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Research Article

Constrained Delaunay Triangulation for Ad Hoc Networks

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Geometric spanners can be used for efficient routing in wireless ad hoc networks. Computation of existing spanners for ad hoc networks primarily focused on geometric properties without considering network requirements. In this paper, we propose a new spanner called constrained Delaunay triangulation (CDT) which considers both geometric properties and network requirements. The CDT is formed by introducing a small set of constraint edges into local Delaunay triangulation (LDel) to reduce the number of hops between nodes in the network graph. We have simulated the CDT using network simulator (ns-2.28) and compared with Gabriel graph (GG), relative neighborhood graph (RNG), local Delaunay triangulation (LDel), and planarized local Delaunay triangulation (PLDel). The simulation results show that the minimum number of hops from source to destination is less than other spanners. We also observed the decrease in delay, jitter, and improvement in throughput.

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1. INTRODUCTION

An ad hoc network is a collection of wireless nodes with no fixed infrastructure. In wireless ad hoc networks, routing is one of the important challenging tasks because of its distributed nature, mobility, limited transmission range, shared communication media, and limited node capabilities. Many researchers proposed various network topologies for efficient routing. These topologies are used to route the packets from source to destination. These network topologies can be broadly divided into three categories: flat, hierarchical, and hybrid network structures. The flat network graphs form connected graph with all nodes in the network, for example, minimum spanning tree (MST), nearest neighbor graph (NNG), relative neighborhood graph (RNG), Gabriel graph (GG), Yao graph (Yao), and Delaunay triangulation (Del) [1–3]. In the hierarchical structures, only some subset of nodes is existed in the network graph based on the hierarchies created by the network. Mainly, hierarchical structures are based on dominating set, connected dominating set, and d-dominating set [4–8]. On the other hand, hybrid structures [9, 10] combine both hierarchical structures and flat structures, and the graph contains the subset of nodes of the original network graph.

Another type of network graph is unit disk graph (UDG) [11] which can be defined as follows. Assume each node has the same and fixed transmission range of one unit, then there exists an edge between two nodes u and v if and only if their Euclidean distance $\|uv\|$ is not more than one unit. The number of edges in UDG is in $O(n^2)$, where n is the number of nodes. The communication cost in this graph is high due to the large number of edges.

To reduce the communication cost over UDG, many researchers derived various connected geometric subgraphs of UDG, called spanner, with linear number of edges. The formal definition of spanners are as follows. A spanning subgraph G^1 is a t-spanner of a graph G if and only if the number of vertices in graph G^1 is equivalent to the number of vertices in graph G, and the length of the shortest path connecting any two vertices in graph G^1 is bounded by a value t to the length of the shortest path connecting the same vertices in the graph G. In other words, a graph G^1 is a t-spanner of graph G if $V(G^1) = V(G)$ and $\forall\, u, v \in V(G)$, $\|\pi_{G^1}(u,v)\| \le t\|\pi_G(u,v)\|$, where $V(G)$ is the vertex set of graph G and $\|\pi_G(u,v)\|$ is the length of the shortest path between two vertices u and v in the graph G.

Computation of any geometric spanner requires position of nodes. So, each node knows its position using some positioning techniques such as global positioning system (GPS). Some of the important spanners are GG [2], RNG [2], Del [12], Yao [3, 13], and variations of Delaunay triangulation [1, 9, 10, 14–17].

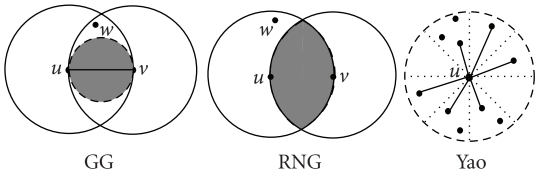

FIGURE 1: Rules of GG, RNG, and Yao graphs.

These spanners are used as underlying network graph by various position-based routing protocols like greedy perimeter stateless routing (GPSR) [18], FACE routing [19], adaptive FACE routing (AFR) [20], greedy FACE greedy (GFG) routing [19], GOAFR$^+$ [21], and Bose's localized routing protocols [22].

1.1. Survey

Recently, many researchers have shown interest in geometric spanners because of its wide range of applications. Some of the spanners often used in wireless networks are briefly discussed below.

The edge $\overline{uv}$ exists between two nodes u and v in GG if there is no node w inside the circle with uv as the diameter as shown in Figure 1. In other words, the edge $\overline{uv} \in E$ if and only if the circle centered at $(u + v)/2$ with the radius $d(u, v)/2$ does not contain any other node inside, where $d(u, v)$ denotes distance between u and v. Bose et al. have given a localized algorithm [19] to construct GG for ad hoc networks. The spanning ratio of GG is bounded by $O(\sqrt{n})$ [2, 23].

Two nodes $u, v \in V$ are connected in RNG if and only if their lune $L_{u,v}$ does not contain any other node $w \in V$. The lune $L_{u,v}$ is defined as the intersection of the two circles of radius $d(u, v)$ centered at p and q, respectively, see Figure 1. The algorithm to construct RNG for wireless networks is given in [18]. Bose et al. [2] have proved that the spanning ratio of RNG is in $O(n)$. RNG is a subgraph of GG.

In Yao graph [3, 13], the transmission range circle of a node u is divided into k equal sectors, where $k \geq 6$, as shown in Figure 1. In each sector, closest node is connected with u by an edge if exists. Ties are broken arbitrarily. The spanning ratio of Yao graph is bounded by a constant. Unlike GG and RNG, Yao graph is not a planar graph.

