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Research Article

Discrete PID-Type Iterative Learning Control for Mobile Robot

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Through studying tracking problems of the wheeled mobile robot, this paper proposed a discrete iterative learning control approach based on PID with strong adaptability, fast convergence, and small error. This algorithm used discrete PID to filter rejection and restrained the influence of interference and noise on trajectory tracking, which made it more suitable for engineering application. The PID-type iterative learning convergence condition and certification procedure are presented. The results of simulation reveal that the PID-type ILC holds the features of simplicity, strong robustness, and high repeating precision and can well meet the control requirement of nonlinear discrete system.

1. Introduction

Iterative learning control (ILC) [1, 2] is an advanced control which can realize repetitive control and antidisturbance, tracking the trajectory with high precision independent of accurate mathematical models for its control. A substantial proportion of inchoate ILC algorithms was applied in the continuous-time systems [3, 4]. Some ILC algorithms have been developed for discrete-time systems, but they are restricted to linear systems [5, 6]. In an ordinary way, the ILC update rules are composed of P-type [7, 8], PD-type [9, 10], and high-order type [11].

Many ILC algorithms [12, 13] have been used in motion controller system and a certain number of simulation results display the effectiveness of these algorithms. Besides, they can help the mobile robot to better track the desired trajectory iteratively. Even under the complicated conditions, the mobile control system could fulfill quite nicely the being stipulated tasks with the aid of ILC. However, for tracking problem of mobile robot, a multitude of researches are heavily dependent on the precision of the robot model and the developed control laws are often complex. Therefore, ILC would be the preferred method of tracking the given trajectories for mobile robot repeatedly, especially in the real complex environment. Ostafew et al. [14] present a mobile robot controller that combines ILC algorithm with a feedback controller to reduce path-tracking errors over repeated traverses along a reference path by the experimental results of mobile robot travelling through challenging terrain including steep hills and sandy turns. In the extreme environments without global localization system (GPS), for a path-repeating mobile robot, ILC can handle unmodelled terrain and rover dynamics. Furgale and Barfoot [15] apply an ILC scheme on the mobile robot to repeat long routes without the need for an accurate global reconstruction. In an urban area and in a planetary analog environment, it can perform path tracking by itself.

Mobile robot is a complex automation system which combines environment sense, dynamical plan with motion control, and performance, whose requirement for the dynamical and static characteristics of mobile robot becomes high. There have been several research works on the control of mobile robots. Jiang and Nijmeijer [16] firstly apply adaptive control to the mobile robot. Buccieri et al. [17] use point stabilization for mobile robot in tracking control. Moosavian and Keymasi [18] designed a Lyapunov-based controller for a wheeled robot. A few articles [19] have showed the probability of realizing perfect tracking for mobile robots by the ILC algorithm. The closed-open-loop P-type iterative learning control scheme [20] for mobile robot system is proposed. Although it can reduce the system steady-state error and improve the control precision, the relative stability and convergence speed of control systems can be whittled down further. Yu and Weili [21] showed the open-loop

D-type ILC laws in trajectory tracking of wheeled robot, but the anti-interference function of this system was not ideal. Panzieri and Ulivi [22] presented an ILC method which takes advantage of the possibility of transforming systems in chained form via feedback for mobile robots to improve the robustness properties of the controller.

The contribution of this paper is applying PID-type ILC algorithm to the nonlinear discrete system of mobile robot. With the continual advances in control theory, PID controller is still the most commonly used controller in the controlling field [23, 24]. This is mainly attributed to its noticeable effectiveness, simple structure, and robustness. So we are keen on using the PID strategy in designing the iterative learning control schemes. PID have some specific advantage roles of each of modes in the ILC algorithm. The P-component has a stabilizer role in the ILC system and causes monotonic convergence and the I-component decreases the effect of nonzero initial errors and increases the convergence rate, while D-term can reduce the effect of disturbance inputs. The effectiveness and superiority of this algorithm are shown by simulation.

This paper is organized as follows. Section 2 gives the problem formulation. In Section 3, the controller design is proposed and its convergence is also discussed. Simulation results are shown in Section 4. Section 5 concludes the paper.

