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Research Article

Optimal Robust Adaptive Fuzzy H_∞ Tracking Control without Reaching Phase for Nonlinear System

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An optimal H_∞ tracking-based indirect adaptive fuzzy controller for a class of perturbed uncertain affine nonlinear systems without reaching phase is being developed in this paper. First a practical Interval Type-2 (IT2) fuzzy system is used in an adaptive scheme to approximate the system using a nonlinear model and to determine the optimal value of the H_∞ gain control. Secondly, to eliminate the trade-off between H_∞ tracking performance and high gain at the control input, a modified output tracking error has been used. The stability is ensured through Lyapunov synthesis and the effectiveness of the proposed method is proved and the simulation is also given to illustrate the superiority of the proposed approach.

1. Introduction

After the approximation-based adaptive fuzzy controller (AFC) of Wang [1] for a class of uncertain affine nonlinear system, many approaches and ideas have been developed in recent years to overcome the difficulty in controller design [2–4].

The primary feature that characterizes the fuzzy logic is its high capacity for representing and modelling the nonlinear systems with imprecise uncertainty, as the universal approximation theorem, by Lee and Tomizuka, illustrates [5].

Many effective adaptive fuzzy control schemes have been developed to incorporate with human expert knowledge and information in a systematic way, which can also guarantee various stability and performance [6]. The most important issue for Fuzzy Logic Systems (FLSs) is how to get a system design with the guarantee of stability and control performance [7–9].

An adaptive fuzzy control system includes uncertainties caused by unmodeled dynamics, Fuzzy Approximation Errors (FAEs), and external disturbance, which cannot be effectively handled by the FLS and may degrade the tracking performance of the closed-loop system [10, 11]. The AFC combined with H_∞ control technique is an effective approach for rejecting those uncertainties, ensuring stability and consistent performance [12–14].

The research of fuzzy model under H_∞ has attracted many attentions in recent years [4, 15], such as the apparent similarities between H_∞ and fuzzy control which motivate considerable research efforts in combining the two approaches for achieving more superior performance.

Moreover, to the best of our knowledge, the control gain needs to be known in all previous H_∞, indirect adaptive fuzzy controller (HIAFC) approaches, such as the arbitrary choice of the gain which does not always give good results, for which we propose in this work a method for extracting automatically the optimum gain from the Lyapunov equation whilst respecting system stability.

The convergence of the system in the initial time needs the appearance of high gain at the control input, and the high gain is unavoidable in all previous H_∞ tracking-based AFC approaches. The best method to solve the problem of the tradeoffs between H_∞ tracking performance and high gain at the control input is to eliminate the reaching phase. During the reaching phase the tracking error cannot be controlled directly and the system response is sensitive to parameter uncertainties. Several methods have been proposed to completely eliminate the reaching phase [16].

This paper focuses on a class of Single-Input Single-Output (SISO) perturbed uncertain affine nonlinear systems involving external disturbances without exact knowledge of dynamic functions. Firstly, we use the type-2 fuzzy technique to determine the optimal value of the H_∞ gain control. Secondly, a modified output tracking error is used to eliminate the reaching phase [4].

The paper is organized as follows: Section 2 presents the problem statement. Section 3 gives the control design strategy. An illustration example is described in Section 4. Finally, the simulation results are being used to demonstrate the effectiveness and performance of the proposed approach.

2. Problem Formulations

Considering the following nth-order SISO affine nonlinear dynamical system, Chen et al. [14]:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= f(\underline{x}, t) + g(\underline{x}, t)\, u(t) + d(t) \\ y &= x_1. \end{aligned} \tag{1}$$

Or equivalently

$$\begin{gathered} x^{(n)} = f(\underline{x}, t) + g(\underline{x}, t)\, u(t) + d(t) \\ y = x, \end{gathered} \tag{2}$$

where $\underline{x} = [x, \dot{x}, \ldots, x^{(n-1)}]^T = [x_1, x_2, \ldots, x_n]^T \in R^n$ is the state vector of the systems which is assumed to be available for measurement, $u \in R$ and $y \in R$ are, respectively, the input and the output of the systems. $f(\underline{x}, t)$ and $g(\underline{x}, t)$ are two functions that are unknown, nonlinear, and continuous; $d(t)$ denotes the external disturbance. For (1) to be controllable, we require that $g(\underline{x}, t) \neq 0$ for $\underline{x}$ in certain controllability region. Assume that the given reference $\underline{y}_r$ is bounded and have up to $(n-1)$ bounded derivatives. The reference vector is denoted as $\underline{y}_r = [y_r, \dot{y}_r, \ldots, y_r^{(n-1)}]^T$. Define the tracking error $e = y_r - y$ and the error vector $\underline{e} = [e_1, e_2, \ldots, e_n]^T = [e, \dot{e}, \ldots, e^{(n-1)}]^T \in R^n$.

