

Research Article

Robust Design of Terminal ILC with H_∞ Mixed Sensitivity Approach for a Thermoforming Oven

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This paper presents a robust design approach for terminal iterative learning control (TILC). This robust design uses the H_∞ mixed-sensitivity technique. An industrial application is described where TILC is used to control the reheat phase of plastic sheets in a thermoforming oven. The TILC adjusts the heater temperature setpoints such that, at the end of the reheat cycle, the surface temperature map of the plastic sheet will converge to the desired one. Simulation results are included to show the effectiveness of the control law.

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1. INTRODUCTION

Up till now, the cycle-to-cycle temperature control of industrial thermoforming machines has been rather primitive. The in-cycle temperature control is performed with traditional PID control of heater temperatures [1, 2]. However, the cycle-to-cycle adjustments of the heater temperature setpoints are made manually by trial and error. Typically, this causes financial losses for thermoformers because of bad setpoint adjustments leading to wasted parts.

For the thermoforming application, the terminal iterative learning control (TILC) algorithm is an efficient cycle-to-cycle control technique to adjust the setpoint temperatures of heaters in the thermoforming oven [3, 4]. Infrared temperature sensors are placed in the oven to measure the surface temperature of the plastic sheet. The TILC adjusts the heater temperature setpoints to control the terminal surface temperature to a desired temperature profile at the end of the cycle.

TILC was first addressed in [5]. There have been various applications of TILC, notably in rapid thermal processing for chemical vapor deposition [5–8]. TILC is derived from iterative learning control (ILC), an approach that has attracted a lot of attention. The main difference between

TILC and ILC is that TILC has access to a measurement of the process output only at the end of the cycle, whereas ILC uses output measurements throughout the cycle. Many papers have been written about ILC as shown in a survey by Moore [9].

This paper is about the use of H_∞ mixed-sensitivity analysis as a tool to design robust TILC controllers. The mixed-sensitivity approach was successfully used by other researchers for ILC, see [10–13]. Recent work [14–16] addressed the robustness problem in the cycle domain using H_∞ optimal iterative learning control based on a super vector approach.

While robust ILC design has been studied extensively, robust TILC has attracted less attention. However, high-order TILC has been proposed in [5, 6, 8, 14–16] to improve the robustness.

Section 2 presents the system to be controlled by TILC. Section 3 introduces H_∞ concepts necessary to carry out the robust design such as weighting functions and their parameters. Section 4 presents the H_∞ mixed-sensitivity method to design a controller. Simulation results, using a TILC designed by H_∞ mixed-sensitivity on a model of a thermoforming machine are given in Section 5. Section 6 concludes.

2. PROBLEM OF SET UP

The TILC algorithm is applied to a continuous, linear time-invariant system. This system may be the linearized model of a thermoforming machine [3, 4] and is represented by

$$\begin{aligned} \dot{x}_k(t) &= Ax_k(t) + Bu_k, \\ y_k(t) &= Cx_k(t), \end{aligned} \quad (1)$$

where t is the time and the subscript k is the cycle number ($k \in \mathbb{N}$). Matrices A , B , and C are time-invariant. The state vector in cycle k is $x_k(t) \in \mathbb{R}^n$, the (constant over one cycle) input vector is $u_k \in \mathbb{R}^m$, and the output vector is $y_k(t) \in \mathbb{R}^p$.

The control task is to update the control input u_k (heater temperature setpoints) after each cycle such that the terminal output $y_k(T)$ (sheet surface temperatures) converges to a desired terminal value y_d at time T . From linear system theory, one can write the solution of (1) at $t = T$ as

$$x_k(T) = e^{AT}x_k(0) + \left(\int_0^T e^{A(T-\tau)} B d\tau \right) u_k. \quad (2)$$

From this terminal state, we calculate the corresponding terminal output as

$$y_k(T) = Ce^{AT}x_k(0) + \left(C \int_0^T e^{A(T-\tau)} B d\tau \right) u_k. \quad (3)$$

Now, we change the notation to emphasize the fact that for the cycle-to-cycle control, cycle k is equivalent to the time argument of a discrete-time system. Equation (3) is rewritten as

$$y_T[k] = \Gamma_0 x_0[k] + \Psi_0 u[k], \quad (4)$$

where $y_T[k] = y_k(T)$, $x_0[k] = x_k(0)$. Matrix $\Gamma_0 \in \mathbb{R}^{p \times n}$ is defined by

$$\Gamma_0 := Ce^{AT}, \quad (5)$$

and matrix $\Psi \in \mathbb{R}^{p \times m}$ by

$$\Psi_0 := C \int_0^T e^{A(T-\tau)} B d\tau. \quad (6)$$

Thus, we can apply discrete-time control on system (4) which will appear like cycle-to-cycle control to the system (1).

