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Research Article

Steady State Analysis of Base Station Buffer Occupancy in a Cellular Mobile System

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The 3rd generation partnership project (3GPP) standards organizations makes great efforts in order to reduce the latency of 5G mobile networks to the least possible extent. Recently, these networks are associated with big buffers to maximize the network utilization and minimize the wasted wireless resources. However, in existence of the TCP congestions, having bottlenecks are still expected on radio access networks (RANs) data paths. Consequently, this influences the network performance and reduces its quality of services (QoSs). Apparently, studying and improving the behavior of buffers deployed at 5G mobile networks devices can contribute to solving these problems (at least by reducing the queuing time at these buffers). In this paper, we study the buffer behavior of base stations in a 5G mobile network at steady state. We consider a cellular mobile network consisting of finite number of users (stations, terminals, and mobiles). At any time-slot, a station may be using the channel (busy) or not using the channel (idle). Since system analysis of cellular mobile networks in general form is rather complex, solutions are always obtained in closed forms or by numerical techniques. A two-dimensional traffic system for cellular mobile networks is presented, and the main performance evaluations are derived. Moreover, different moments of the base station buffer occupancy are calculated. The study reveals that there is a correlation between the state of the mobile stations (busy or idle) and the expected buffers occupancy of the base station. In addition, the results discussions demonstrate some important factors and parameters that affect the base station buffers and the overall network performance. These factors can be further worked on and controlled to obtain the least possible latency in next generation mobile networks.

1. Introduction

Buffer management is such an important network parameter that affects the quality of service of data traffic. In the study of [1], buffer sizing in wireless networks has been studied addressing the unique challenges of wireless environments such as time-varying channel capacity, variable packet interservice time, and packet aggregation. They classified the current state-of-the-art solutions, discuss their limitations, and provide directions for future research in the area. Furthermore, wireless sensor network (WSN) has emerged as the new technology that will have a profound effect in all the fields being wireless in nature. Data packet delivery process in WSN was discussed in [2] with the help of two buffer policies. Because two different priorities (high priority and low priority) are applied at each node. The number of packets to be transmitted by the nodes in the route is decided with two buffer policies, which is single buffer policy and dual buffer policy.

Cellular mobile networks have been affected significantly by the concept of software-defined networking (SDN). The type and the capacity of output buffer, which stores packets temporary, have influenced mainly the average service time of an OpenFlow switch. Reference [3] modeled the handover delay due to the exchange of OpenFlow-related messages in mobile SDN networks. The total delay encountered by a mobile node while in a handover process, to establish a session from the switch in the source eNodeB to the switch in the destination eNodeB, is called the handover delay. Moreover, the study of [4] presented steady state analysis of buffer occupancy for different forwarding strategies in mobile opportunistic network (MON). Actually, depending

on local information exchange to measure buffer occupancy in buffer management in MON had brought overhead. Consequently, to find the mean buffer occupancy, it is better to study the aggregated bulk transfer size using real-life contact traces and find that it follows a log-normal distribution. However, results of this paper help in measuring how fast a node buffer gets depleted when applying different routing algorithms. Thus, helping in designing better buffer management techniques and routing algorithms.

Recently, great attention has been paid to the mobile services especially in cellular systems which has covered urban areas. A lot of topics concerning these systems have been studied, i.e., frequency assignment techniques, channel access methods, transmission quality, standards for interfering with the wired networks, and traffic analysis. Considering the last topic, many performance measures of voice systems have been evaluated with mathematical modelling. Asynchronous time division multiplexing (ATDM) scheme is used for transmitting packets coming from many users on a single channel simultaneously. While waiting for transmissions on the channel, the data packets are stored in the ATDM buffers (statistical multiplexer). The aim of this study is investigating the buffer behavior of random-multiple access base station and cellular mobile networks. These systems are characterized by the fact that a number of mobile stations exchange digital information by using a distributed random access algorithm on a common radio channel. Whenever a given station attempts transmission of a packet to another station, the attempt may be unsuccessful, in which case the packet should be retransmitted. Unsuccessful transmission may occur due to the channel noise, or because of the interfering from another station trying to send a packet over the common channel at the same time, or because the intended receiver is itself in a mode of transmission. Data packets coming from different stations can share a single communication channel through asynchronous time division multiplexing (ATDM) system (or statistical multiplexer [5]). All packets waiting for service are temporarily stored at the buffer of the statistical multiplexer.

This is the organization of the rest of this paper. Section 2 presents the used mathematical model and the main model assumptions. Section 3 introduces the base station buffer analysis at the steady state and the corresponding probability generating function (PGF) is derived. In Section 4, mean base station buffer occupancy at steady state is calculated. Section 5 introduces discussion and comments on the results. Section 6 concludes the study.

2. Mathematical Model

We consider a cellular mobile network with k independent and identical stations (sources, terminals, ...). Data generated by different stations are divided to small fixed size packets and saved in the base station buffer. Packets can be transmitted from the buffer only at the beginning of each slot. Each station alternates between two independent states with arbitrary length: state of transmission (busy) and a state of not transmitting (idle). So we have the following:

λ: probability that a busy station in a given slot will remain busy in the next slot.

$1-\lambda$: probability that a busy station in a given slot will become idle in the next slot.

μ: probability that an idle station in a given slot will remain idle in the next slot.

$1-\mu$: probability that an idle station in a given slot will become busy in the next slot.

where $\lambda+\mu\neq 1$. Actually, this helps to add a type of correlation between different stations. During each slot, a busy station generates a number of packets N with PGF $N(z)$, where this function is independent from one busy station to another. $N(z)$ can be proposed so as to add different levels of the activity of the station.

Let the random variable (RV) c_l represents the number of busy stations during slot. It is obvious that both busy and idle states of the k stations have geometric distributions. So the value of c_{l+1} can be obtained from c_l, as follows:

$$c_{l+1}=\sum_{j=1}^{c_l}A_j+\sum_{j=1}^{k-c_l}B_j, \tag{1}$$

where $\sum_{j=1}^{c_l}A_j$ specifies how many stations are busy in slot l will remain busy in slot $l+1$, and $\sum_{j=1}^{k-c_l}B_j$ specifies how many idle stations in slot l will change to be busy in slot $l+1$ i.e.,

$$c_{l+1}=A_1+A_2+\cdots+A_{c_l}+B_1+B_2+\cdots+B_{k-c_l}. \tag{2}$$

Note that, A_j, B_j are all Bernoulli RVs, where

$A_1=1$, if the first busy station in slot l will remain busy in slot $l+1$.