Assumption 1. For all $\underline{x} \in D$, there exist unknown bounded $\overline{f}(\underline{x}, t), \overline{g}(\underline{x}, t)$, and $\overline{d}(t)$ such that $|f(\underline{x}, t)| \leq \overline{f}(\underline{x}, t)$, $|g(\underline{x}, t)| \leq \overline{g}(\underline{x}, t)$ and $|d(t)| \leq \overline{d}(t)$ hold, where compact set $D \subset R^n$ is a certain controllable region.

During the AFC design, to improve the tracking performance under the external disturbance, an additional H_∞ compensator associated with an attenuation level is usually suggested to apply, Chen et al. [14]. If the prescribed attenuation level is smaller, the tracking performance is better while the control input gain is higher as the output of the H_∞ compensator becomes larger.

To avoid high control input gain, we have modified the following output tracking error Yilmaz and Hurmuzlu [16]:

$$E(t) = e(t) - \psi(t), \tag{3}$$

where (condition 1) $\psi(t)$ is designed to make $E(t)$ small enough at the onset of the motion $t = 0$, and (condition 2) should rapidly vanish as the motion evolves at $t > 0$.

A suggested $\psi(t)$ is given in the following exponential form:

$$\begin{gathered} \psi(t) = \gamma(t)\ \exp(F(t)) \\ \gamma(t) = \sum_{i=0}^{n-1} \frac{1}{i!}\, \gamma^{(i)}(t_0)(t - t_0)^i, \qquad \gamma^{(i)}(t_0) = \frac{d^i \gamma}{dt^i}(t = t_0), \end{gathered} \tag{4}$$

$$F(t) = -\alpha t \tag{5}$$

with $\gamma^{(i)}(t_0)$ $(i = 0, 1, \ldots, n-1)$ is selected to satisfy condition one and $F(t)$ is selected to satisfy condition two. For the selections of $\gamma(t)^{(i)}(t_0)$ one can follow the methods in Yilmaz and Hurmuzlu [16], on the other side α is selected to satisfy condition two [4].

Now, the objective of this paper is to determine the optimal value of the H_∞ gain control, in a way to force $y(t)$ to follow a given bounded reference signal $y_r(t)$.

Let us denote the parameter tracking error $\underline{\Phi} = \underline{\theta} - \underline{\theta}^*$ for some parameter estimate $\underline{\theta}$ and optimal parameter estimate $\underline{\theta}^*$ of Type-2 Fuzzy Logic System (T2FLS). Let w denote the sum of error due to fuzzy modelling approximations. Then our design objective is to impose an adaptive fuzzy control algorithm so that the following asymptotically stable tracking

$$E^{(n)} + k_{n-1} E^{(n-1)} + \cdots + k_0 E = 0 \tag{6}$$

is achieved while $w = 0$ (i.e., in the case of perfect fuzzy approximation and free of external disturbance). While w appears, the following H^∞ tracking performance is requested [17]:

$$\int_0^T \underline{E}^T Q \underline{E}\, dt \leq 2V(0) + \rho^2 \int_0^T w^2\, dt \quad T \in [0 \quad \infty], \tag{7}$$

where $\underline{E} = [E_1, E_2, \ldots, E_n] = [E, \dot{E}, \ldots, E^{(n-1)}]^T$. V is the Lyapunov function, $Q = Q^T > 0$, $\rho \in R^+$ is the prescribed attenuation level, and $w \in L_2[0, T]$.

Q, V, and w will be defined in the next subsection.

Remark 2. (i) The roots of polynomial $\widehat{L}(s) = s^{(n)} + k_{n-1} s^{(n-1)} + \cdots + k_0 s$ in the characteristic equation of (6) are all in the open left-half plane via an adequate choice of coefficients $k_0, k_1, \ldots, k_{n-1}$.

(ii) If the system starts with initial condition $V(0) = 0$, then the H^∞ performance in (7) can be rewritten as

$$\sup_{w \in L_2[0 \quad T]} \frac{\|\underline{E}\|_Q}{\|w\|} \leq \rho, \tag{8}$$

where $\|\underline{E}\|_Q^2 = \int_0^T \underline{E}^T Q \underline{E}\, dt$, and $\|w\|^2 = \int_0^T w^T w\, dt$, that is, the L_2-gain form w to the tracking error $\underline{E}$ must be equal to or less than ρ.

3. Control Design Strategy

3.1. Indirect Adaptive Control Scheme. In this section, we propose a new optimal H_∞ tracking-based indirect adaptive output-feedback fuzzy controller that eliminates the reaching phase, with guaranteed stability of the closed loop system. Based on the combination of the H_∞ optimal control with fuzzy logic control, using fuzzy identifier and fuzzy logic control, the H_∞ control design relies on the solution of an algebraic Riccati equation.