The z -transform of (4) is

$$\hat{y}_T(z) = \Psi_0 \hat{u}(z) + \Gamma_0 \hat{x}_0(z), \quad (7)$$

where $\hat{u}(z)$, $\hat{y}_T(z)$, and $\hat{x}_0(z)$ are the z -transforms of $u[k]$, $y_T[k]$, and $x_0[k]$, respectively.

The following assumptions are made in this paper.

- (A1) The initial state repeats itself. Thus, $x_k(0) = x_0[k] = \chi_0$ is a constant vector for all cycles. This corresponds to the assumption that all plastic sheets are at the same initial temperature before being heated.
- (A2) The desired terminal output is constant for all cycles k : $y_d[k] = \gamma$. Since we want to thermoform plastic sheets to obtain a desired part in a repetitive way, the desired temperature must remain constant.

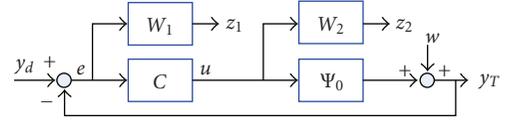


FIGURE 1: Block diagram of the system.

3. PRELIMINARIES ON THE MIXED-SENSITIVITY PROBLEM

The mixed-sensitivity approach is used to design a robust discrete-time controller. This controller has to keep the system stable despite perturbations on the entries of matrix Ψ from the nominal values contained in Ψ_0 . In thermoforming machines, a large part of the uncertainties in the perturbed system Ψ is due to process nonlinearities.

Figure 1 shows the nominal system with the TILC controller $C(z)$ and the weighting functions $W_1(z)$ and $W_2(z)$ in the z -domain. The output disturbance signal $w[k]$ in the block diagram contains the term $\Gamma_0 x_0[k]$ in (4) due to the initial state.

From the closed-loop system shown in Figure 1, we obtain the output sensitivity matrix

$$S = (I + \Psi_0 C)^{-1}, \quad (8)$$

and the complementary sensitivity matrix

$$T = \Psi_0 C (I + \Psi_0 C)^{-1}. \quad (9)$$

Furthermore, we can define the input sensitivity matrix

$$U := C (I + \Psi_0 C)^{-1}. \quad (10)$$

The mixed-sensitivity design for the system is based on finding a stabilizing controller $C(z)$ that minimizes the norm

$$d := \left\| \begin{bmatrix} W_1 S \\ W_2 U \end{bmatrix} \right\|_{\infty}. \quad (11)$$

This norm uses two weighting functions.

(i) W_1 is a diagonal weighting function on the process error signal, with each term on the main diagonal defined as

$$W_{1ii}(z) := \frac{1}{M_{1i}} \left\{ \frac{(2 + M_{1i}\omega_{1i})z - (2 - M_{1i}\omega_{1i})}{(2 + \varepsilon_{1i}\omega_{1i})z - (2 - \varepsilon_{1i}\omega_{1i})} \right\}, \quad (12)$$

where $i \in \{1, 2, \dots, p\}$. To establish a connection to the more widely used Laplace-domain weighting functions in continuous-time H_{∞} control, $W_{1ii}(z)$ was obtained from a bilinear transformation of the continuous-time first-order lag $W_c(s) = (1/M_{1i})((s + M_{1i}\omega_{1i})/(s + \varepsilon_{1i}\omega_{1i}))$. The DC gain of $W_c(s)$ is ε_{1i}^{-1} , its high-frequency gain is M_{1i}^{-1} , and ω_{1i} is a parameter that can shift the frequency range at which the lag is applied. The inverse of $W_1(z)$ provides an upper bound for the shape of the sensitivity function S . The parameters shaping W_1 are shown in Figure 2.

