Hindawi Publishing Corporation
Journal of Control Science and Engineering
Volume 2013, Article ID 327525, 9 pages
http://dx.doi.org/10.1155/2013/327525

Research Article

Integrity Design for Networked Control Systems with Actuator Failures and Data Packet Dropouts

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Received 8 June 2013; Revised 7 October 2013; Accepted 8 October 2013

Academic Editor: Ying Wang

The integrity design problem of fault tolerant control for networked control system (NCS) with actuator failures and data packet dropouts is investigated. The data packet dropouts in both sensor-controller (S-C) and controller-actuator (C-A) links are described by two switches, which can be modeled as a discrete event system with known rate. After introducing the matrix of actuator failure, the closed-loop NCS is developed, which can be viewed as asynchronous dynamical systems (ADSs). Then, the sufficiency of exponential stability for the NCS is obtained based on the theory of ADSs. The output feedback controllers that can guarantee system stability are also proposed. Finally, two numerical examples are given to demonstrate the validity of our proposed approach.

1. Introduction

Along with the rapid development of communication networks, a great amount of effort has been made on fault-tolerant control (FTC) problems for networked control systems (NCSs) recently. NCSs are distributed systems, which are comprised of controlled plants actuators, sensors, and controllers. The essential feature of NCSs is that the information is exchanged through some form of communication networks (as in [1–5]). The use of a shared network to connect spatially distributed devices results in flexible architectures and generally reduces installation and maintenance costs. Consequently, NCSs have been widely applied to many complex control systems, for example, unmanned aerial vehicles, avionics industries, remote surgery, and rapid transit trains.

However, the insertion of networks also brings some new issues, such as network-induced delay (as in [1, 6–12]) and data packet dropout (as in [1, 13–15]), which make NCSs more vulnerable to faults than conventional systems. As we know, research on FTC strives to make the system stable and retain acceptable performance under the system faults. An important part of FTC is the one specializing in actuator faults, FTC techniques dealing with actuator faults are relevant for practical application and have already been the subject of many publications (as in [16–19]). Therefore, a suitable architecture for FTC of NCSs must take the dynamical behavior of network into consideration (as in [8–12, 14, 15, 20–23]).

A wide range of research has recently been reported dealing with problems related to the FTC for NCSs with network-induced delay (as in [8–11]). As compared to the plentiful works on FTC for NCSs with network-induced delay, only a few attention has been paid to the study of FTC for NCSs with data packet dropout (as in [14, 15, 23]). As we know, packet dropouts can be modeled either as stochastic or deterministic phenomena, such as packet dropouts in both S-C link and C-A link are characterized by Bernoulli process in [14]. Finite-state Markovian process is used to model correlated packet dropouts in [23]. Deterministic models for data dropouts have also been proposed, specified in terms of time averages as in [1, 15], such as packet dropouts in the S-C link modeled by ADSs, discussed in [15].

From the above description, considering data packet dropout in both S-C and C-A links, the problem of FTC for NCSs with actuator failures is still a challenging problem. Thus, this paper is devoted to study FTC for NCSs with actuator failures and packet dropout in both S-C and C-A links. According to the method in [1], data packet dropouts

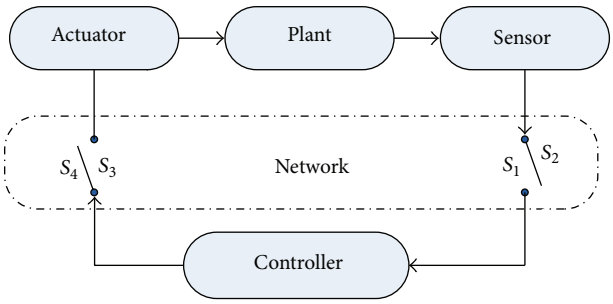


Figure 1: Schematic description of NCSs with random data packet dropouts.

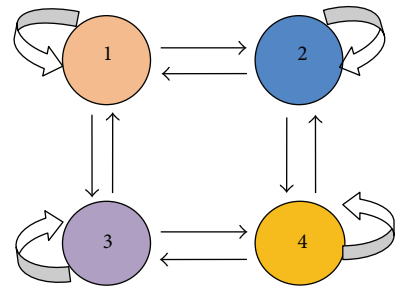


Figure 2: Four-state discrete event system.

in both S-C and C-A links are described by two independent switches, which can be modeled as a discrete event system with known rate. After introducing the matrix of actuator failures, the model of closed-loop NCSs with actuator failures is addressed. It should be mentioned that the closed-loop NCSs can be viewed as ADSs. Then, based on the theory of ADSs, the sufficiency of exponential stability for such NCSs is derived and the output feedback controllers that can guarantee system stability are presented. Finally, two numerical examples are provided to show the effectiveness of the present approach.

