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Research Article

Fault Diagnosis and Fault Tolerant Control for Non-Gaussian Singular Time-Delayed Stochastic Distribution Systems with Disturbance Based on the Rational Square-Root Model

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For the non-Gaussian singular time-delayed stochastic distribution control (SDC) system with unknown external disturbance where the output probability density function (PDF) is approximated by the rational square-root B-spline basis function, a robust fault diagnosis and fault tolerant control algorithm is presented. A full-order observer is constructed to estimate the exogenous disturbance and an adaptive observer is used to estimate the fault size. A fault tolerant tracking controller is designed using the feedback of distribution tracking error, fault, and disturbance estimation to let the postfault output PDF still track desired distribution. Finally, a simulation example is included to illustrate the effectiveness of the proposed algorithms and encouraging results have been obtained.

1. Introduction

In order to improve the reliability of practical stochastic systems, fault diagnosis and fault tolerant control for stochastic dynamic systems has long been one of the important areas of control theory and application [1–4]. Stochastic distribution control (SDC) system is a new branch of stochastic system control in which the output is the non-Gaussian probability density function (PDF) of the system output. The equations of these systems describe the relationship between the input PDF and output PDF of systems rather than the traditional relationship between input and output. SDC theory was proposed by professor Wang [5], which has been applied on some actual processes, for instance, the paper evenness control in the process of paper making, high polymer polymerization process of chemical industry, and flame distribution control. In the framework of non-Gaussian SDC systems, it has very important theoretical significances and deep application prospect to develop fault diagnosis and fault tolerant control technology for complex industrial processes in which the control product quality and the distribution of indirect indicators need to be controlled.

With the study of fault diagnosis for non-Gaussian SDC systems, some fault diagnosis algorithms have been proposed. For fault diagnosis of SDC systems, observer or filter-based methods are mainly used so far [6–10], in which the information of system output PDF and other measured information generate residuals in order to analyze and estimate the change of fault. A stable filter-based residual generator is constructed such that the fault can be detected and diagnosed for general stochastic systems in [6]. In [7], a nonlinear neural network observer is designed for fault diagnosis in which the adaptive tuning rule for network parameters is determined by the Lyapunov stability theorem. In [8] a fault diagnosis algorithm is proposed based on iterative learning observer for SDC system. Otherwise, in [11], a novel fault-estimation observer is designed for Takagi-Sugeno (T-S) fuzzy systems with actuator faults, and the problem of fault tolerant control is addressed. For fault tolerant control, the active fault tolerant control is mainly used so far. Combined with the controller design method without fault, such as optimal control, PI control, sliding mode control, and model reference adaptive control, the controller can be reconfigured or reconstructed when fault occurred. When the target PDF is known, the purpose of fault tolerant control is to make the output PDF of the systems still track a given PDF as close as possible after the fault happened. It has been shown in [8] that an optimal control strategy is designed to reconfigure the controller to compensate the influence of the fault on the system performance. A fault tolerant controller based on PI tracking control is proposed for a time-delayed SDC system in [12] and for singular time-delayed SDC system in [13]. In [14], a fuzzy fault tolerant control scheme is developed to guarantee the closed-loop system to be exponentially stable in mean square. When the target PDF is unknown, introduce the concept of entropy to fault tolerant control of non-Gaussian SDC systems, in which the purpose is to minimize the uncertainty of the system output after fault happened [15, 16].

Although the model descriptions of SDC systems are always the dynamic link relations between the input and the weights of output PDFs, the conventional dynamic links do not meet the practical requirements because of the existence of some algebraic constraint conditions between some state variables. Thus, it is necessary to study the SDC systems in the framework of singular systems [8, 13, 16–18]. Due to the material delivering by conveyor belt, system modeling, and data operation and transmission, time-delay widely exists in actual industrial systems. Time-delay can make system performance degradation and even make systems unstable; at the same time, time-delay will largely reduce the effectiveness of the fault diagnosis and fault tolerant control. The results of fault diagnosis and fault tolerant control for singular time-delayed SDC systems are focused on the model approximated by the linear or square-root B-spline model [13, 18]. The rational square-root B-spline model can guarantee that the weights of feedback control are positive at the same time independent of each other compared with other B-spline models [19]. External disturbance widely exists in various industrial processes and this situation will turn to be very complicated for fault diagnosis [9, 20], but controller design to eliminate the influence of disturbance is not considered. Thus, it is significant to study fault diagnosis and fault tolerant control for non-Gaussian singular time-delayed SDC system with external disturbance based on the rational square-root model approximation.

In this paper, a robust fault diagnosis and fault tolerant control approach is proposed for non-Gaussian singular time-delayed stochastic distribution system with external disturbance based on the rational square-root approximation. The external disturbance is considered and supposed to be generated by a linear exogenous system, and a full-order observer is designed to estimate the disturbance. Then an adaptive observer is constructed to estimate fault information. The gain matrices can be determined by solving the corresponding linear matrix inequalities (LMIs). In order to eliminate the influence of fault and disturbance, a fault tolerant tracking controller is designed to track the desired PDF. An augmentation control input is defined to contain the feedback of output PDF tracking error and the estimation of fault and disturbance, so that the control input of the SDC system can eliminate the influence of fault and disturbance on the system performance, leading to fault tolerant tracking control. Finally, the computer simulation results show the effectiveness of the algorithm.

The main contributions of this paper can be summarized as follows. (1) Comparing with most of the existing studies of SDC system with disturbance, integrated fault diagnosis and fault tolerant control is achieved; rather only fault diagnosis is considered. Besides, the influence of disturbance to the system performance can be rejected by the fault tolerant controller. (2) The rational square-root model is used to approximate the output PDF of the singular time-delayed SDC system, as a contrast, linear, or square-root model is used in the existing studies of singular time-delayed SDC system.

2. Model Description

Denoting $\gamma(y, u(t))$ as the PDF of the system output with y being defined on a known bounded interval $[a, b]$, the continuous singular time-delayed SDC system can be expressed as follows:

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + A_d x(t-\tau) + Bu(t) + NF(t) \\ &\quad + B_d d(t), \\ V(t) &= Dx(t), \\ x(t) &= \varphi(t), \quad t \in [-\delta, 0], \end{aligned} \tag{1}$$

$$\begin{aligned} \sqrt{\gamma(y, u(t))} &= \frac{C(y)V(t)}{\sqrt{V^T(t)\Sigma_1 V(t)}}, \\ \Sigma_1 &= \int_a^b C^T(y)C(y)\,dy, \end{aligned} \tag{2}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input vector, $V(t) \in R^n$ is the weight vector and $F \in R^m$ is the fault vector, and τ is the time-delay term. $\varphi(t)$ is a real valued continuous function. $A \in R^{n\times n}$, $A_d \in R^{n\times n}$, $B \in R^{n\times m}$, $D \in R^{n\times n}$, $B_d \in R^{n\times m}$, $E \in R^{n\times n}$, and $N \in R^{n\times m}$ are system parameter matrices with $\operatorname{rank}(E) = q < n$. Equation (2) represents the static model of the output PDF approximated by the square-root B-spline model. It is denoted that

$$\begin{aligned} C(y) &= [\phi_1(y), \phi_2(y), \ldots, \phi_n(y)], \\ V &= [\omega_1, \omega_2, \ldots, \omega_n]^T \quad (V \neq 0), \end{aligned} \tag{3}$$

where $\phi_i(y)$ $(i = 1, 2, \ldots, n,\ n \geq 2)$ is the prespecified basis function, ω_i $(i = 1, 2, \ldots, n)$ is the approximation weight which is only related to $u(t)$, and n is the number of basis functions. $d(t)$ is the unknown external disturbance which can be supposed to be generated by a linear exogenous system described by

$$\begin{aligned} \dot{\omega}(t) &= W\omega(t), \\ d(t) &= T\omega(t). \end{aligned} \tag{4}$$