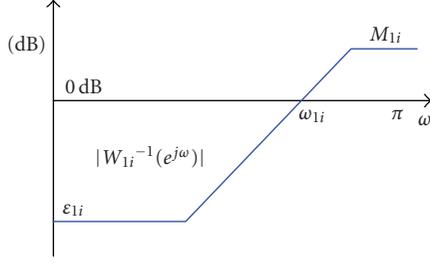


FIGURE 2: Frequency response magnitude of $W_{1i}^{-1}(e^{j\omega})$ and its parameters.

A minimal state-space realization of $W_1(z)$ is given by

$$W_1 = \left[\begin{array}{c|c} A_{W_1} & B_{W_1} \\ \hline C_{W_1} & D_{W_1} \end{array} \right] := D_{W_1} + C_{W_1}(zI - A_{W_1})^{-1}B_{W_1}. \quad (13)$$

All state-space matrices are diagonal. The (i, i) entries on the main diagonal of the state-space matrices are given by

$$A_{W_{1ii}} = \frac{2 - \epsilon_{1i}\omega_{1i}}{2 + \epsilon_{1i}\omega_{1i}}, \quad (14)$$

$$B_{W_{1ii}} = 1, \quad (15)$$

$$C_{W_{1ii}} = \frac{4\omega_{1i}(M_{1i} - \epsilon_{1i})}{M_{1i}(2 + \epsilon_{1i}\omega_{1i})^2}, \quad (16)$$

$$D_{W_{1ii}} = \frac{(2 + M_{1i}\omega_{1i})}{M_{1i}(2 + \epsilon_{1i}\omega_{1i})}, \quad (17)$$

where $i \in \{1, 2, \dots, p\}$.

The parameter ϵ_{1i} is the DC gain of $W_{1i}^{-1}(z)$, that is, $W_{1i}^{-1}(e^{j0}) = W_{1i}^{-1}(1) = \epsilon_{1i}$. To minimize the steady-state error, we need this value to be smaller than 1. M_{1i} is the high-frequency gain of $W_{1i}^{-1}(z)$, that is, $W_{1i}^{-1}(e^{j\pi}) = W_{1i}^{-1}(-1) = M_{1i}$. This value is chosen to be larger than one. As stated earlier, ω_{1i} is a parameter to adjust the crossover frequency of the weighting function.

(ii) W_2 is a diagonal weighting function on the process input signal, with each term on the main diagonal defined as

$$W_{2ii} = M_{2i}^{-1}, \quad (18)$$

where $M_{2i} \in \mathbb{R}_+$ and $i \in \{1, 2, \dots, m\}$. The inverse of W_2 gives an upper bound for the magnitude of the input sensitivity function U on the unit circle. Note that there are no dynamics in W_2 . Weighting function W_2 also expresses the uncertainty level in the system subject to real parametric perturbations in the entries of Ψ .

Since the TILC algorithm must be causal, we need to obtain a controller with a strictly proper transfer function. To do so, the controller $C(z)$ is decomposed in two parts, as shown in Figure 3. A proper controller $\tilde{C}(z)$, designed with the mixed-sensitivity analysis, is combined with a delay $(z^{-1}I_m)$. The minimal state-space realization of the delay $W_{\text{delay}} = z^{-1}I_m$ is given by

$$W_{\text{delay}} = \left[\begin{array}{c|c} A_{W_{\text{delay}}} & B_{W_{\text{delay}}} \\ \hline C_{W_{\text{delay}}} & D_{W_{\text{delay}}} \end{array} \right] = \left[\begin{array}{c|c} 0 & I_m \\ \hline I_m & 0 \end{array} \right]. \quad (19)$$

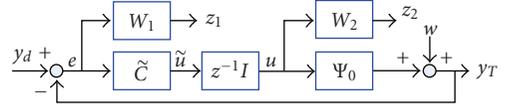


FIGURE 3: Block diagram with the controller $\tilde{C}(z)$ and the delay $z^{-1}I$.

In Figures 1 and 3, the nominal system Ψ_0 is a full-rank real matrix and, therefore, has no dynamics.

4. MIXED-SENSITIVITY PROBLEM OF TILC

To solve the mixed-sensitivity problem we have to minimize the norm in (11) to a value smaller than 1. But first, we must check whether the problem is well posed [17].

