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Research Article

Chaotic Dynamics in Joint Price QoS Game with Heterogeneous Internet Service Providers

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This paper tries to investigate the complex characteristics of the communication market where Internet service providers (ISP) lease network access services and compete to serve a large pool of subscribers. For this purpose, we analyze the dynamics of a mixed duopoly game with two decision parameters: price and quality of service (QoS). We calculate and discuss the stability of each equilibrium solution by using the nonlinear system. A numerical simulation is used to show the flip bifurcation to chaos by the decisions of ISPs with different statuses. We discovered that the Nash equilibrium loses stability when the speed of adjustment and transmission fee increase. We show that the system parameter changes the stability of the communication market. In addition, we use a control method to keep the communication market in a stable state.

1. Introduction

Oligopoly is a market mechanism, with a few players producing homogeneous products. These players consider the market demand and the decisions of their opponents. The Cournot model [1] is a duopoly game between two players, where the player uses naïve expectations. Bertrand in Reference [2] presented the Bertrand model with two players, in which the players' decisions compete with price competition.

In recent years, many works studied the dynamics of games and tremendous efforts have been devoted to investigating the complex nonlinear dynamic systems with bounded rationality behaviors. Several expectations have been proposed such as naïve expectation, adaptive expectation, and bounded rationality. In Reference [3], the authors studied a repeated Bertrand duopoly model with bounded rational players. The authors in reference [4] used a bounded rationality mechanism to study a dynamic game with a dynamical map. In Reference [5], the authors considered the Cournot model of consumer surplus with bounded rationality. They show that the system goes into chaos throughout flip bifurcation. The authors in Reference [6] studied the complex behavior of a duopoly game with two parameters: price and quantity. The authors considered that each player maximizes its expected profit with bounded rationality and adaptive expectation. In Reference [7], the authors investigated the dynamic game of agricultural product supply chain with bounded rationality. The authors in Reference [8] used nonlinear dynamics and game theory to investigate the dynamical behavior of airline bidding games with bounded rationality. In Reference [9], the authors established a dynamic model of a supply chain. The authors used nonlinear dynamic theory to investigate the stability of dynamic models. In addition, the authors controlled the dynamic game process based on an adaptive control method.

In the communication market, many ISPs aim to increase their profit by providing many services for the subscribers (X SMS, Y min Voice, Z Mo Data,. . .). The authors in Reference [10] analyzed the interaction between the ISPs in the communication market, where each ISP chooses price and QoS.

The authors in Reference [11] analyzed the effect of content sponsoring on the price and QoS of ISPs in the communication market. The authors in Reference [12] investigated ISPs' best strategies in terms of quality offered to a big CP in a competitive context. In Reference [13], the authors used S-modular theory to investigate the price and power control competition in wireless networks. The authors in Reference [14] investigated the impact caching in Information Centric Network by building an analytical framework with multiple ISPs for the distribution of popular content. The authors used game theory to study the interaction between ISPs. In Reference [15], the authors investigated a non-neutral communication market where ISPs charge content providers (CP) for content distribution. The authors study the competition among ISP in two cases: (1) competitive case, where the ISP charge CP; (2) cooperative case, where CP cooperates with ISP to optimize their strategies.

The authors in Reference [16, 17] investigated the interactions in price and beaconing duration between the UAV in the communication market. In Reference [18], the authors studied profit-sharing contracts between CP and ISPs in the communication market.

In the literature on the communication market, most papers focused on games between ISPs adopted naive expectations to update their policies. However, we can hardly find a few papers that investigate the Bertrand game with two heterogeneous ISPs in the communication market. The present paper studies the interaction among heterogeneous ISPs in the communication market, where one ISP is rational and the second ISP is adaptative.

In this paper, we investigate a communication market competition and focus on this more realistic problem. The ISPs compete to serve subscribers in terms of QoS and pricing. Based on maximizing the profits, the bounded rationality expectation rule, and adaptative expectation, this paper builds a dynamic game. In addition, we calculate and investigate the stability of the Nash equilibrium points by mathematical analysis. Through numerical results, we explore the effects of system parameters on the stability of the Nash equilibrium point. At last, we use a control method to control the chaos.

The paper is organized as follows: in Section 2, we describe dynamic model. In Section 3, we have presented the analysis and numerical result of the price game with bounded rationality. In Section 4, we have presented the analysis and numerical result of the joint QoS price game. In Section 5, we have presented the conclusion.

2. System Model

We consider a communication market with several subscribers and two ISPs using adaptative expectation and bounded rationality expectation. Each ISP_f chooses a network access price p_f and the QoS q_f. Subscribers' behavior is a function of ISPs' strategies, see (1). The parameters used in this paper are presented in Table 1.

We assume all subscribers need the same type of service, and they achieve their demand by subscribing to one of the ISPs. The number of subscribers in service of ISP_f is affected by both network access price and QoS. We model the number of subscribers served by ISP_f as in References [19, 20]:

Table 1: Summary of notation.

Notation	Description
p_f	Network access price of ISP_f.
q_f	Quality of service of ISP_f.
v_f	Backhaul bandwidth cost.
p_{t_f}	Transmission fee of ISP_f.
σ_f^g	Sensitivity to price p_g.
ς_f^g	Sensitivity to QoS q_g.
d_f	The total potential demand of subscribers.
D_f	Demand ISP_f.
B_f	Backhaul bandwidth.
α_f	Speed of adjustment of the price p_f.
β_f	Speed of adjustment of the QoS q_f.
ε	Speed of adjustment of the price of the adaptive player.
τ	Speed of adjustment of the QoS of the adaptive player.
κ	The controlling parameter

$$D_f = d_f - \sigma_f^f p_f + \varsigma_f^f q_f + \sum_{g=1, f\neq g}^{2} \left(\sigma_f^g p_g - \varsigma_f^g q_g\right), \quad (1)$$

d_f is the potential demand of subscribers. σ_f^g and ς_f^g are the responsiveness to price p_g and QoS q_g of ISP_g. For ISP_f, the number of subscribers D_f increases in QoS q_{s_f} and decreases in q_{s_g}. In addition, D_f is decreasing in network access price p_f and increasing in p_g.

Assumption 1. The sensitivity ς verifies:

$$\sigma_f^f \geq \varsigma_f^g, \ \forall f \neq g \in \{1, 2\}. \quad (2)$$

The utility function of ISP_f is written as follows:

$$\Pi_f = p_f D_f + p_{t_f} D_f - v_f B_f, \quad (3)$$

where $p_f D_f$ is the income from network access. $p_{t_f} D_f$ is the transmission revenue. v_f is the unit cost of bandwidth. B_f is the backhaul bandwidth of ISP_f which is expressed as follows [21]:

$$B_f = D_f + q_f^2. \quad (4)$$

Then, the utility function is given by

$$\Pi_f = \left(p_f + p_{t_f}\right) D_f - v_f \left(D_f + q_f^2\right). \quad (5)$$

3. Price Game

In the communication market, for the sake of becoming entirely rational and having complete knowledge about the communication market, the ISPs need to invest a significant cost, whereas most ISPs are not willing to do this activity. Hence, in this situation, we consider that some ISPs have incomplete knowledge about the market. In other words, the ISPs are boundedly rational. But in the communication market, a few ISPs willing to invest to grasp the market knowledge. We assume that ISPs will adopt expective expectations. In this paper, we study an asymmetric scenario with two ISPs, one ISP adopts expective expectations while the other is boundedly rational. The first-order condition for ISP_f is as follows:

$$\frac{\partial \Pi_f}{\partial p_f} = d_f - 2\sigma_f^f p_f + \varsigma_f^f q_f + \sigma_f^g p_g - \varsigma_f^g q_g - \sigma_f^f p_{t_f} + v_f \sigma_f^f, \tag{6}$$

we assume $c_f = d_f + \varsigma_f^f q_f - \varsigma_f^g q_g - \sigma_f^f p_{t_f} + v_f \sigma_f^f$, then (6) becomes:

$$\frac{\partial \Pi_f}{\partial p_f} = c_f - 2\sigma_f^f p_f + \sigma_f^g p_g, \tag{7}$$

from (7), we obtain the following reaction function for ISP_f:

$$p_f = \frac{c_f + \sigma_f^g p_g}{2\sigma_f^f}. \tag{8}$$

We consider that the ISP_1 uses bounded rationality expectation; hence, he builds its decision based on the expected marginal payoff $\partial \Pi_1 / \partial p_1$. Then, the dynamical equation of ISP_1 is as follows:

$$p_1(t+1) = p_1(t) + \alpha_1 p_1(t) \frac{\partial \Pi(p_1, p_2)}{\partial p_1}, \tag{9}$$

where α_1 is the speed of adjustment.