2. Problem Formulation

Figure 1 shows system model of mobile robot with two wheels [20]. The mobile robot moves in two-dimensional space and $P(k)$ represents the mobile robot's current position. In global coordinates, the current position of $P(k)$ is determined by the coordinates $x_P(k)$ and $y_P(k)$ and $\theta_P(k)$ is the orientation angle of the robot. So, the generalized coordinates of $P(k)$ are defined as $[x_P(k), y_P(k), \theta_P(k)]$. Defining the linear and angular velocities of $P(k)$ in the robot coordinate as $\nu_P(k)$ and $\omega_P(k)$, we can describe the kinematic motion of the mobile robot in the discrete-time domain as

$$\begin{bmatrix} x_P(k+1) \\ y_P(k+1) \\ \theta_P(k+1) \end{bmatrix} = \begin{bmatrix} x_P(k) \\ y_P(k) \\ \theta_P(k) \end{bmatrix} + \Delta T \begin{bmatrix} \cos\theta_P(k) & 0 \\ \sin\theta_P(k) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \nu_P(k) \\ \omega_P(k) \end{bmatrix}, \tag{1}$$

where ΔT is the sampling time. Defining the robot states and velocities, $q(k) = [x_P(k), y_P(k), \theta_P(k)]^T$ and $u_P(k) = [\nu_P(k), \omega_P(k)]^T$, we can express kinematic motion (1) as

$$q(k+1) = q(k) + B(q(k), k)\, u_P(k), \tag{2}$$

where $B(q(k), k) = \Delta T\,[\cos\theta_P(k)\;\; 0;\;\; \sin\theta_P(k)\;\; 0;\;\; 0\;\; 1]$.

As shown in Figure 1, the dotted line is desired trajectory. $P_d(k) = [x_d(k), y_d(k), \theta_d(k)]$ $(1 \le k \le n)$. The trajectory tracking control problem of mobile robots is to determine $u(k) = [\nu(k), \omega(k)]^T$ which makes $x_P(k) \to x_d(k)$, $y_P(k) \to y_d(k)$, and $\theta_P(k) \to \theta_d(k)$. If ILC is applied to the mobile control system, the velocity errors can be uniformly bounded under bounded uncertainties and disturbances that occur in the mobile system dynamics [20, 25, 26]. So we try to find a novel ILC algorithm to make perfect tracking.

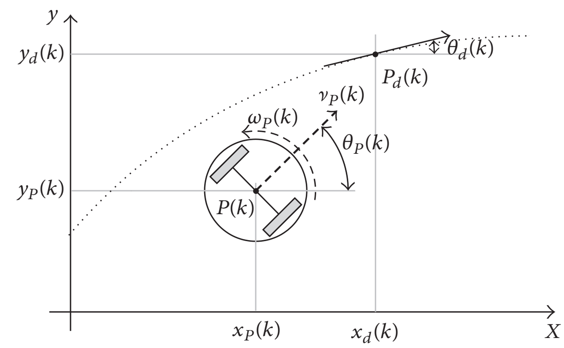


Figure 1: Mobile robot configuration.

Considering state disturbances $\beta(k)$ and output measurement noises $\gamma(k)$, the kinematic model of mobile robot at the ith iteration can be described as

$$q_i(k+1) = q_i(k) + B(q_i(k), k)\, u_i(k) + \beta_i(k), \tag{3}$$

$$y_i(k) = q_i(k) + \gamma_i(k), \tag{4}$$

where i indicates the number of iterations and $k \in [1, n]$ is the discrete-time index. For all $k \in N$, $q_i(k), u_i(k) = [\nu_i(k), \omega_i(k)]^T$, and $y_i(k) = [x_i(k), y_i(k), \theta_i(k)]^T$, $\beta_i(k)$ and $\gamma_i(k)$ are the states, inputs, outputs, state disturbances, and output noises at the ith iteration. If state disturbances $\beta_i(k) = 0$ and output measurement noises $\gamma_i(k) = 0$ $(k \in N)$, the desired trajectory equation can be written as

$$q_d(k+1) = q_d(k) + B(q_d(k), k)\, u_d(k), \tag{5}$$

$$y_d(k) = q_d(k). \tag{6}$$

Furthermore, the matrix $B(q_i(k), k)$ is bounded as $B(q_i(k), k) \le b_B$ and meet Lipschitz conditions:

$$\|B(q_1, k) - B(q_2, k)\| \le L_B \|q_1 - q_2\|, \tag{7}$$

where L_B and b_B are positive constants. Meanwhile, formulas (5) and (6) satisfy the following assumptions:

$$\begin{gathered} \max_{1\le k\le n} \|u_d(k)\| \le b_{u_d}, \\ \max_{1\le i\le\infty}\max_{1\le k\le n} \|\beta_i(k)\| \le b_\beta, \\ \max_{1\le i\le\infty}\max_{1\le k\le n} \|\gamma_i(k)\| \le b_\gamma, \\ \|q_d(0) - q_i(0)\| \le b_{q_0}, \end{gathered} \tag{8}$$

where b_{u_d}, b_β, b_γ, and b_{q_0} are positive constants.