If the system (1) is well known and $g(\underline{x}) \neq 0$ then the control should be designed to have the following idealized control law:

$$u^* = \frac{1}{g(\underline{x},t)}\left(-f(\underline{x},t) + y_r^{(n)} - \psi^{(n)} + \sum_{i=0}^{n-1} k_i E^{(i)} + u_h\right), \tag{9}$$

where $u_h = \mathfrak{R} \cdot \underline{E}^T PB$.

However, in practice the functions $f(\underline{x},t)$ and $g(\underline{x},t)$ are unknown, thus the ideal controller in (9) cannot be realized, and the choice of the H^∞ gain control $\mathfrak{R}$ does not always give good results. In this case the nonlinear functions $f(\underline{x},t)$ and $g(\underline{x},t)$ are approximated using T2-fuzzy systems universal approximation property and by the same technique we determine the optimal H^∞ gain control. Hence, the fuzzy adaptive control law is as follows:

$$u = \frac{1}{\hat{g}(\underline{x},\underline{\theta}_g)}\left(-\hat{f}(\underline{x},\underline{\theta}_f) + y_r^{(n)} - \psi^{(n)} + \sum_{i=0}^{n-1} k_i E^{(i)} + \hat{u}_h\right), \tag{10}$$

where $\hat{u}_h = \widehat{\mathfrak{R}} \cdot \underline{E}^T PB$ defined the auxiliary control employed to attenuate the approximation error of the fuzzy model and to eliminate the external disturbance.

$\hat{f}(\underline{x},\underline{\theta}_f)$, $\hat{g}(\underline{x},\underline{\theta}_g)$, and $\widehat{\mathfrak{R}}(\underline{x},\underline{\theta}_{\mathfrak{R}})$ are the type-2 fuzzy approximation of $f(\underline{x},t)$,$g(\underline{x},t)$, and $\mathfrak{R}$.

3.2. Interval Type-2 Fuzzy Logic System (IT2FLS). For an Interval Type-2 Fuzzy Logic System IT2FLS with M, total number of IF-THEN rules in the rule base, the jth rule can be written as follows:

$$R^j\text{: if } x_1 \text{ is } \mu_{y_1^j} \text{ and} \ldots \text{and } x_n \text{ is } \mu_{y_n^j}, \text{ then } y \text{ is } \theta_j. \tag{11}$$

Equation (11) represents a T2 fuzzy relation between the input and the output spaces of the FLS, where $\mu_{F_1^j}$'s are antecedent type-2 sets, y is the output, and θ_j's are the consequent T2 fuzzy singleton.

Since fuzzy sets are type-2, we need to perform the reduction operation type. This operation will give each function estimated $\hat{y}$ two vector of the fuzzy basis functions [18]

$$\begin{aligned}\hat{Y}_{\cos} &= \left(\theta_y^1,\ldots,\theta_y^M, W_y^1,\ldots,W_y^M\right)\\ &= \int_{\theta_y^1}\cdots\int_{\theta_y^M}\int_{W_y^1}\cdots\int_{W_y^M}\frac{1}{\sum_{i=1}^M w_y^i\theta_y^i/\sum_{i=1}^M w_y^i}\end{aligned} \tag{12}$$

$\hat{Y}_{\cos}$ is the interval set determined by two end points $\hat{y}_r$ and $\hat{y}_l$, and $w_y^i \in W_y^i = [w_{yl}^i, w_{yr}^i]$ is the firing interval.

Accordingly, the firing interval bounds for the ith rule of an IT2FLS with n inputs, w_{yl}^i and w_{yr}^i, can be rewritten as follows:

$$\begin{aligned} w_{yl}^j &= \underline{\mu}_{y_1^j}(x_1)\times\underline{\mu}_{y_2^j}(x_2)\times\cdots\times\underline{\mu}_{y_n^j}(x_n) = \prod_{i=1}^n \underline{\mu}_{y_i^j}(x_i)\\ w_{yr}^i &= \overline{\mu}_{y_1^j}(x_1)\times\overline{\mu}_{y_2^j}(x_2)\times\cdots\times\overline{\mu}_{y_n^j}(x_n) = \prod_{i=1}^n \overline{\mu}_{y_i^j}(x_i).\end{aligned} \tag{13}$$

Using the centre of gravity, the defuzzified crisp output for each output is given by Liang and Mendel [19]:

$$\hat{y} = \frac{\hat{y}_l + \hat{y}_r}{2} \tag{14}$$

$\hat{y}_l$ can be represented as a vector of fuzzy basis functions (FBFs) expansion as follows:

$$\hat{y}_l = \frac{\sum_{i=1}^M w_{yl}^i\theta_y^i}{\sum_{i=1}^M w_{yl}^i} = \sum_{i=1}^M \theta_y^i\xi_{yl}^i = \underline{\theta}_y^T\underline{\xi}_{yl}(\underline{x}) \tag{15}$$

$\underline{\xi}_{yl}^T(\underline{x})$ is the FBF vector of $\hat{y}_l$ such that $\underline{\xi}_{yl}^T(\underline{x}) = [\xi_{yl}^1,\ldots,\xi_{yl}^M]$ whose components are given by

$$\xi_{yl}^i = \frac{w_{yl}^i}{\sum_{i=1}^M w_{yl}^i} \tag{16}$$

$\underline{\theta}_y^T = [\theta_y^1,\ldots,\theta_y^M]$ represents the conclusion of T2FLS.