The dynamical equation of the adaptative player ISP_2 is as follows:

$$p_2(t+1) = (1-\varepsilon) p_2(t) + \varepsilon r_2(p_1(t)), \tag{10}$$

where $\varepsilon \in [0,1]$ is a speed of adjustment. $r_2(p_1) = c_2 + \sigma_2^1 p_1 / 2\sigma_2^2$ is the response function.

Then, the dynamical game with heterogeneous ISPs has the following form:

$$\begin{cases} p_1(t+1) = p_1(t) + \alpha_1 p_1(t) \dfrac{\partial \Pi(p_1, p_2)}{\partial p_1}, \\ p_2(t+1) = (1-\varepsilon) p_2(t) + \varepsilon r_2(p_1(t)). \end{cases} \tag{11}$$

Then (11) becomes

$$\begin{cases} p_1(t+1) = p_1(t) + \alpha p_1(t)\left(c_1 - 2\sigma_1^1 p_1(t) + \sigma_1^2 p_2(t)\right), \\ p_2(t+1) = (1-\varepsilon) p_2(t) + \dfrac{\varepsilon}{2\sigma_2^2}\left(c_2 + \sigma_2^1 p_1(t)\right), \end{cases} \tag{12}$$

where $c_1 = d_1 + \varsigma_1^1 q_1 - \varsigma_1^2 q_2 - \sigma_1^1 p_{t_1} + v_1 \sigma_1^1$, $c_2 = d_2 + \varsigma_2^2 q_2 - \varsigma_2^1 q_1 - \sigma_2^2 p_{t_2} + v_2 \sigma_2^2$

Setting $p_1(t+1) = p_1(t)$ in (12), we obtain the following:

$$\begin{cases} p_1(t)\left(c_1 - 2\sigma_1^1 p_1(t) + \sigma_1^2 p_2(t)\right) = 0, \\ -2\sigma_2^2 p_2(t) + c_2 + \sigma_2^1 p_1(t) = 0. \end{cases} \tag{13}$$

Solving system (12), we get two equilibrium points as follows:

$$\begin{aligned} E_0 &= \left(0, \frac{c_2}{2\sigma_2^2}\right) \\ E_* &= \left(\frac{2c_1\sigma_2^2 + c_2\sigma_1^2}{4\sigma_1^1\sigma_2^2 - \sigma_1^2\sigma_2^1}, \frac{c_1\sigma_2^1 + 2c_2\sigma_2^2}{4\sigma_1^1\sigma_2^2 - \sigma_1^2\sigma_2^1}\right), \end{aligned} \tag{14}$$

where E_0 is a fixed point while E_1 is a Nash equilibrium point. For economic significance, the equilibrium points E_0 and E_1 should be positive. According to Assumption 1, we have $4\sigma_1^1\sigma_2^2 - \sigma_1^2\sigma_2^1 > 0$, so, the equilibrium points E_0 and E_* have economic meaning if $c_1 > 0$ and $c_2 > 0$.

The Jacobian matrix of the dynamic system is as follows (12):

$$J(p_1, p_2) = \begin{pmatrix} 1 + \alpha\left(c_1 - 4\sigma_1^1 p_1 + \sigma_1^2 p_2\right) & \alpha\sigma_1^2 p_1 \\ (1-\varepsilon) & \dfrac{\varepsilon\sigma_2^1}{2\sigma_2^2} \end{pmatrix}. \tag{15}$$

Theorem 1. *The boundary equilibrium point E_0 is unstable.*

The Jacobian matrix (15) at E_0 is as follows:

$$J(E_0) = \begin{pmatrix} 1 + \alpha\left(c_1 + \dfrac{\sigma_1^2 c_2}{2\sigma_2^2}\right) & 0 \\ (1-\varepsilon) & \dfrac{\varepsilon\sigma_2^1}{2\sigma_2^2} \end{pmatrix}, \tag{16}$$

whose eigenvalues are $\mu_1 = 1 + \alpha(c_1 + \sigma_1^2 c_2 / 2\sigma_2^2)$ and $\mu_2 = \varepsilon\sigma_2^1 / 2\sigma_2^2$. We have $|\mu_1| > 1$ and $|\mu_2| < 1$. Therefore, the fixed point E_0 is unstable.

Theorem 2. *The Nash equilibrium point E_1 is locally asymptotically stable.*

The Jacobian at E_1 is as follows:

$$J(E_1) = \begin{pmatrix} 1 + \alpha\left(c_1 - 4\sigma_1^1 p_1^* + \sigma_1^2 p_2^*\right) & \alpha\sigma_1^2 p_1^* \\ (1-\varepsilon) & \dfrac{\varepsilon\sigma_2^1}{2\sigma_2^2} \end{pmatrix}. \tag{17}$$

The Nash equilibrium point E_1 is asymptotically stable if and only if all the roots of the characteristic equation

$$P(\mu) = \mu^2 - a\mu + b = 0, \tag{18}$$

have magnitudes of eigenvalues less than one, in which

(i) $a = 1 + \alpha(c_1 - 4\sigma_1^1 p_1^* + \sigma_1^2 p_2^*) + \varepsilon\sigma_2^1 / 2\sigma_2^2$.

(ii) $b = (\varepsilon\sigma_2^1 / 2\sigma_2^2) + (\alpha\varepsilon\sigma_2^1 / 2\sigma_2^2)(c_1 + \sigma_1^2 p_2^*) - (\alpha p_1^* / 2\sigma_2^2)(4\alpha\varepsilon\sigma_1^1\sigma_2^1 + 2\alpha(1-\varepsilon)\sigma_1^2\sigma_2^2)$.

According to [22], the necessary and sufficient condition for the local stability of the Nash equilibrium point E_1 is as follows:

(1) $1 - a + b > 0$,

(2) $1 + a + b > 0$,

(3) $1 - b > 0$.

Table 2: Setting used for numerical simulations.

$\sigma_1^1 = \varsigma_2^2$	$\sigma_f^g = \varsigma_f^g, f \neq g$	d_1	d_2
0.9	0.1	20	20
$v_1 = v_2$	$p_{t_1} = p_{t_2}$	$q_1 = q_2$	ε
0.5	12	20	0.5

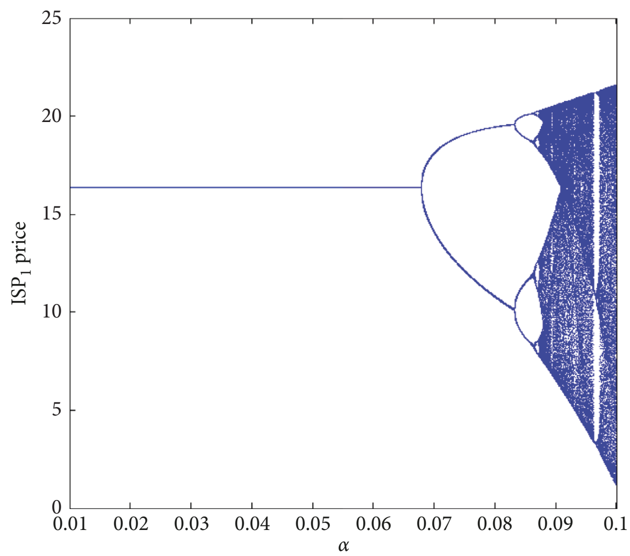

Figure 1: Bifurcation diagrams of p_1 with respect to α.