3. Controller Design and Convergence Analysis

Now we propose a PID-type iterative learning rule as

$$\begin{aligned} u_{i+1}(k) = u_i(k) + G_p(k) e_i(k+1) \\ + G_d(k)\left[e_i(k+1) - e_i(k)\right] \\ + G_I(k)\sum_{l=0}^{k} e_i(l+1), \end{aligned} \tag{9}$$

for the ith iteration, where $e_i(k) = y_d(k) - y_i(k)$ are the output tracking errors, $G_p(k)$, $G_d(k)$, $G_I(k)$ are the proportional, derivative, and integral gain matrices that satisfy $\|G_p(k)\| \le b_{G_p}$, $\|G_d(k)\| \le b_{G_d}$, and $\|G_I(k)\| \le b_{G_I}$ for all $k \in N$ and for some $b_{G_p} > 0$, $b_{G_d} > 0$, and $b_{G_I} > 0$.

Define 1. The λ-norm is defined for a positive real function $f : N \to R$ as

$$\|f(\cdot)\|_\lambda = \sup_{k\in N}\left\{\left(\frac{1}{\lambda}\right)^k \|f(k)\|\right\} \quad \text{for } \lambda \ge 1. \tag{10}$$

Theorem 1. *Consider the time-varying discrete-time system (3) and (4) with the iterative learning rule (9). Let us assume that the assumptions (8) are satisfied and the inequality*

$$\left\|I - \left[G_p(k) + G_d(k) + G_I(k)\right] B\left(q_i(k), k\right)\right\| \le \rho < 1 \tag{11}$$

holds for all $(q_i(k), k) \in R^n \times N$. Given a bounded output $y_d(k)$ and in the absence of state disturbances, output noises, and initial state errors, $u_i(k)$, $q_i(k)$, and $y_i(k)$, respectively, converge to $u_d(k)$, $q_d(k)$, and $y_d(k)$ for all $k \in [1, n]$, as $i \to \infty$. In the presence of disturbances, noises, and errors, $\|u_i(k) - u_d(k)\|$, $\|q_i(k) - q_d(k)\|$, and $\|y_i(k) - y_d(k)\|$ will be ultimately bounded with bounds that are functions of b_β, b_γ, and b_{q_0}.

Proof. Subtracting (5) from (3), we obtain the state errors as follows:

$$\begin{aligned} \Delta q_i(k+1) &= q_d(k+1) - q_i(k+1) \\ &= \left[q_d(k) + B\left(q_d(k), k\right) u_d(k)\right] \\ &\quad - \left[q_i(k) + B\left(q_i(k), k\right) u_i(k) + \beta_i(k)\right] \\ &= \Delta q_i(k) + B\left(q_d(k), k\right) u_d(k) \\ &\quad - B\left(q_i(k), k\right)\left[u_i(k) - u_d(k) + u_d(k)\right] \\ &\quad - \beta_i(k) \\ &= \Delta q_i(k) \\ &\quad + \left[B\left(q_d(k), k\right) - B\left(q_i(k), k\right)\right] u_d(k) \\ &\quad - B\left(q_i(k), k\right)\Delta u_i(k) - \beta_i(k). \end{aligned} \tag{12}$$

Considering (7) and (8), taking norms on both sides of (12) gives

$$\begin{aligned} \left\|\Delta q_i(k+1)\right\| \le \left\|\Delta q_i(k)\right\| + L_B b_{u_d}\left\|\Delta q_i(k)\right\| \\ + b_B\left\|\Delta u_i(k)\right\| + b_\beta. \end{aligned} \tag{13}$$

Let $h_1 = 1 + L_B b_{u_d}$. Then it follows from (13) that

$$\left\|\Delta q_i(k+1)\right\| \le h_1\left\|\Delta q_i(k)\right\| + b_B\left\|\Delta u_i(k)\right\| + b_\beta \tag{14}$$

from which we have

$$\left\|\Delta q_i(k)\right\| \le \sum_{j=0}^{k-1} h_1^{k-1-j}\left[b_B\left\|\Delta u_i(j)\right\| + b_\beta\right] + h_1^k b_{q_0}. \tag{15}$$