Similar to the foregoing we have

$$\hat{y}_r = \frac{\sum_{i=1}^M w_{yr}^i\theta_y^i}{\sum_{i=1}^M w_{yr}^i} = \sum_{i=1}^M \theta_y^i\xi_{yr}^i = \underline{\theta}_y^T\underline{\xi}_{yr}(\underline{x}). \tag{17}$$

Substituting (17) and (15) in (14) then the output of the T2FLS can be given as follows:

$$\hat{y} = \frac{\underline{\theta}_y^T\,\underline{\xi}_{fl} + \underline{\theta}_y^T\,\underline{\xi}_{fr}}{2} = \underline{\theta}_y^T\,\underline{\xi}_y(\underline{x}), \tag{18}$$

where $\underline{\xi}_y(\underline{x}) = (\underline{\xi}_{yl} + \underline{\xi}_{yr})/2$

The previous equation (18) will be used, in an indirect adaptive control, to approximate the unknown system dynamics and to determine the optimal H_∞ gain control.

Therefore the expression (18) can be expressed as:

$$\begin{aligned}\hat{y}_f &= \hat{f}\left(\frac{x}{\underline{\theta}_f}\right) = \underline{\theta}_f^T\xi_f(\underline{x})\\ \hat{y}_g &= \hat{g}\left(\frac{x}{\underline{\theta}_g}\right) = \underline{\theta}_g^T\xi_g(\underline{x})\\ \hat{y}_{\mathfrak{R}} &= \widehat{\mathfrak{R}}\left(\frac{x}{\underline{\theta}_{\mathfrak{R}}}\right) = \underline{\theta}_{\mathfrak{R}}^T\xi_{\mathfrak{R}}(\underline{x}).\end{aligned} \tag{19}$$

Define the compact sets $D = \{ \underline{x} : \|\underline{x}\| \le M_x \}$,

$$\begin{aligned}\Omega_f &= \left\{\underline{\theta}_f \in R^n \ \left\|\underline{\theta}_f\right\| \le M_f\right\},\\ \Omega_g &= \left\{\underline{\theta}_g \in R^n \ \left\|\underline{\theta}_g\right\| \le M_g\right\},\\ \Omega_{\mathfrak{R}} &= \left\{\underline{\theta}_{\mathfrak{R}} \in R^n \ \left\|\underline{\theta}_{\mathfrak{R}}\right\| \le M_{\mathfrak{R}}\right\},\end{aligned} \tag{20}$$

where M_f, M_g, and $M_{\mathfrak{R}}$ are given constants.

The minimum approximation error is defined by

$$\begin{aligned}w = {}& \hat{f}\left(\underline{x},\underline{\theta}_f^*\right) - f\left(\underline{x},t\right) + \left(\hat{g}\left(\underline{x},\underline{\theta}_g^*\right) - g\left(\underline{x},t\right)\right)u\\ &- \left(\widehat{\mathfrak{R}}\left(\underline{x},\underline{\theta}_{\mathfrak{R}}^*\right) - \mathfrak{R}\right)\left(E^T PB\right)^2 - d\left(t\right),\end{aligned} \tag{21}$$

where $\underline{\theta}_f^*, \underline{\theta}_g^*$, and $\underline{\theta}_{\mathfrak{R}}^*$ are an optimal parameter vector defined as

$$\begin{aligned}\underline{\theta}_f^* &= \arg\min_{\theta_f\in\Omega_f}\left(\sup_{x\in D}\left|\hat{f}\left(\underline{x},\underline{\theta}_f\right) - f\left(\underline{x},t\right)\right|\right)\\ \underline{\theta}_g^* &= \arg\min_{\theta_g\in\Omega_g}\left(\sup_{x\in D}\left|\hat{g}\left(\underline{x},\underline{\theta}_g\right) - g\left(\underline{x},t\right)\right|\right)\\ \underline{\theta}_{\mathfrak{R}}^* &= \arg\min_{\theta_{\mathfrak{R}}\in\Omega_{\mathfrak{R}}}\left(\sup_{x\in D}\left|\widehat{\mathfrak{R}}\left(\underline{x},\underline{\theta}_{\mathfrak{R}}\right) - \mathfrak{R}\right|\right).\end{aligned} \tag{22}$$

$$\text{Such that: } \mathfrak{R} > \frac{1}{r}, \tag{23}$$

where r is a positive constant used below.