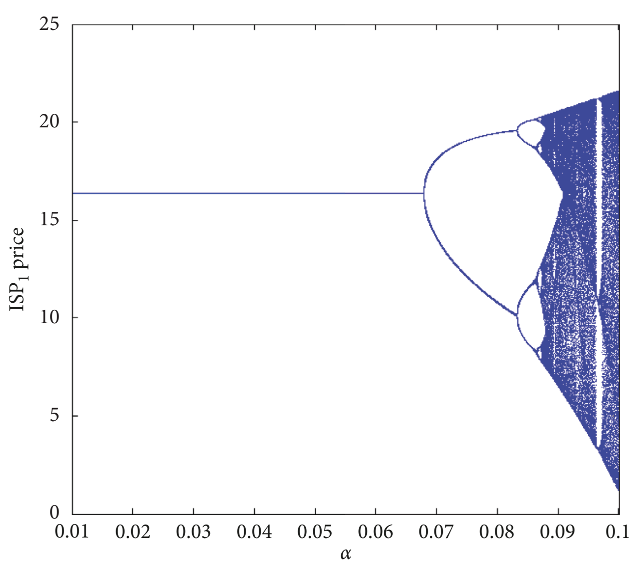

Figure 2: Bifurcation diagrams of p_2 with respect to α.

3.1. Numerical Result. Now, we make numerical simulations to show the dynamic behavior of the communication market by taking the parameter values in Table 2.

Figures 1 and 2 describe the bifurcation diagram of price p_1 and p_2 with the change of the speed adjustment α. When α is small, the Nash equilibrium E_1 is locally stable. As α increased, the Nash equilibrium became unstable.

So now we can make a short conclusion that the system or we can say the communication market can be in a stable

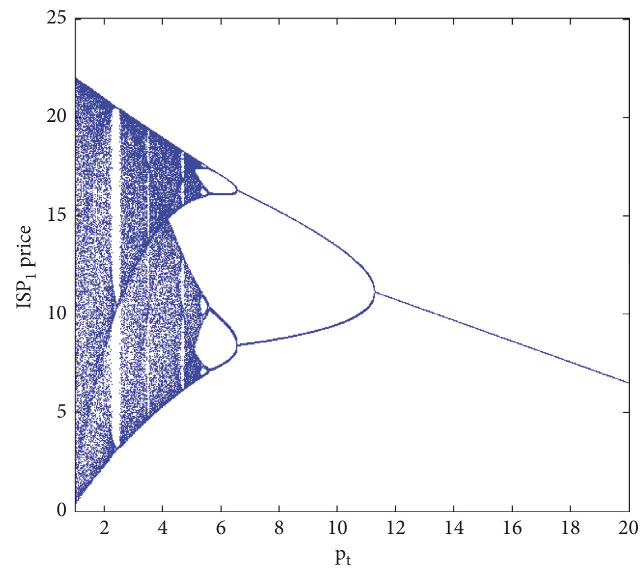

Figure 3: Bifurcation diagrams of p_1 with respect to p_t.

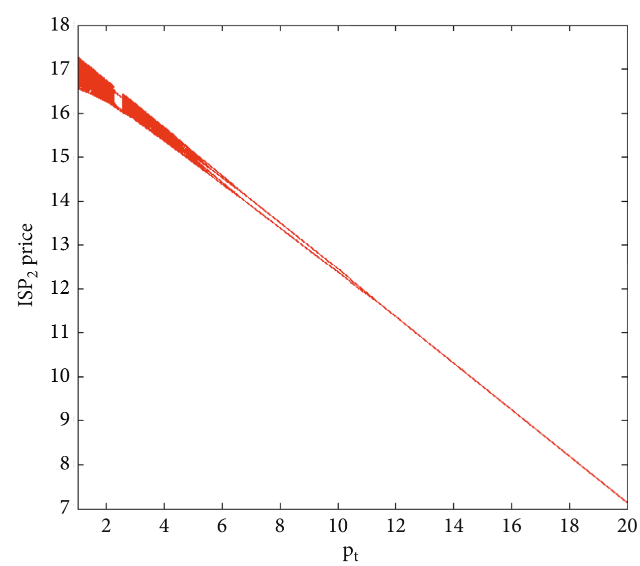

Figure 4: Bifurcation diagrams of p_2 with respect to p_t.

Nash equilibrium, but as the adjustment speed increases, the system will go into chaos, and the price of two ISPs will be unstable, and the market will be in chaos. The ISP$_1$ may not push adjustment speed too fast to keep the market in a stable situation.

Figures 3 and 4 show the bifurcation diagram of prices p_1 and p_2 as a function of the transmission fee p_t. When the transmission fee is small, the market is stable. With the increase in the transmission fee, the market becomes unstable.

The flip bifurcation describes the communication market from a stable state to chaos. According to the previous results, the transmission fee plays an essential role. The communication market is more stable when p_t is less. If the ISPs increase the transmission fee without limit, the stability of the communication market will be destroyed.

3.2. Chaos Control. The numerical results demonstrate that when the adjustment of speed and the transmission fee increases, a chaotic behavior occurs. In this section, we use a control method [23, 24] to control the chaos of system (12). By introducing a control parameter κ, we get the controlled system as follows:

$$\begin{cases} p_1(t+1) = p_1(t) + \dfrac{\alpha p_1(t)}{\kappa}\left(c_1 - 2\sigma_1^1 p_1(t) + \sigma_1^2 p_2(t)\right), \\ p_2(t+1) = (1-\varepsilon)p_2(t) + \dfrac{\varepsilon}{2\sigma_2^2}\left(c_2 + \sigma_2^1 p_1(t)\right). \end{cases} \tag{19}$$

Figures 5 and 6 show the bifurcation diagram of p_1 and p_2 as a function of κ. The dynamical system is in a chaotic state when $\kappa < 0.2$. When $\kappa > 0.45$, system (12) is controlled in a stable state.

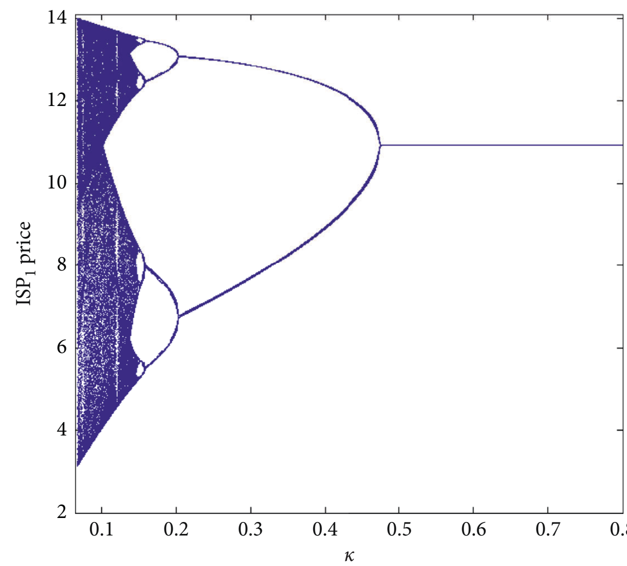

Figure 5: Bifurcation diagrams of p_1 with respect to κ.

4. Joint QoS Price Game

As in the previous section, we consider that the ISP_1 uses bounded rationality expectation; hence, the dynamical equation of ISP_1 is as follows:

$$\begin{aligned} p_1(t+1) &= p_1(t) + \alpha p_1(t)\frac{\partial\Pi(p_1,p_2)}{\partial p_1} \\ q_1(t+1) &= q_1(t) + \beta q_1(t)\frac{\partial\Pi(p_1,p_2,q_1,q_2)}{\partial q_1}, \end{aligned} \tag{20}$$

where α and β are the speeds of adjustment.

The ISP_2 uses adaptative expectation; hence, the price and the QoS of ISP_1 are given as follows:

$$\begin{aligned} p_2(t+1) &= (1-\varepsilon)p_2(t) + \varepsilon r_3(p_1(t),q_1(t),q_2(t)) \\ q_2(t+1) &= (1-\tau)q_2(t) + \tau r_4(p_1(t),p_2(t),q_1(t)), \end{aligned} \tag{21}$$

where ε and τ are the speeds of adjustment of ISP_2.