We can define Delaunay triangulation (Del) [12] of V as the triangulation of all the nodes such that the circumcircle of each triangle should not contain any other node $x \in V$, see Figure 2. Delaunay triangulation is a planar graph. The spanning ratio of Del is 2.42 [24, 25]. Hence, it is a t-spanner of UDG. RNG and GG are subgraphs of Del, but still it maintains linear number of edges in the graph. Del is not suitable for ad hoc networks because it contains longer edges than the transmission range. The unit Delaunay triangulation UDel [15] can be obtained from the Delaunay triangulation Del by removing all the edges longer than one unit. UDel is a t-spanner of UDG [9, 15].

Though the unit Delaunay triangulation (UDel) is suitable for ad hoc networks, it is not known how to construct UDel locally. Li et al. [1] gave a localized algorithm to construct Delaunay triangulation for ad hoc networks called localized Delaunay triangulation (LDel). In LDe1, as shown in Figure 2, the circumcircle of each triangle $\triangle u,\ v,\ w$ does not contain any other node x such that $x \in n_1(u)$, $x \in n_1(v)$, and $x \in n_1(w)$ where $n_1(u)$ denotes the set of *1-hop* neighbors for node u. In this algorithm, first it constructs LDel^1 with one hop neighbors. LDel^1 is a t-spanner of UDG [15], but LDel^1 is not a planar graph. A planarization algorithm is applied on LDel^1 to make it a planar graph, and it is called planarized local Delaunay triangulation (PLDel) or simply localized Delaunay triangulation (LDel). LDel is a planar 2.5-spanner of UDG [1, 15]. It contains all the edges that are in the both UDG and Del.

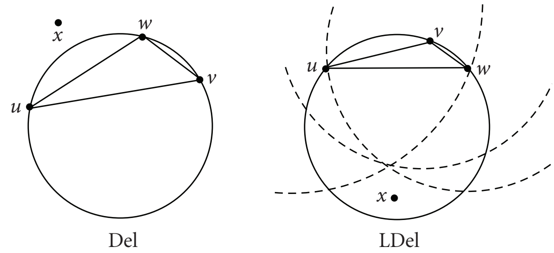

FIGURE 2: Rules of Del and LDel.

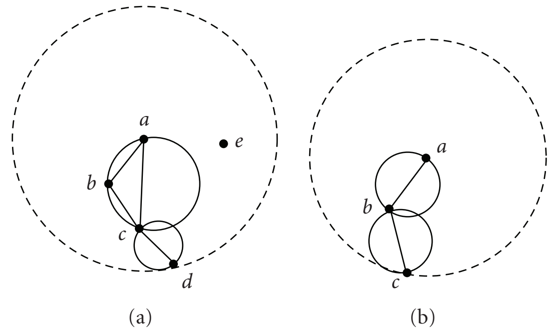

FIGURE 3: More hops in LDel and GG.

Gao et al. [9, 14] proposed a spanner called restricted Delaunay graph (RDG) which combines the node clustering algorithm with Delaunay triangulation graph. Alzoubi et al. [10, 17] proposed another spanner which first constructs the backbone using clustering algorithm followed by the Delaunay triangulation.

LDel contains many desirable geometric properties which are suitable for wireless ad hoc networks, but at the same time some of the problems in LDel motivated us to propose a new spanner called constrained Delaunay triangulation (CDT). The problems include high hop count between the nodes and large packet delays.

Some times, in LDel, packet traversal may take more than one hop even though the nodes are in the communication range of each other. This can happen due to the geometric property of LDel. For example, consider the LDel shown in Figure 3(a), the outer circle indicates the transmission range of node a. Nodes b, c, d, and e are neighbor nodes of a. Even though the node d is in the transmission range of node a, the packet traversal from node a to node d takes more than one hop. Thus, the edge $\overline{ad}$ cannot be placed in the

LDel. The same thing can happen to other spanners also. For example, Gabriel graph (see Figure 3(b)) does not include the edge $\overline{ac}$ in the graph due to its property. In this paper, we propose a new spanner called constrained Delaunay triangulation (CDT) by introducing longer edges to reduce the number of hops and packet delay between the nodes. We call these longer edges as constraints of localized Delaunay triangulation (LDel). The main objectives of the constrained Delaunay triangulation are (1) reducing the number of hops between the nodes in LDel and (2) reducing the end-to-end packet delay. We have simulated CDT in ns-2.28 and compared with PLDel, LDel^1, GG, and RNG using various greedy routing protocols. The simulation results show that CDT give better performance in terms of number of hops, packet delay, jitter, and throughput.

The remaining part of this paper is arranged as fallows. In Section 2, we define constrained Delaunay triangulation followed by the algorithm to construct CDT. Section 3 presents the results of ns2 [26] simulation. We conclude the paper with the future research direction in Section 4.

2. CONSTRAINED DELAUNAY TRIANGULATION (CDT)

Localized Delaunay triangulation (LDel) [1, 15] is popular because of its useful geometric properties and utility in wireless networks. In LDel, even though two nodes exist in the communication range of each other, they do not have a direct edge between the nodes because of the geometric properties. The same things happen in other geometric graphs such as GG and RNG. This leads to longer path from source to destination, hence more number of transmissions and high latency. We are motivated by this problem and proposed CDT which reduces the number of transmissions and latency.

If we have an edge between the nodes a and d in Figure 3(a) or a and c in Figure 3(b), it reduces the number of hops between the nodes and in turn leads to many other benefits such as reduction in the delay, jitter, and energy consumption; but, these type of edges violate the geometric properties of the spanner. So, in order to improve the various network parameters, we introduce the so-called constraint edges into the network graph, but most of other edges satisfy the Delaunay property. This gives us improvement in network parameters and at the same time satisfying various geometric properties.

Constrained Delaunay triangulation is well-studied problem in the area of computational geometry. Chew proposed a divide-and-conquer algorithm for CDT [27]. This algorithm is not suitable for wireless ad hoc networks because it is a centralized algorithm. In this section, we give a localized algorithm to construct CDT for wireless networks.