The rest of the paper is organized as follows. Section 2 gives the formulation of the problem, and the NCSs with data packet dropout is modeled as a discrete event system. The integrity design of FTC for the NCSs is addressed in Section 3. The output-feedback controllers are also presented in Section 3. Two numerical examples to testify the effectiveness of the proposed method are shown in Section 4. Section 5 makes the concluding remarks.

2. Problem Formulation

The LTI discrete-time system is considered as follows:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k, \\ y_k &= Cx_k, \end{aligned} \tag{1}$$

where $x_k \in \mathbf{R}^n$, $y_k \in \mathbf{R}^m$, and $u_k \in \mathbf{R}^p$ denote the state vector, the measurable output vector, and the control input vector, respectively. A, B, and C are known matrices with appropriate dimensions.

Networks can be viewed as unreliable data transmission paths, where packet collision and network node failure occasionally occur. When there is a packet collision, instead of repeated retransmission attempts, it might be advantageous to drop the old packet and transmit a new one. Hence, in this study, we focus on the phenomenon of data packet dropout. Figure 1 shows a typical closed loop of NCSs with data packet dropout. Some assumptions in this paper are as follows.

(1) The controller and the actuator are event-driven.

(2) Data are single-packet transmission, and the dropout packet rate is known.

From Figure 2, it can been seen that data packet dropouts in the network can be treated as switches that close at a certain rate. When these switches are closed (position S_1, S_3), data packets containing y_k, u_k are transmitted successfully, whereas when they are open (position S_2, S_4), the outputs of these switches are held at the previous values $\overline{y}_{k-1}$, u_{k-1}, stored in the buff, and the current data packet is missing. Thus, the dynamic model of this network with data packet dropout is given by

$$\begin{aligned} &S_1\colon \overline{y}_k = y_k, \qquad S_2\colon \overline{y}_k = \overline{y}_{k-1}, \\ &S_3\colon u_k = -K_{\theta_k}\overline{y}_k, \qquad S_4\colon u_k = u_{k-1}, \end{aligned} \tag{2}$$

where θ_k is a four-state time-homogenous Markov chain which takes values in index set $\Upsilon = \{1, 2, 3, 4\}$; the rate at which the event $\theta_k = j$ occurs is defined by the following:

$$\begin{aligned} &p_{ij} = \Pr\{\theta_{k+1} = j \mid \theta_k = i\}, \\ &\sum_{j=1}^{4} p_{ij} = 1, \quad \forall i, j \in \Upsilon, \ p_{ij} \geq 0. \end{aligned} \tag{3}$$

From the above discussion, data packet dropouts in the network can be viewed as two independent switches; that is, we can obtain the network model (2) by analyzing the features of these switches. Meanwhile, it is worth noting that the network model (2) can also be a discrete event system, which contains four events as follows.

Event 1: S_1S_3, no data packets are lost in the whole loop.

Event 2: S_2S_3, data packets are lost in the S-C link.

Event 3: S_1S_4, data packets are lost in the C-A link.

Event 4: S_2S_4, data packets's loss exist in both the S-C and the C-A link.

Actually, sensor failures model can be transformed into actuator failures model by changing format of system with faults. So, in this paper, we only discuss actuator failures model. Usually, when a system is under actuator failures, a matrix $L = \operatorname{diag}\{l_1, l_2 \ldots, l_p\}$ will be introduced into this system between its coefficient matrix and the feedback control matrix to describe actuator failures. We have

$$l_j = \begin{cases} 1, & \text{no actuator fault}, \\ 0, & \text{the } j\text{th actuator has fault}, \\ \in (0,1), & \text{the } j\text{th actuator has partial fault}, \end{cases} \tag{4}$$

where $l_j = 1$ denotes that the jth actuator is in the normal situation, $l_j = 0$ denotes that the jth actuator is in the full fault situation, and $l_j \in (0,1)$ denotes that the jth actuator is in partial fault situation.

Define a new augmented vector $z_k = [x_k^T,\ \bar{y}_{k-1}^T,\ u_{k-1}^T]^T$; then the closed-loop NCS model with actuator failures will be

$$z_{k+1} = \widetilde{\Phi}_{\theta_k} z_k, \tag{5}$$

where

$$\widetilde{\Phi}_1 = \begin{bmatrix} A - BLK_1C & 0 & 0 \\ C & 0 & 0 \\ -LK_1C & 0 & 0 \end{bmatrix}, \qquad \widetilde{\Phi}_2 = \begin{bmatrix} A & 0 & B \\ C & 0 & 0 \\ 0 & 0 & I \end{bmatrix},$$
$$\widetilde{\Phi}_3 = \begin{bmatrix} A & -BLK_3 & 0 \\ 0 & I & 0 \\ 0 & -LK_3 & 0 \end{bmatrix}, \qquad \widetilde{\Phi}_4 = \begin{bmatrix} A & 0 & B \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}. \tag{6}$$

Up till now, the state space expressions for different discrete events of the closed-loop NCS are acquired.