$A_1=0$, if the first busy station in slot l will change to idle in slot $l+1$. The same is applied to other busy stations using RVs $A_2, A_3, \ldots, A_{c_l}$.

And,

$B_1=1$, if the first idle station in slot l will remain idle in slot $l+1$.

$B_1=0$, if the first idle station in slot l will change to busy in slot $l+1$. The same is applied to other idle stations using the RVs $B_2, B_3, \ldots, B_{k-c_l}$.

$$\begin{aligned} c_{l+1} &= \underbrace{(0\ or\ 1)+(0\ or\ 1)+\cdots+(0\ or\ 1)}_{c_l\ terms} \\ &+ \underbrace{(0\ or\ 1)+(0\ or\ 1)+\cdots+(0\ or\ 1)}_{k-c_l\ terms}. \end{aligned} \tag{3}$$

Therefore, the group of RVs $A_j's$ and $B_j's$ can be considered as a group of independent and identically distributed Bernoulli RVs with common PGFs $A(z), B(z)$, respectively. Here,

$$A(z)=1-\lambda+\lambda z, \tag{4}$$

$$B(z)=\mu+(1-\mu)z. \tag{5}$$

If the number of packets entering the buffer during slot l is represented by the RV D_l, hence

$$D_l = N_{1l} + N_{2l} + \cdots + N_{c_l l} = \sum_{i=1}^{c_l} N_{il}, \tag{6}$$

where first busy station generates N_{1l} packets, second busy station generates N_{2l} packets, and so on. These RVs are independent and identically distributed with common PGF $N(z)$. Now, let the number of packets stored in the base station buffer at the beginning of slot $l+1$ be denoted by the RV w_l, then we have

$$w_{l+1} = D_{l+1} + (w_l - 1)^+, \tag{7}$$

where

$$(w_l - 1)^+ = \max(0, (w_l - 1)), \tag{8}$$

and D_{l+1} represents the number of packets entering the base station buffer during slot $l+1$.

3. Steady-State Buffer Analysis

It is obvious from equation (7) that the value of w_{l+1} is not dependent only on w_l, but rather on D_{l+1} also. However, since D_{l+1} is dependent on c_{l+1} (from equation (6)), we assume that after a long time (as $l \longrightarrow \infty$) the distribution of the system state in an arbitrary slot no longer varies with time and we use a two-dimensional Markov chain that describes the base station buffer in terms of the pair (c_l, w_l). Let $S^l(x,z)$ represents the joint PGF of c^l, w^l, so

$$S^l(x,z) \triangleq E\left[x^{c^l} z^{w^l}\right], \tag{9}$$

$$S^{l+1}(x,z) \triangleq E\left[x^{c^{l+1}} z^{w^{l+1}}\right]. \tag{10}$$

Then,

$$S(x,z) \triangleq \lim_{l \longrightarrow \infty} E\left[x^{c^l} z^{w^l}\right]. \tag{11}$$

Using equation (7) in equation (9), then

$$S^{l+1}(x,z) = E\left[x^{c^{l+1}} z^{D_{l+1} + (w_l - 1)^+}\right]. \tag{12}$$

Using D^{l+1} from equation (6) in equation (12), hence

$$\begin{aligned} S^{l+1}(x,z) &= E\left[x^{c^{l+1}}.z^{\sum_{i=1}^{c^{l+1}} N_i^{l+1}}.z^{(w^l-1)^+}\right] \\ &= E\left[x^{c^{l+1}}.\left(\prod_{i=1}^{c_{l+1}} z^{N_i^{l+1}}\right).z^{(w^l-1)^+}\right], \end{aligned} \tag{13}$$

which can be written in the form

$$\begin{aligned} S^{l+1}(x,z) &= E_{c^{l+1}, w_l} \\ &\quad \times \left[x^{c^{l+1}}.\left(\prod_{i=1}^{c_{l+1}} E\left[z^{N_i^{l+1}} \middle| c^{l+1}, w_l\right] N(z)\right).z^{(w^l-1)^+}\right], \end{aligned} \tag{14}$$

from which we can obtain

$$\begin{aligned} S^{l+1}(x,z) &= E\left[x^{c^{l+1}}.\left(\prod_{i=1}^{c_{l+1}} N(z)\right).z^{(w^l-1)^+}\right] \\ &= E\left[x^{c^{l+1}}.(N(z))^{c^{l+1}}.z^{(w^l-1)^+}\right] \\ &= E\left[(xN(z))^{c^{l+1}}.z^{(w^l-1)^+}\right]. \end{aligned} \tag{15}$$

Using c^{l+1} from equation (1), yields

$$\begin{aligned} S^{l+1}(x,z) &= E\left[(xN(z))^{\sum_{j=1}^{c_l} A_j + \sum_{j=1}^{k-c_l} B_j}.z^{(w^l-1)^+}\right] \\ &= E\left[\left(\prod_{j=1}^{c^l} (xN(z))^{A_j}\right) \times \left(\prod_{j=1}^{j=k-c^l} (xN(z))^{B_j}\right).z^{(w^l-1)^+}\right], \end{aligned} \tag{16}$$

which can be manipulated to

$$\begin{aligned} S^{l+1}(x,z) &= E\left[\prod_{j=1}^{c^l} E\left[(xN(z))^{A_j}\right]\right. \\ &\quad \left. \times \prod_{j=1}^{k-c^l} E\left[(xN(z))^{B_j}\right].z^{(w^l-1)^+}\right], \end{aligned} \tag{17}$$

where A_j and B_j are all i.i.d RVs with common distribution, where

$$E\left[z^{A_j}\right] = A(z), E\left[z^{B_j}\right] = B(z). \tag{18}$$

Substituting in equation (17), hence

$$S^{l+1}(x,z) = E\left[\prod_{j=1}^{c^l} A(xN(z)).\prod_{j=1}^{k-c^l} B(xN(z)).z^{(w^l-1)^+}\right]$$
$$= E\left[A(xN(z))^{c^l}.\frac{B(xN(z))^k}{B(xN(z))^{c^l}}.z^{(w^l-1)^+}\right], \tag{19}$$

that can be written in the following form:

$$S^{l+1}(x,z) = B(xN(z))^k.E\left[\left[\frac{A(xN(z))}{B(xN(z))}\right]^{c^l}.z^{(w^l-1)^+}\right]. \tag{20}$$