Remark 1. From literatures [21], many kinds of disturbances in engineering can be described by this model, for example, unknown constant and harmonics with unknown phase and magnitude. In most existing results, the disturbances are restricted to be bounded exogenous signals. The state variable of the exogenous system $\omega(t)$ is the target of estimation, $d(t)$ is the form of disturbance, and W and T are the known parameter matrices with suitable dimensions.

The following assumptions are used throughout this paper.

Assumption 2. (A, D) is observable, and $(W, B_d T)$ is observable.

Assumption 3. The fault and disturbance occurring in SDC system are bounded; that is, $\|F\| \leq M_f$ and $\|d(t)\| \leq M_d$, where M_f and M_d are two positive constants.

Assumption 4. The system (1) is regular and impulse-free; that is, $|sE - A| \neq 0$ and $\operatorname{rank}(E) = \deg(|sE - A|)$.

With the assumptions, there exist two matrices (L_1, L_2) such that the following equation

$$\begin{aligned} L_1 E L_2 &= \begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix}, \\ L_1 A L_2 &= \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-q} \end{bmatrix} \end{aligned} \tag{5}$$

holds. It is assumed that $L_1 A_d L_2 = \begin{bmatrix} A_{d1} & 0 \\ 0 & A_{d2} \end{bmatrix}$ holds simultaneously. Then the SDC system (1) can be transformed as

$$\begin{aligned} \dot{x}_1(t) &= A_1 x_1(t) + A_{d1} x_1(t-\tau) + B_1 u(t) + N_1 F(t) \\ &\quad + B_{d1} T \omega(t), \\ x_2(t) &= -A_{d2} x_2(t-\tau) - B_2 u(t) - N_2 F(t) \\ &\quad - B_{d2} T \omega(t), \\ V(t) &= D_1 x_1(t) + D_2 x_2(t), \\ \dot{\omega}(t) &= W \omega(t), \end{aligned} \tag{6}$$

where $B_1, B_{d1}, N_1 \in R^{q \times m}$, $B_2, B_{d2}, N_2 \in R^{(n-q) \times m}$, $D_1 \in R^{(n-1) \times q}$, and $D_2 \in R^{(n-1) \times (n-q)}$, which can be determined as follows:

$$\begin{aligned} L_2 B &= \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \\ L_2 B_d &= \begin{bmatrix} B_{d1} \\ B_{d2} \end{bmatrix}, \\ D L_1 &= \begin{bmatrix} D_1 & D_2 \end{bmatrix}, \\ L_2 N &= \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}. \end{aligned} \tag{7}$$

3. Fault Diagnosis

In order to reject the disturbance and eliminate the influence to the fault diagnosis, a full-order observer is constructed to estimate the external disturbance. Denoting $s(t) = \begin{bmatrix} x_1(t) \\ \omega(t) \end{bmatrix}$, the full-order system is constructed by augmenting state (1) with exogenous system (4) as follows:

$$\begin{aligned} \dot{s}(t) &= A_0 s(t) + A_{d0} s(t-\tau) + B_0 u(t) + N_0 F(t), \\ x_2(t) &= -A_{d2} x_2(t-\tau) - B_2 u(t) - N_2 F(t) \\ &\quad - B_{d2} T \omega(t), \\ V(t) &= D_1 x_1(t) + D_2 x_2(t) = D_0 s(t) + D_2 x_2(t), \end{aligned} \tag{8}$$

where

$$\begin{aligned} A_0 &= \begin{bmatrix} A_1 & B_{d1} T \\ 0 & W \end{bmatrix}, \\ A_{d0} &= \begin{bmatrix} A_{d1} & 0 \\ 0 & 0 \end{bmatrix}, \\ B_0 &= \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \\ N_0 &= \begin{bmatrix} N_1 \\ 0 \end{bmatrix}, \\ D_0 &= \begin{bmatrix} D_1 & 0 \end{bmatrix}. \end{aligned} \tag{9}$$

The full-order observer is designed as follows:

$$\begin{aligned} \dot{\hat{s}}(t) &= A_0 \hat{s}(t) + A_{d0} \hat{s}(t-\tau) + B_0 u(t) \\ &\quad + K_1 \varepsilon(t), \\ \hat{x}_2(t) &= -A_{d2} \hat{x}_2(t-\tau) - B_2 u(t) - N_2 F(t) \\ &\quad - B_{d2} T \hat{\omega}(t), \\ \widehat{V}(t) &= D_0 \hat{s}(t) + D_2 \hat{x}_2(t), \\ \sqrt{\hat{\gamma}(y, u(t))} &= \frac{C(y) \widehat{V}(t)}{\sqrt{\widehat{V}^T(t) \Sigma_1 \widehat{V}(t)}}, \\ \varepsilon(t) &= \int_a^b \left(\sqrt{\gamma} - \sqrt{\hat{\gamma}} \right) dy, \end{aligned} \tag{10}$$

where K_1 is observer gain matrix, $\varepsilon(t)$ is the residual signal, and

$$\begin{aligned} \varepsilon(t) &= \frac{\Sigma_2 (D_0 s(t) + D_2 x_2(t))}{\sqrt{V^T(t) \Sigma_1 V(t)}} \\ &\quad - \frac{\Sigma_2 (D_0 \hat{s}(t) + D_2 \hat{x}_2(t))}{\sqrt{\widehat{V}^T(t) \Sigma_1 \widehat{V}(t)}} \\ &\quad + \frac{\Sigma_2 (D_0 s(t) + D_2 x_2(t))}{\sqrt{\widehat{V}^T(t) \Sigma_1 \widehat{V}(t)}} \end{aligned}$$

$$-\frac{\Sigma_2\left(D_0 s(t)+D_2 x_2(t)\right)}{\sqrt{\widehat{V}^T(t)\,\Sigma_1\widehat{V}(t)}}$$

$$=\frac{\Sigma_2\left(D_0 e_s(t)+D_2 e_2(t)\right)}{\sqrt{\widehat{V}^T(t)\,\Sigma_1\widehat{V}(t)}}+\frac{\Sigma_2\left(D_0 s(t)+D_2 x_2(t)\right)}{\sqrt{\widehat{V}^T(t)\,\Sigma_1\widehat{V}(t)}}\cdot\left(\frac{\sqrt{\widehat{V}^T(t)\,\Sigma_1\widehat{V}(t)}-\sqrt{V^T(t)\,\Sigma_1 V(t)}}{\sqrt{V^T(t)\,\Sigma_1 V(t)}}\right), \tag{11}$$

where $e_s(t)=s(t)-\widehat{s}(t)$, $\Sigma_2=\int_a^b C(y)dy$, and $e_2(t)=x_2(t)-\widehat{x}_2(t)$.