3. Integrity Design of FTC for the NCS

In this section, the integrity design of FTC for the NCS (5) is discussed. As we know, the integrity design is that system can keep asymptotically stable under some sensors failures or actuators failures and is one of the important means for the FTC. Meanwhile, because system (5) can also be viewed as ADSs, which are systems that incorporate continuous and discrete dynamics, the continuous dynamics are governed by differential or difference equations, whereas the discrete dynamics are governed by finite automata that are driven asynchronously by external discrete events with fixed rates [24, 25]. Then before studying the problem of integrity design, an important lemma about ADSs should be introduced.

Lemma 1 (as in [24]). *Given an ADS as follows:*

$$x(k+1) = f_s(x(k)), \quad s = 1, 2, \ldots, N. \tag{7}$$

Suppose a Lyapunov function $V : R_n \to R_+$ and

$$\beta_1 \|x\|^2 \le V(x) \le \beta_2 \|x\|^2, \tag{8}$$

where $\beta_{1,2} > 0$. The ADS (7) is exponentially stable with the decay rate δ if there exist the scalars $\alpha_1, \alpha_2, \ldots, \alpha_M > 0$, satisfying

$$\alpha_1^{r_1}\alpha_2^{r_2}\cdots\alpha_M^{r_M} > \alpha > 1,$$
$$V(x(k+1)) - V(x(k)) \le \left(\alpha_{s_1}^{-2}\alpha_{s_2}^{-2}\cdots\alpha_{s_{M_s}}^{-2} - 1\right) V(x(k)), \tag{9}$$

where M_s is the number of events to each discrete state, M is the number of events in total, and $r_{1,\ldots,M}$ are corresponding to each event rate.

Remark 2. Note that the $\alpha_{s_j}, j = 1, \ldots, M_s$ belongs to the set $\{\alpha_1, \alpha_2, \ldots, \alpha_M\}$. And Lemma 1 implies that the Lyapunov function $V(x)$ does not have to decrease monotonically at some rate δ along all trajectories, but rather it should decrease at a rate δ on the average.

Theorem 3. *If r_1, r_2 represents the rate of events S_1, S_3, respectively and $\hat{r}_1, \hat{r}_2, \hat{r}_3, \hat{r}_4$ are the rates of events S_1S_3, S_1S_4, S_2S_3, S_2S_4 in NCS (5), then $\hat{r}_1, \hat{r}_2, \hat{r}_3, \hat{r}_4$ can be described as*

$$\hat{r}_1 = r_1 r_2, \qquad \hat{r}_2 = r_1(1 - r_2),$$
$$\hat{r}_3 = (1 - r_1) r_2, \qquad \hat{r}_4 = (1 - r_1)(1 - r_2). \tag{10}$$

Proof. Since the switches in Figure 1 are independent, then S_1, S_2, S_3, S_4 are also independent events. According to the probability theory, the probabilities of these four independent events can be obtained as follows:

$$\Pr(S_1S_3) = \Pr(S_1)\Pr(S_3) = r_1 r_2,$$
$$\Pr(S_1S_4) = \Pr(S_1)\Pr(S_4) = r_1(1 - r_2),$$
$$\Pr(S_2S_3) = \Pr(S_2)\Pr(S_3) = (1 - r_1) r_2, \tag{11}$$
$$\Pr(S_2S_4) = \Pr(S_2)\Pr(S_4) = (1 - r_1)(1 - r_2).$$

Then, Theorem 3 can be proved. □

Theorem 4. *The NCS (5) is exponentially stable with the decay rate δ if there exist $\delta > 0$, $\alpha_i > 0$, $i = 1, 2, 3, 4$, positive definite matrices P, Q, R, and matrices K_1, K_3, satisfying the following:*

$$\alpha_1^{r_1 r_2}\alpha_2^{r_1(1-r_2)}\alpha_3^{(1-r_1)r_2}\alpha_4^{(1-r_1)(1-r_2)} > \delta > 1, \tag{12}$$

$$\begin{bmatrix} -\alpha_1^{-2}P^{-1} & 0 & 0 & P^{-1}A^T - P^{-1}C^TK_1^TL^TB^T & P^{-1}C^T & P^{-1}C^TK_1^TL^T \\ 0 & -\alpha_1^{-2}Q^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha_1^{-2}R^{-1} & 0 & 0 & 0 \\ AP^{-1} - BLK_1CP^{-1} & 0 & 0 & -P^{-1} & 0 & 0 \\ CP^{-1} & 0 & 0 & 0 & -Q^{-1} & 0 \\ LK_1CP^{-1} & 0 & 0 & 0 & 0 & -R^{-1} \end{bmatrix} < 0, \tag{13}$$