However, since a busy station generates at least one packet that cannot leave the buffer before the next slot, the last expression can be written in the following form:

$$\begin{aligned} S^{l+1}(x,z) &= [B[xN(z)]]^k \\ &\quad \times E\left[\left[\frac{A(xN(z)}{B(xN(z)}\right]^{c^l} z^{(w_l-1)^+}\Big| w^l = 0\right]\Pr\left[w^l = 0\right] \\ &\quad + E\left[\left[\frac{A(xN(z)}{B(xN(z)}\right]^{c^l} z^{(w_l-1)^+}\Big| w^l > 0\right]\Pr\left[w^l > 0\right] \\ &= [B[xN(z)]]^k.E\left[\left[\frac{A(xN(z)}{B(xN(z)}\right]^{0}\right]\Pr\left[w^l = 0\right] \\ &\quad + E\left[\left[\frac{A(xN(z)}{B(xN(z)}\right]^{c^l} \frac{z^{w_l}}{z}\Big| w^l > 0\right]\Pr\left[w^l > 0\right], \end{aligned} \tag{21}$$

$$\begin{aligned} S^{l+1}(x,z) &= [B[xN(z)]]^k.E[1]\Pr\left[w^l = 0\right] \\ &\quad + \frac{1}{z}E\left[\left[\frac{A(xN(z)}{B(xN(z)}\right]^{c^l} z^{w^l}\Big| w^l > 0\right]\Pr\left[w^l > 0\right] \\ &= [B[xN(z)]]^k\left\{\left[1-\frac{1}{z}\right]\Pr\left[w^l = 0\right] + \frac{1}{z}S^l\left[\frac{A(xN(z)}{B(xN(z)}, z\right]\right\} \\ &= [B[xN(z)]]^k\left\{\left[\frac{z-1}{z}\right]\Pr\left[w^l = 0\right] + \frac{1}{z}S^l\left[\frac{A(xN(z)}{B(xN(z)}, z\right]\right\}. \end{aligned} \tag{22}$$

At the steady state, $S^l(x,z)$ and S^{l+1}, which are the joint PGFs of the number of busy stations and the number of packets saved in the base station buffer, will converge to $S(x,z)$.

In view of (4) and (5), (22), gives

$$S(x,z) = [\mu + (1-\mu)xN(z)]^k \times\left\{\left[\frac{z-1}{z}\right]\Pr[w=0] + \frac{1}{z}S\left[\frac{1-\lambda+\lambda xN(z)}{\mu+(1-\mu)xN(z)}, z\right]\right\}, \tag{23}$$

which can be written in the following form:

$$zS(x,z) = [\mu + (1-\mu)xN(z)]^k \times\left\{[z-1]\Pr[w=0] + S\left[\frac{1-\lambda+\lambda xN(z)}{\mu+(1-\mu)xN(z)}, z\right]\right\}. \tag{24}$$

If S_0 is the probability of an empty buffer ($S_0 = \Pr[w=0]$), then $S(x,z)$ should satisfy the following:

$$zS(x,z) = [\mu + (1-\mu)xN(z)]^k \tag{25}$$

$$\times\left\{[z-1]S_0 + S\left[\frac{1-\lambda+\lambda xN(z)}{\mu+(1-\mu)xN(z)}, z\right]\right\}. \tag{26}$$

Although no explicit formula for $S(x,z)$ can be obtained, we can derive many results from equation (25) considering that

$$S(x,1) = C(x), \tag{27}$$

where $C(x)$ is the PGF of the number of busy stations, and

$$S(1,z) = L(z), \tag{28}$$

where $L(z)$ is the PGF of the base station buffer occupancy at the steady state. Substituting for $z = 1$ in equation (25) we can get an expression for $C(x)$

$$\begin{aligned} S(x,1) &= [\mu + (1-\mu)xN(1)]^k \\ &\times \left\{ [1-1]S_0 + S\left[\frac{1-\lambda+\lambda xN(1)}{\mu+(1-\mu)xN(1)}, 1\right]\right\}, \end{aligned} \tag{29}$$

$$C(x) = [\mu + (1-\mu)x]^k . \left\{ C\left[\frac{1-\lambda+\lambda x}{\mu+(1-\mu)x}\right]\right\}. \tag{30}$$

However, an explicit formula for $C(x)$ (which represents the number of busy stations at steady state) can be obtained equation (30) knowing that it is a polynomial of degree k, to get

$$C(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots + c_k x^k. \tag{31}$$

Substituting for $C(x)$ from equation (31) in equation (30), gives

$$\begin{aligned} &c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots + c_k x^k \\ &= [\mu + (1-\mu)x]^k \\ &\times \left(D_0 + D_1\left[\frac{1-\lambda+\lambda x}{\mu+(1-\mu)x}\right] + D_2\left[\frac{1-\lambda+\lambda x}{\mu+(1-\mu)x}\right]^2 + \cdots + D_k\left[\frac{1-\lambda+\lambda x}{\mu+(1-\mu)x}\right]^k \right), \end{aligned} \tag{32}$$

which gives a system of $(k+1)$ equations in the $(k+1)$ unknown $c_0, c_1, \ldots, c_k$. Now, let us focus on a specific station of the k stations where the average length of the busy period of this station is $1/1-\lambda$ and the average length of the idle period is $1/1-\mu$, then the probability that this station is busy during any selected slot, is given by

$$\begin{aligned} \frac{1/1-\lambda}{1/1-\lambda+1/1-\mu} &= \frac{1/1-\lambda}{(1-\mu)+(1-\lambda)/(1-\lambda)(1-\mu)} \\ &= \frac{(1-\lambda)(1-\mu)1/1-\lambda}{2-\mu-\lambda} \\ &= \frac{(1-\mu)}{2-\mu-\lambda}. \end{aligned} \tag{33}$$

Let

$$\frac{(1-\mu)}{2-\mu-\lambda} = \omega. \tag{34}$$

Then, a specific station is busy with probability ω and idle with probability $(1-\omega)$. Considering one station i, let v_i be Bernoulli RV represents the number of busy stations (0 or 1), where

$$\begin{aligned} &\Pr[v_i = 1] = \omega, \Pr[v_i = 0] = 1-\omega \\ &E[x^{v_i}] = \sum_{j=0}^{1} \Pr[v_i = j] x^j \\ &= \Pr[v_i = 0]x^0 + \Pr[v_i = 1]x \\ &= 1 - \omega + \omega x. \end{aligned} \tag{35}$$

