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Research Article

Improved Absolute Stability for a Class of Nonlinear Switched Delay Systems via Mode-Dependent Average Dwell Time

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In this paper, the absolute stability for a class of switched delay systems with time-varying uncertainties is analyzed. By constructing an appropriate Lyapunov-Krasovskii functional, a new and less conservative criterion is proposed based on the MDADT method. Besides, the idea of N−segmentation is also utilized to make it more flexible to solve the LMIs. Finally, a numerical example is given to show the feasibility and advantages of the method proposed in this paper.

1. Introduction

Switched system is an important class of hybrid system which can be described by a series of continuous or discrete subsystems with a rule organizing the switching among them. As well known, many kinds of practical systems, such as power systems and chemical procedure control systems, may be properly modeled by the switched systems [1]. The main problem of switched systems is the stability analysis. In the last two decades, there have been increases in interests in the stability analysis for switched systems; see, for example, [2–5] and the references cited therein.

Apart from that, in many physical systems, such as chemical process and electric network, time-delay phenomenon is also unavoidable. Time-delays may lead to instability or even poor performance such as chaos [6]. Recently, the stability of systems with delays has aroused extensive research of scholars (see [7–10] and the references cited therein). For systems with time delay, the most important technique is to construct appropriate Lyapunov-Krasovskii functional.

Consequently, the stability of switched systems with time delay is worth considering. In fact, switched delay systems have a wide range of practical backgrounds in network control [11], chemical reactors [12], drilling system [13], and so on. To cope with the stability and design problem of switched delay systems, a good deal of methods have been proposed in terms of state-dependent switching and time-dependent switching. For instance, in [14], a delay-dependent stability criterion is derived under a state-driven law for switched delay systems. With regard to time-dependent switching, the dwell time (DT) method [15] and the average dwell time (ADT) method [16], as well as the mode-dependent average dwell time (MDADT) method [17], are put forward successively. In particular, for impulsive dynamical networks, in order to describe the frequency of occurrence of impulses, in 2010, by referring to the concept of ADT, Jianquan, Daniel W.C. Ho, and Jinde Cao [18] presented the concept of average impulsive interval. Recently, more and more work has been done on some special switched systems. For instance, for a class of impulsive stochastic differential systems, the stability properties are investigated under Markovian switching [19]. For switched cellular neural networks, by the MDADT approach, the exponential stability is considered in two cases where all subsystems are stable or unstable subsystems exist [20]. For a class of genetic regulatory networks (GRNs) with mixed delays, adaptive feedback control scheme is used to obtain the stability criteria [21].

In addition, since it was raised in 1940s, Lur'e systems have been deeply concerned. The main concern of Lur'e systems is absolute stability which covers systems without delays and those with delays [22–24]. To list a few, [25, 26] survey the absolute stability of Lur'e systems with constant delays and time-varying delays, respectively. From then on, more and more results have appeared to get less conservative stability criteria. Free-matrix-weighing method [27] and improved Lyapunov-Krasovskii functional method [28] are adopted in succession to reduce the conservativeness of previous results.

Integrated with the features of switched systems and Lur'e systems, switched Lur'e systems have also been paid much attention. A number of systems in control communities can be modeled by switched Lur'e systems, such as Hopfield neural network [29] and variable structure system [30]. However, as far as we know, up to now, absolute stability of switched Lur'e systems has not been completely considered.

Recently, [31] studies the absolute stability of switched delay Lur'e systems. By the method of ADT and Lyapunov functional, the efficient delay-dependent condition is obtained in the form of LMIs. However, the system considered in [31] is linear; meanwhile, the Lyapunov function chosen in [31] is too simple and restrictive, as only quadratic term and the integration term of state are included. Therefore, the result obtained in [31] is more conservative, which is one motivation of present work. Also, when a system with delay is considered, one chief objective is to get maximum allowed delay upper bounds, and the bigger, the better. Considering the complexity of switched systems, it is challenging to design a switching law for switched delay systems to get bigger maximum allowed delay upper bounds and better performance, which is the other driving force of this paper. Besides, sometimes, in previous work, such as [32], it is difficult to get the same positive matrix which satisfies the corresponding LMIs.