Then, the dynamical price QoS game has the following form:

$$\begin{cases} p_1(t+1) = p_1(t) + \alpha p_1(t)\dfrac{\partial\Pi(p_1,p_2,q_1,q_2)}{\partial p_1}, \\ p_2(t+1) = (1-\varepsilon)p_2(t) + \varepsilon r_3(p_1(t),q_1(t),q_2(t)), \\ q_1(t+1) = q_1(t) + \beta q_1(t)\dfrac{\partial\Pi(p_1,p_2,q_1,q_2)}{\partial q_1}, \\ q_2(t+1) = (1-\tau)q_2(t) + \tau r_4(p_1(t),p_2(t),q_1(t)), \end{cases} \tag{22}$$

where $r_3(p_1(t),q_1(t),q_2(t)) = (n_2 + \sigma_2^1 p_1(t) + \varsigma_2^2 q_2(t) - \varsigma_2^1 q_1(t)/2\sigma_2^2)$ and $r_4(p_1(t),p_2(t),q_1(t)) = (m_2 + \sigma_2^2 p_2(t)/2v_2)$.

Thus, the dynamical game price QoS game becomes as follows:

$$\begin{cases} p_1(t+1) = p_1(t) + \alpha p_1(t)\left(n_1 - 2\sigma_1^1 p_1(t) + \sigma_1^2 p_2(t)\right) + \varsigma_1^1 q_1(t) - \varsigma_1^2 q_2(t), \\ p_2(t+1) = (1-\varepsilon)p_2(t) + \dfrac{\varepsilon}{2\sigma_2^2}\left(c_2 + \sigma_2^1 p_1(t)\right) + \varsigma_2^2 q_2(t) - \varsigma_2^1 q_1(t), \\ q_1(t+1) = q_1(t) + \beta q_1(t)\left(m_1 + \sigma_1^1 p_1(t) - 2v_1 q_1(t)\right), \\ q_2(t+1) = (1-\tau)q_2(t) + \dfrac{\tau}{2v_2}\left(m_2 + \sigma_2^2 p_2(t)\right), \end{cases} \tag{23}$$

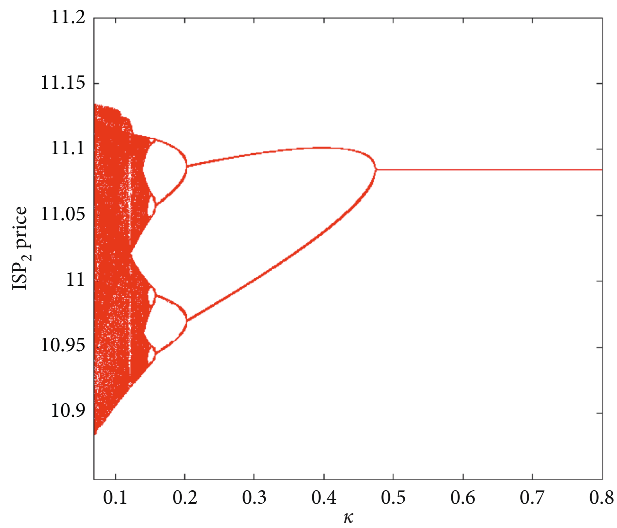

FIGURE 6: Bifurcation diagrams of p_2 with respect to κ.

where $n_1 = d_1 - \sigma_1^1 p_{t_1} + v_1\sigma_1^1$, $n_2 = d_2 - \sigma_2^2 p_{t_2} + v_2\sigma_2^2$, $m_1 = \varsigma_1^1 p_{t_1} - \varsigma_1^1 v_1$ and $m_2 = \varsigma_2^2 p_{t_2} - \varsigma_2^2 v_2$.

There exist four fixed points of (23).

$$E_2 = \left(0, \frac{n_2\varsigma_2^2}{2\sigma_2^2}, 0, \frac{n_2 + 2m_2 + \varsigma_2^2}{4v_2}\right)$$

$$E_3 = \left(0, \frac{2n_2v_1v_2 - m_1v_2\varsigma_2^1 + m_2v_1\varsigma_2^2}{v_1\sigma_2^2\left(4v_2 - \varsigma_2^2\right)}, \frac{m_1}{2v_1}, \frac{2n_2v_1 - m_1\varsigma_2^1 + 4m_2v_1}{2v_1\left(4v_2 - \varsigma_2^2\right)}\right)$$

$$E_4 = \left(\frac{4n_1v_2\sigma_2^2 - n_1\sigma_2^2\varsigma_2^2 + 2n_2v_2\sigma_1^2 - n_2\sigma_2^2\varsigma_1^2 + m_2\sigma_1^2\varsigma_2^2 - 2m_2\sigma_2^2\varsigma_1^2}{8v_2\sigma_1^1\sigma_2^2 - 2v_2\sigma_1^2\sigma_2^1 - 2\sigma_1^1\sigma_2^2\varsigma_2^2 + \sigma_2^1\sigma_2^2\varsigma_1^2}, \frac{2n_1v_2\sigma_2^1 + 4n_2v_2\sigma_1^1 + 2m_2\sigma_1^1\varsigma_2^2 - m_2\sigma_2^1\varsigma_1^2}{8v_2\sigma_1^1\sigma_2^2 - 2v_2\sigma_1^2\sigma_2^1 - 2\sigma_1^1\sigma_2^2\varsigma_2^2 + \sigma_2^1\sigma_2^2\varsigma_1^2}, 0, \frac{n_1\sigma_2^1\sigma_2^2 + 2n_2\sigma_1^1\sigma_2^2 + 4m_2\sigma_1^1\sigma_2^2 - m_2\sigma_1^2\varsigma_2^1}{8v_2\sigma_1^1\sigma_2^2 - 2v_2\sigma_1^2\sigma_2^1 - 2\sigma_1^1\sigma_2^2\varsigma_2^2 + \sigma_2^1\sigma_2^2\varsigma_1^2}\right)$$

$$E_5 = \left(\frac{-4m_2v_1\sigma_2^2\varsigma_1^1 - 2n_2v_1\sigma_2^2\varsigma_1^2 - 2n_1\sigma_2^2v_1\varsigma_2^2 + 8n_1\sigma_2^2v_1v_2 + 4n_2v_1v_2\sigma_1^2 + 2m_2\sigma_1^2v_1\varsigma_2^2 - m_1\sigma_2^2\varsigma_1^1\varsigma_2^2 + 4m_1\sigma_2^2v_2\varsigma_1^1 + m_1\sigma_2^2\varsigma_1^2\varsigma_2^1 - 2m_1v_2\sigma_1^2\varsigma_2^1}{\sigma_1^1\sigma_2^2\varsigma_1^1\varsigma_2^2 - 4v_2\sigma_1^1\sigma_2^2\varsigma_1^1 - 4v_1\sigma_1^1\sigma_2^2\varsigma_2^2 + 16v_1v_2\sigma_1^1\sigma_2^2 - \sigma_1^1\sigma_2^2\varsigma_1^2\varsigma_2^1 + 2v_2\sigma_1^1\sigma_1^2\varsigma_2^1 + 2v_1\sigma_2^1\sigma_2^2\varsigma_1^2 - 4v_1v_2\sigma_1^2\sigma_2^1},\right.$$

$$\frac{2m_1v_2\sigma_2^1\varsigma_1^1 - 2n_2v_2\sigma_1^1\varsigma_1^1 + 8n_2v_1v_2\sigma_1^1 - 4v_2m_1\sigma_1^1\varsigma_2^1 - 2v_2n_1\sigma_1^1\varsigma_2^1 + 4v_2n_1v_1\sigma_2^1 - m_2\sigma_{1^1}\varsigma_1^1\varsigma_2^2 + 4m_2v_1\sigma_1^1\varsigma_2^2 + m_2\sigma_1^1\varsigma_1^2\varsigma_2^1 - 2m_2v_1\sigma_2^1\varsigma_1^2}{\sigma_1^1\sigma_2^2\varsigma_1^1\varsigma_2^2 - 4v_2\sigma_1^1\sigma_2^2\varsigma_1^1 - 4v_1\sigma_1^1\sigma_2^2\varsigma_2^2 + 16v_1v_2\sigma_1^1\sigma_2^2 - \sigma_1^1\sigma_2^2\varsigma_1^2\varsigma_2^1 + 2v_2\sigma_1^1\sigma_1^2\varsigma_2^1 + 2v_1\sigma_2^1\sigma_2^2\varsigma_1^2 - 4v_1v_2\sigma_1^2\sigma_2^1},$$