Previous relation is applied to all k stations. Since all stations are identical and independent and so are the RVs v_i. Therefore, $C(x)$ (the PGF of the total number of busy stations) is given by

$$\begin{aligned} C(x) &= E\left[x^{\sum_{i=1}^{k} v_i}\right] \\ &= \prod_{i=1}^{k} E[x^{v_i}] \\ &= (1-\omega+\omega x)^k. \end{aligned} \tag{36}$$

But we have

$$\omega = \frac{1-\mu}{2-\lambda-\mu},$$
$$1-\omega = \frac{1-\lambda}{2-\lambda-\mu}. \tag{37}$$

So, $C(x)$ can be written as

$$C(x) = \left[\frac{1-\lambda}{2-\lambda-\mu} + \frac{(1-\mu)x}{2-\lambda-\mu}\right]^k = \left[\frac{1-\lambda+(1-\mu)x}{2-\lambda-\mu}\right]^k. \tag{38}$$

Next, we turn the attention to the steady state distribution of the buffer occupancy. Equation (25) can lead us to the following relation

$$x = \frac{1-\lambda+\lambda x N(z)}{\mu+(1-\mu)xN(z)}. \tag{39}$$

Equation (39) is a quadratic equation which has two roots for x in terms of z. One of these roots satisfies that $x = 1$ for $z = 1$. Consider $x = p(z)$ is that root of equation (39). Substituting for this root in equation (39), yields

$$p(z)[\mu+(1-\mu)p(z)N(z)] = 1-\lambda+\lambda p(z)N(z),$$
$$p(1) = 1. \tag{40}$$

When $x = p(z)$ in equation (25), gives

$$zS(p(z),z) = [\mu+(1-\mu)p(z)N(z)]^k \times \{[z-1]S_0 + S(p(z),z)\}, \tag{41}$$

which can be written as

$$\begin{aligned} &zS(p(z),z) - [\mu+(1-\mu)p(z)N(z)]^k S(p(z),z) \\ &= [\mu+(1-\mu)p(z)N(z)]^k [z-1]S_0 \\ &\quad \times \left\{z - [\mu+(1-\mu)p(z)N(z)]^k\right\} S(p(z),z) \\ &= [\mu+(1-\mu)p(z)N(z)]^k [z-1]S_0. \end{aligned} \tag{42}$$

Solving for $S(p(z),z)$, hence

$$S(p(z),z) = \frac{[\mu+(1-\mu)p(z)N(z)]^k [z-1]S_0}{z-[\mu+(1-\mu)p(z)N(z)]^k}. \tag{43}$$

Equation (43) represents the generating function $S(p(z),z)$ in terms of the constant S_0. Now, we proceed to determine the value of S_0 using the normalizing condition $S(1,1) = 1$. Let $p(z) = 1, z = 1$ in equation (43), then

$$\begin{aligned} S(1,1) &= \frac{[\mu+(1-\mu)(1)N(1)]^k [1-1]S_0}{1-[\mu+(1-\mu)(1)N(1)]^k} \\ &= \frac{[\mu+(1-\mu)]^k [0]S_0}{1-[\mu+(1-\mu)]^k} \\ &= \frac{[0]S_0}{1-1} = \frac{0}{0}. \end{aligned} \tag{44}$$

Before applying L'hospital rule, let

$$\begin{aligned} M(z) &= [\mu+(1-\mu)p(z)N(z)]^k, \\ M(1) &= [\mu+(1-\mu)(1)N(1)]^k \\ &= [\mu+(1-\mu)]^k \\ &= 1. \end{aligned} \tag{45}$$

So equation (43) is written as

$$S(p(z),z) = \frac{M(z)[z-1]S_0}{z-M(z)}. \tag{46}$$

Applying L'hospital rule on equation (46), then

$$S(p(z),z) = \frac{M(z)S_0 + M'(z)(z-1)S_0}{1-M'(z)}. \tag{47}$$

Using normalizing condition, yields

$$S(1,1) = \frac{M(1)S_0 + M'(1)(1-1)S_0}{1-M'(1)}$$
$$1 = \frac{S_0}{1-M'(1)}. \tag{48}$$

Hence, we get

$$S_0 = 1 - M'(1). \tag{49}$$

Substituting for the value of S_0 in equation (46), therefore

$$S(p(z),z) = \frac{M(z)[z-1]\left[1-M'(1)\right]}{z-M(z)}. \tag{50}$$

4. Mean Base Station Buffer Occupancy

Although equation (50) does not give an explicit formula for the generating function of the base station buffer occupancy, many steady-state features of the buffer can be derived from it. The base station mean buffer occupancy $\overline{G}$ at the steady

state can be evaluated by finding the first derivative of equation (50) at $z = 1$, where

$$\begin{aligned} S(x,1) &= C(x) \\ \frac{\partial S}{\partial x}(1,1) &= C^{'}(1), \\ S(1,z) &= L(z) \\ \frac{\partial S}{\partial z}(1,1) &= L^{'}(1) = \overline{G}. \end{aligned} \tag{51}$$

Equation (50) leads us to

$$\begin{aligned} S(p(z),1) &= C(p(z)) \\ \frac{\partial S}{\partial p(z)}(p(z),1) &= C^{'}(p(z))p^{'}(z) \\ \frac{\partial S}{\partial p(z)}(1,1) &= C^{'}(1)p^{'}(1). \end{aligned} \tag{52}$$

From equation (38), we find that

$$\begin{aligned} C(p(z)) &= \left[\frac{1-\lambda+(1-\mu)p(z)}{2-\lambda-\mu}\right]^{k} \\ \frac{B}{Bp(z)}C(p(z)) &= k(1-\mu)\left[\frac{1-\lambda+(1-\mu)p(z)}{2-\lambda-\mu}\right]^{k-1} \\ &= C^{'}(p(z)). \end{aligned} \tag{53}$$

When $p(z) = 1$, then

$$\begin{aligned} C^{'}(1) &= k(1-\mu)\left[\frac{1-\lambda+(1-\mu)(1)}{2-\lambda-\mu}\right]^{k-1} \\ &= \frac{k(1-\mu)}{[2-\lambda-\mu]^{k}}[2-\lambda-\mu]^{k-1} \\ &= \frac{k(1-\mu)}{[2-\lambda-\mu]}. \end{aligned} \tag{54}$$