Constrained Delaunay triangulation is a triangulation of the given set of nodes V with the following properties: (1) CDT is a subgraph of UDG; (2) constraint edges exist at two hop away; (3) CDT is a planar graph; (4) the total number of edges in CDT is in $O(n)$; and (5) CDT is a spanner of UDG.

The algorithm to construct CDT is a localized algorithm, that is, it uses only the constant number of neighborhood hop information. In this algorithm, we assume that each node is static and knows its location information. We can broadly divide the CDT algorithm into three parts. First, we construct the planarized local Delaunay triangulation (PLDel) [1, 15] with one hop neighborhood information. For the sake of completeness, we briefly describe the algorithm. Each node u computes Delaunay triangles $\text{Del}(u)$ with its 1-hop neighbors. The Gabriel edges are computed from Delaunay triangles which will never be deleted from local Delaunay triangulation (LDel). Remove all the inconsistent triangles. A triangle $\triangle abc$ is called consistent if the nodes a, b, and c contain the triangle $\triangle abc$. The resulting graph is called LDel^1 which is not planar graph. A separate planarization algorithm is applied on LDel^1 to induce a planarized local Delaunay triangulation (PLDel).

In the second part of CDT algorithm, we find BLACK nodes which are at least two-hop apart. These Black nodes place nonintersecting constraint edges on top of PLDel and mark the other side of the constraint edge to BROWN. The resulting graph is not planar graph. The third part planarizes this graph and forms new triangles with the constraint

Algorithm for CDT

(1) Construct PLDel, set color of each node to WHITE, and broadcast all its 1-hop neighbor information using the packet *Neighbor_Packet*.

(2) Nodes having lowest *id* among its 2-hop neighbors set their color to BLACK.

(3) Each BLACK node chooses a set N of nodes from its 1-hop neighbors using the following method, see Figure 4.

   (a) N = empty

   (b) n_1 = farthest neighbor

   (c) $N = N \cup n_1$

   (d) for $i = 2, 3, \ldots$
   {
   n_i = choose ith farthest neighbor
   if n_i makes more than 60° angle with
   $n_1, n_2, \ldots, n_{i-1}$
   then $N = N \cup n_i$
   }

(4) Each BLACK node adds the constraint edges to the nodes in N and broadcasts these constraint edges information using the message *Constraint_Packet*.

(5) Each WHITE node sets its *color* = BROWN if it is other end of any constrained edges received using *Constraint_Packet*.

(6) Each BROWN node broadcasts its constraint edge information using the control packet *Constraint_Packet*.

(7) All WHITE and BROWN nodes remove edges connected to it which crosses constraint edges, see Figure 5(b). This information is broadcasted using *Edgecross_Packet*.

(8) Each BLACK node places a new edge from the WHITE nodes, from which the edge was deleted in the previous step to form new triangles, see Figure 6.

Algorithm 1

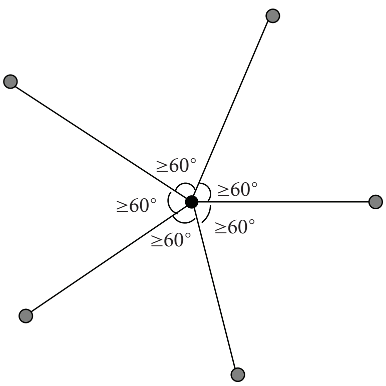

Figure 4: Constraint edges selection.

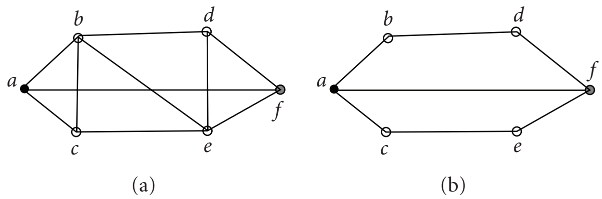

Figure 5: Constraint edge $\overline{af}$ is added to PLDel.

edges. The resulting graph is called constrained Delaunay triangulation (CDT). Please note that the new edges added belong to UDG. Formally, the algorithm is described as shown in Algorithm 1.

Constrained Delaunay triangulation can be maintained efficiently in a distributed fashion. CDT has nice theoretical guarantees. The simulation results show that the performance of CDT is better than PLDel, GG, and RNG.

Theorem 1. *The expected number of BLACK nodes in any 2-hop disk is in $O(1)$.*

Theorem 2. *The expected total number of BLACK and BROWN nodes in a unit disk is in $O(1)$.*

Proof. From the definition of BROWN node, for each BLACK node, the other side of each constraint edge is a BROWN node. Since the maximum number of constraint edges for each BLACK node is 5, the total number of BROWN nodes for a single BLACK node is in $O(1)$. We know that the total number of *BLACK* nodes in unit disk is in $O(1)$, from Theorem 1. Thus, the total number of BROWN nodes in unit disk is in $O(1)$. Hence, the total number of BLACK and BROWN nodes are in $O(1)$. □

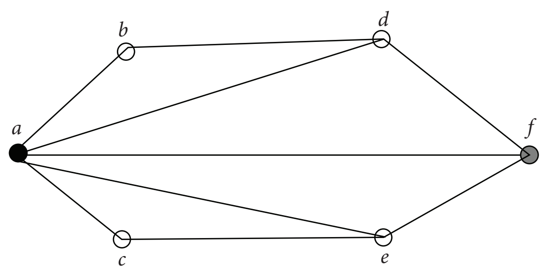

Figure 6: Non-Delaunay triangles $\triangle abd$, $\triangle adf$, $\triangle ace$, and $\triangle aef$ are created in CDT.