$$\begin{bmatrix} -\alpha_2^{-2}P^{-1} & 0 & 0 & P^{-1}A^T & P^{-1}C^T & 0 \\ 0 & -\alpha_2^{-2}Q^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \left(1-\alpha_2^{-2}\right)R^{-1} & R^{-1}B^T & 0 & R^{-1} \\ AP^{-1} & 0 & BR^{-1} & -P^{-1} & 0 & 0 \\ CP^{-1} & 0 & 0 & 0 & -Q^{-1} & 0 \\ 0 & 0 & R^{-1} & 0 & 0 & -R^{-1} \end{bmatrix} < 0, \tag{14}$$

$$\begin{bmatrix} -\alpha_3^{-2}P^{-1} & 0 & 0 & P^{-1}A^T & 0 & 0 \\ 0 & \left(1-\alpha_3^{-2}\right)Q^{-1} & 0 & -Q^{-1}K_3^TL^TB^T & Q^{-1} & -Q^{-1}K_3^TL^T \\ 0 & 0 & \alpha_3^{-2}R^{-1} & 0 & 0 & 0 \\ AP^{-1} & -BLK_3Q^{-1} & 0 & -P^{-1} & 0 & 0 \\ 0 & Q^{-1} & 0 & 0 & -Q^{-1} & 0 \\ 0 & -LK_3Q^{-1} & 0 & 0 & 0 & -R^{-1} \end{bmatrix} < 0, \tag{15}$$

$$\begin{bmatrix} -\alpha_4^{-2}P^{-1} & 0 & 0 & P^{-1}A^T & 0 & 0 \\ 0 & \left(1-\alpha_4^{-2}\right)Q^{-1} & 0 & 0 & Q^{-1} & 0 \\ 0 & 0 & \left(1-\alpha_4^{-2}\right)R^{-1} & R^{-1}B^T & 0 & R^{-1} \\ AP^{-1} & 0 & BR^{-1} & -P^{-1} & 0 & 0 \\ 0 & Q^{-1} & 0 & 0 & -Q^{-1} & 0 \\ 0 & 0 & R^{-1} & 0 & 0 & -R^{-1} \end{bmatrix} < 0. \tag{16}$$

Proof. From Lemma 1 and (10), we can get (12) directly.

Next, for nonzero $x_k, \overline{y}_{k-1}, u_{k-1}$, choose the Lyapunov function as follows:

$$V = x_k^TPx_k + \overline{y}_{k-1}^TQ\overline{y}_{k-1} + u_{k-1}^TRu_{k-1}. \tag{17}$$

For simplicity, let $x_k = x$, $\overline{y}_{k-1} = \overline{y}$, and $u_{k-1} = \widehat{u}$. Then, each discrete event of NCS will be discussed, respectively.

Event 1. If event $S_1S_3 : \overline{y}_k = y_k$, $u_k = -LK_1\overline{y}_k$ occurs, it follows that

$$\begin{aligned} &V\left(z_{k+1}\right) - \alpha_1^{-2}V\left(z_k\right) \\ &\quad = x^TA^TPAx - x^TA^TPBLK_1Cx \\ &\qquad - x^TC^TK_1^TL^TB^TPAx + x^TC^TQCx \\ &\qquad + x^TC^TK_1^TL^TB^TPBLK_1Cx \\ &\qquad + x^TC^TK_1^TL^TRLK_1Cx \\ &\qquad - \alpha_1^{-2}x^TPx - \alpha_1^{-2}\overline{y}^TQ\overline{y} - \alpha_1^{-2}\widehat{u}^TR\widehat{u}. \end{aligned} \tag{18}$$

So, we have

$$V\left(z_{k+1}\right) - \alpha_1^{-2}V\left(z_k\right) = \begin{bmatrix} x^T & \overline{y}^T & \widehat{u}^T \end{bmatrix} H_1 \begin{bmatrix} x \\ \overline{y} \\ \widehat{u} \end{bmatrix}, \tag{19}$$

where

$$H_1 = \begin{bmatrix} \Pi_{11} & 0 & 0 \\ 0 & -\alpha_1^{-2}Q & 0 \\ 0 & 0 & -\alpha_1^{-2}R \end{bmatrix},$$

$$\begin{aligned} \Pi_{11} &= A^TPA + A^TC^TQCA \\ &\quad - C^TK_1^TL^TB^TPA - A^TPBLK_1C \\ &\quad + C^TK_1^TL^TB^TPBLK_1C \\ &\quad + C^TK_1^TL^TRLK_1C - \alpha_1^{-2}P. \end{aligned} \tag{20}$$