On the other hand, it follows from (9) that the control input errors become

$$\begin{aligned} \Delta u_{i+1}(k) &= u_d(k) - u_{i+1}(k) = u_d(k) - u_i(k) \\ &\quad - G_p(k) e_i(k+1) - G_d(k)\left[e_i(k+1) - e_i(k)\right] \\ &\quad - G_I(k)\sum_{l=0}^{k} e_i(l+1) = \Delta u_i(k) - \left[G_p(k) + G_d(k)\right] \\ &\quad \cdot e_i(k+1) + G_d(k) e_i(k) - G_I(k)\sum_{l=0}^{k} e_i(l+1) \\ &\quad = \Delta u_i(k) - \left[G_p(k) + G_d(k)\right]\left[y_d(k+1) - y_i(k\right. \\ &\quad \left. + 1)\right] + G_d(k)\left[y_d(k) - y_i(k)\right] - G_I(k) \\ &\quad \cdot \sum_{l=0}^{k}\left[y_d(l+1) - y_i(l+1)\right] = \Delta u_i(k) - \left[G_p(k)\right. \\ &\quad \left. + G_d(k)\right]\left[q_d(k+1) - q_i(k+1) - \gamma_i(k+1)\right] \\ &\quad + G_d(k)\left[q_d(k) - q_i(k) - \gamma_i(k)\right] - G_I(k) \\ &\quad \cdot \sum_{l=0}^{k}\left[q_d(l+1) - q_i(l+1) - \gamma_i(l+1)\right] = \Delta u_i(k) \\ &\quad - \left[G_p(k) + G_d(k)\right]\left[q_d(k) + B\left(q_d(k), k\right) u_d(k)\right. \\ &\quad \left. - q_i(k) - B\left(q_i(k), k\right) u_i(k) - \beta_i(k) - \gamma_i(k+1)\right] \\ &\quad + G_d(k)\left[q_d(k) - q_i(k) - \gamma_i(k)\right] - G_I(k)\sum_{l=0}^{k}\left[q_d(l)\right. \\ &\quad + B\left(q_d(l), l\right) u_d(l) - q_i(l) - B\left(q_i(l), l\right) u_i(l) \\ &\quad \left. - \beta_i(l) - \gamma_i(l+1)\right] = \Delta u_i(k) - \left[G_p(k) + G_d(k)\right] \\ &\quad \cdot \left\{\Delta q_i(k) + B\left(q_d(k), k\right) u_d(k) - B\left(q_i(k), k\right)\right. \\ &\quad \left. \cdot \left[u_i(k) - u_d(k) + u_d(k)\right] - \beta_i(k) - \gamma_i(k+1)\right\} \\ &\quad + G_d(k)\left[\Delta q_i(k) - \gamma_i(k)\right] - G_I(k)\sum_{l=0}^{k}\left\{\Delta q_i(l)\right. \\ &\quad + B\left(q_d(l), l\right) u_d(l) \end{aligned}$$

$$\begin{aligned}
&- B(q_i(l), l)[u_i(l) - u_d(l) + u_d(l)] - \beta_i(l) \\
&- \gamma_i(l+1)\} = \{I - [G_p(k) + G_d(k)] B(q_i(k), k)\} \\
&\cdot \Delta u_i(k) - G_p(k)\Delta q_i(k) - [G_p(k) + G_d(k)] \\
&\cdot [B(q_d(k), k) - B(q_i(k), k)] u_d(k) + [G_p(k) \\
&+ G_d(k)][\beta_i(k) + \gamma_i(k+1)] - G_d(k)\gamma_i(k) \\
&- G_I(k)\sum_{l=0}^{k}\{\Delta q_i(l) \\
&+ [B(q_d(l), l) - B(q_i(l), l)] u_d(l) \\
&+ B(q_i(l), l)\Delta u_i(l) - \beta_i(l) - \gamma_i(l+1)\}.
\end{aligned} \tag{16}$$

Applying (7) and (8) to (16), we have

$$\begin{aligned}
&\|\Delta u_{i+1}(k)\| \\
&\quad \le \{I - [G_p(k) + G_d(k) + G_I(k)] B(q_i(k), k)\} \\
&\quad \cdot \|\Delta u_i(k)\| \\
&\quad + [(b_{G_p} + b_{G_d} + b_{G_I})(1 + L_B b_{u_d}) + b_{G_d} + b_{G_I}] \\
&\quad \cdot \|\Delta q_i(k)\| + b_{G_I}\sum_{l=0}^{k-2}\|\Delta q_i(l+1)\| \\
&\quad + (b_{G_p} + b_{G_d} + b_{G_I})(b_B + b_\gamma) + b_{G_I}\sum_{l=0}^{k-1} b_\gamma + b_{G_d} b_\gamma.
\end{aligned} \tag{17}$$