Lemma 5 (see [19]). *There exists a constant λ such that the following equation*

$$\sqrt{\widehat{V}^T(t)\,\Sigma_1\widehat{V}(t)}-\sqrt{V^T(t)\,\Sigma_1 V(t)}=\lambda\left(\left\|\widehat{V}(t)\right\|-\left\|V^T(t)\right\|\right) \tag{12}$$

holds, where $\lambda_{\min}(\Sigma_1)/\lambda_{\max}(\Sigma_1)\le\lambda\le\lambda_{\max}(\Sigma_1)/\lambda_{\min}(\Sigma_1)$ and $\lambda_{\max}(\Sigma_1)$ and $\lambda_{\min}(\Sigma_1)$ are the maximum and minimum eigenvalues of matrix Σ_1, respectively.

Then, it can be obtained that

$$\varepsilon(t)=\frac{\Sigma_2\left(D_0 e_s(t)+D_2 e_2(t)\right)}{\sqrt{\widehat{V}^T(t)\,\Sigma_1\widehat{V}(t)}}+\frac{\Sigma_2\left(D_0 s(t)+D_2 x_2(t)\right)}{\sqrt{\widehat{V}^T(t)\,\Sigma_1\widehat{V}(t)}}\frac{\lambda_1\left(\left\|\widehat{V}(t)\right\|-\left\|V^T(t)\right\|\right)}{\sqrt{V^T(t)\,\Sigma_1 V(t)}}. \tag{13}$$

The full-order observation error dynamic system can be formulated as follows:

$$\begin{aligned}\dot{e}_s(t)&=\dot{s}(t)-\dot{\widehat{s}}(t)\\&=A_0 e_s(t)+A_{d0}e_s(t-\tau)+N_0F\\&\quad-L_3\Sigma_2D_0e_s(t)-L_3\Sigma_2D_2e_2(t)\\&\quad+L_3\Sigma_2V\left(\frac{\lambda_1\left(\|V\|-\left\|\widehat{V}\right\|\right)}{\sqrt{V^T\Sigma_1V}}\right),\end{aligned} \tag{14}$$

where $L_3=K_1/\sqrt{\widehat{V}^T(t)\Sigma_1\widehat{V}(t)}$ is the gain matrix to be determined later.

The purpose of fault diagnosis is to estimate the size of fault. The fault diagnosis observer is constructed as follows:

$$\begin{aligned}\dot{\widehat{x}}_1(t)&=A_1\widehat{x}_1(t)+A_{d1}\widehat{x}_1(t-\tau)+B_1u(t)\\&\quad+N_1\widehat{F}(t)+B_{d1}T\widehat{\omega}(t)+K_2\varepsilon(t),\\\widehat{x}_2(t)&=-A_{d2}\widehat{x}_2(t-\tau)-B_2u(t)-N_2\widehat{F}(t)\\&\quad-B_{d2}T\widehat{\omega}(t),\\\widehat{V}(t)&=D_1\widehat{x}_1(t)+D_2\widehat{x}_2(t),\\\sqrt{\widehat{\gamma}(y,u(t))}&=\frac{C(y)\widehat{V}(t)}{\sqrt{\widehat{V}^T(t)\,\Sigma_1\widehat{V}(t)}},\\\dot{\widehat{F}}&=-\Gamma_1\widehat{F}+\Gamma_2\varepsilon(t),\end{aligned} \tag{15}$$

where $e_1(t)=x_1(t)-\widehat{x}_1(t)$. Then the observation error dynamic system is obtained as follows:

$$\begin{aligned}\dot{e}_1(t)&=A_1e_1(t)+A_{d1}e_1(t-\tau)+N_1\widetilde{F}(t)\\&\quad+B_{d1}T\widetilde{\omega}(t)-L_4\Sigma_2D_1e_1-L_4\Sigma_2D_2e_2\\&\quad+L_4\Sigma_2V\left(\frac{\lambda_2\left(\|V\|-\left\|\widehat{V}\right\|\right)}{\sqrt{V^T\Sigma_1V}}\right),\end{aligned} \tag{16}$$

$$\begin{aligned}\dot{\widehat{F}}&=-\Gamma_1\widehat{F}+L_5\Sigma_2D_1e_1+L_5\Sigma_2D_2e_2\\&\quad-L_5\Sigma_2V\left(\frac{\lambda_2\left(\|V\|-\left\|\widehat{V}\right\|\right)}{\sqrt{V^T\Sigma_1V}}\right),\end{aligned} \tag{17}$$

where $\widetilde{F}=F-\widehat{F}$, $\widetilde{\omega}=\omega-\widehat{\omega}$. $L_4=K_2/\sqrt{\widehat{V}^T(t)\Sigma_1\widehat{V}(t)}$, $L_5=\Gamma_2/\sqrt{\widehat{V}^T(t)\Sigma_1\widehat{V}(t)}$, and Γ_1 are gain matrices to be determined later.

Combing (14), (16), and (17), the following augmentation error dynamic system can be obtained as

$$\begin{aligned}\begin{pmatrix}\dot{e}_s(t)\\\dot{e}_1(t)\\\dot{\widehat{F}}(t)\end{pmatrix}&=\overline{A}\begin{pmatrix}e_s(t)\\e_1(t)\\\widehat{F}(t)\end{pmatrix}+\begin{pmatrix}A_{d0}e_s(t-\tau)\\A_{d1}e_1(t-\tau)\\0\end{pmatrix}\\&\quad+\begin{pmatrix}N_0F(t)\\N_1F(t)\\0\end{pmatrix}+\begin{pmatrix}-L_3\Sigma_2D_2e_2\\-L_4\Sigma_2D_2e_2\\L_5\Sigma_2D_2e_2\end{pmatrix}\\&\quad+\begin{pmatrix}L_3\Sigma_2h_1\\L_4\Sigma_2h_2\\-L_5\Sigma_2h_2\end{pmatrix},\end{aligned} \tag{18}$$

where

$$\overline{A} = \begin{bmatrix} A_0 - L_3\Sigma_2 D_0 & 0 & 0 \\ \overline{T} & A_1 - L_4\Sigma_2 D_1 & -N_1 \\ 0 & L_5 & -\Gamma_1 \end{bmatrix}, \quad \overline{T} = \begin{bmatrix} 0 & B_{d1}T \end{bmatrix}, \tag{19}$$

$$h_i = V\left(\frac{\lambda_i\left(\|V\| - \|\widehat{V}\|\right)}{\sqrt{V^T\Sigma_1 V}}\right), \quad i = 1, 2.$$