From theoretical point of view, the constraint edges in CDT do not affect the link quality, as the definition of UDG says that every node in its vicinity receives the transmitted packet [1, 9–11, 14–17]. On the other hand, the quality of signal strength decreases with the increase in distance to the receiving node [28–30]. In this case, the constraint edges may affect the link quality, as the constraint edges are considerably long. In addition to the distance, link quality also depends on walls, buildings, mountains, and weather conditions, which might obstruct signal propagation. We can use the following models for link quality [31, 32]. In our proposed method, we can consider distance parameter for link quality, while selecting the constrained edges. In other words, consider the link quality as a parameter for choosing the constraint edges.

In the remaining part of the section, we analyze the communication cost in terms of the number of transmissions. In addition to the messages of planarized local Delaunay triangulation [1, 15], CDT uses the following control messages:

(1) *neighbor_packet*. This packet is used to broadcast the 1-hop neighborhood information;

(2) *constraint_packet*. This packet is used to broadcast the constraint edge information;

(3) *edgecross_packet*. This packet is used to broadcast the edgecross information.

The message *neighbor_packet* is broadcasted at most once by each node in the network graph. So, the communication cost for this message is in $O(n)$ bits, where n is the total number of nodes. Only BLACK and BROWN nodes use the message *constraint_packet*. Each BLACK and BROWN nodes broadcast the message *constraint_packet* at most once. The communication complexity for the message *constraint_packet* is in $O(n)$. The message *edgecross_packet* is used by only WHITE and BROWN nodes. Each WHITE and BROWN node uses *edgecross_packet* at most once in CDT. So, the message complexity for this packet is in $O(n)$ bits. Hence follows the theorems.

Theorem 3. *The communication cost of constructing CDT from PLDel is in $O(n)$.*

Theorem 4. *The communication complexity to construct CDT is in $O(n \log n)$.*

Proof. We know that the message complexity for constructing PLDel is in $O(n \log n)$ [1]. The total number of message exchanges by CDT, excluding PLDel, is in $O(n)$. Thus, the total communication cost of constructing the CDT is in $O(n \log n)$. Hence, Theorem 4 the is proved. □

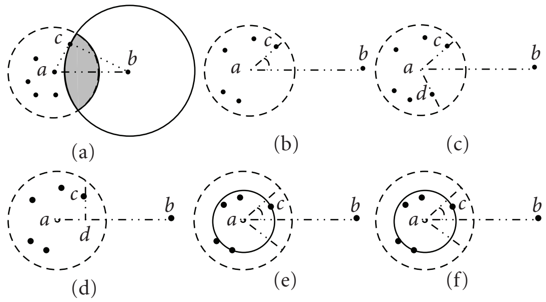

Figure 7: Greedy routing algorithms.

3. SIMULATION AND PERFORMANCE

In order to evaluate the performance of the proposed spanner, we have done various experimental work using networks simulator (nanoseconds 2.28). We have simulated GG, RNG, LDel[1], and PLDel, and compared with our CDT spanner. The following routing protocols are simulated to run on these spanners.

(1) *Greedy routing (Grdy)*. As shown in Figure 7(a), let the node b be the destination node, then the current node a selects a node c as the next relay node in its transmission range such that the distance $\|\overline{cb}\|$ is the smallest among its neighbors. Here, the circle with the center at a represents the transmission range of node a. Please note that if there exists a node in the shaded area, then the node c cannot be next relay node.

(2) *Compass routing (Cmp)*. Assume that the node b is the destination node. The current node a sends packet to the next relay node c which makes the smallest angle, that is, $\angle cab$ is the smallest among all a's neighbors, see Figure 7(b).

(3) *Random compass routing (RandCmp)*. It is similar to the compass routing. As shown in Figure 7(c), let a and b be the current and destination nodes, respectively. We compute the two nodes c and d, above and below the line $\overline{ab}$, respectively. Here, the angle $\angle cab$ and $\angle dab$ are the minimum angles above and below the line ab, respectively, among a's neighbors. The current node a sends packet to the destination node, by randomly selecting, either node c or node d as the next relay node.

(4) *Most forward routing (MFR)*. That is shown in Figure 7(d), the current node a selects the next relay node c to transmit the packet to the destination node b such that the distance $\|\overline{db}\|$ is the smallest among a's neighbors.

(5) *Nearest neighbor routing (NN)*. Let a and b be the current and destination nodes, respectively, as shown in Figure 7(e). For a particular angle α, node a selects the next relay node c such that $\angle cab \leq \alpha$, and c is the nearest neighbor of a.

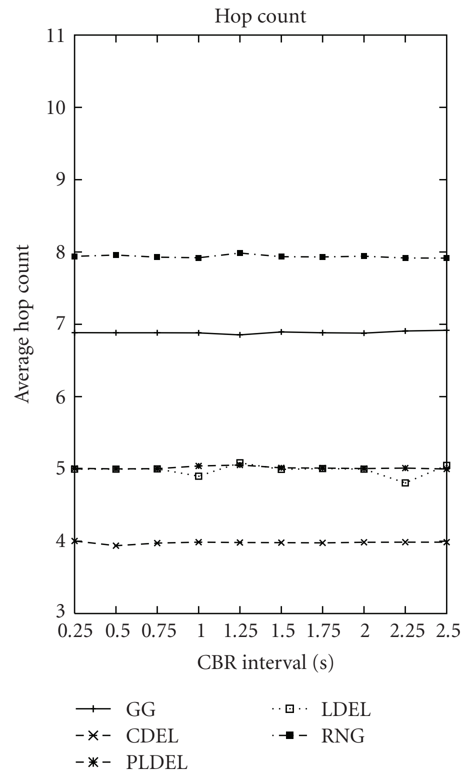

Figure 8: Average hop count.

