

## Research Article

# Aeroelastic Dynamic Response and Control of an Aeroelastic System with Hysteresis Nonlinearities

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Received 11 October 2014; Accepted 6 February 2015

Academic Editor: Ai-Guo Wu

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A state feedback control law based on the sliding mode control method is derived for the aeroelastic response and flutter suppression of a two-dimensional airfoil section with hysteresis nonlinearity in pitch. An observer is constructed to estimate the unavailable state variables of the system. With the control law designed, nonlinear effect of time delay between the control input and actuator is investigated by a numerical approach. The closed-loop system including the observer and nonlinear controller is asymptotically stable. The simulation results show that the observer can give precise estimations for the plunge displacement and the velocities in pitch and plunge and that the controller is effective for flutter suppression. The time delay between the control input and actuator may jeopardize the control performance and cause high-frequency vibrations.

## 1. Introduction

Under the condition of a certain flight, aeroelastic systems exhibit a variety of phenomena including instability, limit cycle, and even chaotic vibration [1–3]. Flutter instability can jeopardize aircraft structure and its performance. A number of investigators have considered control problems for such systems and designed controllers for flutter suppression. Kurdila et al. [4] gave a good summary of nonlinear control methods for high-energy limit cycle oscillations. Mukhopadhyay [5] presented a historical perspective on analysis and flutter control of aeroelastic systems. In recent years, a large number of control strategies have been developed for the flutter suppression [6–13], such as nonlinear adaptive control [6], and global robust control using output feedback [7]. Wells [8] carried out a control design of the reconfigurable flight control systems using the sliding mode control (SMC) method. In [9], adaptive decoupled fuzzy sliding-mode control laws have been implemented for suppressing flutter and reducing the vibrational level in subcritical flight speed range. For the two-degree-of-freedom aeroelastic system with uncertainties, the higher order sliding mode control laws were designed by Defoort et al. [10, 11]. Degaki et al. [12, 13] applied sliding mode controller for suppressing two-dimensional flow flutter problems in which structural nonlinearities are considered.

The active feedback control involves many technical problems, one of which is the unavoidable presence of a time delay between the controller and actuators [14]. Time delay feedback control has received much attention in recent years [15]. In [16], the flutter instability of actively controlled airfoils involving a time-delayed feedback control related to the aeroelasticity of 2D lifting surfaces is considered via Pontryagin's approach in conjunction with Stépán's theorems. As indicated in [17], the actuators may input energy at the moment when the controlled system does not need it. The time delay is very detrimental, because redundant energy may be inputted into the controlled system, which can lead to a reduction of the control performance and even cause instability of the dynamical system. Zhao [18, 19] presents a systematic study on aeroelastic stability of a two-dimensional airfoil with single or multiple time delays in the feedback control loops and investigates the effects of time delay on the flutter instability of an actively controlled airfoil. Huang et al. [20] reveal the effect of input time delay on the stability of a controlled high-dimensional aeroelastic system and present a new optimal control law to suppress the flutter.

In the previous research, the flutter control was studied without considering nonlinearity such as hysteresis in the airfoil [6–13]. In this paper, hysteresis nonlinearity in pitch

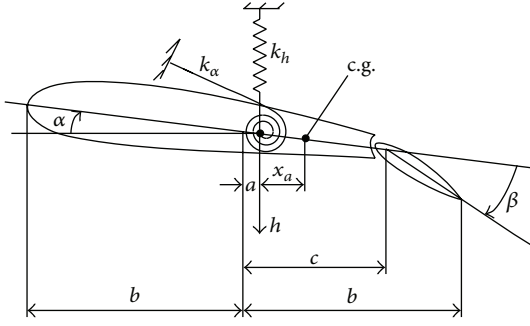


FIGURE 1: Schematic of airfoil section with a control surface.

has been considered in the design of a control law for the flutter control of nonlinear aeroelastic systems by state variable feedback. The model represents a prototypical aeroelastic wing section which has been traditionally used for the theoretical and experimental study of two-dimensional aeroelastic behavior. The purpose of the paper is to investigate the effect of hysteresis on the dynamic response and flutter suppression with the control law designed. In addition, the effect of time delay between the actuator control input and the control surface action is also investigated.

## 2. Aeroelastic Model and Control Problem

The prototypical aeroelastic wing section is shown in Figure 1. The governing equations of motion are provided by [21, 22]

$$\begin{aligned} & \begin{bmatrix} m & mx_a b \\ mx_a b & I_a \end{bmatrix} \begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} c_h & 0 \\ 0 & c_a \end{bmatrix} \begin{bmatrix} \dot{h} \\ \dot{\alpha} \end{bmatrix} \\ & + \begin{bmatrix} k_h & 0 \\ 0 & k_a(\alpha) \end{bmatrix} \begin{bmatrix} h \\ \alpha \end{bmatrix} = \begin{bmatrix} -L \\ \overline{M} \end{bmatrix}, \end{aligned} \quad (1)$$

where  $h$  is the plunge displacement and  $\alpha$  is the pitch angle. The parameter  $m$  is the mass of the wing;  $I_a$  is the moment of inertia;  $b$  is the semichord of the wing;  $x_a$  is the nondimensionalized distance of the center of mass from the elastic axis;  $c_a$  and  $c_h$  are the pitch and plunge damping coefficients, respectively. The parameters  $\overline{M}$  and  $L$  are the aerodynamic lift and moment. Assuming a quasi-steady aerodynamic model, the aerodynamic lift and moment are given by

$$\begin{aligned} L &= \rho U^2 b c_{l\alpha} \\ & \cdot \left[ \alpha + \left( \frac{\dot{h}}{U} \right) + \left( \frac{1}{2} - \alpha \right) b \left( \frac{\dot{\alpha}}{U} \right) \right] + \rho U^2 b c_{l\beta} \beta, \\ \overline{M} &= \rho U^2 b^2 c_{m\alpha} \\ & \cdot \left[ \alpha + \left( \frac{\dot{h}}{U} \right) + \left( \frac{1}{2} - \alpha \right) b \left( \frac{\dot{\alpha}}{U} \right) \right] + \rho U^2 b^2 c_{m\beta} \beta, \end{aligned} \quad (2)$$

where  $a$  is the nondimensionalized distance from the mid-chord to the elastic axis,  $c_{l\alpha}$  and  $c_{m\alpha}$  are the lift and moment coefficients per angle of attack,  $c_{l\beta}$  and  $c_{m\beta}$  are lift and moment