Select the reference output as follows:

$$s_\infty = C_1 e_s(t) + C_2 e_1(t) + C_3 e_1(t-\tau) + C_4\widetilde{F} + C_5 e_2(t). \tag{20}$$

For the constants $\mu_1 > 0$ and $\mu_2 > 0$, the generalized H_∞ performance is denoted as follows:

$$\begin{aligned} J_\infty &= \|s_\infty\|^2 - \mu_1^2\|F(t)\|^2 - \mu_2^2\|e_2(t)\|^2 - \delta(Q_1, Q_2, Q_3, Q_4), \\ \delta(Q_1, Q_2, Q_3, Q_4) &= e_s^T(0) Q_1 e_s(0) + e_1^T(0) Q_2 e_1(0) + \int_{-\tau}^{0}\varphi_1^T Q_3\varphi_1 d\alpha + \int_{-\tau}^{0}\varphi_2^T Q_4\varphi_2 d\alpha. \end{aligned} \tag{21}$$

Theorem 6. *For the matrices $C_i > 0$ $(i = 1, 2, 3, 4, 5)$ and $Q_i > 0$ $(i = 1, 2, 3, 4)$ and constants $\mu_i > 0$ $(i = 1, 2)$, there exist positive definite symmetric matrices P_1 and P_2, positive definite matrices R_1 and R_2, and gain matrices L_3, L_4, L_5, and Γ_1 satisfying $P_1 \le Q_1$, $P_2 \le Q_2$, $R_1 \le Q_3$, $R_2 \le Q_4$, and the following LMI, and then the augmenting error dynamic system (18) is stable and satisfies $J_\infty < 0$*

$$\Phi_1 = \begin{bmatrix} \xi_{11} & P_1 A_{d0} & \overline{T}^T P_2^T & 0 & P_1 L_3\Sigma_2 & 0 & P_1 N_0 & 0 & -P_1 L_3\Sigma_2 D_2 \\ * & -R_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \xi_{33} & P_2 A_{d1} & 0 & P_2 L_4\Sigma_2 & P_2 N_1 & \xi_{38} & -P_2 L_4\Sigma_2 D_2 \\ * & * & * & -R_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & \Sigma_2^T L_5 & 0 \\ * & * & * & * & * & * & -\mu_1^2 & 0 & 0 \\ * & * & * & * & * & * & * & -2\Gamma_1 & L_5\Sigma_2 D_2 \\ * & * & * & * & * & * & * & * & -\mu_2^2 \end{bmatrix} + \begin{bmatrix} C_1 & 0 & C_2 & C_3 & 0 & 0 & C_4 & -C_4 & C_5 \end{bmatrix}^T \begin{bmatrix} C_1 & 0 & C_2 & C_3 & 0 & 0 & C_4 & -C_4 & C_5 \end{bmatrix} < 0, \tag{22}$$

where

$$\begin{aligned} \xi_{11} &= (A_0 - L_3\Sigma_2 D_0)^T P_1 + P_1(A_0 - L_3\Sigma_2 D_0) + R_1 + m_1 I, \\ \xi_{33} &= (A_1 - L_4\Sigma_2 D_1)^T P_2 + P_2(A_1 - L_4\Sigma_2 D_1) + R_2 + m_2 I, \\ \xi_{38} &= D_1^T\Sigma_2^T L_5^T - P_2 N_1. \end{aligned} \tag{23}$$

Proof. Select the following Lyapunov functions π_1, π_2, and π_3 as follows:

$$\begin{aligned} \pi_1 &= e_s^T P_1 e_s + \int_0^t \left[m_1 e_s^T(\alpha) e_s(\alpha) - h_1^T h_1\right] d\alpha + \int_{t-\tau}^{t} e_s^T(\alpha) R_1 e_s(\alpha) d\alpha, \\ \pi_2 &= e_1^T P_2 e_1 + \int_0^t \left[m_2 e_1^T(\alpha) e_1(\alpha) - h_2^T h_2\right] d\alpha + \int_{t-\tau}^{t} e_1^T(\alpha) R_2 e_1(\alpha) d\alpha, \\ \pi_3 &= \widehat{F}^T\widehat{F}. \end{aligned} \tag{24}$$

It can be obtained that the first-order derivatives of π_1, π_2, and π_3 are given as follows:

$$\begin{aligned} \dot{\pi}_1 = {} & e_s^T\left[(A_0 - L_3\Sigma_2 D_0)^T P_1 + P_1(A_0 - L_3\Sigma_2 D_0) + R_1 + m_1 I\right] e_s + 2e_s^T P_1 A_{d0} e_s(t-\tau) + 2e_s^T P_1 N_0 F \\ & - 2e_s^T P_1 L_3\Sigma_2 D_2 e_2 + 2e_s^T P_1 L_3\Sigma_2 h_1 - h_1^T h_1 - e_s^T(t-\tau) R_1 e_s(t-\tau), \end{aligned}$$

$$\begin{aligned}\dot{\pi}_2 &= e_1^T\left[\left(A_1 - L_4\Sigma_2 D_1\right)^T P_2 + P_2\left(A_1 - L_4\Sigma_2 D_1\right)\right.\\&\left.+ R_2 + m_2 I\right] e_1 + 2e_1^T P_2 \overline{T} e_s + 2e_1^T P_2 N_1 F\\&+ 2e_1^T P_2 A_{d1} e_1(t-\tau) - 2e_1^T P_2 L_4 \Sigma_2 D_2 e_2\\&- 2e_1^T P_2 N_1 \widehat{F} + 2e_1^T P_2 L_4 \Sigma_2 h_2 - h_2^T h_2 - e_1^T(t-\tau)\\&\cdot R_2 e_1(t-\tau),\\ \dot{\pi}_3 &= -2\Gamma_1 \widehat{F}^T \widehat{F} + 2e_1^T D_1^T \Sigma_2^T L_5^T \widehat{F} + 2\widehat{F}^T L_5 \Sigma_2 D_2 e_2\\&+ 2\widehat{F}^T L_5 \Sigma_2 h_2.\end{aligned} \tag{25}$$

Denote $\pi = \pi_1 + \pi_2 + \pi_3$, and consider an auxiliary function as the performance index

$$J_{\text{aux}} = \int_0^t \left(\|s_\infty\|^2 - \mu_1^2 \|F(\alpha)\|^2 - \mu_2^2 \|e_2(\alpha)\|^2 + \dot{\pi}(\alpha)\right) d\alpha. \tag{26}$$