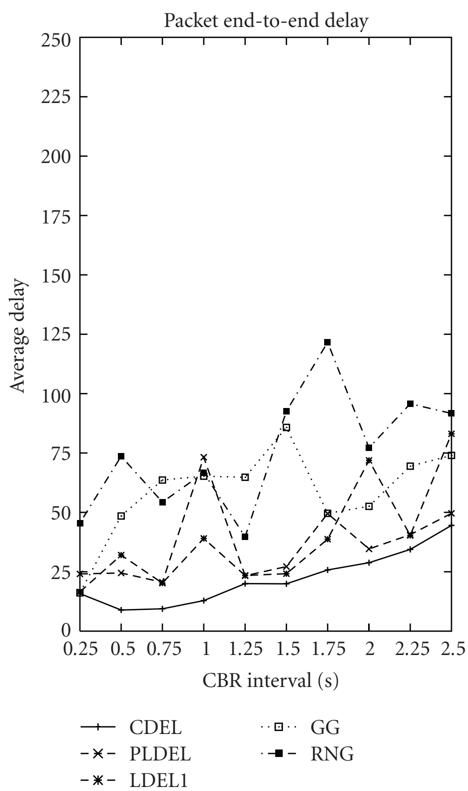

Figure 9: Average delay in μsec.

(6) *Farthest neighbor routing (FN)*. In this routing protocol, for the given angle α, as shown in Figure 7(f), the current node a selects the next relay node c such that $\angle cab \leq \alpha$, and c is the farthest neighbor of node a. Here, b is the destination node.

The greedy routing, compass routing, and random compass routing provide guarantee delivery of packets if the underlying network topology is a Delaunay triangulation [18, 19].

3.1. Performance metrics

In this paper, we have considered the following metrics to evaluate the performance of different spanners.

(1) *Hop count*. This metric represents the total number of hops traveled by a data packet from source to destination on the underlying network graph.

(2) *Delivery ratio*. The delivery ratio (DR) is the ratio of the total number of packets received successfully to the total packets sent by the source.

(3) *Delay*. Delay metric is the total time spent by a packet during the data transmission from source to destination.

(4) *Standard delay*. Standard delay is the standard deviation in the delay of a packet from source to destination. The following mathematical formula is used to calculate the standard delay (S_d):

$$S_d = \sqrt{\frac{1}{m} \times \sum_{i=1}^{m} (a_i - \overline{a})}, \quad (1)$$

where a_i denotes delay of i^{th} packet, $\overline{a}$ is average delay, and m is total number of packets.

(5) *Throughput*. Throughput is the total number of packets transmitted per unit time from source to destination.

In the simulation results, we have presented average, minimum, and maximum of the above metrics.

3.2. Simulation model and results

Important parameters of the simulation are shown in Table 1. All experiments are conducted on ten different node scenarios. For each scenario, we have considered ten different connection patterns, by randomly choosing source nodes at one side and destination nodes at the other side. In our simulation model, we do not consider the link quality, as we assumed that the channel is error free.

The first experiment is on delivery ratio. The average delivery ratio of ten different scenarios with six different routing protocols is shown in Table 2. The delivery ratio of CDT is better than RNG and GG because CDT is denser graph than RNG and GG. There is not much difference among CDT, PLDel, and LDel[1] because many edges are common among them.

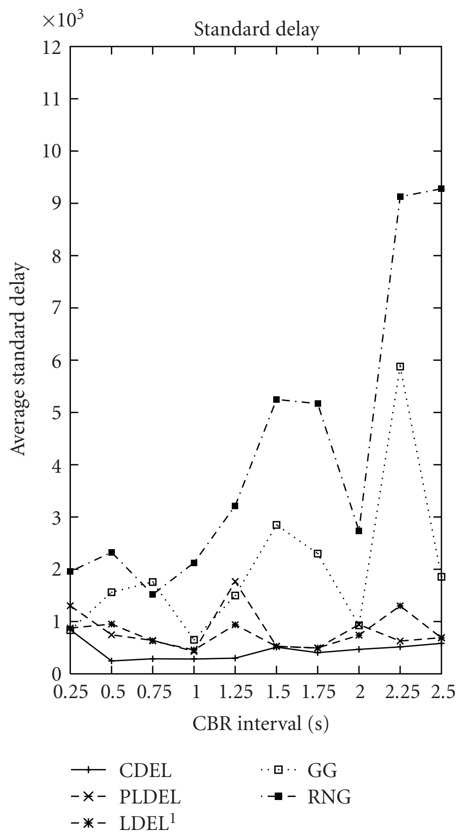

Figure 10: Standard delay in μsec.

Table 1: Simulation parameters.

Grid size	$200 \times 200\text{m}^2$
Number of nodes	100
Transmission range	40 m
Propagation model	TwoRay Ground
MAC layer type	IEEE 802.11
CBR packet size	512 bytes
Simulation run time	350 sec
Antenna	OmniAntenna
Antenna height	1.5 m
Max packet in ifq	100
Frequency	914e+6 Hz
System loss factor	1.0
Recv power threshold	3.652e-10 w
Carrier sense threshold	1.559e-11
Capture threshold	10.0 db
Initial energy	300 J

In the second experiment, we have computed the minimum hop count between source and destination on different network topologies. The source node sends the packet to the destination greedily on five different network topologies. The average hop count at different CBR intervals are shown in Figure 8. It clearly indicates that CDT has less average hop count than the other spanners. This happens

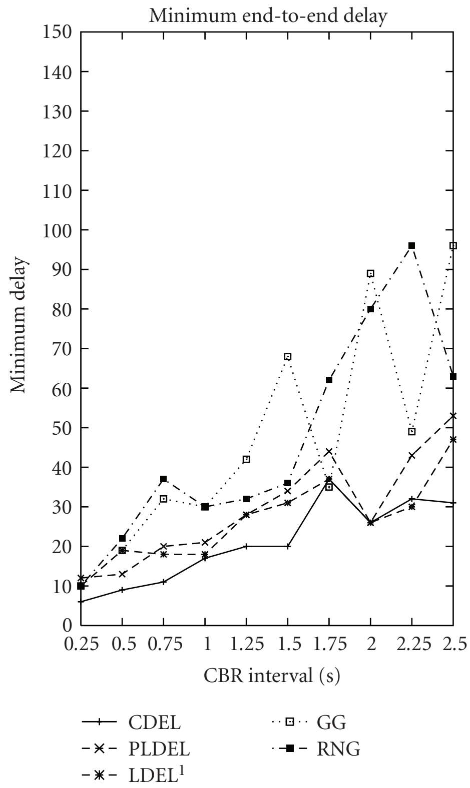

Figure 11: Minimum delay in μsec.