By Schur complement, $H_1 < 0$ is equivalent to

$$\begin{aligned} &\begin{bmatrix} -\alpha_1^{-2}P & 0 & 0 \\ 0 & -\alpha_1^{-2}Q & 0 \\ 0 & 0 & -\alpha_1^{-2}R \end{bmatrix} \\ &\quad + \begin{bmatrix} A^T - C^TK_1^TL^TB^T & C^T & C^TK_1^TL^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &\quad \times \begin{bmatrix} -P^{-1} & 0 & 0 \\ 0 & -Q^{-1} & 0 \\ 0 & 0 & -R^{-1} \end{bmatrix}^{-1} \begin{bmatrix} A - BLK_1C & 0 & 0 \\ C & 0 & 0 \\ LK_1C & 0 & 0 \end{bmatrix} < 0. \end{aligned} \tag{21}$$

Then,

$$\begin{bmatrix} -\alpha_1^{-2}P & 0 & 0 & A^T - C^T K_1^T L^T B^T & C^T & C^T K_1^T L^T \\ 0 & -\alpha_1^{-2}Q & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha_1^{-2}R & 0 & 0 & 0 \\ A - BLK_1C & 0 & 0 & -P^{-1} & 0 & 0 \\ C & 0 & 0 & 0 & -Q^{-1} & 0 \\ LK_1C & 0 & 0 & 0 & 0 & -R^{-1} \end{bmatrix} < 0. \tag{22}$$

Next, after pre- and postmultiplying inequality (22) by $\mathrm{diag}\{P^{-1}, Q^{-1}, R^{-1}, I, I, I\}$, we have (13). Therefore, the following inequality holds:

$$V(z_{k+1}) - \alpha_1^{-2} V(z_k) < 0. \tag{23}$$

Then, according to the Lemma 1, the NCS (5) in discrete events $\theta_k = 1$ is exponentially stable.

Event 2. If event $S_2S_3 : \overline{y}_k = \overline{y}_{k-1},\ u_k = -LK_3\overline{y}_k$ occurs, then we get

$$\begin{aligned} &V(x(k+1)) - \alpha_3^{-2} V(x(k)) \\ &\quad = x^T A^T PAx - x^T A^T PBLK_3\overline{y} \\ &\qquad - \overline{y}^T K_3^T L^T B^T PAx + \overline{y}^T K_3^T L^T B^T PBLK_3\overline{y} \\ &\qquad + \overline{y}^T Q\overline{y} + \overline{y}^T K_3^T L^T RLK_3\overline{y} \\ &\qquad - \alpha_3^{-2} x^T Px - \alpha_3^{-2}\overline{y}^T Q\overline{y} - \alpha_3^{-2}\widehat{u}^T R\widehat{u}. \end{aligned} \tag{24}$$

So, we have

$$V(z_{k+1}) - \alpha_3^{-2} V(z_k) = \begin{bmatrix} x^T & \overline{y}^T & \widehat{u}^T \end{bmatrix} H_3 \begin{bmatrix} x \\ \overline{y} \\ \widehat{u} \end{bmatrix}, \tag{25}$$

where

$$H_3 = \begin{bmatrix} A^T PA - \alpha_3^{-2}P & -A^T PBLK_3 & 0 \\ -K_3^T L^T B^T PA & \Pi_{33} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{26}$$

where $\Pi_{33} = K_3^T L^T B^T PBLK_3 + Q + K_3^T L^T RLK_3 - \alpha_2^{-2}Q$.

Applying Schur complement theory, $H_3 < 0$ is equivalent to

$$\begin{bmatrix} -\alpha_3^{-2}P & 0 & 0 & A^T & 0 & 0 \\ 0 & \left(1-\alpha_3^{-2}\right)Q & 0 & -K_3^T L^T B^T & I & -K_3^T L^T R \\ 0 & 0 & \alpha_2^{-2}R & 0 & 0 & 0 \\ A & -BLK_3 & 0 & -P^{-1} & 0 & 0 \\ 0 & I & 0 & 0 & -Q^{-1} & 0 \\ 0 & -LK_3 & 0 & 0 & 0 & -R^{-1} \end{bmatrix} < 0. \tag{27}$$

Next, after pre- and postmultiplying inequality (27) by $\mathrm{diag}\{P^{-1}, Q^{-1}, R^{-1}, I, I, I\}$, we have (15). Therefore, the inequality $V(z_{k+1}) - \alpha_3^{-2}V(z_k) < 0$ holds.