In this paper, the absolute exponential stability of switched Lur'e systems with delays is investigated. The main contribution of this paper is as follows. (1) Employing the integration of state derivative, an appropriate Lyapunov-Krasovskii functional is constructed to get new stability criteria which are less conservative. (2) The switching law is designed based on MDADT method to reduce the conservativeness. (3) The criteria proposed in this paper are more flexible than the previous results [31], involving those criteria obtained in [25, 31]. On top of that, compared with those in [32], the method derived in this paper can also be used to get less conservative results for general switched delay systems.

The rest of this paper is organized as follows. The problem is stated in Section 2. Section 3 shows the main results and a simulation example is given in Section 4. In the end of the paper, conclusions are presented.

Notations. Throughout this paper, R denotes the real number set; N denotes the nonnegative integer set; R^n is the Euclid space with dimension n; $R^{m\times n}$ is the real matrices set with dimension $m \times n$. For the two given symmetric matrices P and Q, we use $P > Q$ ($P \geq Q$) to denote that $P - Q$ is a positive definite (positive semidefinite) matrix. "A^T" denotes the transpose of matrix A; I denotes the identity matrix with appropriate dimension. For any matrix W and two symmetric matrices P, Q, we use $*$ to denote the symmetric part of the symmetric matrix.

2. Problem Statement

Consider the following system

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t-\tau) + C_{\sigma(t)}\omega(t) \\ z(t) &= D_{\sigma(t)}x(t) + E_{\sigma(t)}x(t-\tau) \\ \omega(t) &= -\varphi(t, z(t)) \\ x(t) &= \phi(t), \quad \forall t \in [-\tau, 0] \end{aligned} \tag{1}$$

where $x(t) \in R^n$, $\omega(t) \in R^m$, $z(t) \in R^m$ are the state vector, input vector, and output vector, respectively; $\tau > 0$ is system delay; $\phi(\cdot)$ is a continuous vector-valued function and $A_{\sigma(t)}$, $B_{\sigma(t)} \in R^{n\times n}$, $C_{\sigma(t)} \in R^{n\times m}$, $D_{\sigma(t)}$, $E_{\sigma(t)} \in R^{m\times n}$ are all constant matrices; $\sigma(t)$: $[0,\infty) \longrightarrow M = \{1, 2, \ldots, m\}$ is the switching signal; $\varphi(t, z(t)) : [0,\infty] \times R^m \longrightarrow R^m$ is a nonlinear vector-valued function which is continuous about t and globally Lipchitz at $z(t)$ with $\varphi(t, 0) = 0$. Besides $\varphi(t, z(t))$ also satisfies

$$\begin{aligned} &[\varphi(t, z(t)) - K_1 z(t)]^T [\varphi(t, z(t)) - K_2 z(t)] \leq 0 \\ &\qquad \forall t \geq 0,\ \forall z(t) \in R^m \end{aligned} \tag{2}$$

where $K = K_2 - K_1$ is a symmetric positive definite matrix with constant matrices K_1, K_2. The nonlinear function $\varphi(t, z(t))$ which satisfies (2) is said to belong to $[K_1, K_2]$. Initially, we will give some definitions and lemmas needed in this paper.

Definition 1 (see [22]). System described by (1) and (2) is called to be absolutely exponentially stable in $[K_1, K_2]$ if the zero solutions of system (1) are all globally exponentially stable under some switching signal $\sigma(t)$.

Definition 2 (see [15]). For a switching signal $\sigma(t)$ and any $T \geq t \geq 0$, let $N_{\sigma i}(T, t)$ be the switching numbers, that is, the i^{th} subsystem is activated over the interval $[t, T]$, and $T_i(T, t)$ denote the total running time of the i^{th} subsystem over $[t, T]$, $i \in M$. We say that $\sigma(t)$ has a mode-dependent average dwell time (MDADT) τ_{ai} if there exist positive numbers N_{0i} (chatter bounds) and τ_{ai} such that

$$N_{\sigma i}(T, t) \leq N_{0i} + \frac{T_i(T, t)}{\tau_{ai}}, \quad \forall T \geq t \geq 0. \tag{3}$$

Lemma 3 (see [25]). *For any symmetric positive definite matrix $M > 0$, scalar $\gamma > 0$, and function $\omega : [0, \gamma] \longrightarrow R^n$, if the integrations related are well defined, the following holds.*

$$\gamma \int_0^\gamma \omega^T(s) M \omega(s)\, ds \geq \left(\int_0^\gamma \omega(s)\, ds\right)^T M \left(\int_0^\gamma \omega(s)\, ds\right) \tag{4}$$

