} = (t-1)/2$. Therefore, using equation (3), $Mo(v_{(t-1)/2}, v_{(t+1)/2}) = 2(n-t+1)/2 = (n-t+1)/2$.

Case (ii) (for $xy \in E_2^4(C_{n,t})$): let $x = v_i$ and $y = v_{i+1}$ for $1 \leq i \leq (t-3)/2$, by means of Figure 2, $\eta(v_i, v_{i+1}) = \{v_1, \dots, v_i, v_{(t+2i+3)/2}, \dots, v_n\}$ and $\eta(v_{i+1}, v_i) = \{v_{i+1}, \dots, v_{(t+2i-1)/2}\}$. Hence, $\eta_{v_i, v_{i+1}} = (2n-t-1)/2$ and $\eta_{v_{i+1}, v_i} = (t-1)/2$. Therefore, in view of equation (3), $Mo(v_i, v_{i+1}) = n - t$.

Case (iii) (for $xy \in E_3^4(C_{n,t})$): let $x = v_{(n+t-1)/2}$ and $y = v_{(n+t-1)/2}$, the resolving neighborhoods of $(v_{(n+t-1)/2}, v_{(n+t+1)/2})$ and $(v_{(n+t-1)/2}, v_{(n+t+1)/2})$ are $\{v_{(t+1)/2}, \dots, v_{(n+t-1)/2}\}$ and $\{v_{(n+t+1)/2}, \dots, v_n\}$, respectively. Therefore, $\eta_{v_{(n+t-1)/2}, v_{(n+t+1)/2}} = n/2$ and $\eta_{v_{(n+t+1)/2}, v_{(n+t-1)/2}} = (n-t+1)/2$. Hence, equation (3) implies

$$Mo(v_{(n+t-1)/2}, v_{(n+t+1)/2}) = 2(t-1)/2 = (t-1)/2.$$

Case (iv) (for $xy \in E_4^4(C_{n,t})$): let $x = v_{(n+t-3)/2}$ and $y = v_{(n+t-1)/2}$, we have $\eta(v_{(n+t-3)/2}, v_{(n+t-1)/2}) = \{v_1, \dots, v_{(n+t-3)/2}\}$ and $\eta(v_{(n+t-1)/2}, v_{(n+t-3)/2}) = \{v_{(n+t-1)/2}, \dots, v_{n-1}\}$. Hence, $\eta_{v_{(n+t-3)/2}, v_{(n+t-1)/2}} = (n+t-3)/2$ and $\eta_{v_{(n+t-1)/2}, v_{(n+t-3)/2}} = (n-t+1)/2$. Therefore, applying equation (3), $Mo(v_{(n+t-3)/2}, v_{(n+t-1)/2}) = t - 2$.

Case (v) (for $xy \in E_5^4(C_{n,t})$): let $x = v_i$ and $y = v_{i+1}$ where $t \leq i \leq (n+t-5)/2$, the resolving neighborhoods of (v_i, v_{i+1}) and (v_{i+1}, v_i) are $\{v_1, \dots, v_i, v_{(n-t+2i+5)/2}, \dots, v_n\}$ and $\{v_{i+1}, \dots, v_{(n-t+2i+1)/2}\}$, respectively. Hence, $\eta_{v_i, v_{i+1}} = (n+t-3)/2$ and $\eta_{v_{i+1}, v_i} = (n+t-1)/2$. Therefore, using equation (3), $Mo(v_i, v_{i+1}) = t - 2$.

Case (vi) (For $xy \in E_6^4(C_{n,t})$): Let $x = v_1$ and $y = v_t$, then resolving neighborhoods are $\eta(v_1, v_t) = \{v_1, \dots, v_{t/2}, v_{(n+t+3)/2}, \dots, v_n\}$ and $\eta(v_t, v_1) = \{v_{(t+3)/2}, \dots, v_{(n+t-1)/2}\}$. Hence, $\eta_{v_1, v_t} = (n-2)/2$ and $\eta_{v_t, v_1} = (n-2)/2$. Therefore, in view of equation (3), $Mo(v_1, v_t) = 0$. \square

In the following theorem, the Mostar index of cycle with a chord graph $C_{n,t}$ is computed.