Substituting from equation (54) in equation (52), hence

$$\frac{\partial S}{\partial p(z)}(1,1) = \frac{k(1-\mu)}{[2-\lambda-\mu]}p^{'}(1). \tag{55}$$

Now, we proceed to get the mean buffer occupancy $\overline{G}$ of the base station from the relation

$$\begin{aligned} \frac{\partial S}{\partial p(z)}(p(z),z) + \frac{\partial S}{\partial z}(p(z),z) &= S^{'}(p(z),z) \\ \frac{\partial S}{\partial p(z)}(1,1) + \frac{\partial S}{\partial z}(1,1) &= S^{'}(1,1) \\ C^{'}(1)p^{'}(1) + \overline{G} &= S^{'}(1,1). \end{aligned} \tag{56}$$

Since $C^{'}(1)$ has been specified, we need also to specify both $S^{'}(1,1)$ and $p^{'}(1)$ to substitute in the previous relation, and find $\overline{G}$. Since equation (56) is the first derivative of equation (50) evaluated at $z = 1$, we can get $S^{'}(1,1)$, from equation (50), as follows:

$$\begin{aligned} \lim_{z\longrightarrow 1}\frac{BS}{Bz}(p(z),z) &= \lim_{z\longrightarrow 1}\frac{M(z)[z-1]\left[1-M^{'}(1)\right]}{z-M(z)}[z-M(z)]\left(\begin{array}{c}\left[1-M^{'}(1)\right][z-1]M^{'}(z) \\ +\left[1-M^{'}(1)\right]M(z)\end{array}\right) \\ &= \lim_{z\longrightarrow 1}\frac{-M(z)[z-1]\left[1-M^{'}(1)\right]\left[1-M^{'}(z)\right]}{[z-M(z)]^{2}}. \end{aligned} \tag{57}$$

So

$$c[1-M(1)]\left(\begin{array}{c}\left[1-M^{'}(1)\right][1-1]M^{'}(1)\\ +\left[1-M^{'}(1)\right]M(1)\end{array}\right)$$

$$\frac{\partial S}{\partial z}(1,1)=\frac{-M(z)[1-1]\left[1-M^{'}(1)\right]\left[1-M^{'}(1)\right]}{[1-M(1)]^{2}}$$

$$=\frac{0}{0}. \tag{58}$$

Before applying L'hospital rule on equation (50), let us consider the following

$$u(z)=M(z)[z-1]\left[1-M^{'}(1)\right], v(z)=z-M(z). \tag{59}$$

So, $S(p(z),z)$ is written as

$$S(p(z),z)=\frac{u(z)}{v(z)}, \tag{60}$$

where

$$\begin{aligned}
&u^{'}(z)=\left[1-M^{'}(1)\right][z-1]M^{'}(z)+\left[1-M^{'}(1)\right]M(z)\\
&u^{''}(z)=\left[1-M^{'}(1)\right]\left([z-1]M^{''}(z)+2M^{'}(z)\right)\\
&v^{'}(z)=1-M^{'}(z)\\
&v^{''}(z)=-M^{''}(z),\\
&u(1)=0v(1)=0\\
&u^{'}(1)=1-M^{'}(1)v^{'}(1)=1-M^{'}(1)\\
&u^{''}(1)=2M^{'}(1)\left[1-M^{'}(1)\right]v^{''}(1)=-M^{''}(1).
\end{aligned} \tag{61}$$

Now, the desired derivative becomes

$$\begin{aligned}
\lim_{z\longrightarrow 1}\frac{B}{Bz}S(p(z),z)&=\lim_{z\longrightarrow 1}\frac{B}{Bz}\frac{u(z)}{v(z)}\\
&=\lim_{z\longrightarrow 1}\frac{v(z)u^{'}(z)-u(z)v^{'}(z)}{v^{2}(z)}=\frac{0}{0}\left(\text{use L}^{'}\text{hospital}\right)\\
&=\lim_{z\longrightarrow 1}\frac{v(z)u^{''}(z)-u(z)v^{''}(z)}{2vv^{'}(z)}=\frac{0}{0}\left(\text{use L}^{'}\text{hospital}\right)\\
&=\lim_{z\longrightarrow 1}\frac{v(z)u^{'''}(z)+v^{'}(z)u^{''}(z)-u^{'}(z)v^{''}(z)-u(z)v^{'''}(z)}{2v^{'}(z)^{2}+2vv^{''}(z)}\\
&=\frac{v^{'}(1)u^{''}(1)-u^{'}(1)v^{''}(1)}{2v^{'}(1)^{2}}.
\end{aligned} \tag{62}$$

Substituting for the values of $u^{'}(1), v^{'}(1), u^{''}(1)$, and $v^{''}(1)$, we conclude

$$\lim_{z\longrightarrow 1}\frac{B}{Bz}S(p(z),z)=\frac{c\left[1-M'(1)\right]2M'(1)\left[1-M'(1)\right]-\left[1-M'(1)\right]\left[-M''(1)\right]}{2\left[1-M'(1)\right]^2}$$
$$=\frac{c2M'(1)\left[1-M'(1)\right]^2+M''(1)\left[1-M'(1)\right]}{2\left[1-M'(1)\right]^2}$$
$$=2M''(1)+\frac{M''(1)}{2\left[1-M'(1)\right]}. \tag{63}$$

Second, to find $p'(1)$, from equation (40), we have

$$p(z)=\frac{1-\lambda+\lambda p(z)N(z)}{\mu+(1-\mu)p(z)N(z)}. \tag{64}$$

Taking the first derivative with respect to z, then

$$p'(z)=\frac{c[\mu+(1-\mu)p(z)N(z)]\begin{bmatrix}\lambda p(z)N'(z)\\+\lambda p'(z)N(z)\end{bmatrix}-1-\lambda+\lambda p(z)N(z)\begin{bmatrix}(1-\mu)p(z)N'(z)\\+(1-\mu)p'(z)N(z)\end{bmatrix}}{[\mu+(1-\mu)p(z)N(z)]^2}. \tag{65}$$