Lemma 4 (see [26]). *For given matrices $Q = Q^T$, H, E with appropriate dimensions,*

$$Q + HF(t)E + E^T F^T(t) H^T < 0 \tag{5}$$

holds for all $F(t)$ satisfying $F^T(t)F(t) \leq I$ if and only if there exists $\varepsilon > 0$ such that

$$Q + \varepsilon^{-1} H H^T + \varepsilon E^T E < 0. \tag{6}$$

3. Main Results

In the previous work [31], the Lyapunov function used is of the following form.

$$
\begin{aligned}
V(t,x(t)) &= V_{\sigma(t)}(t,x(t)) \\
&= x^T(t)P_{\sigma(t)}x(t) \\
&\quad + \int_{t-\tau}^{t} e^{\alpha(s-t)}x^T(s)Q_{\sigma(t)}x(s)\,ds
\end{aligned} \tag{7}
$$

Inspired by the method in [33] and letting $h = \tau/n$, we choose the following Lyapunov-Krasovskii functional:

$$
\begin{aligned}
V(t,x(t)) &= V_{\sigma(t)}(t,x(t)) \\
&= x^T(t)P_{\sigma(t)}x(t) \\
&\quad + \sum_{i=1}^{n}\int_{t-ih}^{t-(i-1)h} e^{\alpha_{\sigma(t)}(s-t)}x^T(s)Q_{i\sigma(t)}x(s)\,ds \\
&\quad + \sum_{i=1}^{n}\int_{-ih}^{-(i-1)h}\int_{t+\theta}^{t} e^{\alpha_{\sigma(t)}(s-t)}\dot{x}(s)^T hR_{i\sigma(t)}\dot{x}(s)\,ds\,d\theta
\end{aligned} \tag{8}
$$

where $P_{\sigma(t)} = P_{\sigma(t)}^T > 0$, $Q_{i\sigma(t)} = Q_{i\sigma(t)}^T > 0$, and $R_{i\sigma(t)} = R_{i\sigma(t)}^T > 0, i = 1,2,3,\ldots,n$, are positive definite matrices to be determined.

As in [31], the case when the nonlinear function $\varphi(t,z(t)) \in [0,K]$ will be firstly considered and we have the following result.

Theorem 5. *For given $h > 0$, $\alpha_j > 0$, $\mu_j \ge 1$, $j \in M$, system (1) is absolutely exponentially stable in sector $[0,K]$ with MDADT $\tau_{aj} > \tau_{aj}^* = (\ln\mu_j)/\alpha_j$ if there exist a real number $\varepsilon > 0$ and symmetric matrices $P_j > 0$, $Q_{ij} > 0$, $R_{ij} > 0$, $i = 1,2,3,\ldots,n$, $j \in M$ such that the following LMIs hold*

$$
\Upsilon = \begin{pmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} \\ * & \Upsilon_{22} & \Upsilon_{23} \\ * & * & \Upsilon_{33} \end{pmatrix} < 0 \tag{9}
$$

where

$$
\begin{aligned}
\Upsilon_{11} &= \begin{pmatrix} \Upsilon_{11}^1 & P_jB_j & P_jC_j - \varepsilon D_j^T K^T \\ * & \Upsilon_{22}^1 & -\varepsilon E_j^T K^T \\ * & * & -2\varepsilon I \end{pmatrix} \\
\Upsilon_{11}^1 &= P_j^T A_j + A_j^T + P_j + Q_{1j} - R_{1j} \\
\Upsilon_{22}^1 &= -e^{-\alpha_j nh}Q_{nj} - e^{-\alpha_j(n-1)h}R_{nj} \\
\Upsilon_{12} &= \begin{pmatrix} R_{1j} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & e^{-\alpha_j(n-1)j}R_{nj} \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix} \\
\Upsilon_{13} &= h\Gamma^T\sum_{i=1}^{n}R_{ij}, \quad \Gamma = \begin{pmatrix} A_j & B_j & C_j \end{pmatrix} \\
\Upsilon_{22} &= \begin{pmatrix} S_1 & R_{2j} & & & \\ R_{2j} & S_2 & & & \\ & & \ddots & & \\ & & & S_{n-2} & R_{(n-1)j} \\ & & & R_{(n-1)j} & S_{n-1} \end{pmatrix} \\
S_i &= e^{-\alpha_j ih}\left(Q_{(i+1)j} - Q_{ij}\right) - e^{-\alpha_j(i-1)h}\left(R_{ij} + R_{(i+1)j}\right) \\
\Upsilon_{33} &= -e^{-\alpha_j(i-1)h}\sum_{i=1}^{n}R_{ij}, \quad i = 1,2,\ldots,n-1, \\
P_l &\le \mu_l P_m, \\
Q_{il} &\le \mu_l Q_{im}, \\
R_{il} &\le \mu_l R_{im}, \\
&\qquad \forall l, m \in M.
\end{aligned} \tag{10}
$$