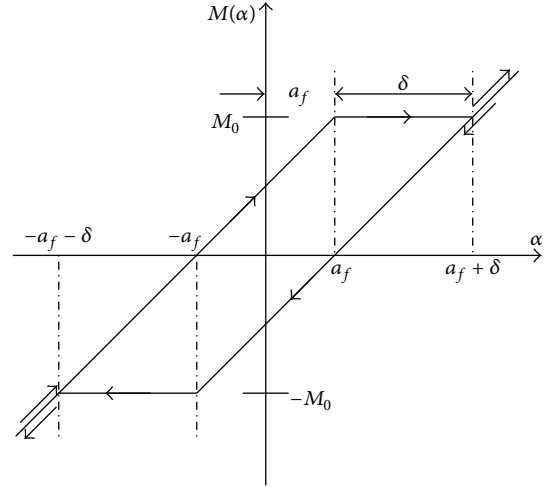


FIGURE 2: General sketch of hysteresis stiffness.

coefficients per control surface deflection  $\beta$ , and  $k_a(\alpha)$  and  $k_h$  are the pitch and plunge stiffness coefficients, respectively. The structural nonlinearities are represented by the nonlinear functions  $M(\alpha)$ . In this paper, we investigate system (1) for a hysteresis model in pitch, where  $M(\alpha)$  is illustrated in Figure 2 and given by [23]

$$\begin{aligned} M(\alpha) &= k_\alpha(\alpha) \alpha \\ &= \begin{cases} \alpha - \alpha_f + M_0, & \alpha < \alpha_f \uparrow, \\ \alpha + \alpha_f - M_0, & \alpha < -\alpha_f \downarrow, \\ M_0, & \alpha_f \leq \alpha \leq \alpha_f + \delta \uparrow, \\ -M_0, & -\alpha_f - \delta \leq \alpha \leq -\alpha_f \downarrow, \\ \alpha - \alpha_f - \delta + M_0, & \alpha > \alpha_f + \delta \uparrow, \\ \alpha + \alpha_f + \delta - M_0, & \alpha < -\alpha_f - \delta \downarrow, \end{cases} \end{aligned} \quad (3)$$

where  $\uparrow$  and  $\downarrow$  represent the motion in the increasing and decreasing  $\alpha$  direction, respectively.  $M_0$ ,  $\delta$ , and  $\alpha_f$  are constants.

A hysteresis model is a piecewise linear system whose state space consists of several linear regions, each of which is governed by a linear subsystem. Let the vector  $x \in \mathbb{R}^4$  be given by  $x = [h, \alpha, \dot{h}, \dot{\alpha}]$ ; then, the preceding equations can be written in a state-space form given by

$$\dot{x} = A(x)x + B\beta_c(t), \quad (4)$$

where  $\beta_c$  is the command input. The definition of matrix  $A(x)$  and  $B$  is given by

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_1 & -k_2 U^2 - p(x_2) & -c_1 & -c_2 \\ -k_3 & -k_4 U^2 - q(x_2) & -c_3 & -c_4 \end{bmatrix}, \quad (5)$$

$$B = [0 \ 0 \ b_3 U^2 \ b_4 U^2]^T,$$

where the parameters are given as follows:

$$\begin{aligned}
d &= m(I_a - mx_a^2 b^2), \\
k_1 &= \frac{I_a k_h}{d}, \quad k_2 = \frac{(I_a \rho b c_{la} + mx_a b^3 \rho c_{ma})}{d}, \\
k_3 &= \frac{-mx_a b k_h}{d}, \quad k_4 = \frac{-(mx_a b^2 \rho c_{la} + mb^2 \rho c_{ma})}{d}, \\
p(x_2) &= \frac{-mx_a b k_\alpha(x_2)}{d}, \quad q(x_2) = \frac{mk_\alpha(x_2)}{d}, \\
c_1 &= \frac{[I_a(c_h + \rho U b c_{la}) + mx_a \rho U b^3 c_{ma}]}{d}, \\
c_2 &= \left[ I_a \rho U b^2 c_{la} \left( \frac{1}{2} - a \right) - mx_a b c_a \right. \\
&\quad \left. + mx_a \rho U b^4 c_{ma} \left( \frac{1}{2} - a \right) \right] \cdot (d)^{-1}, \\
c_3 &= \frac{-m(x_a b c_h + x_a \rho U b^2 c_{la} + \rho U b^2 c_{la})}{d}, \\
c_4 &= m \left[ c_a - x_a \rho U b^3 c_{la} \left( \frac{1}{2} - a \right) \right. \\
&\quad \left. - \rho U b^3 c_{ma} \left( \frac{1}{2} - a \right) \right] \cdot (d)^{-1}, \\
b_3 &= \frac{-(I_a \rho b c_{l\beta} + mx_a \rho b^3 c_{m\beta})}{d}, \\
b_4 &= \frac{-(mx_a b^2 \rho c_{l\beta} + m \rho b^3 c_{m\beta})}{d}.
\end{aligned} \tag{6}$$

If a time delay  $\tau$  exists between the control input and actuator, (4) becomes

$$\dot{x} = A(x)x + B\beta_c(t - \tau). \tag{7}$$

### 3. State Variable Feedback Control Law

In this section, a nonlinear flutter control law based on the sliding mode control (SMC) method [24] is designed. Consider the nominal linear model of an uncertain system, given by