$$\frac{-2m_2\sigma_1^1\sigma_2^2\varsigma_1^2 - n_2\sigma_1^1\sigma_2^2\varsigma_1^2 - n_1\sigma_2^2\sigma_1^1\varsigma_2^2 + 4n_1\sigma_2^2v_2\sigma_1^1 - 2m_1\sigma_2^2\sigma_1^1\varsigma_2^2 + 8m_1\sigma_2^2v_2\sigma_1^1 + m_1\sigma_2^2\sigma_2^1\varsigma_1^2 + 2n_2v_2\sigma_1^1\sigma_1^2 - 2m_1v_2\sigma_1^2\sigma_2^1 + m_2\sigma_1^2\sigma_1^1\varsigma_2^2}{\sigma_1^1\sigma_2^2\varsigma_1^1\varsigma_2^2 - 4v_2\sigma_1^1\sigma_2^2\varsigma_1^1 - 4v_1\sigma_1^1\sigma_2^2\varsigma_2^2 + 16v_1v_2\sigma_1^1\sigma_2^2 - \sigma_1^1\sigma_2^2\varsigma_1^2\varsigma_2^1 + 2v_2\sigma_1^1\sigma_1^2\varsigma_2^1 + 2v_1\sigma_2^1\sigma_2^2\varsigma_1^2 - 4v_1v_2\sigma_1^2\sigma_2^1},$$

$$\left.\frac{-2m_2\sigma_1^1\sigma_2^2\varsigma_1^1 + 8m_2v_1\sigma_1^1\sigma_2^2 + m_1\sigma_1^1\sigma_2^2\varsigma_1^1 - n_2\sigma_1^1\sigma_2^2\varsigma_1^1 + 4n_2v_1\sigma_1^1\sigma_2^2 + n_2\sigma_1^1\sigma_1^2\varsigma_2^1 - 2m_2v_1\sigma_1^2\sigma_2^1 - 2m_1\sigma_1^1\sigma_2^2\varsigma_2^1 - n_1\sigma_1^1\sigma_2^2\varsigma_2^1 + 2n_1v_1\sigma_2^1\sigma_2^2}{\sigma_1^1\sigma_2^2\varsigma_1^1\varsigma_2^2 - 4v_2\sigma_1^1\sigma_2^2\varsigma_1^1 - 4v_1\sigma_1^1\sigma_2^2\varsigma_2^2 + 16v_1v_2\sigma_1^1\sigma_2^2 - \sigma_1^1\sigma_2^2\varsigma_1^2\varsigma_2^1 + 2v_2\sigma_1^1\sigma_1^2\varsigma_2^1 + 2v_1\sigma_2^1\sigma_2^2\varsigma_1^2 - 4v_1v_2\sigma_1^2\sigma_2^1}\right), \tag{24}$$

where E_2, E_3, and E_4 are fixed points, while E_5 is a Nash equilibrium point.

The Jacobian matrix of system (23) has the following form:

$$J(p_1, p_2, q_1, q_2) = \begin{pmatrix} 1 + \alpha\left(n_1 - 4\sigma_1^1 p_1 + \sigma_1^2 p_2 + \varsigma_1^1 q_1 - \varsigma_1^2 q_2\right) & \alpha\sigma_1^2 p_1 & \alpha\varsigma_1^1 p_1 & -\alpha\varsigma_1^2 p_1 \\ \dfrac{\varepsilon\sigma_2^1}{2\sigma_2^2} & (1-\varepsilon) & -\dfrac{\varepsilon\varsigma_2^1}{2\sigma_2^2} & \dfrac{\varepsilon\varsigma_2^2}{2\sigma_2^2} \\ \beta\sigma_1^1 q_1 & 0 & 1 + \beta\left(m_1 - 4v_1 q_1 + \sigma_1^1 p_1\right) & 0 \\ 0 & \dfrac{\tau\sigma_2^2}{2v_2} & 0 & (1-\tau) \end{pmatrix}. \quad (25)$$

Theorem 3. *The fixed point E_2 is unstable.*

The Jacobian matrix (25) at E_2 is as follows:

$$J(E_2) = \begin{pmatrix} 1 + \alpha\left(n_1 + \sigma_1^2 p_2 - \varsigma_1^2 q_2\right) & 0 & 0 & 0 \\ \dfrac{\varepsilon\sigma_2^1}{2\sigma_2^2} & (1-\varepsilon) & -\dfrac{\varepsilon\varsigma_2^1}{2\sigma_2^2} & \dfrac{\varepsilon\varsigma_2^2}{2\sigma_2^2} \\ 0 & 0 & 1 + \beta m_1 & 0 \\ 0 & \dfrac{\tau\sigma_2^2}{2v_2} & 0 & (1-\tau) \end{pmatrix}, \quad (26)$$

whose eigenvalues are $\mu_1 = 1 + \alpha(n_1 + \sigma_1^2 p_2 - \varsigma_1^2 q_2)$, $\mu_2 = 1 - \varepsilon$, $\mu_3 = 1 + \beta m_1$, and $\mu_4 = 1 - \tau$. We have $|\mu_1| > 1$, $|\mu_2| < 1$, $|\mu_3| > 1$ and $|\mu_4| < 1$. Therefore, the fixed point E_2 is unstable.

Theorem 4. *The fixed point E_3 is unstable.*

The Jacobian matrix (25) at E_3 is as follows:

$$hskip^{!}{''}substring - after(preceding - sibling :: comment()[starts - with(.,' hskip')],' hskip'){''}pt > \ J(E_3) = \begin{pmatrix} 1 + \alpha\left(n_1 + \sigma_1^2 p_2 + \varsigma_1^1 q_1 - \varsigma \right. \\ \dfrac{\varepsilon\sigma_2^1}{2\sigma_2^2} \\ \beta\sigma_1^1 q_1 \\ 0 \end{pmatrix} \quad (27)$$

whose eigenvalues are $\mu_1 = 1 + \alpha(n_1 + \sigma_1^2 p_2 + \varsigma_1^1 q_1 - \varsigma_1^2 q_2)$, $\mu_2 = 1 - \varepsilon$, $\mu_3 = 1 + \beta(m_1 - 4v_1 q_1)$, and $\mu_4 = 1 - \tau$. We have $|\mu_1| > 1$, $|\mu_2| < 1$, $|\mu_3| < 1$, and $|\mu_4| < 1$. Therefore, the fixed point E_3 is unstable.

Theorem 5. *The fixed point E_4 is unstable.*

The Jacobian matrix (25) at E_4 is as follows:

$$J(E_4)=\begin{pmatrix} 1+\alpha\left(n_1-4\sigma_1^1p_1+\sigma_1^2p_2-\varsigma_1^2q_2\right) & \alpha\sigma_1^2p_1 & \alpha\varsigma_1^1p_1 & -\alpha\varsigma_1^2p_1 \\ \dfrac{\varepsilon\sigma_2^1}{2\sigma_2^2} & (1-\varepsilon) & -\dfrac{\varepsilon\varsigma_2^1}{2\sigma_2^2} & \dfrac{\varepsilon\varsigma_2^2}{2\sigma_2^2} \\ 0 & 0 & 1+\beta\left(m_1+\sigma_1^1p_1\right) & 0 \\ 0 & \dfrac{\tau\sigma_2^2}{2v_2} & 0 & (1-\tau) \end{pmatrix}, \tag{28}$$

whose eigenvalues are $\mu_1=1+\alpha(n_1-4\sigma_1^1p_1+\sigma_1^2p_2-\varsigma_1^2q_2)$, $\mu_2=1-\varepsilon$, $\mu_3=1+\beta(m_1+\sigma_1^1p_1)$, *and* $\mu_4=1-\tau$. *We have* $|\mu_1|>1$, $|\mu_2|<1$, $|\mu_3|>1$, *and* $|\mu_4|<1$. *Therefore, the fixed point* E_4 *is unstable.*