Proof. Differentiating function (8) about time t along the solutions of system (1), we will get the following.

$$
\begin{aligned}
&\dot{V}(t,x(t)) + \alpha_j V(t,x(t)) = x^T(t)\left(A_j^T P_j + P_jA_j + \alpha_jP_j\right) \\
&\quad\cdot x(t) + 2x^T(t)P_jC_j\omega(t) + 2x^T(t)P_jB_jx(t-\tau) \\
&\quad + x^T(t)Q_{1j}x(t) \\
&\quad \cdot \sum_{i=1}^{n}e^{-\alpha_j(i-1)h}x(t-(i-1)h)^T Q_{ij}x(t-(i-1)h) \\
&\quad - \sum_{i=1}^{n}e^{-\alpha_j ih}x(t-ih)^T Q_{ij}x(t-ih) + h^2\sum_{i=1}^{n}\dot{x}^T(t)R_{ij}\dot{x}(t) \\
&\quad - \sum_{i=1}^{n}\int_{t-ih}^{t-(i-1)h}e^{\alpha_j(s-t)}\dot{x}(s)^T hR_{ij}\dot{x}(s)\,ds = x^T(t) \\
&\quad \cdot \left(A_j^TP_j + P_jA_j + \alpha_jP_j + Q_{1j}\right)x(t) + 2x^T(t)P_jC_j\omega(t) \\
&\quad + 2x^T(t)P_jB_jx(t-\tau) \\
&\quad + \sum_{i=1}^{n-1}e^{-\alpha_j ih}x^T(t-ih)\left(Q_{(i+1)j} - Q_{ij}\right)x(t-ih) \\
&\quad - e^{-\alpha_j ih}x^T(t-\tau)Q_{nj}x(t-\tau) + h^2\sum_{i=1}^{n}\dot{x}^T(t)R_{ij}\dot{x}(t) \\
&\quad - \sum_{i=1}^{n}\int_{t-ih}^{t-(i-1)h}e^{\alpha_j(s-t)}\dot{x}(s)^T hR_{ij}\dot{x}(s)\,ds
\end{aligned} \tag{11}
$$

By use of Lemma 3, we can get

$$
-\sum_{i=1}^{n}\int_{t-ih}^{t-(i-1)h}e^{\alpha_j(s-t)}\dot{x}(s)^T hR_{ij}\dot{x}(s)\,ds
$$

$$\leq -\sum_{i=1}^{n} e^{-\alpha_j (i-1)h} \left[x\left(t-(i-1)h\right)-x\left(t-ih\right)\right]^T R_{ij} \cdot \left[x\left(t-(i-1)h\right)-x\left(t-ih\right)\right] \tag{12}$$

and from (1), it is true that

$$\sum_{i=1}^{n} \dot{x}^T(t) R_{ij} \dot{x}(t) = q^T(t) \begin{pmatrix} A_j^T \\ B_j^T \\ C_j^T \end{pmatrix} \sum_{i=1}^{n} R_{ij} \begin{pmatrix} A_j & B_j & C_j \end{pmatrix} \tag{13}$$

where

$$q^T(t) = \left(x^T(t) \;\; x^T(t-\tau) \;\; \omega^T(t)\right). \tag{14}$$

From (1) and considering $\varphi(t, z(t)) \in [0, K]$, the following holds.

$$-\omega^T(t)\,\omega(t) - \omega^T K\left[D_j x(t) + E_j x\left(t-\gamma(t)\right)\right] \geq 0 \tag{15}$$