Then it can be obtained that $\|s_\infty\|^2 - \mu_1^2\|F(t)\|^2 - \mu_2^2\|e_2(t)\|^2 + \dot{\pi} = q^T\Phi_1 q$, where

$$q^T = \left[e_s^T(t)\;\; e_s^T(t-\tau)\;\; e_1^T(t)\;\; e_1^T(t-\tau)\;\; h_1^T(t)\;\; h_2^T(t)\;\; F^T(t)\;\; \widehat{F}^T(t)\;\; e_2^T(t)\right]^T. \tag{27}$$

From the Schur complement lemma, Φ_1 equals inequality (22), leading to $J_{\text{aux}} < 0$. Then

$$\begin{aligned}J_{\text{aux}} &= \int_0^t \left(\|s_\infty\|^2 - \mu_1^2\|F(t)\|^2 - \mu_2^2\|e_2(t)\|^2\right) d\tau\\&+ \pi(t) - \pi(0) \le \|s_\infty\|^2 - \mu_1^2\|F(t)\|^2 - \mu_2^2\|e_2(t)\|^2\\&- \left[e_s^T(0) P_1 e_s(0) + e_1^T(0) P_2 e_1(0)\right.\\&\left.+ \int_{-\tau}^0 \varphi_1^T R_1 \varphi_1 d\alpha + \int_{-\tau}^0 \varphi_2^T R_2 \varphi_2 d\alpha\right],\end{aligned} \tag{28}$$

where when $t \in [-\tau, 0]$, $\widehat{s} = 0$, $\widehat{x}_1 = 0$, $s = \varphi_1(t)$, and $x_1 = \varphi_2(t)$. It can be obtained that

$$\begin{aligned}&J_\infty < J_{\text{aux}} + \delta\left(P_1 - Q_1, P_2 - Q_2, R_1 - Q_3, R_2 - Q_4\right),\\&\delta\left(P_1 - Q_1, P_2 - Q_2, R_1 - Q_3, R_2 - Q_4\right)\\&= \left[e_s^T(0) P_1 e_s(0) + e_1^T(0) P_2 e_1(0)\right.\\&\left.+ \int_{-\tau}^0 \phi_1^T R_1 \phi_1 d\alpha + \int_{-\tau}^0 \phi_2^T R_2 \phi_2 d\alpha\right]\\&- \left[e_s^T(0) Q_1 e_s(0) + e_1^T(0) Q_2 e_1(0)\right.\\&\left.+ \int_{-\tau}^0 \phi_1^T Q_3 \phi_1 d\alpha + \int_{-\tau}^0 \phi_2^T Q_4 \phi_2 d\alpha\right].\end{aligned} \tag{29}$$

Since $\delta(P_1 - Q_1, P_2 - Q_2, R_1 - Q_3, R_2 - Q_4) < 0$, $J_\infty < 0$ is proved. This completes the proof. □

4. Fault Tolerant Control

After estimating the fault and disturbance, it is necessary to design a fault tolerant controller to make the postfault output PDF still track the desired PDF. A given desired PDF can be described as follows:

$$\sqrt{\gamma_g(y)} = \frac{C(y) V_g}{\sqrt{V_g^T \Sigma_1 V_g}}, \quad \forall y \in [a, b], \tag{30}$$

where V_g is the expected weight vector of desired PDF $\gamma_g(y)$. Let

$$\begin{aligned}e_2(t) &= V(t) - V_g,\\ ED^{-1}\dot{e}_2(t) &= ED^{-1}\left(\dot{V}(t) - \dot{V}_g\right) = E\dot{e}_m(t)\\&= E\left(\dot{x}(t) - \dot{x}_g\right).\end{aligned} \tag{31}$$

Then the tracking error dynamic system can be expressed as follows:

$$\begin{aligned}E\dot{e}_m(t) &= Ae_m(t) + A_d e_m(t-\tau) + Bu(t) + NF(t)\\&+ B_d d(t) + \left(A + A_d\right) x_g.\end{aligned} \tag{32}$$

Denote

$$\begin{aligned}\overline{E} &= L_1 E L_2 = \begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix},\\ L_2^{-1} e_m(t) &= \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} = \xi(t).\end{aligned} \tag{33}$$

Then an augmentation control input containing the system input and the information of desired PDF is constructed as $U(t) = u(t) + (B^T B)^{-1} B^T (A + A_d) x_g$, where the former item is the reconstructed control input and the latter is the extended information of desired PDF. The tracking error dynamic system can be rewritten as follows:

$$\begin{aligned}\overline{E}\dot{\xi}(t) &= L_1 A L_2 \xi(t) + L_1 A_d L_2 \xi(t-\tau) + L_1 BU(t)\\&+ L_1 NF(t) + L_1 B_d d(t).\end{aligned} \tag{34}$$

Assume that $U(t)$ is the feedback of the tracking error, fault, and disturbance, then it can be obtained that

$$\begin{aligned} U(t) &= \Gamma_3 \int_a^b \left(\sqrt{\gamma(y, u(t))} - \sqrt{g(y)} \right) dy + \Gamma_4 F(t) \\ &\quad + \Gamma_5 d(t) \\ &= \frac{\Gamma_3 \Sigma_2 D L_2 \xi(t)}{\sqrt{V_g^T \Sigma_1 V_g}} + \frac{\Gamma_3 \Sigma_2 V(t)}{\sqrt{V_g^T \Sigma_1 V_g}} \\ &\quad \cdot \frac{\lambda_3 \left(\|V_g\| - \|V(t)\| \right)}{\sqrt{V^T(t) \Sigma_1 V(t)}} + \Gamma_4 F(t) + \Gamma_5 d(t) \\ &= L_6 \Sigma_2 D L_2 \xi(t) + L_6 \Sigma_2 h_3 + \Gamma_4 F(t) + \Gamma_5 d(t), \end{aligned} \tag{35}$$

where $L_6 = \Gamma_3/\sqrt{V_g^T \Sigma_1 V_g}$ and $h_3 = (\lambda_3(\|V_g\| - \|V(t)\|)/\sqrt{V^T(t)\Sigma_1 V(t)})V(t)$.

Theorem 7. *For the given constant η_i $(i = 1, \ldots, 4)$ and a small positive constant λ, suppose that there exist positive definite symmetric matrices P and matrices Γ_3, Γ_4, and Γ_5 such that the following LMI holds, and then the tracking error dynamic system (34) is stable.*

$$\Phi_4 = \begin{bmatrix} M_4 & A_d P^T & B L_6 \Sigma_2 L_2^{-1} P^T & \frac{1}{\eta_2} B\Gamma_4 & \frac{1}{\eta_4} B\Gamma_5 \\ * & -R_1 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{36}$$

$$M_4 = AP^T + PA^T + PL_2^{-T} R_1 L_2^{-1} P^T + 2BL_6 \Sigma_2 D P^T + PL_2^{-T} \alpha_3 L_2^{-1} P^T + \frac{1}{\eta_1^2} NN^T + \frac{1}{\eta_3^2} B_d B_d^T + \lambda I,$$

$$\Gamma_3 = L_6 \sqrt{V_g^T \Sigma_1 V_g}.$$

Proof. Select the following Lyapunov function as follows:

$$\begin{aligned} \pi_4 &= \xi^T \overline{E}^T \overline{P}^{-T} \xi + \int_{t-\tau}^t \xi^T(s) R_1 \xi(s)\, ds \\ &\quad + \int_0^t \left(\alpha_3 \xi^T \xi - h_3^T h_3 \right) ds, \end{aligned} \tag{37}$$

where $h_3 = (\lambda_3(\|V_g\| - \|V(t)\|)/\sqrt{V^T(t)\Sigma_1 V(t)})V(t)$. It can be obtained that $h_3^T h_3 \leq (\lambda_3 \|DL_2\|/\sqrt{\|\Sigma_1\|})^2 \xi^T(t)\xi(t)$, and denote $\alpha_3 = (\lambda_3\|DL_2\|/\sqrt{\|\Sigma_1\|})^2$; then $\pi_4 \geq 0$ holds. The first-order derivative of π_4 can be obtained as follows:

$$\begin{aligned} \dot{\pi}_4 &= \xi^T \left[\overline{P}^{-1} L_1 A L_2 + L_2^T A^T L_1^T \overline{P}^{-T} + R_1 \right. \\ &\quad \left. + 2\overline{P}^{-1} L_1 B L_6 \Sigma_2 D L_2 \right] \xi + \alpha_3 \xi^T \xi \\ &\quad + 2\xi^T \overline{P}^{-1} L_1 A_d L_2 \xi(t-\tau) - h_3^T h_3 + 2\xi^T \left(\overline{P}^{-1} L_1 N \right. \\ &\quad \left. + \overline{P}^{-1} L_1 B\Gamma_4 \right) F + 2\xi^T \overline{P}^{-1} L_1 B_d d(t) \\ &\quad + 2\xi^T \overline{P}^{-1} L_1 B\Gamma_5 d(t) - \xi^T(t-\tau) R_1 \xi(t-\tau) \\ &\quad + 2\xi^T \overline{P}^{-1} L_1 B L_6 \Sigma_2 h_3 \leq \xi^T \left[\overline{P}^{-1} L_1 A L_2 \right. \\ &\quad \left. + L_2^T A^T L_1^T \overline{P}^{-T} + R_1 + 2\overline{P}^{-1} L_1 B L_6 \Sigma_2 D L_2 + \alpha_3 \right] \xi \\ &\quad + \eta_4^2 d^T(t) d(t) + 2\xi^T \overline{P}^{-1} L_1 A_d L_2 \xi(t-\tau) \\ &\quad - \xi^T(t-\tau) R_1 \xi(t-\tau) + 2\xi^T \overline{P}^{-1} L_1 B L_6 \Sigma_2 h_3 \\ &\quad - h_3^T h_3 + \frac{1}{\eta_1^2} \xi^T \overline{P}^{-1} L_1 N \left(\overline{P}^{-1} L_1 N \right)^T \xi + \eta_1^2 F^T F \\ &\quad + \frac{1}{\eta_2^2} \xi^T \overline{P}^{-1} L_1 B\Gamma_4 \left(\overline{P}^{-1} L_1 B\Gamma_4 \right)^T \xi + \eta_2^2 F^T F + \frac{1}{\eta_3^2} \\ &\quad \cdot \xi^T \overline{P}^{-1} L_1 B_d \left(\overline{P}^{-1} L_1 B_d \right)^T \xi + \eta_3^2 d^T(t) d(t) + \frac{1}{\eta_4^2} \\ &\quad \cdot \xi^T \overline{P}^{-1} L_1 B\Gamma_5 \left(\overline{P}^{-1} L_1 B\Gamma_5 \right)^T \xi = q^T \Phi_1 q + \left(\eta_1^2 \right. \\ &\quad \left. + \eta_2^2 \right) F^T F + \left(\eta_3^2 + \eta_4^2 \right) d^T(t) d(t), \end{aligned} \tag{38}$$

where $q^T = [\xi^T(t) \;\; \xi^T(t-\tau) \;\; h_3^T]$ and

$$\Phi_1 = \begin{bmatrix} M_1 & \overline{P}^{-1} L_1 A_d L_2 & \overline{P}^{-1} L_1 B L_6 \Sigma_2 \\ * & -R_1 & 0 \\ * & * & -I \end{bmatrix},$$

$$\begin{aligned} M_1 &= \overline{P}^{-1} L_1 A L_2 + L_2^T A^T L_1^T \overline{P}^{-T} + R_1 + \alpha_3 \\ &\quad + 2\overline{P}^{-1} L_1 B L_6 \Sigma_2 D L_2 \\ &\quad + \frac{1}{\eta_1^2} \overline{P}^{-1} L_1 N \left(\overline{P}^{-1} L_1 N \right)^T \\ &\quad + \frac{1}{\eta_2^2} \overline{P}^{-1} L_1 B\Gamma_4 \left(\overline{P}^{-1} L_1 B\Gamma_4 \right)^T \\ &\quad + \frac{1}{\eta_3^2} \overline{P}^{-1} L_1 B_d \left(\overline{P}^{-1} L_1 B_d \right)^T \\ &\quad + \frac{1}{\eta_4^2} \overline{P}^{-1} L_1 B\Gamma_5 \left(\overline{P}^{-1} L_1 B\Gamma_5 \right)^T. \end{aligned} \tag{39}$$

Substituting $\overline{P} = L_1 P L_2^{-T}$ and $\overline{P}^{-1} = L_2^T P^{-1} L_1^{-1}$ into Φ_1 and premultiplying $\text{diag}(PL_2^{-T} \;\; I \;\; I)$ and postmultiplying $\text{diag}(L_2^{-1}P^T \;\; I \;\; I)$ to the left and the right side of Φ_1, it can be obtained that

$$\Phi_2 = \begin{bmatrix} M_2 & A_d P^T & BL_6\Sigma_2 L_2^{-1}P^T \\ * & -R_1 & 0 \\ * & * & -I \end{bmatrix},$$

$$\begin{aligned} M_2 &= AP^T + PA^T + PL_2^{-T}R_1L_2^{-1}P^T + 2BL_6\Sigma_2DP^T \\ &\quad + PL_2^{-T}\alpha_3L_2^{-1}P^T + \frac{1}{\eta_1^2}NN^T + \frac{1}{\eta_2^2}B\Gamma_4\Gamma_4^TB^T \\ &\quad + \frac{1}{\eta_3^2}B_dB_d^T + \frac{1}{\eta_4^2}B\Gamma_5\Gamma_5^TB^T. \end{aligned} \tag{40}$$

Adding λI to M_2, then

$$\begin{aligned} \Phi_3 &= \begin{bmatrix} M_3 & A_d P^T & BL_6\Sigma_2 L_2^{-1}P^T \\ * & -R_1 & 0 \\ * & * & -I \end{bmatrix}, \\ M_3 &= M_2 + \lambda I. \end{aligned} \tag{41}$$

When $\Phi_3 < 0$, it can be acquired that $\dot{\pi}_4 \le -\lambda\xi^T\xi + (\eta_1^2 + \eta_2^2)F^TF + (\eta_3^2 + \eta_4^2)d^T(t)d(t)$. According to the Schur complement lemma, $\Phi_3 < 0$ is equivalent to $\Phi_4 < 0$. Thus, when $\Phi_4 < 0$ holds,

$$\|\xi\|^2 > \frac{M_f^2\left(\eta_1^2 + \eta_2^2\right) + M_d^2\left(\eta_3^2 + \eta_4^2\right)}{\lambda} \tag{42}$$

is satisfied, and then $\dot{\pi}_4 < 0$, leading to

$$\|\xi\|^2 \le \frac{M_f^2\left(\eta_1^2 + \eta_2^2\right) + M_d^2\left(\eta_3^2 + \eta_4^2\right)}{\lambda}, \tag{43}$$

and the tracking error dynamic system (34) is stable.