Theorem 6. *The Nash equilibrium point* E_5 *is locally asymptotically stable.*

The Jacobian matrix (25) at E_5 *is as follows:*

$$J(E_5)=\begin{pmatrix} 1+\alpha\left(n_1-4\sigma_1^1p_1^*+\sigma_1^2p_2^*+\varsigma_1^1q_1^*-\varsigma_1^2q_2^*\right) & \alpha\sigma_1^2p_1^* & \alpha\varsigma_1^1p_1^* & -\alpha\varsigma_1^2p_1^* \\ \dfrac{\varepsilon\sigma_2^1}{2\sigma_2^2} & (1-\varepsilon) & -\dfrac{\varepsilon\varsigma_2^1}{2\sigma_2^2} & \dfrac{\varepsilon\varsigma_2^2}{2\sigma_2^2} \\ \beta\sigma_1^1q_1^* & 0 & 1+\beta\left(m_1-4v_1q_1^*+\sigma_1^1p_1^*\right) & 0 \\ 0 & \dfrac{\tau\sigma_2^2}{2v_2} & 0 & (1-\tau) \end{pmatrix}. \tag{29}$$

The characteristic polynomial of Jacobian matrix (29) is as follows:

$$P(\mu)=\mu^4-a\mu^3+b\mu^2-c\mu+d=0, \tag{30}$$

where

(i) $a=4+\alpha n_1-4\alpha\sigma_1^1p_1^*+\alpha\sigma_1^2p_2^*+\alpha\varsigma_1^1q_1^*-\alpha\varsigma_1^2q_2^*-\varepsilon+\beta m_1-4\beta v_1q_1^*+\beta\sigma_1^1p_1^*-\tau$,

(ii) $b=2+(1-\varepsilon)(2+\alpha n_1+\beta m_1)+\beta m_1+\alpha n_1+\alpha\beta n_1 m_1+(2+\alpha n_1-\varepsilon+\beta m_1)(1-\tau)-(\tau\varepsilon\varsigma_2^2\sigma_2^2/4v_2\sigma_2^2)-4\alpha(1-\varepsilon)\sigma_1^1p_1^*-(\varepsilon\alpha\sigma_2^1\sigma_1^2/2\sigma_2^2)p_1^*+\beta\sigma_1^1p_1^*-4\alpha\sigma_1^1 p_1^*+\alpha\beta n_1\sigma_1^1p_1^*-4\alpha\beta m_1\sigma_1^1p_1^*+16\alpha\beta v_1\sigma_1^2q_1^*p_1^*-4\alpha \beta(\sigma_1^1)^2(p_1^*)^2+\alpha\beta\sigma_1^1\sigma_1^2p_2^*p_1^*+\alpha\beta\varsigma_1^1\sigma_1^1q_1^*p_1^*-4\alpha(1-\tau)\sigma_1^1p_1^*+\beta(1-\varepsilon)\sigma_1^1p_1^*+\beta(1-\tau)\sigma_1^1p_1^*-\alpha\beta\varsigma_1^2\sigma_1^1q_2^* p_1^*+\alpha(1-\varepsilon)\sigma_1^2p_2^*+\alpha\sigma_1^2p_2^*+\alpha\beta m_1\sigma_1^2p_2^*-4\alpha\beta v_1 \sigma_1^2q_1^*p_2^*+\alpha(1-\tau)\sigma_1^2p_2^*+\alpha(1-\varepsilon)\varsigma_1^1q_1^*-4\beta v_1q_1^*+\alpha\sigma_1^1q_1^*-4\alpha\beta n_1v_1q_1^*+\alpha\beta\sigma_1^1m_1q_1^*-4\alpha\beta v_1\sigma_1^1q_1^*+4\alpha \beta v_1\sigma_1^2q_2^*q_1^*+\alpha(1-\tau)\sigma_1^1q_1^*-4\beta(1-\varepsilon)v_1q_1^*-4\alpha\beta\sigma_1^1 (q_1^*)^2-\alpha(1-\varepsilon)\varsigma_1^2q_2^*-\alpha\sigma_1^2q_2^*-m_1\alpha\beta\sigma_1^2q_2^*-\alpha(1-\tau)\sigma_1^2q_2^*$,

(iii) $c=(1+\alpha(n_1-4\sigma_1^1p_1^*+\sigma_1^2p_2^*+\varsigma_1^1q_1^*-\varsigma_1^2q_2^*))(1-\varepsilon)(1+\beta(m_1-4v_1q_1^*+\sigma_1^1p_1^*))-(\varepsilon\alpha\sigma_1^2\sigma_2^1p_1^*/2\sigma_2^2)(1+\beta (m_1-4v_1q_1^*+\sigma_1^1p_1^*))-\beta\sigma_1^1q_1^*(\varepsilon\alpha\varsigma_2^1\sigma_1^2p_1^*+\alpha2\sigma_2^2\varsigma_1^1p_1^* (1-\varepsilon)/2\sigma_2^2)+(1+\alpha(n_1-4\sigma_1^1p_1^*+\sigma_1^2p_2^*+\varsigma_1^1q_1^*-\varsigma_1^2q_2^*)) (1-\tau)(1+\beta(m_1-4v_1q_1^*+\sigma_1^1p_1^*))-\beta\alpha\varsigma_1^1\sigma_1^1(1-\tau)q_1^* p_1^*+(1-\tau)(1-\varepsilon)(1+\beta(m_1-4v_1q_1^*+\sigma_1^1p_1^*))-(\tau\varepsilon\sigma_2^2 \varsigma_2^2/4v_2\sigma_2^2)(1+\beta(m_1-4v_1q_1^*+\sigma_1^1p_1^*))$

(iv) $d=(1+\alpha(n_1-4\sigma_1^1p_1^*+\sigma_1^2p_2^*+\varsigma_1^1q_1^*-\varsigma_1^2q_2^*))((1-\tau)(1-\varepsilon)(1+\beta(m_1-4v_1q_1^*+\sigma_1^1p_1^*)))-(\tau\varepsilon\sigma_2^2\varsigma_2^2/4v_2\sigma_2^2)(1+\beta(m_1-4v_1q_1^*+\sigma_1^1p_1^*))-(\varepsilon\sigma_2^1/2\sigma_2^2)(\alpha\sigma_1^2p_1^*(1-\tau)(1+\beta(m_1-4v_1q_1^*+\sigma_1^1p_1^*)))+(\tau\alpha\sigma_2^2\varsigma_1^2p_1^*/2v_2)(1+\beta (m_1-4v_1q_1^*+\sigma_1^1p_1^*))+\beta\sigma_1^1q_1^*(-\alpha\sigma_1^2p_1^*(1-\tau)(\varepsilon\varsigma_2^1/2\sigma_2^2) -\alpha\varsigma_1^1p_1^*(1-\varepsilon)(1-\tau)+(\tau\alpha\varepsilon/4v_2)(\varsigma_1^1\varsigma_2^2-\varsigma_2^1\varsigma_1^2)p_1^*)$.

According to [22], the Nash equilibrium point E_5 *is local stability if*

(1) $1+a+b+c+d>0$,
(2) $1-a+b-c+d>0$,
(3) $(1-d)(1-d^4)-b(1-d)^2+(a-c)(c-a\,d)>0$,
(4) $3+3\,d>b$,
(5) $|d|<1$.

4.1. Numerical Result. We present in this section some various numerical investigations of dynamic systems (22). We just focus on the Nash equilibrium E_5 which is more importantly endowed with economic implications.