From Figure 3, we can construct the open-loop generalized plant between the inputs and the outputs assuming that the controller has been removed. This generalized plant is

$$\begin{bmatrix} z_1 \\ z_2 \\ e \end{bmatrix} = \begin{bmatrix} W_1 & -W_1\Psi_0 W_{\text{delay}} \\ 0 & W_2 W_{\text{delay}} \\ I & -\Psi_0 W_{\text{delay}} \end{bmatrix} \begin{bmatrix} y_d \\ \tilde{u} \end{bmatrix} := P \begin{bmatrix} y_d \\ \tilde{u} \end{bmatrix} \quad (20)$$

and is used for the minimization of (11).

To simplify the analysis, we can rewrite plant P with its own state-space realization as follows:

$$P = \left[\begin{array}{c|c} \left[\begin{array}{cc} A_{W_{\text{delay}}} & 0 \\ -B_{W_1}\Psi_0 C_{W_{\text{delay}}} & A_{W_1} \end{array} \right] & \left[\begin{array}{c} 0 \\ B_{W_1} \end{array} \right] \left[\begin{array}{c} B_{W_{\text{delay}}} \\ B_{W_1}\Psi_0 D_{W_{\text{delay}}} \end{array} \right] \\ \hline \left[\begin{array}{cc} -D_{W_1}\Psi_0 C_{W_{\text{delay}}} & C_{W_1} \\ W_2 C_{W_{\text{delay}}} & 0 \end{array} \right] & \left[\begin{array}{c} D_{W_1} \\ 0 \end{array} \right] \left[\begin{array}{c} D_{W_1}\Psi_0 D_{W_{\text{delay}}} \\ W_2 D_{W_{\text{delay}}} \end{array} \right] \\ \hline \left[\begin{array}{cc} -\Psi_0 C_{W_{\text{delay}}} & 0 \end{array} \right] & \begin{bmatrix} I_p & \Psi_0 D_{W_{\text{delay}}} \end{bmatrix} \end{array} \right]. \quad (21)$$

With the definitions given in the previous section, we can write

$$P = \left[\begin{array}{c|c} \left[\begin{array}{cc} 0 & 0 \\ -\Psi_0 & A_{W_1} \end{array} \right] & \left[\begin{array}{c} 0 \\ I_p \end{array} \right] \left[\begin{array}{c} I_m \\ 0 \end{array} \right] \\ \hline \left[\begin{array}{cc} -D_{W_1}\Psi_0 & C_{W_1} \\ W_2 & 0 \end{array} \right] & \left[\begin{array}{c} D_{W_1} \\ 0 \end{array} \right] \left[\begin{array}{c} 0 \\ 0 \end{array} \right] \\ \hline \left[\begin{array}{cc} -\Psi_0 & 0 \end{array} \right] & \begin{bmatrix} I_p & 0 \end{bmatrix} \end{array} \right]. \quad (22)$$

To simplify the notation in the remaining part of the paper, we can rewrite (22) as

$$P = \left[\begin{array}{c|c} \left[\begin{array}{c} A \\ \hline C_1 \\ C_2 \end{array} \right] & \left[\begin{array}{cc} B_1 & B_2 \\ \hline D_{11} & D_{12} \\ D_{21} & D_{22} \end{array} \right] \end{array} \right]. \quad (23)$$

It is important to note that all matrices appearing in (22) are full rank.

Before going further, we need the following lemma.

Lemma 1. *Suppose all ε_{1i} are different from 0. Then, all of the eigenvalues of A_{W_1} lie strictly inside the unit circle.*

Proof. Since A_{W_1} is diagonal, its eigenvalues are equal to the entries on the diagonal. Thus, the eigenvalues of A_{W_1} are equal to $A_{W_{1i}}$, with $1 \leq i \leq p$. Because both ε_{1i} and ω_{1i} are greater than 0 for all i , we have $2 - \varepsilon_{1i}\omega_{1i} < 2$ and $2 + \varepsilon_{1i}\omega_{1i} > 2$. This implies that the eigenvalues $A_{W_{1i}}$ as given by (14) are strictly lower than 1, for all $i = 1, 2, \dots, p$. Hence, all eigenvalues of A_{W_1} are strictly inside the unit circle. \square

We also need the following standard lemma on the suboptimal discrete-time H_∞ problem [17].