Letting $h_2 = (b_{G_p}+b_{G_d}+b_{G_I})(1+L_B b_{u_d}) = (b_{G_p}+b_{G_d}+b_{G_I})h_1$, $h_3 = h_2 + b_{G_d} + b_{G_I}$, and $b_1 = (b_{G_p} + b_{G_d} + b_{G_I})(b_B + b_\gamma) + b_{G_I}\sum_{l=0}^{k-1} b_\gamma + b_{G_d} b_\gamma$, we have

$$\begin{aligned}
\|\Delta u_{i+1}(k)\| &\le \rho\|\Delta u_i(k)\| + h_3\|\Delta q_i(k)\| \\
&\quad + b_{G_I}\sum_{l=0}^{k-2}\|\Delta q_i(l+1)\| + b_1.
\end{aligned} \tag{18}$$

From (11) and (15), we get

$$\begin{aligned}
&\|\Delta u_{i+1}(k)\| \le \rho\|\Delta u_i(k)\| \\
&\qquad + h_3\sum_{j=0}^{k-1} h_1^{k-1-j}[b_B\|\Delta u_i(j)\| + b_\beta] \\
&\qquad + h_3 h_1^k b_{q_0} \\
&\qquad + b_{G_I}\sum_{l=0}^{k-2}\sum_{j=0}^{l} h_1^{l-j}[b_B\|\Delta u_i(j)\| + b_\beta] \\
&\qquad + b_{G_I}\sum_{l=0}^{k-2} h_1^{l+1} b_{q_0} + b_1.
\end{aligned} \tag{19}$$

Multiplying both sides of (19) by $(1/\lambda)^k$ to compute the λ-norm, we have

$$\begin{aligned}
&\|\Delta u_{i+1}(k)\|\left(\frac{1}{\lambda}\right)^k \le \rho\|\Delta u_i(k)\|\left(\frac{1}{\lambda}\right)^k + \left(\frac{h_1}{\lambda}\right)^k h_3 b_{q_0} \\
&\quad + \left(\frac{h_3}{\lambda}\right)\sum_{j=0}^{k-1}\left(\frac{h_1}{\lambda}\right)^{k-1-j}\left[b_B\|\Delta u_i(j)\|\left(\frac{1}{\lambda}\right)^j\right. \\
&\quad \left. + b_\beta\left(\frac{1}{\lambda}\right)^j\right] + \left(\frac{1}{\lambda}\right)^{k-l} b_{G_I}\sum_{l=0}^{k-2}\sum_{j=0}^{l}\left(\frac{h_1}{\lambda}\right)^{l-j} \\
&\quad \cdot\left[b_B\|\Delta u_i(j)\|\left(\frac{1}{\lambda}\right)^j + b_\beta\left(\frac{1}{\lambda}\right)^j\right] + \left(\frac{1}{\lambda}\right)^{k-l-1} \\
&\quad \cdot b_{G_I}\sum_{l=0}^{k-2}\left(\frac{h_1}{\lambda}\right)^{l+1} b_{q_0} + \left(\frac{1}{\lambda}\right)^k b_1.
\end{aligned} \tag{20}$$

Let $m_1 = \sum_{l=0}^{k-2}[1 - (h_1/\lambda)^{l+1}]$ and $m_2 = \sum_{l=0}^{k-2}(h_1/\lambda)^{l+1}$. Taking $\lambda > \max\{1, h_1\}$ and noting that the λ-norm of a constant is itself the constant, we have

$$\begin{aligned}
&\|\Delta u_{i+1}(k)\|_\lambda \\
&\quad \le \left\{\rho + b_B h_3\left\{\frac{[1 - (h_1/\lambda)^n]}{(\lambda - h_1)}\right\} + \frac{(b_B b_{G_I} m_1)}{(\lambda - h_2)}\right\} \\
&\quad \cdot \|\Delta u_i(k)\|_\lambda + \left[\frac{b_\beta}{(\lambda - h_1)}\right] \\
&\quad \cdot \left\{h_3\left[1 - \left(\frac{h_1}{\lambda}\right)^n\right] + b_{G_I} m_1\right\} + b_{G_I} b_{q_0} m_2 + b_1.
\end{aligned} \tag{21}$$

Then (21) implies

$$\|\Delta u_{i+1}(k)\|_\lambda \le \tilde{\rho}\|\Delta u_i(k)\|_\lambda + \varepsilon, \tag{22}$$

where $\tilde{\rho} = \rho + b_B h_3\{[1-(h_1/\lambda)^n]/(\lambda-h_1)\} + (b_B b_{G_I} m_1)/(\lambda-h_1)$ and $\varepsilon = [b_\beta/(\lambda-h_1)]\{h_3[1-(h_1/\lambda)^n] + b_{G_I} m_1\} + b_{G_I} b_{q_0} m_2 + b_1$.