According to (12)-(15), for a scalar $\varepsilon > 0$, it yields that

$$\begin{aligned}
&\dot{V}(t, x(t)) + \alpha V(t, x(t)) \\
&\quad\leq x^T(t)\left(A_j^T P_j + P_j A_j + \alpha_j P_j + Q_{1j}\right) x(t) \\
&\qquad + 2x^T(t) P_j C_j \omega(t) + 2x^T(t) P_j B_j x(t-\tau) \\
&\qquad - e^{-\alpha_j ih} x^T(t-\tau) Q_{nj} x(t-\tau) \\
&\qquad + \sum_{i=1}^{n-1} e^{-\alpha_j ih} x^T(t-ih)\left(Q_{(i+1)j} - Q_{ij}\right) x(t-ih) \\
&\qquad - \sum_{i=1}^{n} e^{-\alpha_j (i-1)h} \left[x\left(t-(i-1)h\right)-x\left(t-ih\right)\right]^T R_{ij} \\
&\qquad \cdot \left[x\left(t-(i-1)h\right)-x\left(t-ih\right)\right] - 2\varepsilon\omega^T(t)\,\omega(t) \\
&\qquad - 2\varepsilon\omega^T K\left[D_j x(t) + E_j x\left(t-\gamma(t)\right)\right] \\
&\qquad + h^2 q^T(t) \begin{pmatrix} A_j^T \\ B_j^T \\ C_j^T \end{pmatrix} \sum_{i=1}^{n} R_{ij} \begin{pmatrix} A_j & B_j & C_j \end{pmatrix} \\
&\quad = \eta^T(t)\left[\Psi + h^2 \Gamma^T \sum_{i=1}^{n} R_{ij} \Gamma\right] \eta(t)
\end{aligned} \tag{16}$$

where

$$\Psi = \begin{pmatrix} \Upsilon_{11} & \Upsilon_{12} \\ * & \Upsilon_{22} \end{pmatrix}$$
$$\eta(t) = \left(x^T(t) \;\; x^T(t-\tau) \;\; \omega^T(t) \;\; \varsigma^T(t)\right)^T \tag{17}$$

with

$$\varsigma(t) = \left(x^T(t-h) \;\; x^T(t-2h) \;\; \cdots \;\; x^T\left(t-(n-1)h\right)\right)^T. \tag{18}$$

By Schur complement, it yields that $\Psi + h^2\Gamma^T \sum_{i=1}^{n} R_{ij}\Gamma < 0$ is equivalent to inequality (9). Then it can be concluded from [17] that system (1) is absolutely exponentially stable.

When $n = 1$, the following corollary can be derived from Theorem 5. □

Corollary 6. *For given $h > 0$, $\alpha_j > 0$, $\mu_j \geq 1, j \in M$, system (1) is absolutely exponentially stable in sector $[0, K]$ with MDADT $\tau_{aj} > \tau_{aj}^* = (\ln \mu_j)/\alpha_j$ if there exist a real number $\varepsilon > 0$ and symmetric matrices $P_j > 0$, $Q_{1j} > 0$, $R_{1j} > 0$, $j \in M$ such that the following LMIs hold*

$$\begin{pmatrix} \Omega_{11} & P_j B_j + R_{1j} & P_j C_j - \varepsilon D^T K^T & \tau A_j R_{1j} \\ * & -e^{-\alpha_j \tau} Q_{1j} - R_{1j} & -\varepsilon E^T K^T & \tau B_j R_{1j} \\ * & * & -2\varepsilon I & \tau C_j R_{1j} \\ * & * & * - R_{1j} & \end{pmatrix} < 0 \tag{19}$$

where

$$\begin{aligned}
\Omega_{11} &= P_j A_j + A_j^T P_j + Q_{1j} - R_{1j} \\
P_i &\leq \mu_i P_j, \\
Q_{1i} &\leq \mu_i Q_{1j}, \\
R_{1i} &\leq \mu_i R_{1j}, \\
&\qquad \forall i, j \in M.
\end{aligned} \tag{20}$$

Remark 7. In fact, Corollary 6 is the same as Proposition 3 in [25] when $j = 1, m = 1$; also it is Theorem 1 in [31] with $R_{1j} = 0$. So Proposition 3 in [25] and Theorem 1 in [31] are included in Theorem 5 in this paper.