Lemma 2. *The suboptimal H_∞ problem corresponding to the generalized plant in (20) and consisting of computing a controller yielding a norm (11) less than some desired $\gamma > 0$ has a solution if the following four conditions [17] hold:*

- (1) (A, B_2) is stabilizable;
- (2) (C_2, A) is detectable;
- (3) $\begin{bmatrix} A - I_n e^{j\theta} & B_2 \\ C_1 & D_{12} \end{bmatrix}$ is full column rank for all $\theta \in [0, 2\pi]$;
- (4) $\begin{bmatrix} A - I_n e^{j\theta} & B_1 \\ C_2 & D_{21} \end{bmatrix}$ is full row rank for all $\theta \in [0, 2\pi]$.

Lemmas 1 and 2 lead to the following theorem.

Theorem 1. *The H_∞ mixed-sensitivity problem of the system Ψ_0 controlled by a TILC algorithm with the mixed-sensitivity function defined by (8) and (10) can be solved.*

Proof. Since the matrix A is Hurwitz, the first two conditions in Lemma 2 are fulfilled. The eigenvalues of the delay are equal to 0 and the eigenvalues of A_{W_1} are strictly inside the unit circle as shown in Lemma 1.

The third condition in Lemma 2 concerns the matrix

$$\begin{bmatrix} A - I_n e^{j\theta} & B_2 \\ C_1 & D_{12} \end{bmatrix} = \begin{bmatrix} -I_m e^{j\theta} & 0 & I_m \\ -\Psi_0 & A_{W_1} - I_p e^{j\theta} & 0 \\ -D_{W_1} \Psi_0 & C_{W_1} & 0 \\ W_2 & 0 & 0 \end{bmatrix}. \quad (24)$$

The column rank of this matrix must be equal to the rank of $A - I_n e^{j\theta}$ plus the number of inputs m . Since matrix A is Hurwitz with all eigenvalues having their amplitude below 1, and C_{W_1} and D_{W_2} are full rank, the rank of (24) is equal to $m+n$ for all $\theta \in [0, 2\pi]$. Thus, the third condition is satisfied.

The fourth condition is related to the rank of

$$\begin{bmatrix} A - I_n e^{j\theta} & B_1 \\ C_2 & D_{21} \end{bmatrix} = \begin{bmatrix} -I_m e^{j\theta} & 0 & 0 \\ -\Psi_0 & A_{W_1} - I_n e^{j\theta} & I_p \\ -\Psi_0 & 0 & I_p \end{bmatrix}. \quad (25)$$

The row rank of this matrix must be equal to the rank of $A - I_n e^{j\theta}$ plus the number of outputs p . For the same reasons as above, the rank of (25) is equal to $n+p$, for all $\theta \in [0, 2\pi]$, the number of rows of (25). Thus, the fourth condition is fulfilled.

In conclusion, because all conditions of Lemma 2 are satisfied, the H_∞ mixed-sensitivity approach can be used to design a robustly stable TILC algorithm. \square

Theorem 1 shows that the H_∞ mixed-sensitivity approach can be used to compute the TILC controller and this can be done using design tools like Matlab's "dthinflmi" or "dthinfric" functions.

The next section presents a TILC design for a thermoforming machine. This design was tested on a thermoforming oven model.

5. SIMULATION RESULTS

To test the effectiveness of the mixed-sensitivity approach, we take as an example a design based on a model of a thermoforming machine.

Only the heating phase of the thermoforming process is considered here. Molding is not considered since the purpose of the TILC algorithm is to heat the plastic sheet up to a desired surface temperature map, before the molding phase.

Linearizing the model of a thermoforming oven [3] configured with four heater zones and four infrared temperature sensors around the operating point, we obtain

$$\Psi_0 = \begin{bmatrix} 0.0336 & 0.3340 & 0.0121 & 0.1202 \\ 0.3340 & 0.0336 & 0.1202 & 0.0121 \\ 0.0121 & 0.1202 & 0.0336 & 0.3340 \\ 0.1202 & 0.0121 & 0.3340 & 0.0336 \end{bmatrix}. \quad (26)$$

We choose as design parameter for the weighting function W_1 : $M_{1i} = 10$, $\omega_{1i} = 0.1$ and $\varepsilon_{1i} = 0.01$ for $i = 1, \dots, 4$. This will shape the sensitivity function of the closed-loop system such that the system responds relatively quickly, that is, in a few cycles. For a thermoforming machine, we want to get acceptable heater temperature setpoints as fast as possible in order to limit the number of wasted parts.