Substituting for $z=1$, we get

$$p'(1)=\frac{c[\mu+(1-\mu)p(1)N(1)]\begin{bmatrix}\lambda p(1)N'(1)\\+\lambda p'(1)N(1)\end{bmatrix}-[1-\lambda+\lambda p(1)N(1)]\begin{bmatrix}(1-\mu)p(1)N'(1)\\+(1-\mu)p'(1)N(1)\end{bmatrix}}{[\mu+(1-\mu)p(1)N(1)]^2}$$
$$=\left[\lambda N'(1)+\lambda p'(1)\right]-\left[(1-\mu)N'(1)+(1-\mu)p'(1)\right]. \tag{66}$$

The previous result has been approved using Mathematica program [6] in calculating $p'(1)$. Solving for $p'(z)$, therefore

$$p'(1)-\lambda p'(1)+(1-\mu)p'(1)=\lambda N'(1)-(1-\mu)N'(1)$$
$$p'(1)[2-\lambda-\mu]=[\lambda-1+\mu]N'(1), \tag{67}$$

from which we find

$$p'(1)=\frac{\lambda-1+\mu}{2-\lambda-\mu}N'(1). \tag{68}$$

Using equations (63) and (54) in equation (56), we get

$$\frac{k(1-\mu)}{[2-\lambda-\mu]}p'(1)+\overline{G}=M'(1)+\frac{M''(1)}{2\left[1-M'(1)\right]}. \tag{69}$$

After using the value of $p'(1)$ from equation (68) in equation (69), then

$$\frac{k(1-\mu)(\lambda-1+\mu)}{[2-\lambda-\mu]^2}N'(1)+\overline{G}=M'(1)+\frac{M''(1)}{2\left[1-M'(1)\right]}. \tag{70}$$

Solving for $\overline{G}$, then

$$\overline{G}=M'(1)+\frac{M''(1)}{2\left[1-M'(1)\right]}+\frac{k(1-\mu)(\lambda-1+\mu)}{[2-\lambda-\mu]^2}N'(1), \tag{71}$$

where the values of $M'(1)$ and $M''(1)$ can be obtained from equation (45) in terms of known parameters on one hand and the derivatives of $p(z)$ at $z=1$ on the other hand. From equation (45), we have

$$M'(z)=k[\mu+(1-\mu)p(z)N(z)]^{k-1}\times\left[(1-\mu)p'(z)N(z)+(1-\mu)p(z)N'(z)\right], \tag{72}$$

$$=k[\mu+(1-\mu)]^{k-1}\left[(1-\mu)p'(1)+(1-\mu)N'(1)\right]$$
$$=k\left[(1-\mu)p'(1)+(1-\mu)N'(1)\right], \tag{73}$$

which agrees with the result of Mathematica program when used to calculate $M'(1)$. Substituting for the value of $p'(1)$ from equation (68) in equation (73), we get

$$M'(1)=k\left[(1-\mu)\frac{\lambda-1+\mu}{2-\lambda-\mu}N'(1)+(1-\mu)N'(1)\right]$$
$$=k(1-\mu)\left[\frac{\lambda-1+\mu+2-\lambda-\mu}{2-\lambda-\mu}\right]N'(1)$$
$$=k(1-\mu)\left[\frac{1}{2-\lambda-\mu}\right]N'(1)$$
$$=\frac{k(1-\mu)}{2-\lambda-\mu}N'(1), \tag{74}$$

which has been approved with the result of Mathematica. Now, to find $M''(1)$, we proceed as follows:

$$\begin{aligned} M''(z) = {} & k(k-1)[\mu + (1-\mu)p(1)N(1)]^{k-2} \\ & \times\left[(1-\mu)p'(1)N(1) + (1-\mu)p(1)N'(1)\right] \\ & \times\left[(1-\mu)p'(1)N(1) + (1-\mu)p(1)N'(1)\right] \\ & + k[\mu + (1-\mu)p(1)N(1)]^{k-1} \\ & \times\begin{bmatrix} (1-\mu)p'(1)N'(1) + (1-\mu)p''(1)N(1) \\ +(1-\mu)p(1)N''(1) + (1-\mu)p'(1)N'(1) \end{bmatrix}, \end{aligned} \tag{75}$$

which gives

$$\begin{aligned} M''(1) = {} & k(k-1)\left[(1-\mu)p'(1) + (1-\mu)N'(1)\right] \\ & \times\left[(1-\mu)p'(1) + (1-\mu)N'(1)\right], \end{aligned} \tag{76}$$

$$\begin{aligned} & + k\begin{bmatrix} (1-\mu)p'(1)N'(1) + (1-\mu)p''(1) \\ +(1-\mu)N''(1) + (1-\mu)p'(1)N'(1) \end{bmatrix} \\ = {} & k(k-1)\left[(1-\mu)p'(1) + (1-\mu)N'(1)\right]^2 \end{aligned} \tag{77}$$

$$\begin{aligned} & + k\Big[2(1-\mu)p'(1)N'(1) \\ & + (1-\mu)p''(1) + (1-\mu)N''(1)\Big], \end{aligned} \tag{78}$$

(the last result was verified again using Mathematica program). Now, we need to find the value of $p''(1)$, from equation (65), and using Mathematica for simplicity, we will get

$$\begin{aligned} p''(z) = {} & -\frac{\begin{pmatrix} 2\left(\lambda p(z)N'(z) + \lambda N(z)p'(z)\right)\times \\ \left((1-\mu)p(z)N'(z) + (1-\mu)N(z)p'(z)\right) \end{pmatrix}}{(\mu + (1-\mu)N(z)p(z))^2} \\ & 2(1-\lambda + \lambda N(z)p(z))\times \\ & + \frac{\left((1-\mu)p(z)N'(z) + (1-\mu)N(z)p'(z)\right)^2}{(\mu + (1-\mu)N(z)p(z))^3} \\ & + \frac{2\lambda N'(z)p'(z) + \lambda p(z)N''(z) + \lambda N(z)p''(z)}{(\mu + (1-\mu)N(z)p(z))} \\ & (1-\lambda + \lambda N(z)p(z))\times \\ & - \frac{\begin{pmatrix} 2(1-\mu)p'(z)N'(z) + (1-\mu)N(z)p''(z) \\ +(1-\mu)p(z)N''(z) \end{pmatrix}}{(\mu + (1-\mu)N(z)p(z))^2}. \end{aligned} \tag{79}$$