3.3. H_∞ Tracking Performance Design in Indirect Adaptive Fuzzy System.

Choose the H_∞ compensator u_h as

$$u_h = \widehat{\mathfrak{R}}\underline{E}^T PB, \tag{24}$$

where P is the solution of the following Riccati equation:

$$A^T P + PA + Q - PB\left(\frac{2}{r} - \frac{1}{\rho^2}\right)B^T P = 0, \tag{25}$$

where $Q > 0$, ρ is prescribed attenuation level and r is positive constant verified $r < 2\rho^2$.

Theorem 3. *If we select the following adaptive fuzzy control law in the nonlinear system* (1)

$$\begin{aligned}u = \frac{1}{\underline{\theta}_g^T\xi_g\left(\underline{x}\right)}\Bigg(&-\underline{\theta}_f^T\xi_f\left(\underline{x}\right) + y_r^{(n)} - \psi^{(n)} + \sum_{i=0}^{n-1} k_i E^{(i)}\\ &+\underline{\theta}_{\mathfrak{R}}^T\xi_{\mathfrak{R}}\left(\underline{x}\right)\underline{E}^T PB\Bigg),\end{aligned} \tag{26}$$

where

$$\dot{\theta}_f = \begin{cases}-\gamma_1\underline{E}^T PB\xi_f\left(x\right) & \text{if } \left(\left\|\underline{\theta}_f\right\| < M_f \text{ or } \left\|\underline{\theta}_f\right\| = M_f,\ \underline{E}^T PB\ \xi_f\left(x\right) > 0\right)\\ -\gamma_1\underline{E}^T PB\xi_f\left(x\right) + \gamma_1\underline{E}^T PB\xi_f\left(x\right) \times\dfrac{\underline{\theta}_f^T\underline{\theta}_f\xi_f\left(x\right)}{\left\|\theta_f\right\|^2} & \text{otherwise}\end{cases} \tag{27}$$

$$\dot{\theta}_g = \begin{cases}-\gamma_2\underline{E}^T PB\xi_g\left(x\right)u & \text{if } \left(\left\|\underline{\theta}_g\right\| < M_g \text{ or } \left\|\underline{\theta}_g\right\| = M_g,\ \underline{E}^T PB\ \xi_g\left(x\right)u > 0\right)\\ -\gamma_2\underline{E}^T PB\xi_g\left(x\right)u + \gamma_2\underline{E}^T PB\xi_g\left(x\right)u \times\dfrac{\underline{\theta}_g^T\ \underline{\theta}_g\xi_g\left(x\right)}{\left\|\theta_g\right\|^2} & \text{otherwise}\end{cases} \tag{28}$$

$$\dot{\theta}_{\mathfrak{R}} = \begin{cases}\gamma_3\left(\underline{E}^T PB\right)^2\xi_{\mathfrak{R}}(x) & \text{if } \left(\left\|\underline{\theta}_{\mathfrak{R}}\right\| < M_{\mathfrak{R}} \text{ or } \left\|\underline{\theta}_{\mathfrak{R}}\right\| = M_{\mathfrak{R}},\ \underline{E}^T PB\xi_{\mathfrak{R}}(x) > 0\right)\\ \gamma_3\left(\underline{E}^T PB\right)^2\xi_{\mathfrak{R}}(x) - \gamma_3\left(\underline{E}^T PB\right)^2\xi_{\mathfrak{R}}(x) \times\dfrac{\underline{\theta}_{\mathfrak{R}}^T\ \underline{\theta}_{\mathfrak{R}}\xi_{\mathfrak{R}}(x)}{\left\|\theta_{\mathfrak{R}}\right\|^2} & \text{otherwise.}\end{cases} \tag{29}$$

With $P = P^T \ge 0$ is the solution of the Riccati equation (25)*, then the H^∞ tracking performance in* (7) *is achieved for a prescribed attenuation level ρ.*

Proof. We have

$$x^{(n)} = f\left(\underline{x},t\right) + g\left(\underline{x},t\right)u + d\left(t\right). \tag{30}$$

And the equation of the control already proposed

$$u = \frac{1}{\hat{g}\left(\underline{x},\underline{\theta}_g\right)}\left(-\hat{f}\left(\underline{x},\underline{\theta}_f\right) + y_r^{(n)} - \psi^{(n)} + \sum_{i=0}^{n-1} k_i E^{(i)} + \hat{u}_h\right], \tag{31}$$

where

$$\hat{u}_h = \widehat{\mathfrak{R}}\underline{E}^T PB. \tag{32}$$

And $\widehat{\mathfrak{R}} = \underline{\theta}_{\mathfrak{R}}^T\xi_{\mathfrak{R}}$.