If event 3 and event 4 occur, that is, S_1S_4: $\overline{y}_k = y_k,\ u_k = u_{k-1}$ and S_2S_4: $\overline{y}(k) = \overline{y}_{k-1},\ u_k = u_{k-1}$, then we get

$$\begin{aligned} &V(z(k+1)) - \alpha_2^{-2} V(z(k)) \\ &\quad = x^T A^T PAx + x^T A^T PB\widehat{u} + \widehat{u}^T B^T PAx + \widehat{u}^T B^T PB\widehat{u} \\ &\qquad + \widehat{u}^T R\widehat{u} + x^T C^T QCx - \alpha_2^{-2} x^T Px - \alpha_2^{-2}\overline{y}^T Q\overline{y} \\ &\qquad - \alpha_2^{-2}\widehat{u}^T R\widehat{u}, \end{aligned}$$

$$\begin{aligned} &V(x(k+1)) - \alpha_4^{-2} V(x(k)) \\ &\quad = x^T A^T PAx + x^T A^T PB\widehat{u} + \widehat{u}^T B^T PAx + \widehat{u}^T B^T PB\widehat{u} \\ &\qquad + \widehat{u}^T R\widehat{u} + \overline{y}^T Q\overline{y} - \alpha_4^{-2} x^T Px - \alpha_4^{-2}\widehat{u}^T R\widehat{u} - \alpha_4^{-2}\overline{y}^T Q\overline{y}, \end{aligned}$$

$$H_2 = \begin{bmatrix} A^T PA + C^T QC - \alpha_2^{-2}P & 0 & A^T PB \\ 0 & -\alpha_2^{-2}Q & 0 \\ B^T PA & 0 & B^T PB + R - \alpha_2^{-2}R \end{bmatrix},$$

$$H_4 = \begin{bmatrix} A^T PA - \alpha_4^{-2}P & 0 & A^T PB \\ 0 & Q - \alpha_4^{-2}Q & 0 \\ B^T PA & 0 & B^T PB + R - \alpha_4^{-2}R \end{bmatrix}. \tag{28}$$

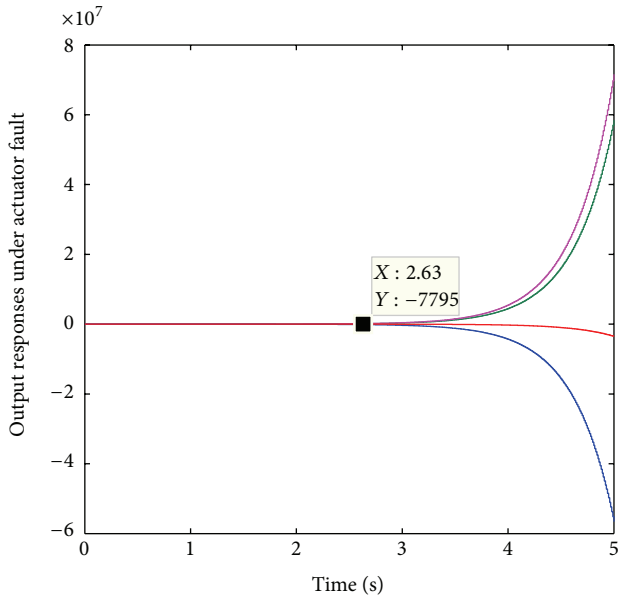


FIGURE 3: Output of the system with data packet dropout and actuator fault.

Similar to the above process, (14) and (16) can be obtained. Then, the following inequalities hold:

$$V(x_{k+1}) - \alpha_2^{-2} V(x_k) < 0,$$
$$V(x_{k+1}) - \alpha_4^{-2} V(x_k) < 0. \tag{29}$$

At last, form Lemma 1 and Theorem 3, the conclusion that the NCS (5) is exponentially stable with the decay rate δ is made. □

Remark 5. From Theorem 4, it can be found that the NCS (5) is exponentially stable. According to the characteristic of trivial solution of system (5), we can get that the exponential stability of NCS (5) is equivalent to asymptotic stability, which corresponds with the requirement of integrity design for FTC.

4. Numerical Examples

In this section, two numerical examples are given to illustrate the performance of the proposed approach.

Example 1. Consider the following linear model of a helicopter (as in [19]):

$$\dot{x}(t) = \overline{A}x(t) + \overline{B}u(t),$$
$$y(t) = \overline{C}x(t), \tag{30}$$

where $x = [V_h, V_v, q, \theta]^T$ is system state, $u = [\delta_c, \delta_l]^T$ is control input, V_h is horizontal velocity, V is vertical velocity, q is pitch rate, θ is angle of pitch, δ_c is blade control, and the matrices of model (30) are

$$\overline{A} = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.1555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681 & -0.707 & 1.420 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$
$$\overline{B} = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.52 & 4.49 \\ 0 & 0 \end{bmatrix}, \quad \overline{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{31}$$