Figures 7-10 show the bifurcation diagram with the α and β. In all these figures, the Nash equilibrium points E_5

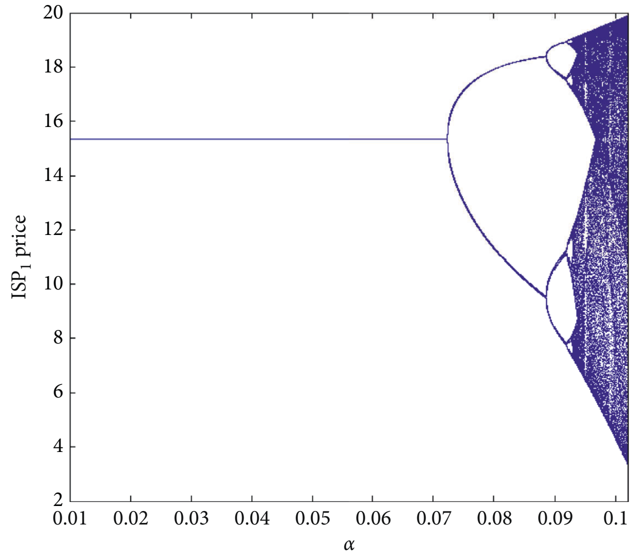

Figure 7: Bifurcation diagrams of p_1 with respect to α.

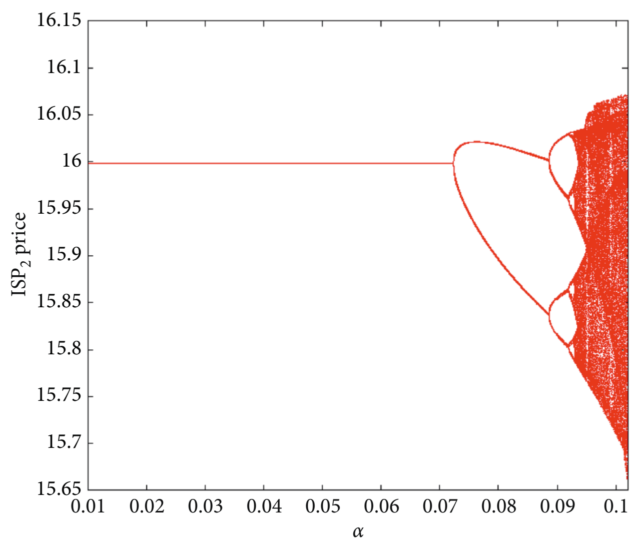

Figure 8: Bifurcation diagrams of p_2 with respect to α.

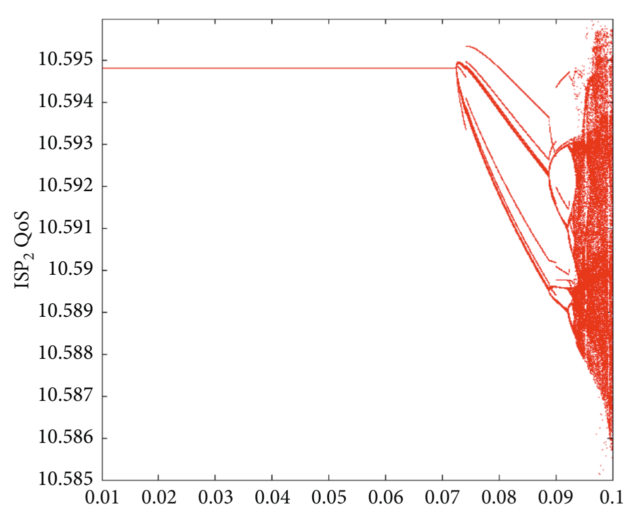

Figure 9: Bifurcation diagrams of q_2 with respect to α.

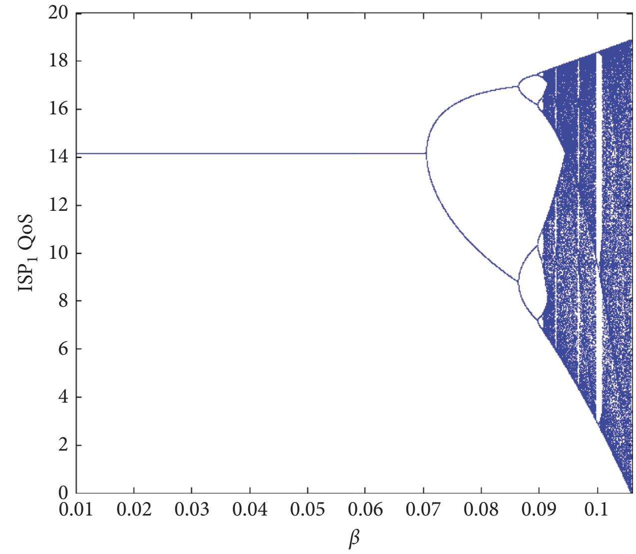

Figure 10: Bifurcation diagrams of q_1 with respect to β.

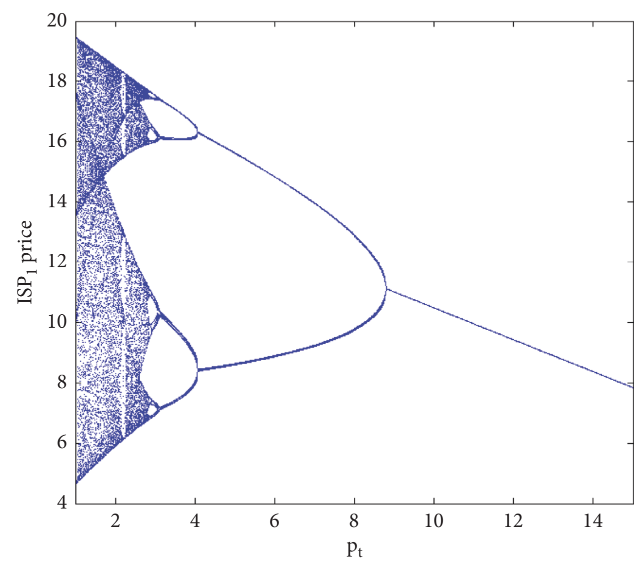

Figure 11: Bifurcation diagrams of p_1 with respect to p_t.

are locally stable for small values of the α and β. When the α and β increase, a 2-cycle, 4-cycle, and chaotic behaviors occur.

Figures 11 and 12 show the bifurcation diagrams of price with respect to the transmission fee p_t. In Figures 11 and 12, the Nash equilibrium E_5 is locally stable only when $p_t > 8.5$. The dynamic system is (23) in bifurcation or chaos if the transmission fee is small.

Figure 13 illustrates the bifurcation diagram of QoS as a function of the transmission fee p_t. Figure 13 shows that the ISP QoS are locally stable for $p_t \leq 3$. Within $pt > 3$, a chaotic phenomenon occurs.

4.2. Chaos Control. We introduce control parameters Υ and Φ in the dynamic system (22), then we have the following:

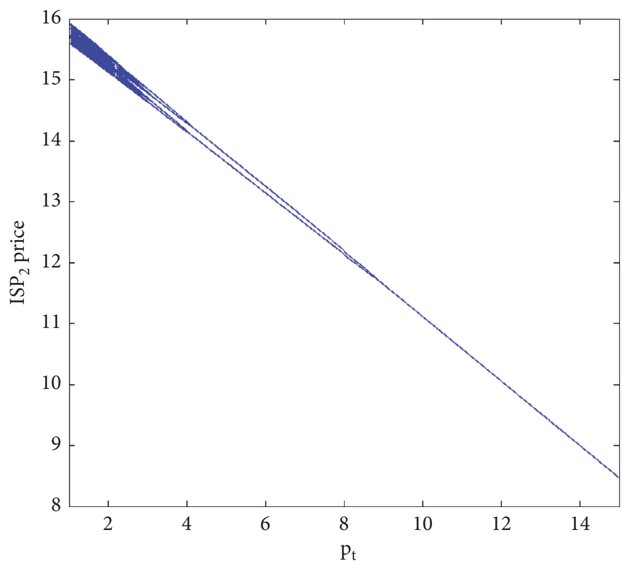

Figure 12: Bifurcation diagrams of p_2 with respect to p_t.