Remark 8. In fact, if we set $Q_{ij} = Q_j$, $R_{ij} = 0$, the Lyapunov-Krasovskii functional (8) will reduce to corresponding Lyapunov function (7) ([31, 32]), so (7) can be seen as a special case of (8) which considers both the integration of state and the integration of state's derivative. As is well known, when a system is considered, the character of the state's derivative also plays an important role in the stability analysis. Besides, sometimes, it is difficult to get the same Q_j in [31]. Fortunately, in our result, Q_{ij} is allowed to be different. Therefore, the LMIs obtained in this paper is more flexible and general than those obtained by ADT method in [31]. Meanwhile, the method can also be used for the stability analysis of general switched delay system to derive less conservative conditions than those in [32] ($Q_{ij} = Q_j, R_{ij} = R_j$,).

Similarly, when $\varphi(t, z(t)) \in [K_1, K_2]$, we have the following result.

Corollary 9. *For given $h > 0$, $\alpha_j > 0$, $\mu_j \geq 1, j \in M$, system (1) is absolutely exponentially stable in sector $[K_1, K_2]$ with MDADT $\tau_{aj} > \tau_{aj}^* = (\ln \mu_j)/\alpha_j$ if there exist a real number*

$\varepsilon > 0$ and symmetric matrices $P_j > 0$, $Q_{ij} > 0$, $R_{ij} > 0$, $i = 1, 2, 3, \ldots, n$, $j \in M$ such that the following LMIs hold

$$\widehat{\Upsilon} = \begin{pmatrix} \widehat{\Upsilon}_{11} & \Upsilon_{12} & \widehat{\Upsilon}_{13} \\ * & \Upsilon_{22} & 0 \\ * & * & \Upsilon_{33} \end{pmatrix} < 0 \tag{21}$$

where

$$\begin{gathered} \widehat{\Upsilon}_{11} = \begin{pmatrix} \widehat{\Upsilon}_{11}^1 & P_j\left(B_j - C_jK_1E_j\right) & \widehat{\Upsilon}_{11}^3 \\ * & \Upsilon_{22}^1 & -\varepsilon E_j^T\left(K_2 - K_1\right)^T \\ * & * & -2\varepsilon I \end{pmatrix} \\ \widehat{\Upsilon}_{11}^1 = P_j^T\left(A_j - C_jK_1D_j\right) + \left(A_j - C_jK_1D_j\right)^T + P_j + Q_{1j} - R_{1j} \\ \widehat{\Upsilon}_{11}^3 = P_jC_j - \varepsilon D_j^T\left(K_2 - K_1\right)^T \\ \widehat{\Upsilon}_{13} = h\widehat{\Gamma}^T\sum_{i=1}^{n} R_{ij} \\ \widehat{\Gamma} = \left(A_j - C_jK_1D_j \quad B_j - C_jK_1E_j \quad C_j\right) \\ P_l \le \mu_l P_m, \\ Q_{il} \le \mu_l Q_{im}, \\ R_{il} \le \mu_l R_{im}, \\ \forall l, m \in M \end{gathered} \tag{22}$$

and Υ_{12}, Υ_{22}, Υ_{33}, Υ_{22}^1 are defined in Theorem 5.

Proof. Using the transform in [34] and Theorem 5, it is easy to get this Corollary.

Except the absolute stability of system (1), we will also consider the more general system as follows:

$$\begin{gathered} \dot{x}(t) = \left(A_{\sigma(t)} + \triangle A(t)\right)x(t) + \left(B_{\sigma(t)} + \triangle B(t)\right)x(t-\tau) + C_{\sigma(t)}\omega(t) \\ z(t) = D_{\sigma(t)}x(t) + E_{\sigma(t)}x(t-\tau) \\ \omega(t) = -\varphi(t, z(t)), \\ x(t) = \phi(t), \quad \forall t \in [-\tau, 0] \end{gathered} \tag{23}$$

with

$$\left[\triangle A(t) \quad \triangle B(t)\right] = LF(t)\left[E_a \quad E_b\right] \tag{24}$$

where L, E_a, E_b are constant matrices with appropriate dimensions and for any t, $F(t)$ satisfying