Let us make recursive formula (22) and choose λ large enough so that $\tilde{\rho} \approx \rho < 1$. Then we have

$$\lim_{i\to\infty}\|\Delta u_i\|_\lambda \le \frac{\varepsilon}{(1-\rho)}. \tag{23}$$

Similarly, multiplying both sides of (15) by $(1/\lambda)^k$ gives

$$\begin{aligned}
&\|\Delta q_i(k)\|\left(\frac{1}{\lambda}\right)^k \le \left(\frac{1}{\lambda}\right) \\
&\quad \cdot \sum_{j=0}^{k-1}\left(\frac{h_1}{\lambda}\right)^{k-1-j}\left[b_B\|\Delta u_i(j)\|\left(\frac{1}{\lambda}\right)^j + b_\beta\left(\frac{1}{\lambda}\right)^j\right] \\
&\quad + \left(\frac{h_1}{\lambda}\right)^k b_{q_0}.
\end{aligned} \tag{24}$$

Because of $b_B(1/\lambda)^j \le b_B$ and $h_1/\lambda < 1$, we have

$$\|\Delta q_i\|_\lambda \le (b_B\|\Delta u_i\|_\lambda + b_\beta)\left\{\frac{[1 - (h_1/\lambda)^n]}{(\lambda - h_1)}\right\} + b_{q_0}. \tag{25}$$

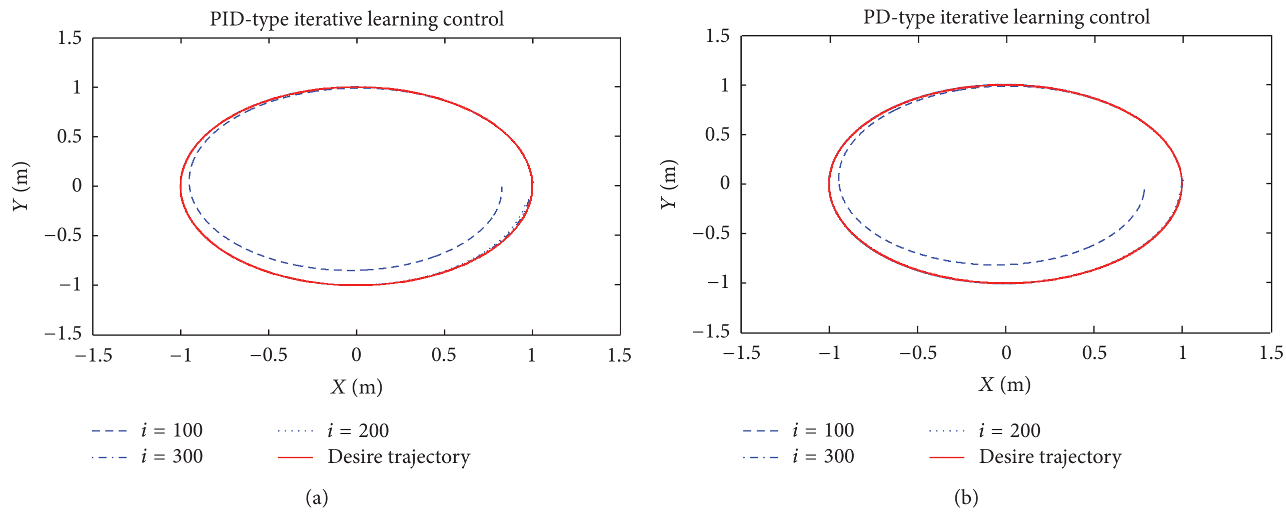


Figure 2: Desired and actual position trajectories.

Substituting (23) into (25), we obtain

$$\lim_{i\to\infty}\|\Delta q_i\|_\lambda \le b_B\left\{\frac{\left[1-(h_1/\lambda)^n\right]}{(\lambda-h_1)}\right\}\left[\frac{\varepsilon}{(1-\rho)}\right] + b_\beta\left\{\frac{\left[1-(h_1/\lambda)^n\right]}{(\lambda-h_1)}\right\}+b_{q_0}. \tag{26}$$

Subtracting (4) from (6) gives

$$\Delta y_i(k) = y_d(k) - y_i(k) = q_d(k) - \left[q_i(k)+\gamma_i(k)\right] = \Delta q_i(k) - \gamma_i(k). \tag{27}$$

Multiplying both sides of (27) by $(1/\lambda)^k$ results in

$$\|\Delta y_i\|_\lambda = \|\Delta q_i\|_\lambda + b_\gamma. \tag{28}$$

Substituting (26) into (28), we obtain

$$\lim_{i\to\infty}\|\Delta y_i\|_\lambda \le b_B\left\{\frac{\left[1-(h_1/\lambda)^n\right]}{(\lambda-h_1)}\right\}\left[\frac{\varepsilon}{(1-\rho)}\right] + b_\beta\left\{\frac{\left[1-(h_1/\lambda)^n\right]}{(\lambda-h_1)}\right\}+b_{q_0}+b_\gamma. \tag{29}$$