Assume that the sampling period is $T = 0.1$ s and the initial state $x_0 = [250 \quad 50 \quad 10 \quad 8]^T$, then the matrices of zero-order holding system (30) are given by

$$A = \begin{bmatrix} 0.9964 & 0.002596 & 0.001039 & -0.01596 \\ 0.004513 & 0.9037 & -0.01879 & -0.3834 \\ 0.009762 & 0.03388 & 0.9383 & 0.1303 \\ 0.0004922 & 0.001741 & 0.09677 & 1.007 \end{bmatrix},$$
$$B = \begin{bmatrix} 0.04424 & 0.01687 \\ 0.3407 & -0.7249 \\ -0.5278 & 0.4214 \\ -0.02678 & 0.02151 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}. \tag{32}$$

Suppose the rate of packet dropout is 30% in the S-C link and the rate of packet dropout is 20% in the C-A link; then, we have $r_1 = 0.7$ and $r_2 = 0.8$.

Denote actuator failure matrix L as follows:

$$L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{33}$$

From Figure 3, it can be seen that the system is unstable when the data packet dropout and actuator faults happen. Then, using the proposed approach in Theorem 4 and taking $\alpha_1 = 3.1641, \alpha_2 = 0.3461,$ and $\alpha_3 = 0.6121, \alpha_4 = 0.6188$, the controllers can be obtained as follows:

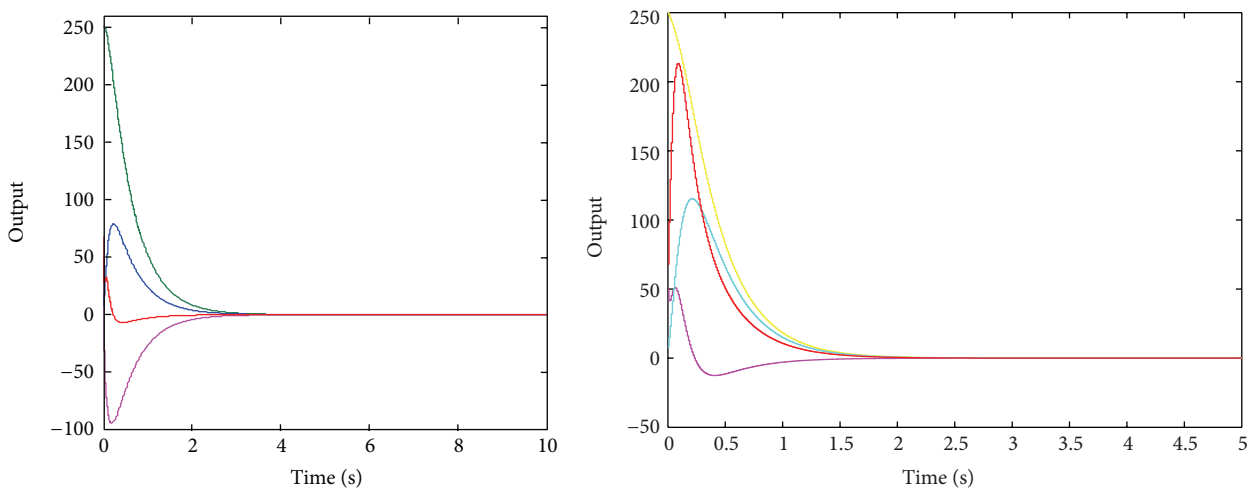
$$K_1 = \begin{bmatrix} 0.6129 & 0.3626 & -1.480 & -2.5049 \\ -0.3064 & -1.0651 & 0.2590 & 0.0393 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -1.1034 & -0.1149 & 0.8454 & 1.0289 \\ -0.1382 & 1.5734 & -0.1126 & -0.574 \end{bmatrix}. \tag{34}$$

Besides, from Figure 4, we can see that controllers can guarantee system stability when the data packet dropout and actuator faults happen. Therefore, the simulation result of Example 1 demonstrates the effectiveness of our presented method.

Example 2. Consider the discrete system as follows (as in [13]):