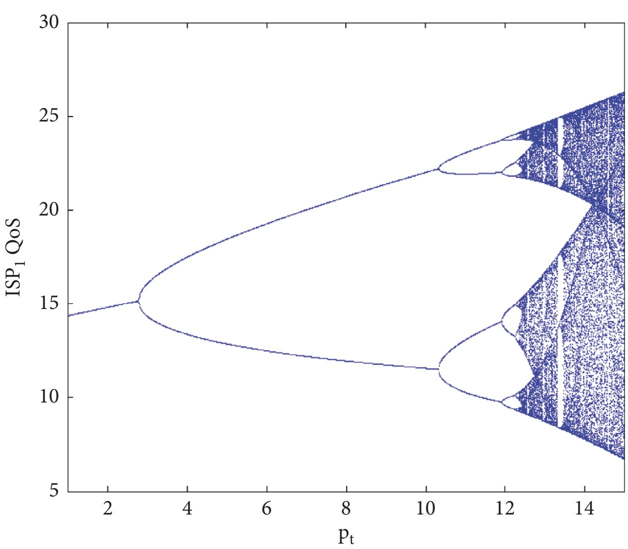

Figure 13: Bifurcation diagrams of q_1 with respect to p_t.

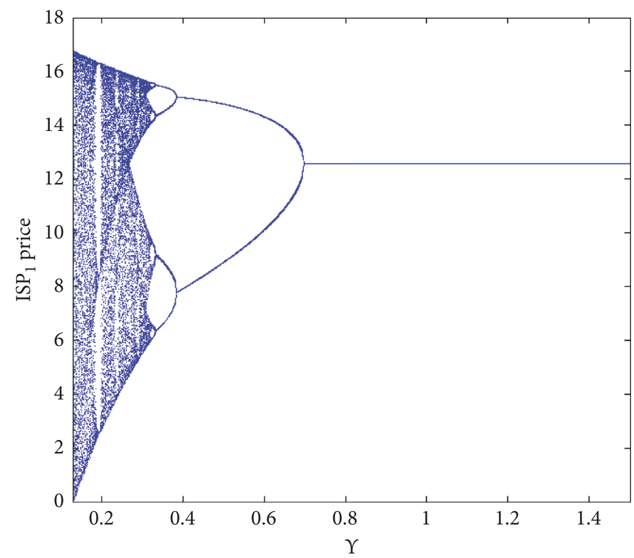

Figure 14: Bifurcation diagrams of p_1 with respect to Υ.

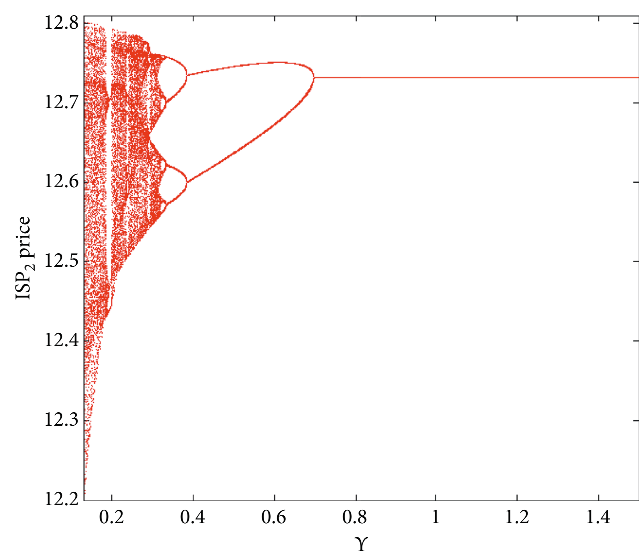

Figure 15: Bifurcation diagrams of p_2 with respect to Υ.

$$\begin{cases} p_1(t+1) = p_1(t) + \dfrac{\alpha p_1(t)}{\Upsilon}\left(n_1 - 2\sigma_1^1 p_1(t) + \sigma_1^2 p_2(t)\right) + \varsigma_1^1 q_1(t) - \varsigma_1^2 q_2(t), \\ p_2(t+1) = (1-\varepsilon)p_2(t) + \dfrac{\varepsilon}{2\sigma_2^2}\left(c_2 + \sigma_2^1 p_1(t) + \varsigma_2^2 q_2(t) - \varsigma_2^1 q_1(t)\right), \\ q_1(t+1) = q_1(t) + \dfrac{\beta q_1(t)}{\Phi}\left(m_1 + \sigma_1^1 p_1(t) - 2v_1 q_1(t)\right), \\ q_2(t+1) = (1-\tau)q_2(t) + \dfrac{\tau}{2v_2}\left(m_2 + \sigma_2^2 p_2(t)\right). \end{cases} \quad (31)$$

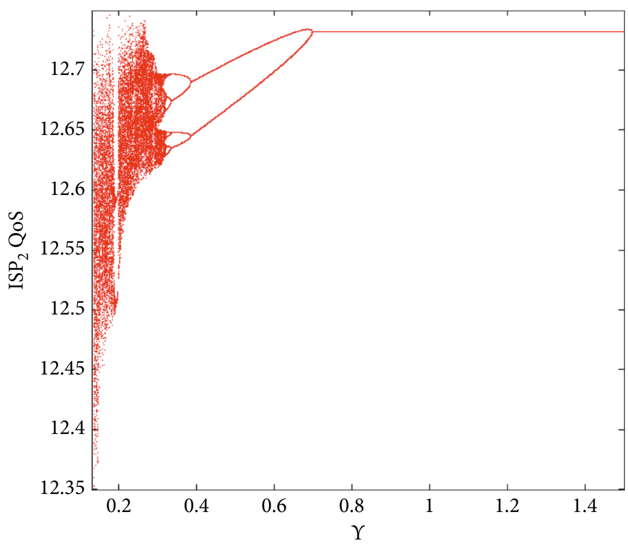

Figure 16: Bifurcation diagrams of q_2 with respect to Υ.

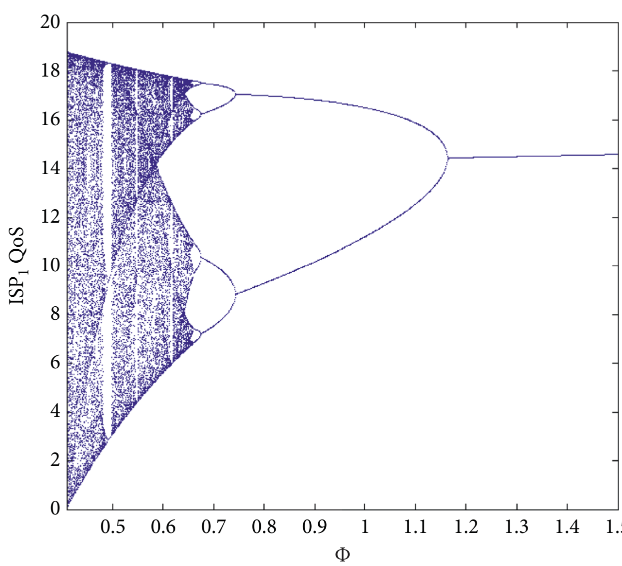

Figure 17: Bifurcation diagrams of q_1 with respect to Φ.

Figures 14-17 illustrate the bifurcation diagram of price as a function of control parameters. When the control parameters increase, the controlled systems (31) are controlled from the chaotic state to a stable state.

So now we can make a short conclusion that the communication market can be in a stable Nash equilibrium, but with the changing of the value system parameters, the system will go into chaos, the policies of the ISPs will be unstable, and the communication market will be in chaos. The ISPs need to choose a low value of system parameters or apply a control method to stabilize the chaotic behaviors to keep the communication market in a stable situation.

5. Conclusion

In this paper, a communication market that consists of two ISPs is studied. We analyzed the dynamics of a Bertrand duopoly game. Each ISP maximizes its utility by using bounded rationality expectation and adaptative expectation. The existence and the stability of the equilibrium point of this dynamic Bertrand duopoly game are investigated. We showed numerically that the model gives chaotic and unpredictable trajectories. The main result is that a high value of transmission fee and a high value of the speed of adjustment may destabilize the communication market. But we also showed that for lower values of transmission fee and speed of adjustment, the communication market is stable. In addition, a control method is used to control the system. This paper presents guidance for ISPs to choose their policies.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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