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{8}$$

where  $\text{rank}(B) = m$  and  $(A, B)$  is a controllable pair. Define an associated switching function

$$\sigma(t) = Sx(t). \tag{9}$$

This system can be transformed into regular form via a change of coordinates defined by an orthogonal matrix  $T_r$  such that

$$z(t) = T_r x(t), \tag{10}$$

where  $T_r$  is found by a QR decomposition of the input distribution matrix; that is,

$$T_r \bar{B} = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}. \tag{11}$$

Then, define

$$T_r \bar{A} T_r^T = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \tag{12}$$

$$S T_r^T = [S_1 \ S_2].$$

The system can be expressed in the well-known regular form

$$\begin{aligned}
\dot{z}_1(t) &= A_{11} z_1(t) + A_{12} z_2(t), \\
\dot{z}_2(t) &= A_{21} z_1(t) + A_{22} z_2(t) + B_2 u(t), \\
\sigma(t) &= S_1 z_1(t) + S_2 z_2(t).
\end{aligned} \tag{13}$$

During the sliding motion, the switching function must be identically zero, so

$$S_1 z_1(t) + S_2 z_2(t) = 0. \tag{14}$$

As mentioned before, the regular form approach of sliding mode control is used for flutter and limit cycle oscillation (LCO) suppression. Controller design process can be found in [13]. The feedback control law is finally given by

$$\beta(t) = -(SB)^{-1} SAx + \rho v, \tag{15}$$

where  $\rho$  is the gain,  $v$  is an approximation of the signum function; that is,

$$v = \text{sat}\left(\frac{\sigma}{\varepsilon}\right) = \begin{cases} \frac{\sigma}{|\sigma|} & \text{if: } \sigma \geq \varepsilon \\ \frac{\sigma}{\varepsilon} & \text{otherwise,} \end{cases} \tag{16}$$

where  $\varepsilon$  is the so-called boundary layer.

Substituting the control law (15) in (4) gives the closed-loop system

$$\begin{aligned}
\dot{x} &= A(x)x + B \left( -(SB)^{-1} SAx + \rho v \right) \\
&= A(x)x - B(SB)^{-1} SAx + B\rho \text{sat}\left(\frac{Sx}{\varepsilon}\right).
\end{aligned} \tag{17}$$

The performance of the closed-loop system depends on the matrix  $A(x)$  and the gains  $\rho$  and  $\varepsilon$ .

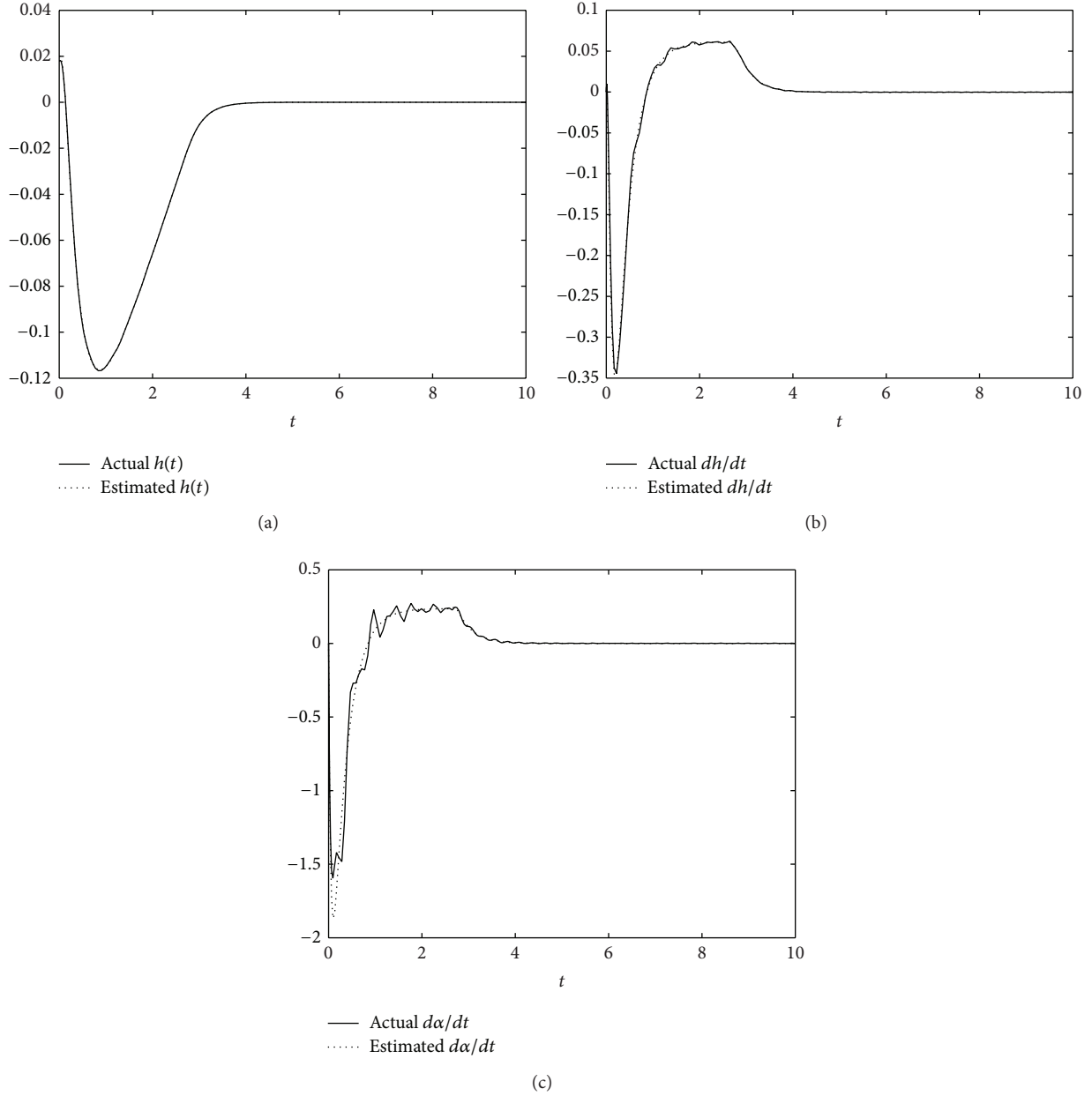


FIGURE 3: Performance of the estimator as a function of time: (a)  $h(t)$ ; (b)  $dh/dt$ ; (c)  $d\alpha/dt$ .