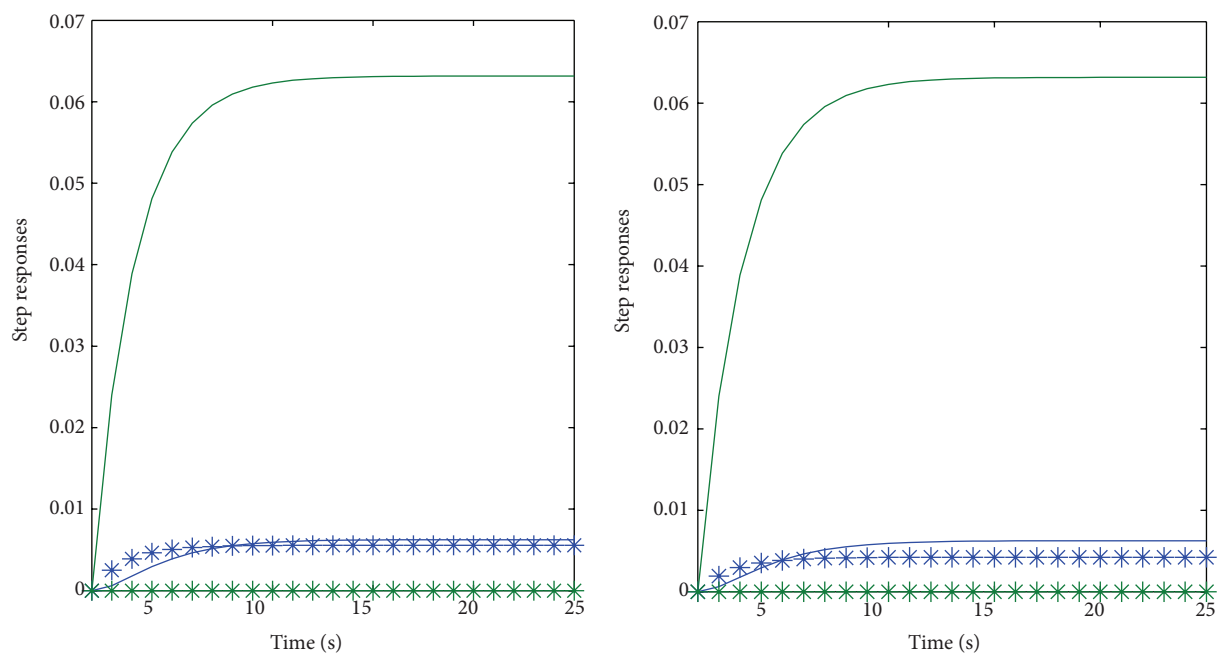
$$x(k+1) = \begin{bmatrix} 0.55 & 0.035 \\ 0 & 0.62 \end{bmatrix} x(k) + \begin{bmatrix} 0.006 \\ 0.24 \end{bmatrix} u(k),$$
$$y(k) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} x(k), \tag{35}$$

with the sampling period $T = 0.3$ s.



(a) Event 1 (b) Event 3

Figure 4: Output of the closed-loop NCS with data packet dropout and actuator fault.



(a) Event 1 (b) Event 3

Figure 5: Step responses under different discrete events.

When data packet dropouts occur in the communication channels, a controller presented in [13] can stabilize the closed-loop NCS. But, when the closed-loop NCS with actuator failure matrix $L = [1 \quad 0]$, the suitable controller and α_i cannot be found using method in [13]. Comparing with method in [13], for given $r_1 = 0.6, r_2 = 0.7, \alpha_1 = 1.574, \alpha_2 = 0.913, \alpha_3 = 0.705$, and $\alpha_4 = 0.72$, the suitable output feedback controllers can be found by the proposed approach in this paper:

$$K_1 = \begin{bmatrix} 0.0151 & 0.4880 \\ 0 & 0 \end{bmatrix}, \qquad K_3 = \begin{bmatrix} 0.3171 & 0.0049 \\ 0 & 0 \end{bmatrix}. \tag{36}$$

Figure 5 shows the step responses of NCS (5) in events 1 and 3. The case without data packet dropout and actuator failure is displayed in Figure 5(a) as the solid lines, controllers can switch immediately between two different control loops, and the system is exponentially stable. When an actuator fault occurs, one of the trajectories becomes zero as shown in Figure 5(a) by the star parts. Similarly, Figure 5(b) shows that the proposed controllers can guarantee the stability even though both the data packet dropout and actuator failure happen. From a comparison between Figures 5(a) and 5(b), we can see that system performance in event 3 is not as good as that in event 1 because of the data packet dropout. Hence, the simulation results of Example 2 imply that the desired goal is well achieved.

5. Conclusion

The problem of FTC for NCSs with actuator failures and data packet dropout in both S-C and C-A links is discussed in this paper. These data packet dropouts are described by two independent switches, which can be modeled as a discrete event system with known rate. Introducing the matrix of actuator failures, the model of NCS with actuator failures is addressed as ADSs. Then, based on the theory of ADSs, the sufficiency of exponential stability for such NCS is derived and the output feedback controllers guaranteed system performance are presented. Finally, two numerical examples are exploited to show the effectiveness of the proposed method.

Acknowledgments

The authors would like to acknowledge the National Natural Science Foundation of P. R. China under Grant no. 61174044, the Independent Innovation Fund of Shandong University (2012JC005), and the Shandong Province Natural Science Foundation under Grant no. ZR2010FM016. The authors also wish to thank the reviewers for their valuable suggestions.

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