We select for W_2 the parameters $M_{2i} = 10$, $i = 1, \dots, 4$. This weighting function will shape the U function to limit the actuator efforts and reduce the risk of large overshoot and saturation. On a thermoforming machine, an overshoot in heater temperature can cause the plastic sheet to sag too much and damage the thermoforming oven.

The mixed-sensitivity approach leads us to the following controller (with $d = 0.5305$):

$$C(z) = \begin{bmatrix} k_1(z) & k_2(z) & k_3(z) & k_4(z) \\ k_2(z) & k_1(z) & k_4(z) & k_3(z) \\ k_3(z) & k_4(z) & k_1(z) & k_2(z) \\ k_4(z) & k_3(z) & k_2(z) & k_1(z) \end{bmatrix} m^{-1}(z), \quad (27)$$

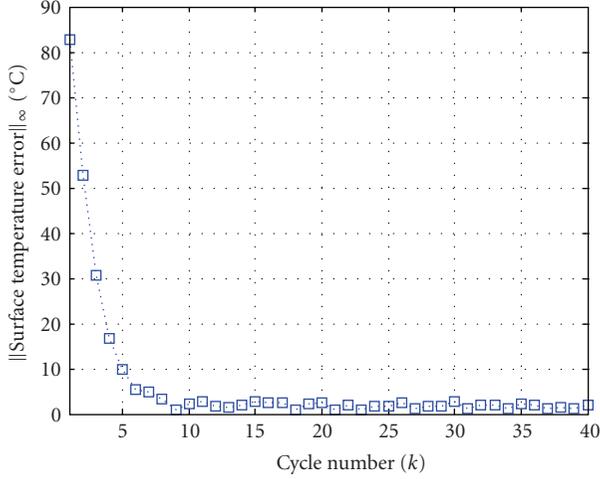


FIGURE 4: Error on terminal surface temperature (1st simulation).

where

$$\begin{aligned}
 k_1(z) &= -0.7417(z + 0.1288)(z - 0.0441) \\
 &\quad \times (z^2 - 0.2309z + 0.0134), \\
 k_2(z) &= 2.3929(z + 0.1262)(z - 0.1184) \\
 &\quad \times (z - 0.1009)(z - 0.1559), \\
 k_3(z) &= 0.6750(z + 0.1297)(z - 0.1402) \\
 &\quad \times (z - 0.1033)(z - 0.0161), \\
 k_4(s) &= -1.1938(z + 0.1271)(z - 0.1175) \\
 &\quad \times (z^2 - 0.2067z + 0.0114), \\
 m(z) &= (z - 0.9993)(z - 0.1301)(z - 0.1074) \\
 &\quad \times (z - 0.0949)(z + 0.1255).
 \end{aligned} \tag{28}$$

This discrete-time controller corresponds to a fifth-order TILC to control the thermoforming oven heater temperature setpoints.

This controller was implemented and tested on the nonlinear model of the thermoforming machine. In this model, the initial sheet temperature is 27°C and subject to a slow variation of 10°C. The measurement noise in the simulation was a Gaussian white noise with standard deviation equal to 1°C, which is representative of infrared sensor noise.

Figure 4 shows the behavior of the infinity norm (maximum component) of the terminal surface temperature error. The desired terminal temperatures are 150°C and 160°C at the end of a 3-minute heating cycle. The error becomes smaller than 5°C at the 7th iteration (or cycle) and seems to remain within this bound thereafter.

For high-density polyethylene (HDPE) thermoplastic sheets, a terminal temperature kept inside a $\pm 10^\circ\text{C}$ margin of the desired temperature is acceptable for forming and the risk of getting a defective part is low [1, 2].