#### 4. State Estimator

In general, not all of the states are available online and the feedback law must be based on an estimate of the states. Assuming that only the pitch angle  $\alpha$  as a function of time can be directly measured, the output equation is given by

$$y = \alpha = [0 \ 1 \ 0 \ 0] x = Cx. \quad (18)$$

To design an observer based on SMC, it is required that the system be observable. For the pointwise observability, it is required that the following matrix has a full rank for all times:

$$E(x) = \begin{bmatrix} C^T & (CA(x))^T & (CA^2(x))^T & (CA^3(x))^T \end{bmatrix}. \quad (19)$$

The determinant of the matrix  $E(x)$

$$\det(E(x)) = -k_3^2 + c_3 c_1 k_3 - c_3^2 k_1. \quad (20)$$

As long as the value of the determinant is not zero, it means that the matrix is nonsingular, the observability of the system is only related to the system parameters. If  $\hat{x}(t)$  is the estimate of the state then the observer dynamics is given by

$$\dot{\hat{x}} = A(\hat{x}) \hat{x} + B\beta(t) + K(y - C\hat{x}), \quad (21)$$

where

$$K = -P_0 C^T V^{-1}. \quad (22)$$

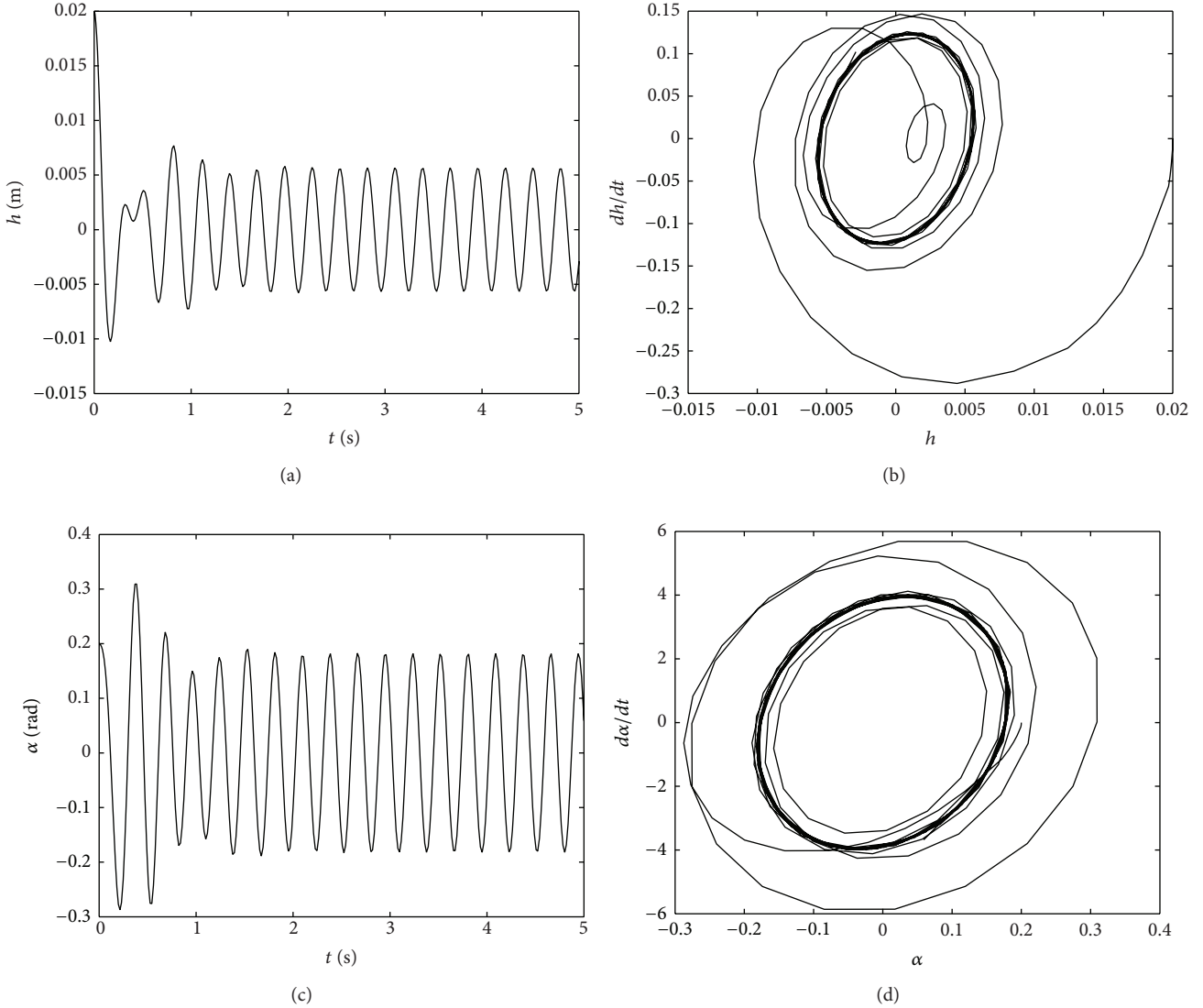


FIGURE 4: Open-loop responses:  $U = 13$  m/s. (a) Plunge. (b) Phase plane plot  $h - \dot{h}$ . (c) Pitch. (d) Phase plane plot  $\alpha - \dot{\alpha}$ .

The matrix  $P_0$  is the positive definite solution of the algebraic Riccati equation

$$A(\hat{x})P_0 + P_0A^T(\hat{x}) - P_0C^TV^{-1}CP_0 + Q_0 = 0, \quad (23)$$

where  $V^{-1}$  and  $Q_0$  are constants.