Theorem 3. For $xy \in E(C_{n,t})$ where $n \geq 9$ and $t \geq 3$, then

$$Mo(C_{n,t}) = \begin{cases} 2(t-2)(n-t), & \text{if } n = \text{odd}, \\ 2(n-t)(t-2), & \text{if } n = \text{even}, t = \text{odd}, \\ 2(n-t)(t-2) - 2, & \text{if } n = \text{even}, t = \text{even}, \end{cases} \quad (21)$$

Proof.

Case (i) $n = \text{odd}, t = \text{odd}$: in view of Lemma 3 and equation (2), we have

$$\begin{aligned}
Mo(C_{n,t}) &= \sum_{xy \in E_1^1(C_{n,t})} |n_{xy} - n_{yx}| + \sum_{xy \in E_2^1(C_{n,t})} |n_{xy} - n_{yx}| \\
&+ \sum_{xy \in E_3^1(C_{n,t})} |n_{xy} - n_{yx}| + \sum_{xy \in E_4^1(C_{n,t})} |n_{xy} - n_{yx}| \\
&+ \sum_{xy \in E_5^1(C_{n,t})} |n_{xy} - n_{yx}| + \sum_{xy \in E_6^1(C_{n,t})} |n_{xy} - n_{yx}|, \tag{22} \\
&= 1 \times 0 + (t-2) \times (n-t) + 2 \times \frac{(t-2)}{2} + 2 \times (t-2) \\
&+ (n-t-3) \times (t-2) + 1 \times 0, \\
&= 2(n-t)(t-2).
\end{aligned}$$

Case (ii) $n = \text{odd}$, $t = \text{even}$: in view of Lemma 4 and equation (2), we have

$$\begin{aligned}
Mo(C_{n,t}) &= \sum_{xy \in E_1^2(C_{n,t})} |n_{xy} - n_{yx}| + \sum_{xy \in E_2^2(C_{n,t})} |n_{xy} - n_{yx}| \\
&+ \sum_{xy \in E_3^2(C_{n,t})} |n_{xy} - n_{yx}| + \sum_{xy \in E_4^2(C_{n,t})} |n_{xy} - n_{yx}| \\
&+ \sum_{xy \in E_5^2(C_{n,t})} |n_{xy} - n_{yx}| + \sum_{xy \in E_6^2(C_{n,t})} |n_{xy} - n_{yx}|, \tag{23} \\
&= 2 \times \frac{n-t}{2} + (t-3) \times (n-t) + 1 \times 0 + 2 \times (t-2) \\
&+ (n-t-2) \times (t-2) + 1 \times 0, \\
&= 2(n-t)(t-2).
\end{aligned}$$

Case (iii) $n = \text{even}$, $t = \text{odd}$: in view of Lemma 5 and equation (2), we have

$$\begin{aligned}
Mo(C_{n,t}) &= \sum_{xy \in E_1^3(C_{n,t})} |n_{xy} - n_{yx}| + \sum_{xy \in E_2^3(C_{n,t})} |n_{xy} - n_{yx}| \\
&+ \sum_{xy \in E_3^3(C_{n,t})} |n_{xy} - n_{yx}| + \sum_{xy \in E_4^3(C_{n,t})} |n_{xy} - n_{yx}| \\
&+ \sum_{xy \in E_5^3(C_{n,t})} |n_{xy} - n_{yx}| + \sum_{xy \in E_6^3(C_{n,t})} |n_{xy} - n_{yx}|, \tag{24} \\
&= 1 \times 0 + (t-2) \times (n-t) + 2 \times \frac{(t-2)}{2} + 2 \times (t-2) \\
&+ (n-t-3) \times (t-2) + 1 \times 0, \\
&= 2(n-t)(t-2).
\end{aligned}$$