Utilising (3) and substituting (30) into (31), the output error dynamics may be expressed as

$$E^{(n)} = -\sum_{i=0}^{n-1} k_i E^{(i)} + \hat{f}\left(\underline{x}, \underline{\theta}_f\right) - f\left(\underline{x}, t\right) + \left(\hat{g}\left(\underline{x}, \underline{\theta}_g\right) - g\left(\underline{x}, t\right)\right) u - \hat{u}_h - d\left(t\right). \tag{33}$$

The error dynamics can be represented by

$$\underline{\dot{E}} = A\underline{E} + B\left[\hat{f}\left(\underline{x}, \underline{\theta}_f\right) - f\left(\underline{x}, t\right) + \left(\hat{g}\left(\underline{x}, \underline{\theta}_g\right) - g\left(\underline{x}, t\right)\right) u - \hat{u}_h - d\left(t\right)\right], \tag{34}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & \vdots \\ \vdots & & & \ddots & \\ \vdots & \vdots & & & 1 \\ -k_0 & -k_1 & \cdots & \cdots & -k_n \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}. \tag{35}$$

Consider the following Lyapunov function:

$$V = \frac{1}{2}\underline{E}^T P \underline{E} + \frac{1}{2\gamma_1}\Phi_f^T \Phi_f + \frac{1}{2\gamma_2}\Phi_g^T \Phi_g + \frac{1}{2\gamma_3}\Phi_{\mathfrak{R}}^T \Phi_{\mathfrak{R}}, \tag{36}$$

where

$$\Phi_f = \underline{\theta}_f - \underline{\theta}_f^*, \qquad \Phi_g = \underline{\theta}_g - \underline{\theta}_g^*, \qquad \Phi_{\mathfrak{R}} = \underline{\theta}_{\mathfrak{R}} - \underline{\theta}_{\mathfrak{R}}^*. \tag{37}$$

$$\dot{V} = \frac{1}{2}\underline{E}^T\left(A^T P + PA\right)\underline{E} + \underline{E}^T PB \times \left[\hat{f}\left(\underline{x}, \underline{\theta}_f\right) - f\left(\underline{x}, t\right) + \left(\hat{g}\left(\underline{x}, \underline{\theta}_g\right) - g\left(\underline{x}, t\right)\right) u - \hat{u}_h - d\left(t\right)\right] + \frac{1}{\gamma_1}\Phi_f^T \dot{\Phi}_f + \frac{1}{\gamma_2}\Phi_g^T \dot{\Phi}_g + \frac{1}{\gamma_3}\Phi_{\mathfrak{R}}^T \dot{\Phi}_{\mathfrak{R}}. \tag{38}$$

Utilizing (25) and (32) into (38)

$$\begin{aligned} \dot{V} = & -\frac{1}{2}\underline{E}^T Q \underline{E} + \frac{\left(\underline{E}^T PB\right)^2}{r} - \frac{\left(\underline{E}^T PB\right)^2}{2\rho^2} \\ & + \left[w - \mathfrak{R}\underline{E}^T PB\right]\underline{E}^T PB \\ & + \frac{1}{\gamma_1}\Phi_f^T\left(\dot{\Phi}_f + \gamma_1 \xi_f\left(x\right)\underline{E}^T PB\right) \\ & + \frac{1}{\gamma_2}\Phi_g^T\left(\dot{\Phi}_g + \gamma_2 \xi_g\left(x\right)\underline{E}^T PBu\right) \\ & + \frac{1}{\gamma_3}\Phi_{\mathfrak{R}}^T\left(\dot{\Phi}_{\mathfrak{R}} - \gamma_3\, \xi_{\mathfrak{R}}\left(x\right)\left(\underline{E}^T PB\right)^2\right) - \left(\underline{E}^T PB\right)^2 \mathfrak{R}, \end{aligned} \tag{39}$$

where w is defined in (21), the $\dot{V}$ can be written as

$$\begin{aligned} \dot{V} = & -\frac{1}{2}\underline{E}^T Q \underline{E} - \frac{1}{2}\left(\frac{\left(\underline{E}^T PB\right)}{\rho} - \rho w\right)^2 + \frac{1}{2}\left(\rho w\right)^2 \\ & + \left(\frac{1}{r} - \mathfrak{R}\right)\left(\underline{E}^T PB\right)^2 + \frac{1}{\gamma_1}\Phi_f^T\left(\dot{\Phi}_f + \gamma_1 \underline{E}^T PB\ \xi_f\left(x\right)\right) \\ & + \frac{1}{\gamma_2}\Phi_g^T\left(\dot{\Phi}_g + \gamma_2 \underline{E}^T PB\ \xi_g\left(x\right) u\right) \\ & + \frac{1}{\gamma_3}\Phi_{\mathfrak{R}}^T\left(\dot{\Phi}_{\mathfrak{R}} - \gamma_3\left(\underline{E}^T PB\right)^2 \xi_{\mathfrak{R}}\left(x\right)\right). \end{aligned} \tag{40}$$