The terminal temperatures on the top surface of the sheet (IR_{T2} and IR_{T5}) and on the bottom surface (IR_{B2} and IR_{B5}), shown in Figure 5, converge smoothly to the desired terminal

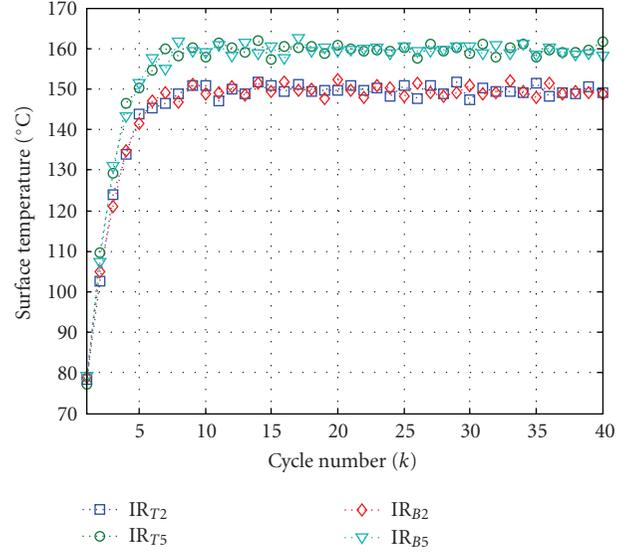


FIGURE 5: Sheet terminal surface temperatures (1st simulation).

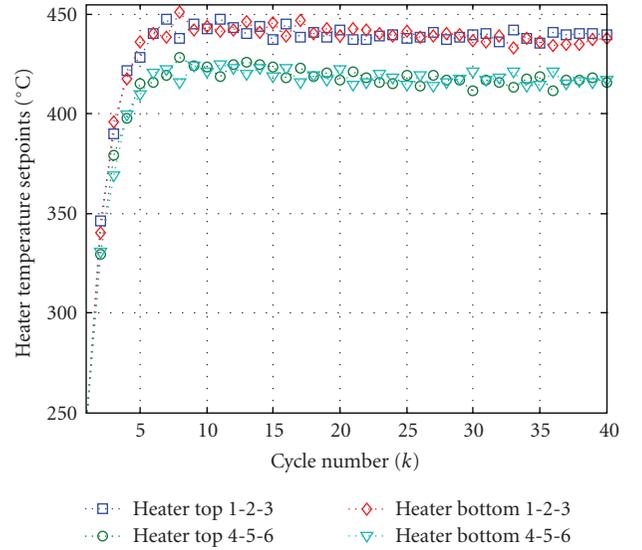


FIGURE 6: Heater temperature setpoints (1st simulation).

temperatures, despite the variation of the initial condition. This variation combined with the noise in the measurements of the surface temperature explains the slight variations in the temperature measurements.

The evolution of heater zone temperatures is shown in Figure 6. Note that the oven model has 6 independent heater zones on top that can be grouped together, for example, here zones 1, 2, and 3 are controlled as a single zone and similarly for zones 4, 5, and 6. The same applies for the bottom heater zones. We can see a smooth behavior of the zone setpoints. The system has to compensate for variation in initial condition (here the initial surface temperature of the sheet).

A second simulation was performed with the same parameters, except for the ambient air temperature which

was set 10°C higher. The rate of the convergence in the second simulation was nearly the same as that obtained in the first simulation. The heater temperature setpoints converged to lower values, since the ambient air temperature is higher. The simulation results showed that the TILC controller can adapt to seasonal changes in temperature and slow variation in the initial temperature of the plastic sheet.

Convergence to the desired terminal temperature is slower than the so-called deadbeat response provided by the TILC in [4]. On the other hand, results have shown that the controller presented here is more robust to perturbations than the deadbeat TILC. This is an illustration of the tradeoff between performance and robustness in feedback control systems.

6. CONCLUSION

The simulation results of the previous section demonstrate the effectiveness of the TILC algorithm for sheet reheat. We can argue that the TILC controlled thermoforming machine will keep the surface temperature profile of the plastic sheet to the desired one and adapt to slow temperature variations that inevitably happen in a thermoforming facility. For a given plastic sheet and temperature map, the heater temperature setpoints will be different on a hot summer day than on a cold winter day. Even when there is a long delay between the processing of two successive batches of sheets, the system will learn again new heater setpoints and converge to the desired temperature profile, as shown experimentally in [4].

In future work, we will test the system with nonfeasible temperature map for nonsquare or rank-deficient Ψ_0 to see how the TILC law can manage. Finally, on a more theoretical aspect of TILC, we will analyze the robust performance of the system with μ -analysis.

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