Consequently, in the absence of $\beta_i(k)$ and $\gamma_i(k)$ and with perfect repeatability or with $b_\beta = 0$, $b_\gamma = 0$, and $b_{q_0} = 0$, the error bounds become zero. Otherwise, the errors become ultimately bounded with error bounds that are functions of b_β, b_γ, and b_{q_0} as given in (23), (26), and (29). □

4. Simulation

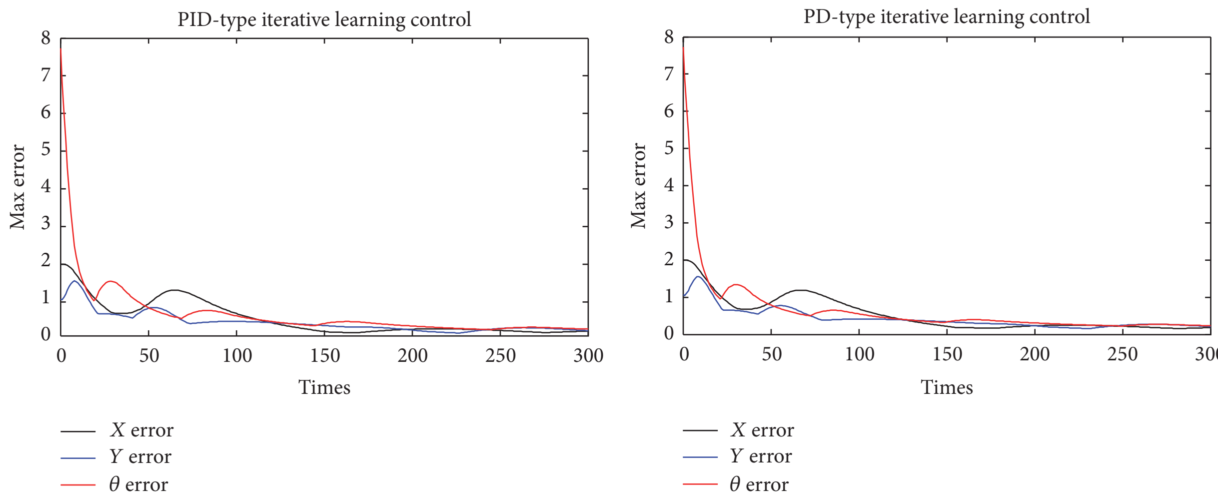
To investigate the validity of the proposed ILC algorithm and compare the performance of the proposed ILC algorithm with that of conventional PD-type ILC algorithm, we performed MATLAB simulations under the bounded disturbances and noises. The desired trajectory is $P_d = [x_d(t)\ \ y_d(t)\ \ \theta_d(t)]^T = [\cos(\pi t)\ \ \sin(\pi t)\ \ \pi t+\pi/2]^T$. The sample period ΔT is set to 0.001 s. The whole task period is $i = 1,\ldots,300$. The initial states are $u_0(0) = 0$ and $q_k(0) = (0,0,0)^T$. The state disturbances term is $\beta_i(t) = 0.01[\sin(40\pi t) + 0.05\,\mathrm{random}(0,8)\ \ \sin(40\pi t) + 0.05\,\mathrm{random}(0,8)\ \ 0.5\sin(40\pi t) + 0.05\,\mathrm{random}(0,4)]^T$. The output measurement noises are Gaussian white noise ($n(t) \sim N(0,\delta^2)$), $\gamma_i(t) = 0.05n(t)$. According to convergence condition (12), we choose the gain matrices $G_p(k) = 0.11\left[\cos\theta(k)\ \ \sin\theta(k)\ \ 0;\ \ 0\ \ 0\ \ 1\right]$, $G_I(k) = 0.001G_p(k)$, and $G_d(k) = 2.1G_p(k)$. Simulation results are shown in Figures 2 and 3.

As can be seen from Figure 2, the output trajectory of the mobile robot converges to the given desired trajectory by increasing the iterations number. For more detailed comparison results, one can refer to Figure 3, from which it is easy to see that the provided PID-type ILC algorithm is better than the PD-type ILC algorithm for nonlinear discrete system in terms of tracking errors. Meanwhile, one can clearly see that the convergence rate of the presented ILC in this paper is faster than PD-type ILC.