$$F^T(t)F(t) \le I. \tag{25}$$

For system (23), Corollary 10 can be easily derived from Corollary 9. □

Corollary 10. *For given $h > 0$, $\alpha_j > 0$, $\mu_j \ge 1$, $j \in M$, system (23) is absolutely exponentially stable in sector $[K_1, K_2]$ with MDADT $\tau_{aj} > \tau_{aj}^* = (\ln \mu_j)/\alpha_j$ if there exist real numbers $\varepsilon > 0$, $\lambda > 0$ and symmetric matrices $P_j > 0$, $Q_{ij} > 0$, $R_{ij} > 0$, $i = 1, 2, 3, \ldots, n$, $j \in M$ such that the following LMIs hold*

$$\begin{bmatrix} \widehat{\Upsilon}_{11} & \Upsilon_{12} & \widehat{\Upsilon}_{13} & \widehat{P}_jL & \lambda\widehat{E} \\ * & \Upsilon_{22} & 0 & 0 & 0 \\ * & * & \Upsilon_{33} & h\sum_{i=1}^{n} R_{ij}L & 0 \\ * & * & * & -\lambda I & 0 \\ * & * & * & * & -\lambda I \end{bmatrix} < 0 \tag{26}$$

where

$$\begin{gathered} \widehat{P}_j = \begin{bmatrix} P_j \\ 0 \\ 0 \end{bmatrix}, \\ \widehat{E} = \begin{bmatrix} E_a^T \\ E_b^T \\ 0 \end{bmatrix} \\ P_l \le \mu_l P_m, \\ Q_{il} \le \mu_l Q_{im}, \\ R_{il} \le \mu_l R_{im}, \\ \forall l, m \in M \end{gathered} \tag{27}$$

and Υ_{12}, Υ_{22}, Υ_{33} are defined in Theorem 5 and $\widehat{\Upsilon}_{11}$, $\widehat{\Upsilon}_{13}$ are defined in Corollary 9.

Proof. Replacing A_j and B_j in (21) with $A_j + LF(t)E_a$ and $B_j + LF(t)E_b$, making use of Schur complement, Lemma 4, and Corollary 9, it is easy to get Corollary 10. □

Remark 11. The results of this paper are based on LMI technology. In further research, when nonconvex matrix inequality conditions (nonlinear coupling) are encountered, the Chang-Yang decoupling method ([35]) can be used for nonlinear coupling.

4. Numerical Example

Consider uncertain system (23) with the following parameters:

$$A_1 = \begin{pmatrix} -2 & 0 \\ 0.5 & -0.9 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix},$$

$$C_1 = \begin{pmatrix} -0.2 \\ -0.3 \end{pmatrix}$$
$$D_1 = \begin{pmatrix} 0.46 & 0.8 \end{pmatrix},$$
$$E_1 = \begin{pmatrix} 0 & 0 \end{pmatrix},$$
$$K_1 = 0.2,$$
$$K_2 = 0.5$$
$$A_2 = \begin{pmatrix} -3 & 0.5 \\ 0.8 & -0.8 \end{pmatrix},$$
$$B_2 = \begin{pmatrix} -1.5 & 0 \\ -1.2 & -1 \end{pmatrix},$$
$$C_1 = \begin{pmatrix} -0.4 \\ -0.8 \end{pmatrix}$$
$$D_2 = \begin{pmatrix} 0.5 & 0.3 \end{pmatrix},$$
$$E_2 = \begin{pmatrix} 0.2 & 0.5 \end{pmatrix},$$
$$L = \begin{pmatrix} \beta & 0 \\ 0 & \beta \end{pmatrix}, \quad \beta \geq 0,$$
$$E_a = E_b = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{28}$$

for $n = 2$, $\varepsilon = 0.3$, $\lambda = 0.5$, $\alpha_1 = 0.8$, $\alpha_2 = 0.5$, $\tau = 0.763$, $\mu_1 = 2$, $\mu_2 = 1.9$, $\beta = 0.5$. Using the Matlab software, the LMI in Corollary 10 is solvable, and we can get the following.