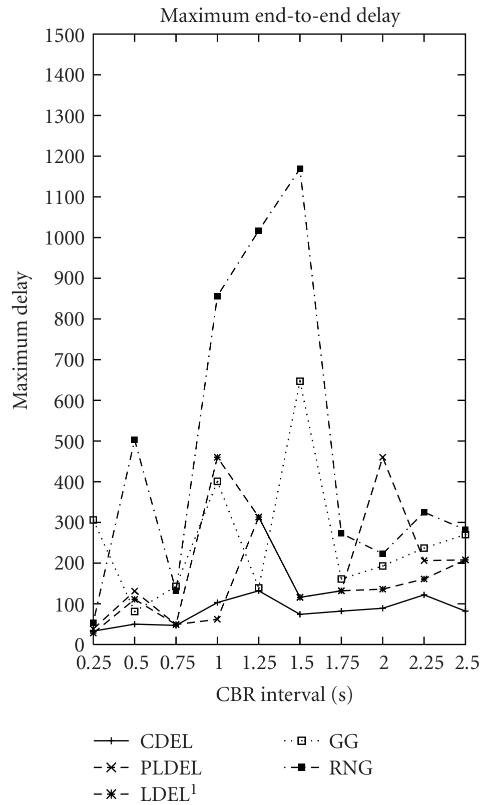

Figure 12: Maximum delay in μsec.

Table 2: Delivery ratios.

	RNG	GG	LDel	PLDel	CDT
NN	40.9	77.8	99.9	99.9	98.4
FN	43.5	78.1	92.8	91.9	95.4
MFR	73.7	86.9	99.8	99.8	99.8
Compass	90.0	97.8	99.9	99.9	99.9
RandComp	90.0	97.9	99.9	99.9	99.9
Greedy	72.8	93.9	100	100	100

because CDT contains constraint edges which make packet transmission from source to destination with fewer hops. Since the constraint edges are not available in RNG, GG, LDel[1], and PLDel, they take more hops.

In third experiment we have checked the end-to-end packet delay on different network topologies. Here, we consider only the packets that reach the destination. The source node sends the packets greedily to the destination on five different spanners at different transfer rates. Figure 9 shows the average delays. We can observe that CDT contains less average delay compared to the other spanners. This is again because of constraint edges in CDT, which make the packets to reach destination with less number of transmissions.

In the fourth experiment, we have considered the standard deviation in delay, called standard delay (S_d). Standard delay is related to one of the quality of service (QoS) parameter called jitter. One can see from Figure 10, the standard deviation of the end-to-end packet delay on the

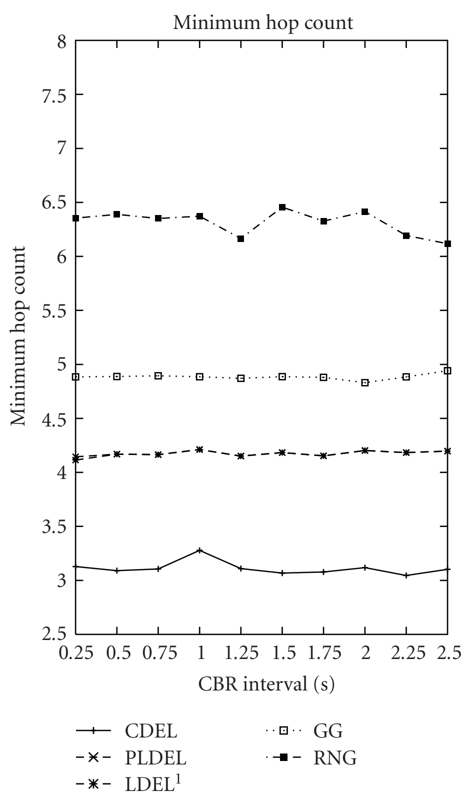

Figure 13: Minimum hop count.

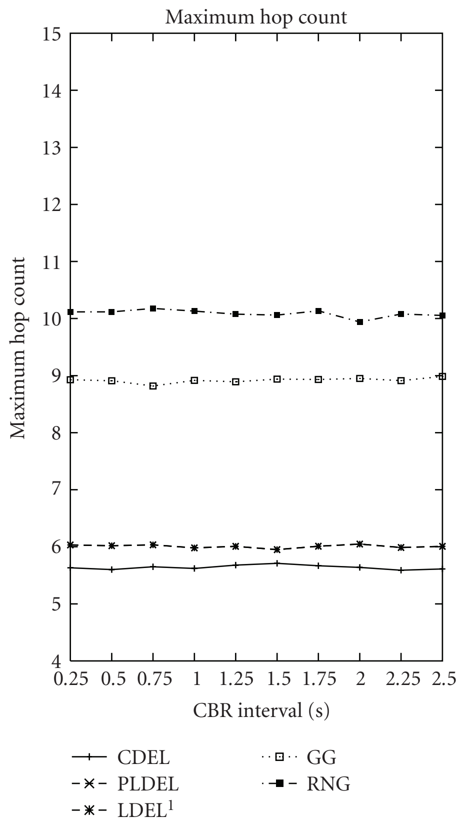

FIGURE 14: Maximum hop count.