By substituting $\widehat{F}(t)$ and $\widehat{d}(t)$ into (35), the practical fault tolerant tracking controller can be formulated as follows:

$$\begin{aligned} u(t) &= U(t) - \left(B^TB\right)^{-1}B^T\left(A + A_d\right)\widehat{x}_g \\ &= L_6\Sigma_2DL_2\xi(t) + L_6\Sigma_2h_3 + \Gamma_4\widehat{F}(t) + \Gamma_5\widehat{d}(t) \\ &\quad - \left(B^TB\right)^{-1}B^T\left(A + A_d\right)\widehat{x}_g. \end{aligned} \tag{44}$$

□

5. A Simulation Example

To illustrate the effectiveness of the proposed algorithms, a SDC system whose output PDF can be approximated by the following B-spline functions $\phi_i(y)$, $i = 1, 2, 3$, is considered as

$$\begin{aligned} \phi_1(y) &= 0.5(y-2)^2 I_1 + \left(-y^2 + 7y - 11.5\right) I_2 \\ &\quad + 0.5(y-5)^2 I_3, \\ \phi_2(y) &= 0.5(y-3)^2 I_2 + \left(-y^2 + 9y - 19.5\right) I_3 \\ &\quad + 0.5(y-6)^2 I_4, \\ \phi_3(y) &= 0.5(y-4)^2 I_3 + \left(-y^2 + 11y - 29.5\right) I_4 \\ &\quad + 0.5(y-7)^2 I_5, \end{aligned} \tag{45}$$

where I_i $(i = 1, 2, \ldots, 5)$ is an interval function defined as

$$I_i = \begin{cases} 1, & y \in [i+1, i+2] \\ 0, & \text{otherwise}, \end{cases} \quad i = 1, 2, \ldots, 5. \tag{46}$$

The system parameter matrices of the SDC system (1) are given as follows:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$A = \begin{bmatrix} -3.2 & 0.9 & -0.2 \\ 0.3 & -3.51 & -0.1 \\ 0 & 0 & -1 \end{bmatrix},$$

$$A_d = \begin{bmatrix} -0.1 & -0.5 & 0.12 \\ 0.2 & -0.25 & 0.06 \\ 0 & 0 & 0.6 \end{bmatrix},$$

$$H = \begin{bmatrix} 0.1 \\ 0.3 \\ 0.3 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 1 & -0.01 \\ 0 & 2 & -0.02 \\ 0.002 & 0.005 & 0.1 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$B_d = \begin{bmatrix} 0.1 & 0.1 & -0.01 \\ 0 & 0.2 & -0.02 \\ 0.002 & 0.005 & 0.1 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 1.0 & 0 & -0.2 \\ 0 & 1.0 & -0.1 \\ 0 & 0 & 1.0 \end{bmatrix},$$
$$L_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (47)$$

The exogenous disturbance can be described by (4), and the corresponding parameter matrices are given as follows:

$$T = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$W = \begin{bmatrix} 0 & 4 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}. \quad (48)$$

The time-delay τ is selected as $\tau = 2$ s and the sampling time is chosen as 0.1 s. It is assumed that the fault is constructed as follows:

$$F = \begin{cases} 0, & t < 5\,\text{s} \\ 0.5, & t > 5\,\text{s}. \end{cases} \quad (49)$$

The reference output is denoted such that

$$C_1 = [0.1 \quad 0.1 \quad 0.1 \quad 0.1 \quad 0.1]$$
$$C_2 = C_3 = [0.2 \quad 0.2], \quad (50)$$
$$C_4 = C_5 = 0.1.$$

It can be calculated from the LMIs of Theorems 6 and 7 that

$$P_1 = \begin{bmatrix} 1.8716 & 0.1950 & -0.5180 & 0.5660 & 0.002 \\ 0.1950 & 2.0251 & -0.0773 & 0.1075 & 0.003 \\ -0.5180 & -0.0773 & 6.8373 & -0.0841 & -0.0026 \\ 0.5660 & 0.1075 & -0.0841 & 6.9108 & 0.0017 \\ 0.0002 & 0.0003 & -0.0026 & 0.0017 & 0.0956 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 1.160 & 0.0599 \\ 0.0599 & 2.1566 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 7.0146 & -0.0797 & -1.4354 & 1.8075 & -0.0037 \\ -0.0797 & 7.5148 & 0.0394 & 0.1264 & -0.0035 \\ -1.4354 & 0.0394 & 1.3237 & -0.3724 & 0.0004 \\ 1.8075 & 0.1264 & -0.3724 & 1.4959 & -0.0011 \\ -0.0037 & -0.0035 & 0.0004 & -0.0011 & 1.1477 \end{bmatrix},$$

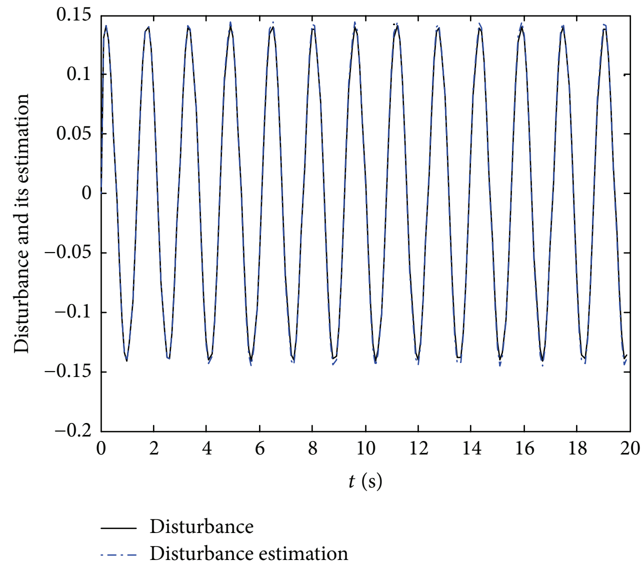

Figure 1: Disturbance and its estimation.