If the system in (4) and (18) is pointwise observable in the linear sense, that is, the matrix in (19) has a full rank for all times, then the preceding equation can be solved uniquely for the positive definite matrix  $P_0$ .

## 5. Results and Discussion

In this section, numerical results for the control of the aeroelastic system are obtained. The values for the system parameters are taken from [25] and listed as follows:  $b = 0.135$  m,  $m = 12.387$  kg,  $I_a = 0.065$ ,  $x_a = 0.3267$ ,  $k_h = 2844.4$  N/m,  $c_h = 27.43$  N·s/m,  $c_a = 0.036$  N·s,  $\rho = 1.225$  kg/m<sup>3</sup>,  $a = -0.68$ ,

$c_{1\alpha} = 6.28$ ,  $c_{1\beta} = 3.358$ ,  $c_{m\alpha} = -1.1304$ , and  $c_{m\beta} = -0.635$ . The nonlinear parameter in pitch stiffness is  $M_0 = 0.25^\circ$ ,  $\delta = 0.5^\circ$ , and  $a_f = 0.25^\circ$ . For the specific data given above, the determinant of the observability matrix  $E(x)$  is equal to  $-3,487,750.89 (\neq 0)$ . The matrix has a full rank for all times and, therefore, it is possible to design an online observer. The weighting matrix and the scalar function in the performance index are selected as  $Q_0 = \text{diag}(1, 10, 1, 10)$  and  $V = 1000$ , respectively. The observer gain  $K$  is equal to  $[0.0112 \quad -0.4161 \quad 0.0702 \quad -0.0816]^T$ . The performance of the estimator is shown in Figure 3. It is thus obvious that the estimator can quickly make an accurate estimate of nondirect measurement variables.

*Case 0.* Simulation is performed for the open-loop system ( $\beta = 0$ ) with the initial conditions  $\alpha(0) = 0.1$  (rad),  $h(0) = 0.01$  (m),  $\dot{\alpha}(0) = 0$  (rad/s), and  $\dot{h}(0) = 0$  and for the values of

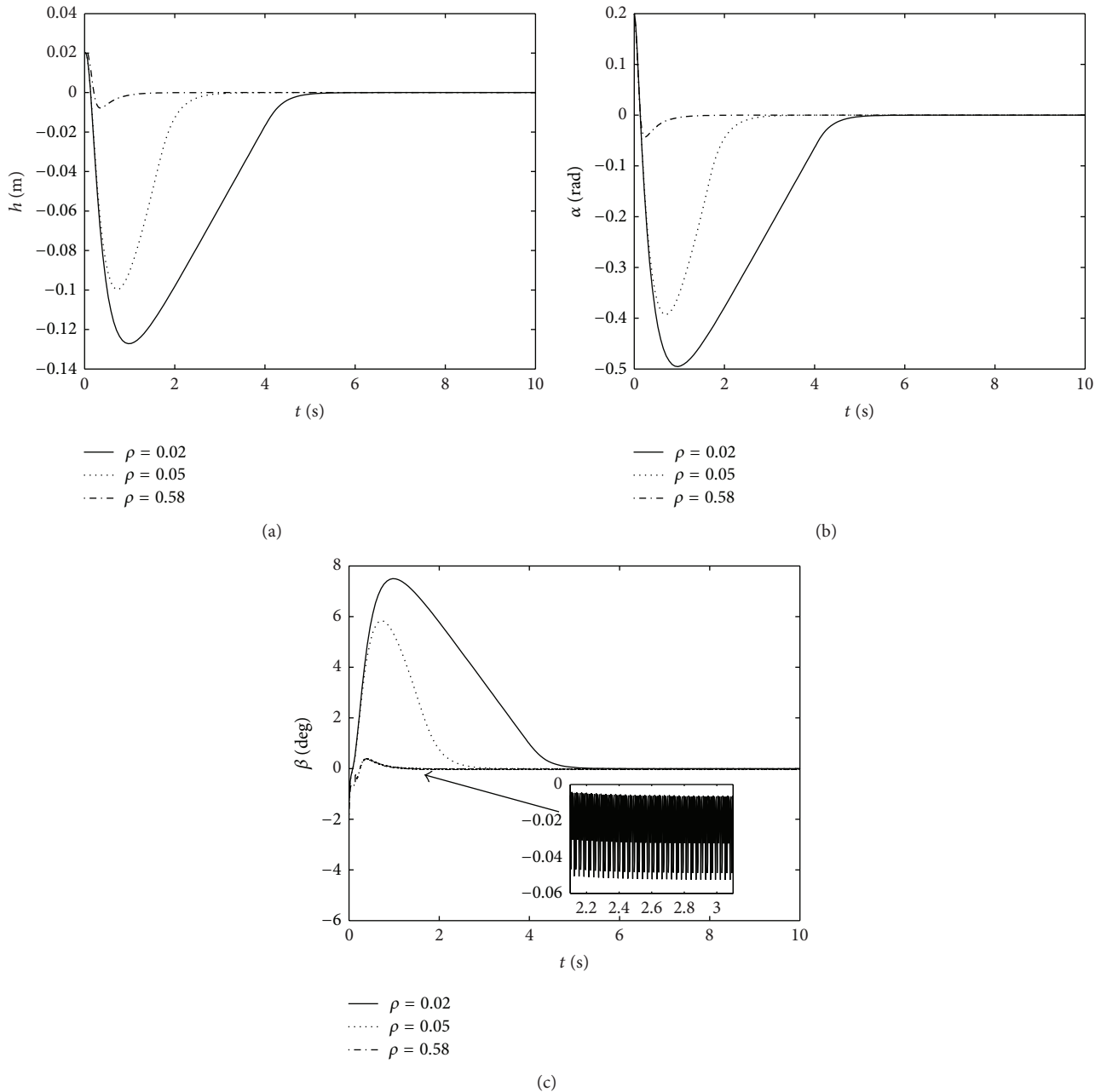


FIGURE 5: Nonlinear flutter control:  $U = 13$  m/s,  $\varepsilon = 0.009$ . (a) Plunge. (b) Pitch. (c) Control input.