These simulation results illustrate that the output trajectory using PID-type ILC algorithm tends to the desired trajectory through less iterations than that using traditional PD-type ILC algorithm. However, due to the existence of disturbances, noises, and errors, tracking error is ultimately bounded and is only in a small range.

5. Conclusions

In this work, a PID-type ILC algorithm has been applied to the nonlinear discrete system of mobile robot. Conditions for guaranteeing the convergence and robustness of proposed ILC algorithm are presented. This algorithm is proposed to facilitate uniform tracking error convergence analysis under the effect of system disturbances and noises. A simulation



(a) (b)

Figure 3: Max absolute values of the tracking error.

result further verifies the theoretical results. The future work aims to apply the proposed algorithm to actual mobile robot control.

Competing Interests

The authors declare that there is not conflict of interests regarding the publication of this paper.

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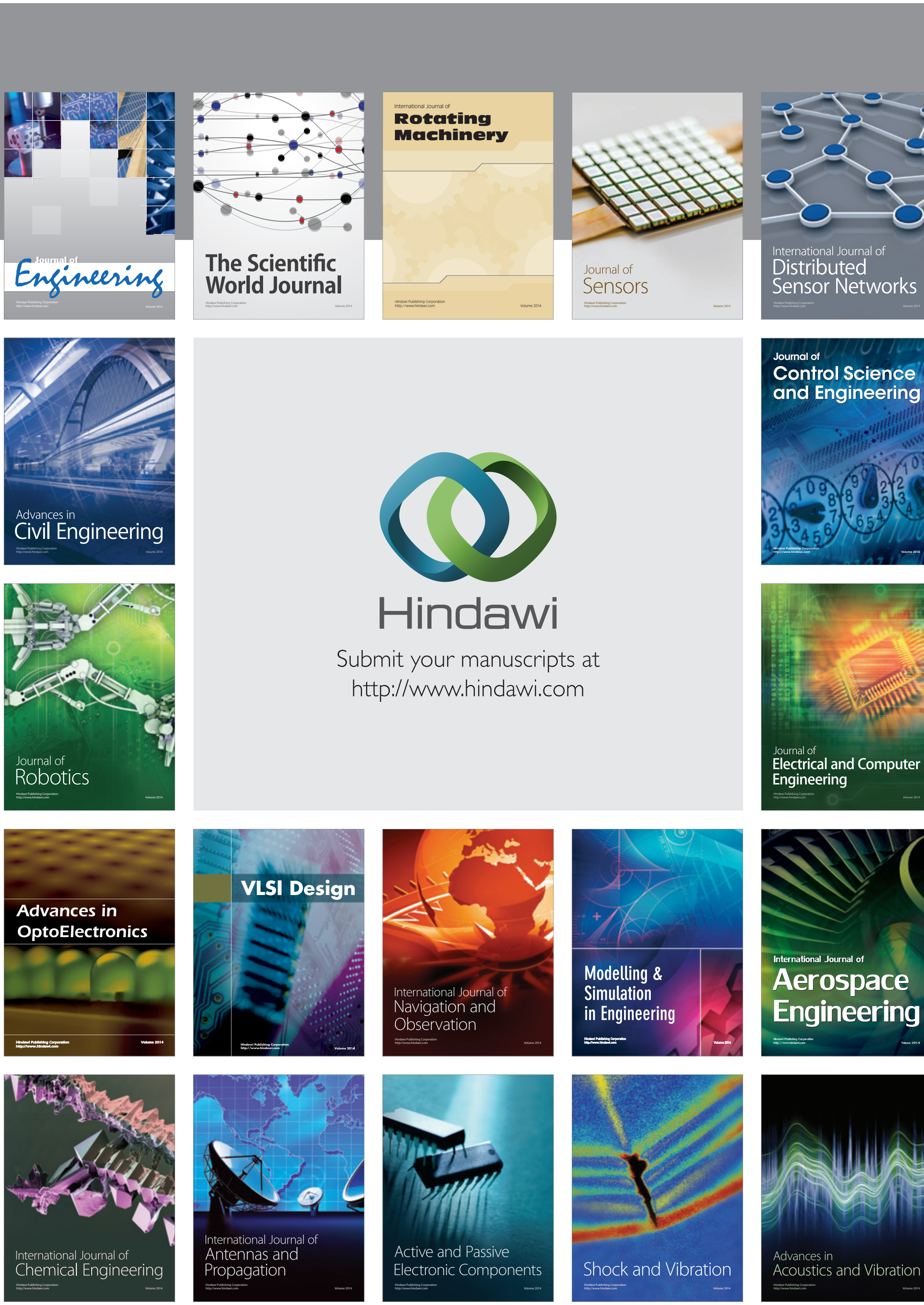
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