$$R_2 = \begin{bmatrix} 2.6380 & -0.5060 \\ -0.5060 & 7.5992 \end{bmatrix},$$
$$L_3 = [0.0090 \quad -0.0316 \quad 0.0109 \quad -0.0143 \quad 0]^T,$$
$$L_4 = [0.2435 \quad -0.0223]^T,$$
$$L_5 = 0.1038,$$
$$\Gamma_1 = 0.01,$$
$$\Gamma_2 = -0.7,$$
$$P = \begin{bmatrix} 0.1201 & 0.0134 & 0 \\ 0.0134 & 0.1206 & 0 \\ 0 & 0 & 0.3652 \end{bmatrix},$$
$$\Gamma_3 = [-0.0169 \quad -0.2712 \quad -3.1352],$$
$$\Gamma_4 = [0.5301 \quad -0.1530 \quad 0.1035],$$
$$\Gamma_5 = \begin{bmatrix} 0.2905 & -0.0639 & 0.0098 \\ -0.0639 & 0.0669 & 0.0206 \\ 0.0098 & 0.0206 & 0.8149 \end{bmatrix}. \quad (51)$$

The disturbance and its estimation have been shown in Figure 1. The fault diagnosis result has been presented in Figure 2. It can be seen that the desired fault diagnosis result has been obtained and the influence of disturbance has been rejected. The initial PDF, the desired PDF, and the final PDF with fault tolerant control are given in Figure 3. In Figures 4 and 5, the three-dimensional mesh plot shows the changes of the output PDF without and with fault tolerant control, respectively. Before fault occurs, the fault tolerant controller has not been reconstructed, so the disturbance cannot be

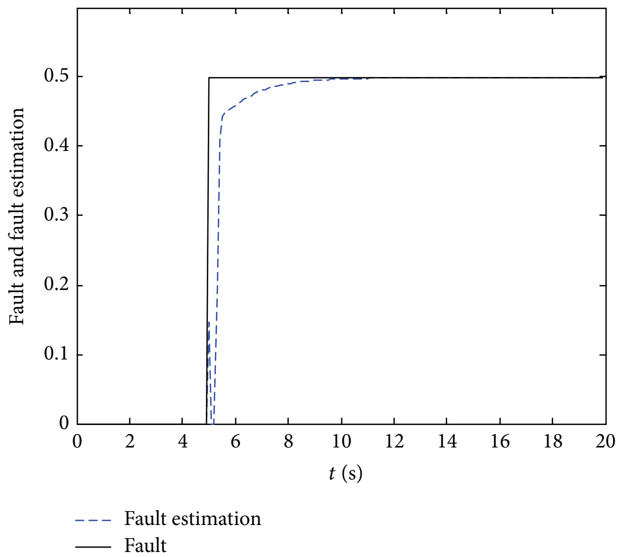

FIGURE 2: Fault and its estimation.

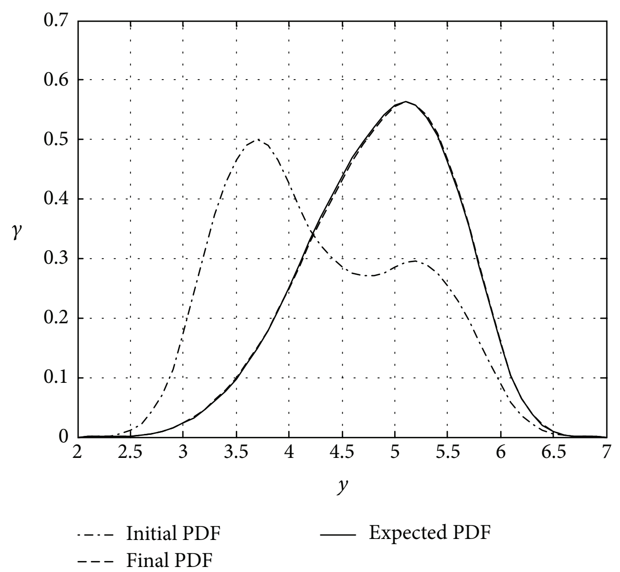

FIGURE 3: The output PDF with fault tolerant control when fault occurs.

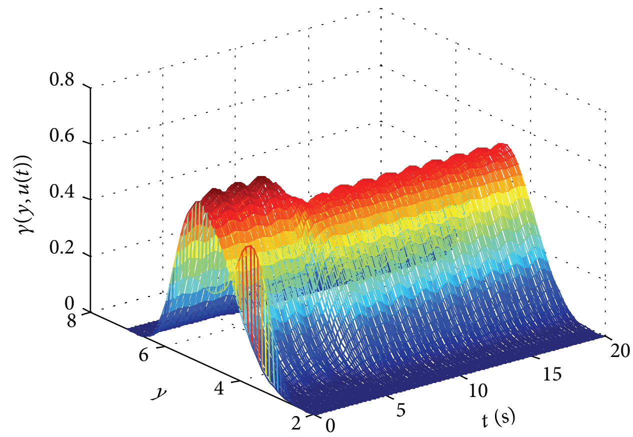

FIGURE 4: The output PDF of the whole process without fault tolerant control.

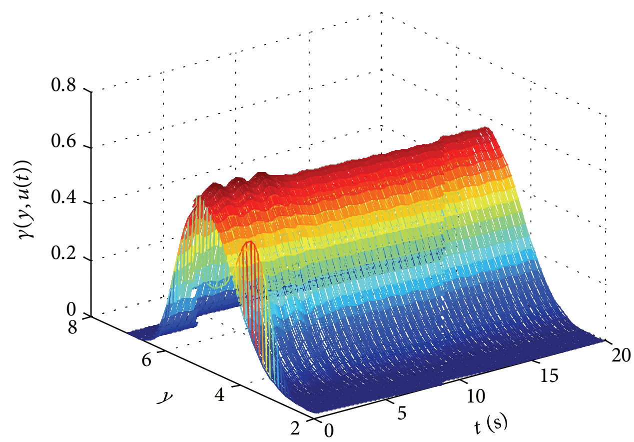

FIGURE 5: The output PDF of the whole process with fault tolerant control.

rejected. The influence of disturbance is shown in Figure 4. By comparing Figures 4 and 5, the effectiveness of the fault tolerant control can be seen.

6. Conclusions

In this paper, a fault diagnosis and fault tolerant control algorithm is given for the non-Gaussian singular time-delayed SDC system based on the rational square-root B-spline approximation model. The external disturbance is taken into consideration. A full-order observer is designed to estimate the disturbance, and then an adaptive observer is constructed to estimate the fault. Using the feedback of output PDF tracking error and the estimation of fault and disturbance, the augmentation control input is designed to make the postfault PDF still track the desired PDF. The Lyapunov stability theorem and LMI method are used to analyze the stability of the augmentation observation error dynamic system and tracking error dynamic system, and H_∞ performance of fault diagnosis is guaranteed. The simulations have further confirmed the efficiency of the proposed fault diagnosis and fault tolerant control algorithm.

Competing Interests

The authors declare that they have no competing interests.

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