$U = 13$  m/s. The open-loop eigenvalues of the system are in the right half plane at  $2.9046 + j18.3698$  and  $2.9046 - j18.3698$  and the remaining eigenvalues are in the left half plane. Thus, the origin  $x = 0$  is locally unstable. From Figure 4, it is seen that, for the chosen initial condition, after an initial transient, the pitch angle and the plunge displacement trajectories converge to limit cycles.

*Case 1.* Now the closed-loop system (17) including the nonlinear control law (15) is simulated. The parameter  $U$  and the initial conditions of case 0 are retained. Figures 5 and 6 illustrate the performance of the regular form of the sliding

mode control using different gains  $\rho$  and  $\varepsilon$ , respectively. From the results shown in Figure 5, it can be observed that the control law designed in the closed-loop system makes the plunge displacement and the pitch angle converge. If the gain  $\rho$  changes, the shape of the response characteristics will also change. When the gain  $\rho = 0.02$ , the settling time for the stabilization of both the plunge displacement and the pitch angle is of the order of 5 seconds, which is fast, and the maximum control magnitude for stabilization is 7.5 (deg). With the increase of the gain, the settling time for the stabilization is smaller and the maximum control magnitude for stabilization is also smaller. Until  $\rho = 0.58$ , the high-frequency chattering

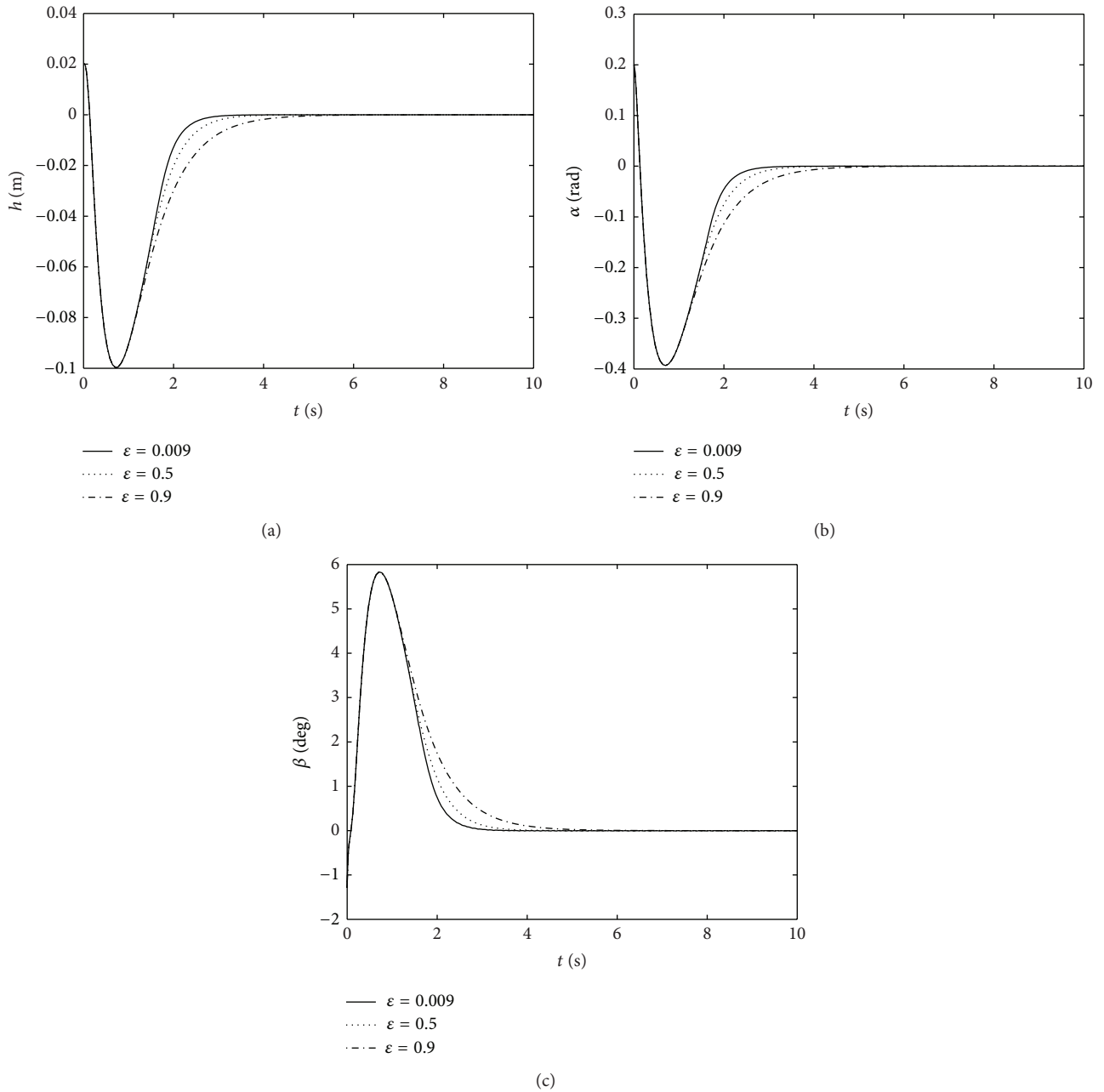


FIGURE 6: Nonlinear flutter control:  $U = 13$  m/s,  $\rho = 0.05$ . (a) Plunge. (b) Pitch. (c) Control input.

of the control signal occurred. From the analysis results, it is found that the choice of larger  $\rho$  gives faster convergence and requires smaller control input. As shown in Figure 6, with the increase of the gain  $\epsilon$ , the settling time for the stabilization is larger, but the maximum control magnitude for stabilization has no changes, so it can be seen that the choice of larger  $\rho$  gives slower convergence, which will cause the performance of the system to become noticeably degraded. Hence, the effects of the gains  $\rho$  and  $\epsilon$  on the response characteristics should be seriously considered.