Substituting for $z = 1$, then

$$\begin{aligned} p''(1) = {} & -\frac{\begin{pmatrix} 2\left(\lambda p(1)N'(1) + \lambda N(1)p'(1)\right)\times \\ \left((1-\mu)p(1)N'(1) + (1-\mu)N(1)p'(1)\right) \end{pmatrix}}{(\mu + (1-\mu)N(1)p(1))^2} \\ & c2(1-\lambda + \lambda N(1)p(1)) \\ & + \frac{\left((1-\mu)p(1)N'(1) + (1-\mu)N(1)p'(1)\right)^2}{(\mu + (1-\mu)N(1)p(1))^3} \\ & + \frac{2\lambda N'(1)p'(1) + \lambda p(1)N''(1) + \lambda N(1)p''(1)}{(\mu + (1-\mu)N(1)p(1))} \\ & c(1-\lambda + \lambda N(1)p(1))\times \\ & - \frac{\begin{pmatrix} 2(1-\mu)p'(1)N'(1) + (1-\mu)N(1)p''(1) \\ +(1-\mu)p(1)N''(1) \end{pmatrix}}{(\mu + (1-\mu)N(1)p(1))^2}, \end{aligned} \tag{80}$$

which gives

$$\begin{aligned} p''(1) = {} & -\frac{\begin{pmatrix} 2\left(\lambda N'(1) + \lambda p'(1)\right)\times \\ \left((1-\mu)N'(1) + (1-\mu)p'(1)\right) \end{pmatrix}}{(\mu + (1-\mu))^2} \\ & + \frac{2(1-\lambda+\lambda)\left((1-\mu)N'(1) + (1-\mu)p'(1)\right)^2}{(\mu + (1-\mu))^3} \\ & + \frac{2\lambda N'(1)p'(1) + \lambda N''(1) + \lambda p''(1)}{(\mu + (1-\mu))} \\ & - \frac{(1-\lambda+\lambda)\begin{pmatrix} 2(1-\mu)p'(1)N'(1) \\ +(1-\mu)p''(1) + (1-\mu)N''(1) \end{pmatrix}}{(\mu + (1-\mu))^2} \\ = {} & -\begin{pmatrix} 2\left(\lambda N'(1) + \lambda p'(1)\right)\times \\ \left((1-\mu)N'(1) + (1-\mu)p'(1)\right) \end{pmatrix} \\ & + 2\left((1-\mu)N'(1) + (1-\mu)p'(1)\right)^2 \\ & + 2\lambda N'(1)p'(1) + \lambda N''(1) + \lambda p''(1) \\ & - \begin{pmatrix} 2(1-\mu)p'(1)N'(1) + (1-\mu)p''(1) \\ +(1-\mu)N''(1) \end{pmatrix}. \end{aligned} \tag{81}$$

Simplification of the last relation with Mathematica, we get

$$\begin{aligned} p''(1) = {} & 2\lambda N'(1)p'(1) - 2(1-\mu)p'(1)N'(1) \\ & - 2\Big(\lambda N'(1) + \lambda p'(1)\Big) \\ & \times \Big((1-\mu)N'(1) + (1-\mu)p'(1)\Big) \\ & + 2\Big((1-\mu)N'(1) + (1-\mu)p'(1)\Big)^2 \\ & + \lambda N''(1) - (1-\mu)N''(1) + \lambda p''(1) \\ & - (1-\mu)p''(1). \end{aligned} \tag{82}$$

Using Mathematica to solve for $p''(1)$, then

$$p''(1) = \frac{1}{2-\lambda-\mu}\left\{\begin{array}{c} 2\lambda N'(1)p'(1) - 2(1-\mu)p'(1)N'(1) \\ -2\Big(\lambda N'(1) + \lambda p'(1)\Big)\Big((1-\mu)N'(1) + (1-\mu)p'(1)\Big) \\ +2\Big((1-\mu)N'(1) + (1-\mu)p'(1)\Big)^2 + \lambda N''(1) - (1-\mu)N''(1) \end{array}\right\}. \tag{83}$$

Simplification of the previous relation, gives

$$p''(1) = \frac{1}{2-\lambda-\mu}\left\{(-1+\lambda+\mu)\Big(2(-1+\mu)N'(1)^2 + 2(-1+2\mu)N'(1)p'(1) + 2(-1+\mu)p'(1)^2 + N''(1)\Big)\right\}. \tag{84}$$

Substituting from equation (68) for the value of $p'(1)$, hence

$$p''(1) = \frac{1}{2-\lambda-\mu}\left\{(-1+\lambda+\mu)\left(\begin{array}{c} 2(-1+\mu)N'(1)^2 + \dfrac{2(-1+\mu)(-1+\lambda+\mu)^2 N'(1)^2}{(2-\lambda-\mu)^2} \\ +\dfrac{2(-1+\lambda+\mu)(-1+2\mu)N'(1)^2}{2-\lambda-\mu} + N''(1) \end{array}\right)\right\}. \tag{85}$$

After some manipulation, we get

$$p''(1) = \frac{1}{(-2+\lambda+\mu)^3}\left\{(-1+\lambda+\mu)\times\left(\begin{array}{c} 2\Big(3+\lambda^2-4\mu+\mu^2+\lambda(-3+2\mu)\Big)N'(1)^2 \\ -(-2+\lambda+\mu)^2 N''(1) \end{array}\right)\right\}. \tag{86}$$

Now, we return to equation (78) to find the final formula for $M''(1)$. Substituting from equations (68) and (86) in equation (78) for the values of $p'(1)$ and $p''(1)$, we get

$$M''(1) = k(k-1)\left[\begin{array}{c}(1-\mu)N'(1)\\ +\dfrac{(1-\mu)(-1+\lambda+\mu)N'(1)}{2-\lambda-\mu}\end{array}\right]^2$$
$$+k\left(\frac{2(1-\mu)(-1+\lambda+\mu)N'(1)^2}{(2-\lambda-\mu)}+(1-\mu)N''(1)+\frac{1}{(-2+\lambda+\mu)^3}\right.$$
$$\left.\cdot\left\{(1-\mu)(-1+\lambda+\mu)\times\left[\begin{array}{c}2\left(3+\lambda^2-4\mu+\mu^2+\lambda(-3+2\mu)\right)N'(1)^2\\ -(-2+\lambda+\mu)^2N''(1)\end{array}\right]\right\}\right). \tag{87}$$