Case (iv) $n = \text{even}$, $t = \text{even}$: by using Lemma 6 and equation (2),

$$\begin{aligned}
 Mo(C_{n,t}) &= \sum_{xy \in E_1^4(C_{n,t})} |n_{xy} - n_{yx}| + \sum_{xy \in E_2^4(C_{n,t})} |n_{xy} - n_{yx}| \\
 &+ \sum_{xy \in E_3^4(C_{n,t})} |n_{xy} - n_{yx}| + \sum_{xy \in E_4^4(C_{n,t})} |n_{xy} - n_{yx}| \\
 &+ \sum_{xy \in E_5^4(C_{n,t})} |n_{xy} - n_{yx}| + \sum_{xy \in E_6^4(C_{n,t})} |n_{xy} - n_{yx}|, \quad (25) \\
 &= 2 \times \frac{n-t+1}{2} + (t-3) \times (n-t) + 2 \times \frac{(t-1)}{2} + 2 \times (t-2) \\
 &+ (n-t-3) \times (t-2) + 1 \times 0, \\
 &= 2(n-t)(t-2) - 2.
 \end{aligned}$$

□

Now, we present a numerical example in support of the above theorem.

Examples 2. Consider a cycle with a chord graph $C_{14,7}$; then, the sets $E_1^4(C_{14,7}) = \{v_3v_4, v_4v_5\}$, $E_2^4(C_{14,7}) = \{v_1v_2, v_2v_3, v_5v_6, v_6v_7\}$, $E_3^4(C_{14,7}) = \{v_{10}v_{11}, v_{11}v_{12}\}$, $E_4^4(C_{14,7}) = \{v_9v_{10}, v_{12}v_{13}\}$, $E_5^4(C_{14,7}) = \{v_7v_8, v_8v_9, v_{13}v_{14}, v_{14}v_1\}$, and $E_6^4(C_{14,7}) = \{v_1v_7\}$ form the edge partition of $C_{14,7}$. Now for respective edges xy, st, uv, pq, ef , and jk in these partitions, Lemma 6 implies that $\eta_{xy} = 7$, $\eta_{yx} = 3$, and $Mo(xy) = 4$; $\eta_{st} = 10$, $\eta_{ts} = 3$, and $Mo(st) = 7$; $\eta_{uv} = 7$, $\eta_{vu} = 4$, and $Mo(uv) = 3$; $\eta_{pq} = 9$, $\eta_{qp} = 4$, and $Mo(pq) = 5$; $\eta_{ef} = 9$, $\eta_{fe} = 4$, and $Mo(ef) = 5$; $\eta_{jk} = 6$, $\eta_{kj} = 6$ and $Mo(jk) = 0$. Therefore, using Theorem 3, $Mo(C_{14,7}) = 1 \times 0 + 4 \times 5 + 2 \times 5 + 2 \times 3 + 4 \times 7 + 2 \times 4 = 70$.

4. Conclusion

In this article, by considering cycle-related graphs, we give explicit expressions of the Mostar index of the Jahangir graph and the cycle with a chord graph. The paper is concluded by the following open problems:

Open Problem 1: compute the Mostar index of certain classes of cycle-related structures such as convex polytopes Q_n , R_n , and A_n

Open Problem 2: compute the Mostar index of some rotationally symmetric graphs and S_n

Data Availability

All the data are included within this article. However, the reader may contact the corresponding author for more details of the data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