By consideration of the update law (27), (28), (23), and (29), $\dot{V}$ can be written as

$$\dot{V} \le -\frac{1}{2}\underline{E}^T Q \underline{E} + \frac{1}{2}\left(\rho w\right)^2. \tag{41}$$

Integrating the above equality from $t = 0$ to T yields ($0 \le T \le \infty$):

$$V\left(T\right) - V\left(0\right) \le -\frac{1}{2}\int_0^T \underline{E}^T Q \underline{E} dt + \frac{1}{2}\rho^2 \int_0^T w^2 dt. \tag{42}$$

Since $V(T) \ge 0$ the above inequality implies the following inequality:

$$\int_0^T \underline{E}^T Q \underline{E}\, dt \le 2V_L\left(0\right) + \rho^2 \int_0^T w^2\, dt, \quad T \in \left[0 \quad \infty\right]. \tag{43}$$

Hence, the inequality (7) holds. This completes the proof of theorem. So the system is stable and the error will asymptotically converge to zero; that is, H_∞ a performance is achieved. □

4. An Illustrative Example

4.1. The Dynamic Model. In this section consider a single-link robot manipulator governed by the following dynamic model [20]:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{d_f x_2}{ml^2} - \frac{g_v \cos\left(x_1\right)}{l} + \left(\frac{1}{ml^2}\right) u + d\left(t\right) \\ y &= x_1, \end{aligned} \tag{44}$$

where x_1: Position, x_2: Velocity, $f(\underline{x}, t)$: Nonlinear term depending on $\underline{x}$, m, d_f: Mass and damping, l: Length of the manipulator, g_v: is the gravitational acceleration and $d(t) = 0.1 * \text{rand}\, n$ is the disturbance.

We assume that the position x_1 and the velocity x_2 are available for measurements, where $m = 2\,\text{kg}$, $l = 1\,\text{m}$, $g_v = 9.8\,\text{m/s}^2$, and $d_f = 1.0\,\text{kg}\,\text{m}^2/\text{s}$.

4.2. Controller Parameters Design

Step 1. In the first step, we need to define the type-2 fuzzy sets for modelling the unknown functions entering into the creation of the control law and to determine the optimal value of the H_∞ gain control. The choice of the number of fuzzy sets and constant M_f, M_g, and $M_{\mathfrak{R}}$ are related to knowledge of expert on the system.

The fuzzy membership functions are chosen as

$$\begin{gathered}
\overline{\mu}_{F_i^1}(x_i) = \exp\left(-\left(\frac{x_i - cn_i}{2 * \mathrm{sig}_i}\right)^2\right), \\
\overline{\mu}_{F_i^2}(x_i) = \exp\left(-\left(\frac{x_i - cp_i}{2 * \mathrm{sig}_i}\right)^2\right), \\
\overline{\mu}_{F_i^3}(x_i) = 1 - \mu_{F_i^1}(x_i) - \mu_{F_i^2}(x_i) \\
\underline{\mu}_{F_i^1}(x_i) = 0.85 * \exp\left(-\left(\frac{x_i - cn_i}{2 * \mathrm{sig}_i}\right)^2\right), \\
\underline{\mu}_{F_i^2}(x_i) = 0.85 * \exp\left(-\left(\frac{x_i - cp_i}{2 * \mathrm{sig}_i}\right)^2\right), \\
\underline{\mu}_{F_i^3}(x_i) = 0.85 * \left(1 - \mu_{F_i^1}(x_i) - \mu_{F_i^2}(x_i)\right) \\
(i = 1, 2),
\end{gathered} \tag{45}$$

such as $cp_1 = -cn_1 = \pi/3$, $cp_2 = -cn_2 = \pi$, $\mathrm{sig}_1 = 5$, and $\mathrm{sig}_2 = 1.53$ with $M_f = 6$, $M_g = 1$, and $M_{\mathfrak{R}} = 10$.

Step 2. Determine parameters of the modified error E in (3).

Choose $\alpha = 5$ in (5).

To determine $\gamma(t)$ in (4) one can follow the method in Yilmaz and Hurmuzlu, and one can make

$$\begin{gathered}
\psi(0) = \gamma(0) = y_r(0) - x_1(0) \\
\dot{\psi}(0) = -\alpha\gamma(0) + \dot{\gamma}(0) = \dot{y}_r(0) - x_2(0)
\end{gathered} \tag{46}$$

with

$$y_r(0) = 0, \qquad \dot{y}_r(0) = \frac{\pi^2}{3}, \qquad x_1(0) = \frac{\pi}{6}, \qquad x_2(0) = \pi. \tag{47}$$

Thus, one gets

$$\gamma(0) = \frac{\pi}{6}, \qquad \dot{\gamma}(0) = \left(-\frac{\pi}{6}\right)\alpha + \frac{\pi^2}{3} - \pi. \tag{48}$$

Step 3. Design parameters of the control law.