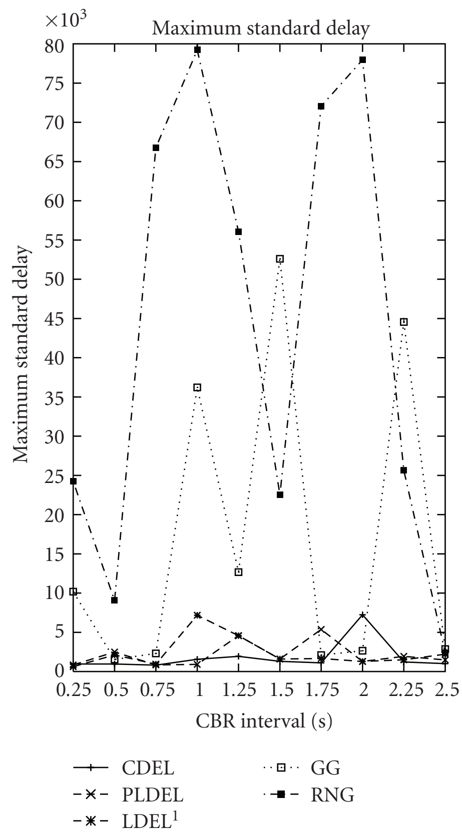

FIGURE 16: Maximum standard delay in μsec.

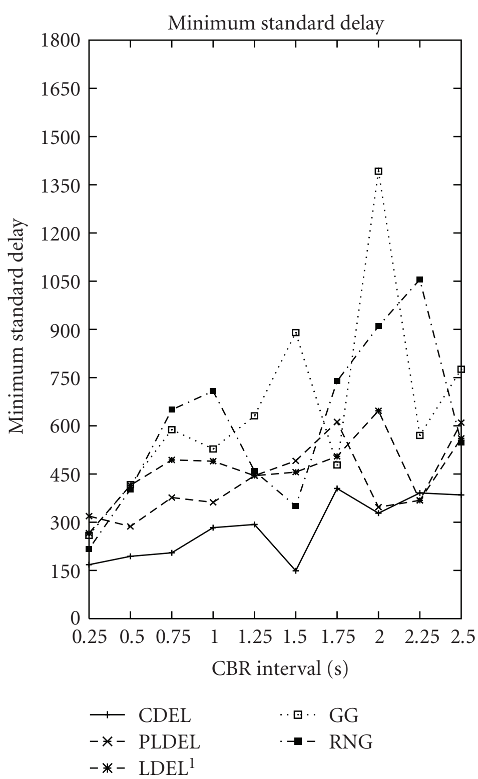

FIGURE 15: Minimum standard delay in μsec.

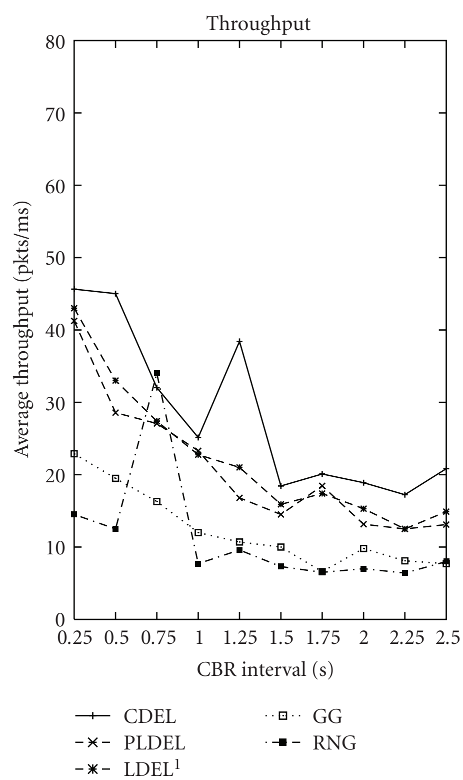

FIGURE 17: Average throughput.

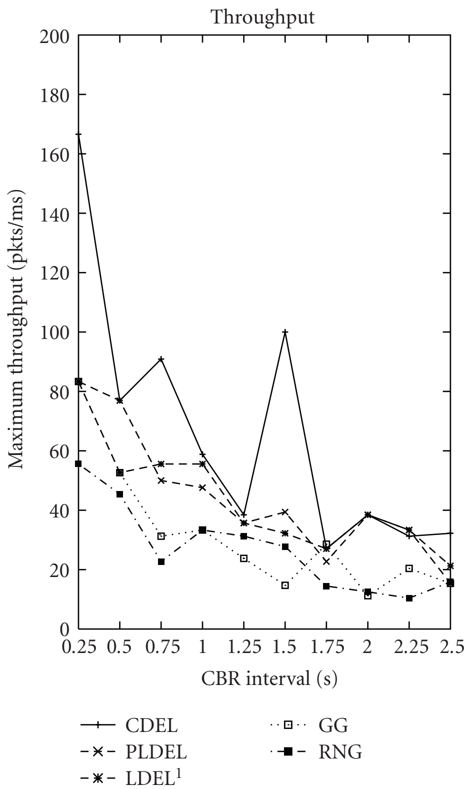

Figure 18: Maximum throughput.

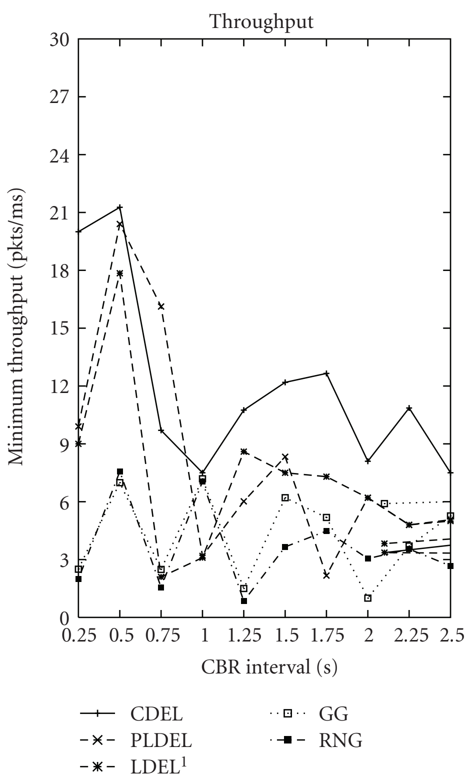

Figure 19: Minimum throughput.

spanner CDT is lesser than the other spanners. This could happen because the total number of transmissions on the spanner CDT is less which makes the variation to the average delay will also be less.