- [1] N. Trinajstić, *Chemical Graph Theory*, Vol. 1, CRC Press, Boca Raton, FL, USA, 1983.
- [2] R. Todeschini and V. Consonni, *Handbook of Molecular Descriptors*, Wiley-VCH, Weinheim, Germany, 2000.
- [3] E. Estrada, *The Structure of Complex Networks*, Oxford University Press, New York, NY, USA, 2011.
- [4] K. Roy, J. C. Dearden, Advances in QSAR modeling," *The Use of Topological Indices in QSAR and QSPR Modeling*, vol. 24, pp. 57–88, Springer, Cham, Switzerland, 2017.
- [5] A. Balaban, I. Motoc, D. Bonchev, and O. Mekenyan, "Topological indices for structure-activity correlations," *Steric Effects in Drug Design*, vol. 114, pp. 21–56, 1983.
- [6] J. Devillers and A. T. Balaban, Eds., *Topological Indices and Related Descriptors in QSAR and QSPR*, Gordon and Breach, Reading, UK, 1999.
- [7] I. Gutman, B. Rušćić, N. Trinajstić, and C. F. Wilcox, "Graph theory and molecular orbitals. XII. Acyclic polyenes," *The Journal of Chemical Physics*, vol. 62, no. 9, pp. 3399–3405, 1975.
- [8] M. Randić, "Novel molecular descriptor for structure-property studies," *Chemical Physics Letters*, vol. 211, no. 4, pp. 478–483, 1993.
- [9] A. Dobrynin, R. Entringer, and I. Gutman, "Wiener index of trees, theory and applications," *Acta Applicandae Mathematica*, vol. 66, no. 3, pp. 211–249, 2011.
- [10] H. Wiener, "Structural determination of paraffin boiling points," *Journal of the American Chemical Society*, vol. 69, no. 1, pp. 17–20, 1947.
- [11] H. Wiener, "Correlation of heats of isomerization and differences in heats of vaporization of isomers among the paraffin hydrocarbons," *Journal of the American Chemical Society*, vol. 69, no. 11, pp. 2636–2638, 1947.

- [12] I. Gutman, "A formula for the Wiener number of trees and its extension to graphs containing cycles," *Graph Theory Notes*, N. Y. vol. 27, no. 9, pp. 9–15, 1994.
- [13] H. P. Schultz, "Topological organic chemistry. 1. Graph theory and topological indices of alkanes," *Journal of Chemical Information and Modeling*, vol. 29, no. 3, pp. 227–228, 1989.
- [14] A. T. Balaban, "Topological indices based on topological distances in molecular graphs," *Pure and Applied Chemistry*, vol. 55, no. 2, pp. 199–206, 1983.
- [15] K. Xu, M. Liu, K. C. Das, I. Gutman, and B. Furtula, "A survey on graphs extremal with respect to distance-based topological indices," *MATCH Communications in Mathematical and in Computer Chemistry*, vol. 71, no. 3, pp. 461–508, 2014.
- [16] F. Asif, Z. Zahid, S. Zafar, M. R. Farahani, and W. Gao, "On topological properties of some convex polytopes by using line operator on their subdivisions," *Hacettepe Journal of Mathematics and Statistics*, vol. 49, no. 1, pp. 136–146, 2020.
- [17] M. Imran, S. Baby, H. M. A. Siddiqui, and M. K. Shafiq, "On the bounds of degree-based topological indices of the Cartesian product of F-sum of connected graphs," *Journal of Inequalities and Applications*, vol. 305, 2017.
- [18] T. Došlić, I. Martinjak, R. Škrekovski, S. Tipurić-Spužević, and I. Zubac, "Mostar index," *Journal of Mathematical Chemistry*, vol. 56, pp. 2995–3013, 2018.
- [19] M. Arockiaraj, J. Clement, and N. Tratnik, "Mostar indices of carbon nanostructures and circumscribed donut benzenoid systems," *International Journal of Quantum Chemistry*, vol. 119, no. 24, Article ID e26043, 2019.
- [20] F. Hayat and B. Zhou, "On Mostar index of trees with parameters," *Filomat*, vol. 33, no. 19, pp. 6453–6458, 2019.
- [21] S. Huang, S. Li, and M. Zhang, "On the extremal Mostar indices of hexagonal chains," *MATCH Communications in Mathematical and in Computer Chemistry*, vol. 84, no. 1, pp. 249–271, 2020.
- [22] A. Tepeh, "Extremal bicyclic graphs with respect to Mostar index," *Applied Mathematics and Computation*, vol. 355, no. 1, p. 319+324, 2019.
- [23] M. Imran, M. Siddiqui, M. Naeem, and M. Iqbal, "On topological properties of symmetric chemical structures," *Symmetry*, vol. 10, no. 5, p. 173, 2018.
- [24] M. F. Nadeem, M. Azeem, and H. M. A. Siddiqui, "Comparative study of Zagreb indices for capped semi-capped and uncapped carbon nanotubes," *Polycyclic Aromatic Compounds*, pp. 1–18, 2021.