Using Mathematica to simplify the previous equation, then

$$M''(1) = \frac{1}{(-2+\lambda+\mu)^3}\left\{ck(-1+\mu)\times\left(\begin{array}{c}\left(-4+5\lambda-2\lambda^2+5\mu c-3\lambda\mu-\mu^2+k(-1+\mu)\times(-2+\lambda+\mu)\right)N'(1)^2\\ +(-2+\lambda+\mu)^2N''(1)\end{array}\right)\right\}$$
$$=\frac{k(-1+\mu)}{(-2+\lambda+\mu)}\times\left[N''(1)+\frac{\begin{array}{c}c-4+5\lambda-2\lambda^2+5\mu-3\lambda\mu-\mu^2\\ +k(-1+\mu)(-2+\lambda+\mu)\end{array}}{(-2+\lambda+\mu)^2}\times N'(1)^2\right]. \tag{88}$$

Higher moments of the buffer occupancy can also be obtained, using the same way; however, this is going to lead to complicated mathematical derivations.

5. Discussion of the Result

The obtained results for the steady-state distribution of the number of busy stations shows that it depends only on the value of the parameter ω. This result may lead us to say that the steady-state buffer behavior of the base station is determined only from the value of ω. This section will be used to discuss this point. We consider that each station is busy with probability ω and is idle with probability $1-\omega$, independently from slot to slot. So

$$\left.\begin{array}{l}\Pr\,[\text{number of busy slots} = g] = (1-\omega)\omega^{g-1},\\ \text{mean number of busy slots} = \dfrac{1}{(1-\omega)}.\\ \Pr\,[\text{number of idle slots} = g] = \omega(1-\omega)^{g-1},\\ \text{mean number of idle slots} = \dfrac{1}{\omega}.\end{array}\right] \tag{89}$$

In such case, the average activity of the station will also be ω if the mean numbers of busy and idle slots are both multiplied by a same factor l, i.e., if λ and μ are selected, such that

$$\begin{aligned}\text{mean busy slots}\,\frac{1}{1-\lambda} &= \frac{l}{1-\omega},\\ \text{mean idle slots}\,\frac{1}{1-\mu} &= \frac{l}{\omega}.\end{aligned} \tag{90}$$

To demonstrate the importance of the parameter l, let us use the following situation. Suppose that the busy stations, every busy slot, generate one message per busy slot. Therefore, the number of packets generated by the busy stations equal to the message length (in packets). Assuming a geometric distribution for the message length and assuming that the random variable N represents the number of packets generated by a station in a given slot (message length), we then get

$$\Pr\,[N = g] = (1-\psi)\psi^{g-1}, \tag{91}$$

where ψ is the probability that the message not finished and $(1-\psi)$ is the probability that the message is finished, then

$$
\begin{aligned}
N(z) &= \sum_{g=1}^{\infty} (1-\psi)\psi^{g-1} z^{g} \\
&= (1-\psi) z \sum_{l=0}^{\infty} \psi^{l} z^{l} \\
&= \frac{(1-\psi)z}{1-\psi z}, \\
\overline{N} &= \frac{B}{Bz} N(z)\Big|_{z=1} \\
&= \frac{B}{Bz}\left(\frac{(1-\psi)z}{1-\psi z}\right)\Bigg|_{z=1} \\
&= \frac{(1-\psi)(1-\psi)-(1-\psi)(-\psi)}{(1-\psi)^{2}} \\
&= \frac{(1-\psi)^{2}+\psi(1-\psi)}{(1-\psi)^{2}} \\
&= \frac{1}{(1-\psi)},
\end{aligned}
\tag{92}
$$

where $\overline{N}$ is the mean message length. In such case, the mean buffer occupancy of the base station, at the steady state, can be obtained in the form

$$
\begin{aligned}
\overline{G} = {} & \frac{k\omega}{2(1-\psi)(1-\psi-k\omega)} \\
& \times [2(1-\psi)-(3k-1)\omega+2l(\psi+(k-1)\omega)].
\end{aligned}
\tag{93}
$$

6. Conclusion

The study and analysis of base stations buffers behaviors in 5G and next generations mobile networks can contribute to reducing the network latency and improving the network performance and the QoS. In this paper, the buffer behavior of base stations of 5G mobile networks at steady state is investigated. The network includes a base station and a finite number of mobile stations. Each mobile station alternates between two independent states with arbitrary length: state of transmission (busy) and a state of no transmission (idle). A two-dimensional Markov chain has been used to derive the probability generating function corresponding to buffer occupancy at the steady state. Mean buffer occupancy of the base station of the cellular mobile network at the steady state is also calculated. The results show a type of dependency between the activity level of the mobile stations (busy or idle) and the expected buffer occupancy of the base station. Moreover, expressions resulted from the analysis have listed factors and parameters that affect the base stations buffer behavior. These factors can be studied and analyzed to further reduce the latency and improve the QoS of next generation mobile networks.

Data Availability

No underlying data were collected or produced in this study.

Conflicts of Interest

The author declares that there are no conflicts of interest.

References

[1] A. Showail, K. Jamshaid, and B. Shihada, "Buffer sizing in wireless networks: challenges, solutions, and opportunities," *IEEE Communications Magazine*, vol. 54, no. 4, pp. 130–137, 2016.

[2] R. Ranjith and H. N. Vishwas, "Analysis study of data buffer policies in a fuzzy logic based WSN using cluster head," in *Proceedings of the 2017 International Conference On Smart Technologies For Smart Nation (SmartTechCon)*, Bengaluru, India, August 2017.

[3] S. Panev and P. Latkoski, "Performance analysis of handover delay and buffer capacity in mobile OpenFlow-based networks," *International Journal of Communication Systems*, vol. 33, no. 3, 2020.

[4] S. Batabyal, P. Bhaumik, S. Chattopadhyay, and S. Misra, "Steady-state analysis of buffer occupancy for different forwarding strategies in mobile opportunistic network," *IEEE Transactions on Vehicular Technology*, vol. 68, no. 7, pp. 6951–6963, 2019.

[5] Z. Wang, C. Nian, W. Liu, M. Han, and M. Lin, "A bandwidth statistical multiplexing and control method for satellite broadcasting," *Scientific Programming*, vol. 2020, Article ID 8841006, 10 pages, 2020.

[6] Wolfram Research Inc, *Mathematica, Version 13.1*, Wolfram Research Inc, Champaign, IL, USA, 2022.