The control parameters for simulation are chosen as follows: $k_0 = 1$, $k_1 = 2$, $\gamma_1 = 0.002$, $\gamma_2 = 0.0001$, $\gamma_3 = 0.001$, $Q = 5$ eye (2), and $\rho = 0.1$.

The solution to Riccati equation for Q is

$$P = \begin{bmatrix} 5.0503 & 0.0503 \\ 0.0503 & 0.0513 \end{bmatrix}. \tag{49}$$

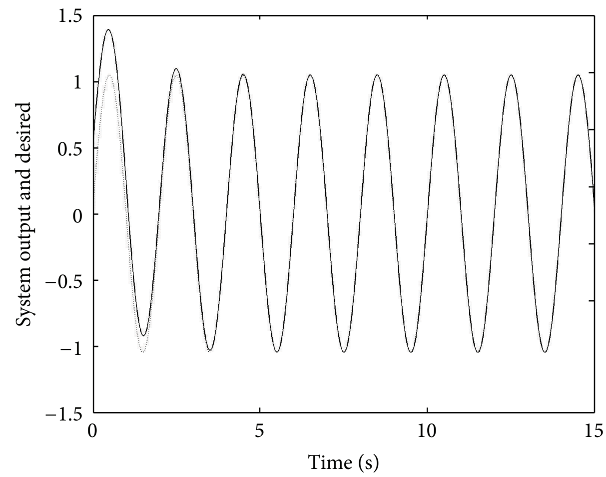

Figure 1: Reponses of the $y(t)$ and the $y_r(t)$.

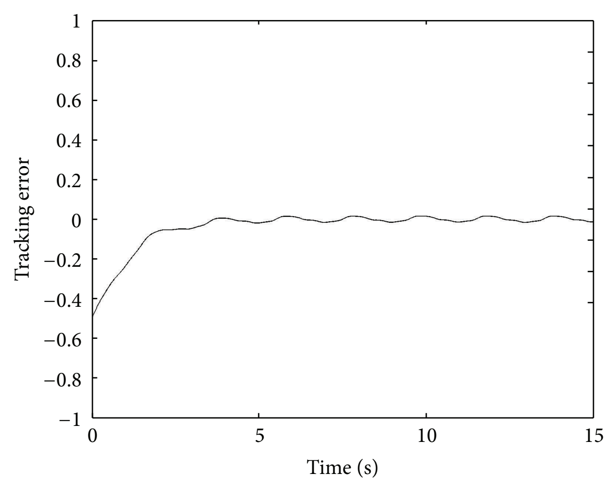

Figure 2: The tracking error.

4.3. Simulation Result. Simulation result is presented to validate performance and robustness of the proposed approaching that we have been using T-S fuzzy logic to determine automatically the gain of the H_∞ control and modifying the output tracking error to eliminate the reaching phase.

Three fuzzy sets for each input have been found sufficient for an efficient system design. Fuzzy sets for inputs x_1 and x_2 are defined according to the membership functions presented forward in Step 1.

The sampling time is defined as 10 ms and the running time as 15 s.

Figure 1 present a responses of the output $y(t)$ versus the desired output $y_r(t)$.

Figures 2 and 3 present successively the tracking error and the control signal that we apply the proposed method.

5. Conclusions

In this paper, we have proposed a new method to determine the optimal value of the H_∞ gain control based on type-2

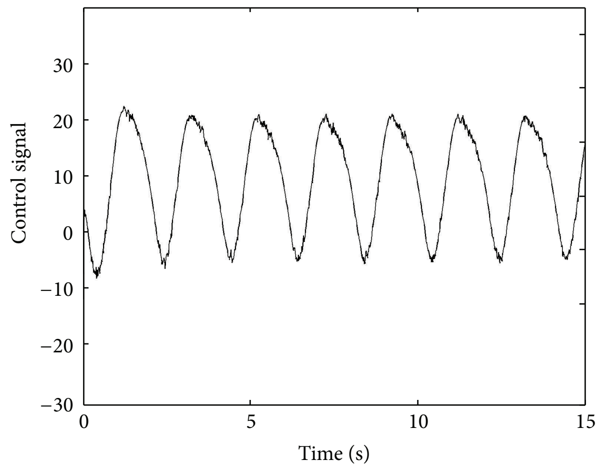

Figure 3: The control signal $u(t)$.

fuzzy logic systems, and to eliminate the trade-off between H_∞ tracking performance and high gain at the control input, we have used the modification in the output tracking error.

The parameters of the dynamics systems are estimated by using the fuzzy model. Furthermore, the parameters can be tuned on-line by the adaptive law based on Lyapunov synthesis.

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