*Case 2.* In the above analysis, time delays in control loops are ignored. In this section, the effect of time delay on

an aeroelastic system is investigated. First, a comparison was made for the control input time histories of the closed-loop system at  $U = 10$  m/s with and without time delay. From the results shown in Figure 7(a), it can be observed that the control law designed in the previous section makes the system response converge without any time delay. If a time delay  $\tau$  between the control input and actuator occurred at time  $t$ , the control input  $\beta(t - \tau)$  would be derived from the previous state  $x(t - \tau)$  at time  $\tau$  before the present state. The control input  $\beta(t - \tau)$  would drive the system to produce a deflection angle, which would cause oscillations of the system state and control input. We observe that this vibration was convergent when  $\tau = 0.001$  s. With the time delay

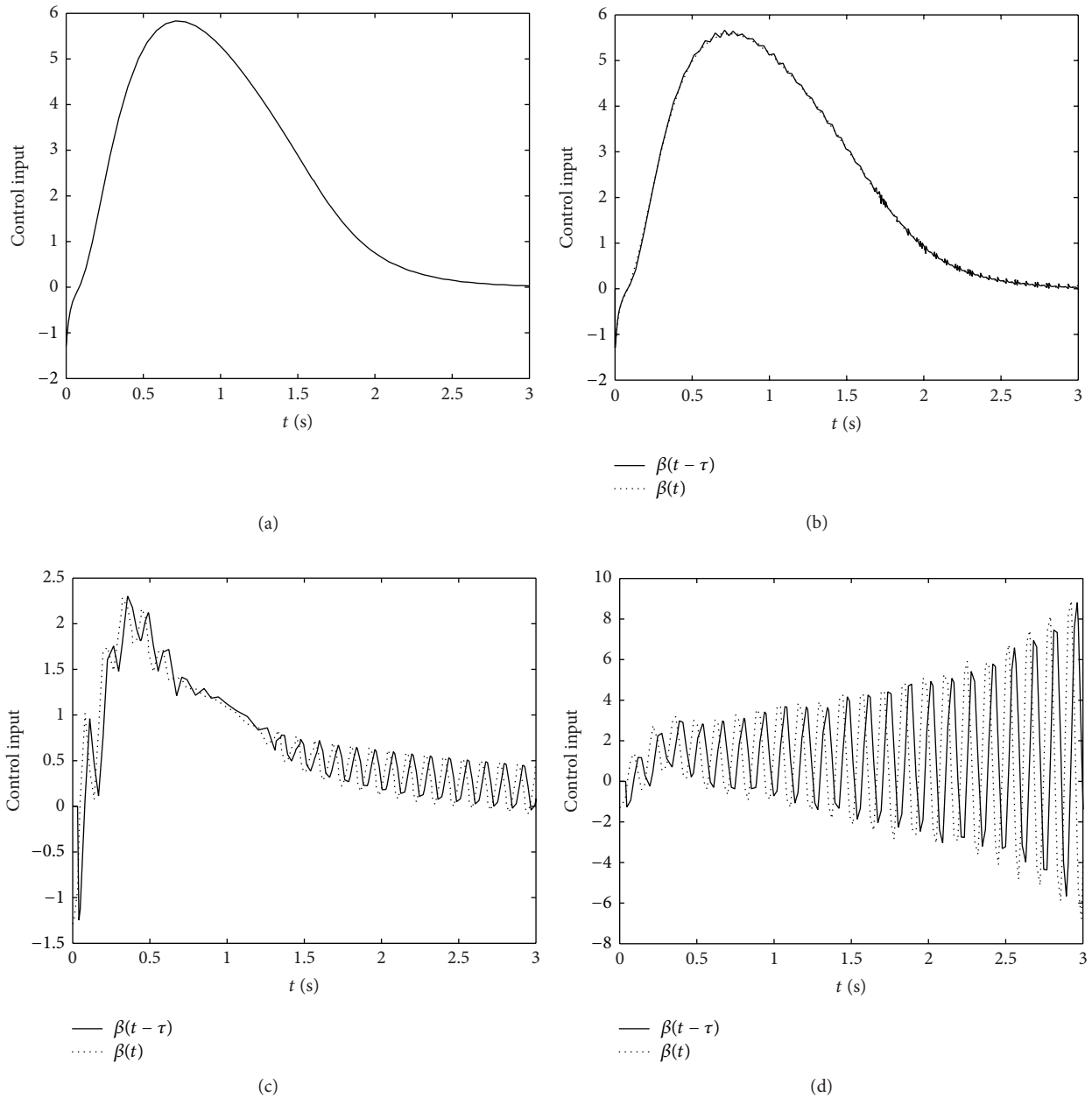


FIGURE 7: Time histories of control input at  $U = 10$  m/s and (a)  $\tau = 0$  s; (b)  $\tau = 0.001$  s; (c)  $\tau = 0.032$  s; (d)  $\tau = 0.041$  s.

increasing, a high-frequency vibration of small amplitude arose in the control input  $\beta(t)$ , but the vibration is still convergent. However, the vibration becomes divergent when  $\tau \geq 0.041$  s. From the analysis results, it can be seen that the time delay will produce an additional motion in the system responses. At the flow velocity  $U = 20$  m/s, the time histories of pitch and plunge responses are shown in Figure 8. If a time delay was set between the control input and actuator, the system response behaved differently. It is obvious that the pitch and plunge responses does not converge when the time delay  $\tau = 0.042$  s. The results indicate that the time delay between the control input and actuator may impair

the performance of a designed control law and cause instability of the system. Bifurcation diagrams of closed-loop system response as function of time delay are given in Figure 9. We can see that the amplitudes of plunge and pitch responses are quite small until the time delay is higher than 0.035 s, which means that a quite small time delay may lead to high-frequency vibration of the system, but the control law designed is still effective to suppress the flutter. Increasing the time delay further, closed-loop response amplitudes become large rapidly. The plunge and pitch amplitudes are almost the same with response amplitudes when  $\tau = 0.042$  s. And now the designed controller has no effect for flutter suppression.