The minimum and maximum values of the packet end-to-end delay for different network topologies are shown in Figures 11 and 12, respectively, at different transfer rates for the five network topologies. We can observe that the minimum and maximum packet's end-to-end delays of CDT are lesser than the other spanners.

The graphs for minimum and maximum hop counts are shown in Figures 13 and 14, respectively. Due to the constraint edges in CDT, the minimum and maximum hop counts are lesser in CDT than the other spanners.

Figures 15 and 16 show the minimum and maximum standard delays, respectively. From the experiments, we have seen that the minimum and maximum standard delays are lesser in CDT.

We have done the final experiment on throughput. Figures 17, 18 and 19 show that the constrained Delaunay triangulation has better throughput compared to other spanners. This can happen because the constraint edges in CDT make the packets to reach the destination with fewer number of transmissions than in PLDel, LDel, GG, and RNG, which reduces the collisions, and latency thereby increase the throughput.

4. CONCLUSION

The existing geometric spanners GG, RNG, LDel[1], and PLDel have smaller edges due to their geometric properties, which increase the hop count and delay. We have proposed CDT by introducing larger constrained edges to reduce hop count and delay. The experimental results substantiate our claim. Moreover, CDT is most suitable geometric spanner for multimedia and real-time applications because of lower standard delays. It would be interesting to study these spanners under various mobility conditions.

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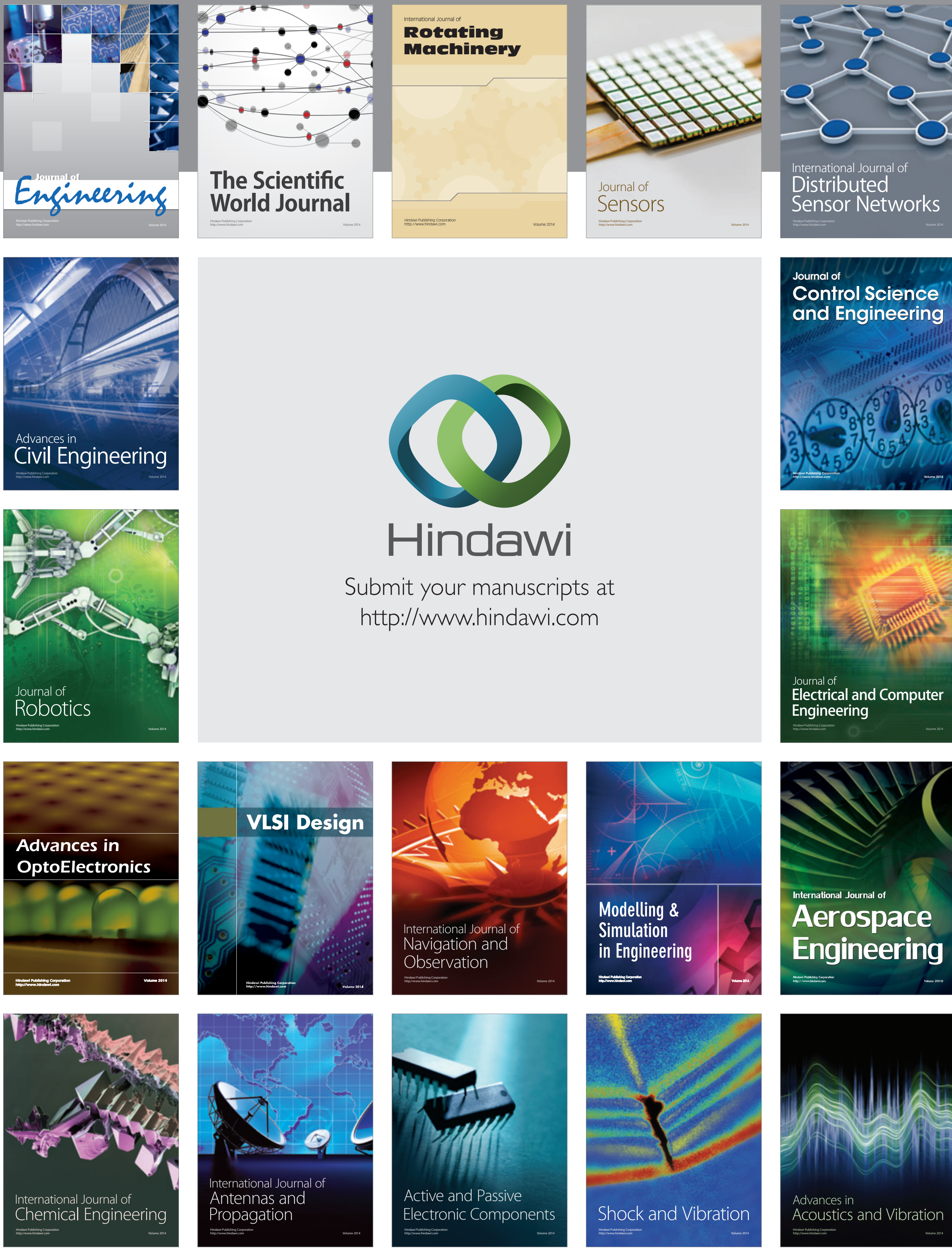
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