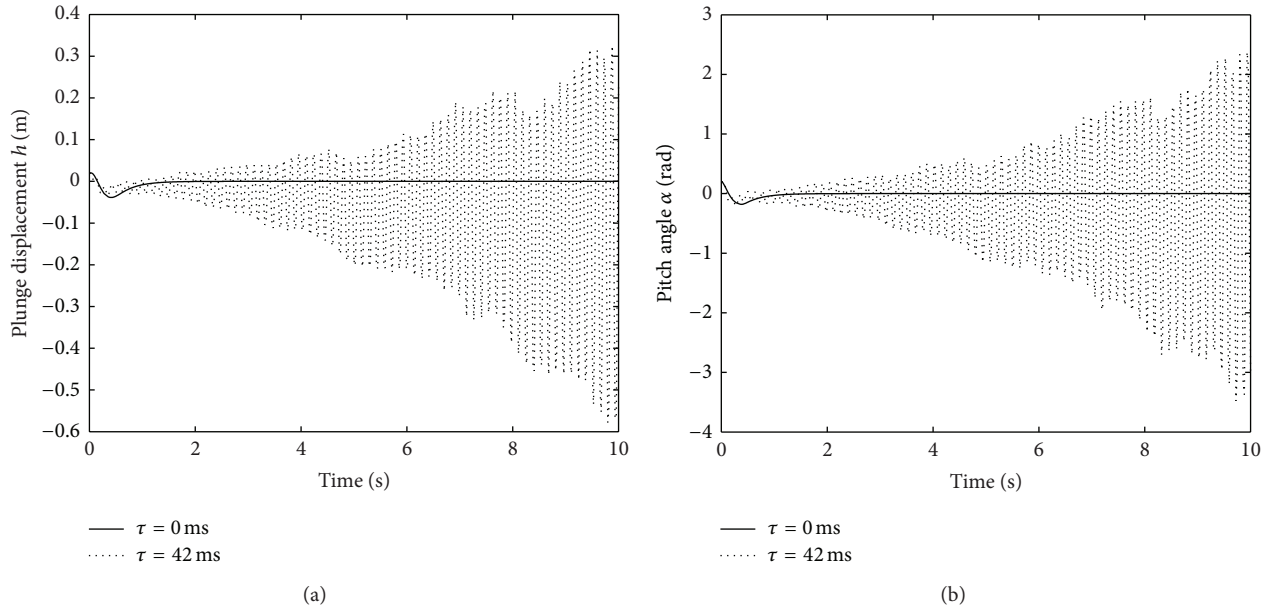


FIGURE 8: Time histories: (a) plunge and (b) pitch at  $U = 20$  m/s.

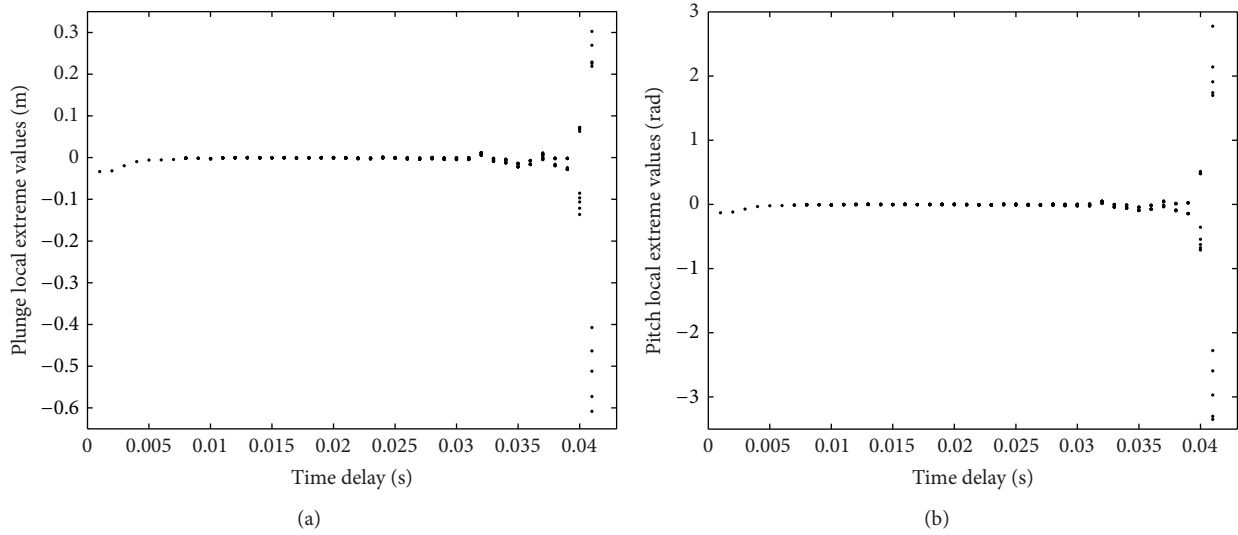


FIGURE 9: Bifurcation diagrams of (a) plunge and (b) pitch response as function of time delay at  $U = 20$  m/s.

**6. Conclusion**

Based on the state space model of a two-dimensional airfoil section with hysteresis nonlinearity, a feedback control law based on the estimated states was designed by using the sliding mode control method and applied for dynamic response suppression in this paper. An observer was constructed to estimate the unavailable state variables of the system. The effects of time delay between the control input and actuator on the aeroelastic responses has been investigated.

The nonlinear control law accomplishes asymptotic regulation of the pitch and plunge motion to the system equilibrium at zero deflections. Simulation results show that

the observer can give precise estimations for the plunge displacement and the velocities in pitch and plunge, and, in the closed-loop system, the designed controller is effective in flutter suppression. The system response is sensitive to the time delay between the control input and actuator. The bifurcation diagram of system response as function of time delay indicates that a small time delay may lead to high-frequency vibration. And with the time delay increasing, the system responses become divergent in the end.

**Conflict of Interests**

The authors declare that there is not any conflict of interests related to this paper.

## Acknowledgment

This work is supported by the Aeronautical Science Foundation of China (no. 20125853035).

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