$$P_1 = \begin{pmatrix} 0.1021 & -0.0010 \\ -0.0010 & 0.0212 \end{pmatrix},$$
$$P_2 = \begin{pmatrix} 0.0107 & 0.0002 \\ 0.0002 & 0.0347 \end{pmatrix}$$
$$Q_{11} = \begin{pmatrix} 0.0035 & 0.0004 \\ 0.0004 & 0.0034 \end{pmatrix},$$
$$Q_{12} = \begin{pmatrix} 0.0040 & 0.0003 \\ 0.0003 & 0.0034 \end{pmatrix}$$
$$Q_{21} = \begin{pmatrix} 0.0025 & 0.0001 \\ 0.0001 & 0.0031 \end{pmatrix},$$
$$Q_{22} = \begin{pmatrix} 0.0020 & 0.0004 \\ 0.0004 & 0.0038 \end{pmatrix}$$
$$R_{11} = \begin{pmatrix} 1.6069 & -0.0107 \\ -0.0107 & 1.0008 \end{pmatrix},$$
$$R_{12} = \begin{pmatrix} 1.2019 & -0.0107 \\ -0.0107 & 1.1002 \end{pmatrix}$$
$$R_{21} = \begin{pmatrix} 1.2100 & -0.0120 \\ -0.0120 & 1.0008 \end{pmatrix},$$
$$R_{22} = \begin{pmatrix} 1.3019 & -0.0237 \\ -0.0237 & 1.1022 \end{pmatrix} \tag{29}$$

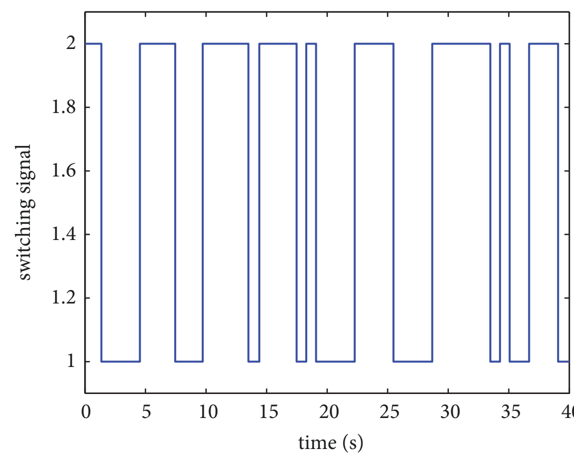

FIGURE 1: Switching signal.

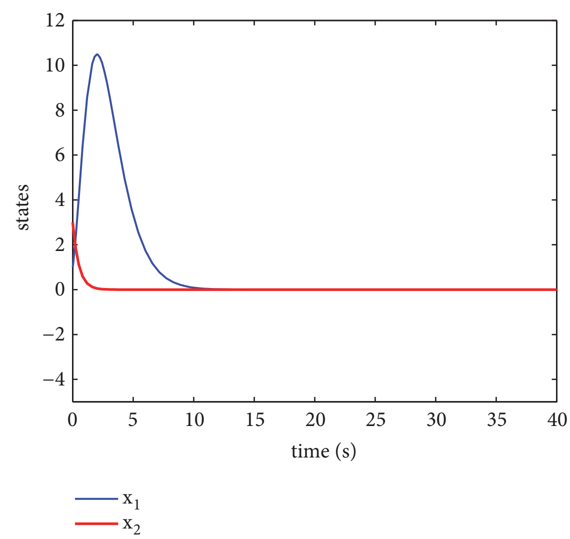

FIGURE 2: The response curve of the states in example.

TABLE 1: Comparison with the result in [31].

	$\beta = 0.00$	$\beta = 0.05$	$\beta = 0.10$
Corollary 2 [31]	2.2845	2.2352	2.0243
Corollary 10, n=2	3.0079	2.6738	2.1654
Corollary 10, n=3	3.1085	2.7602	2.4731

The simulation is shown in Figures 1 and 2.

Table 1 lists the maximum allowed time-delay bounds obtained by Corollary 10 for various β in comparison with those obtained by Corollary 2 in [31]. Clearly, the results in this paper are better than those in [31]. Besides, by setting $m = 1$, Corollary 10 reduces to Theorem 2 in [28].

5. Conclusions

In this article, we considered the stability of switched Lur'e systems. In order to derive less conservative criteria, we introduce an appropriate Lyapunov-Krasovskii functional by the method of delay length segmentation ([33]). Besides, the MDADT method is also used to gain more applicable results. In further research, nonlinear switched Lur'e systems with time-varying delays will be considered; in particular, we will consider if the concept "average impulsive interval" [18] can be used in our further study.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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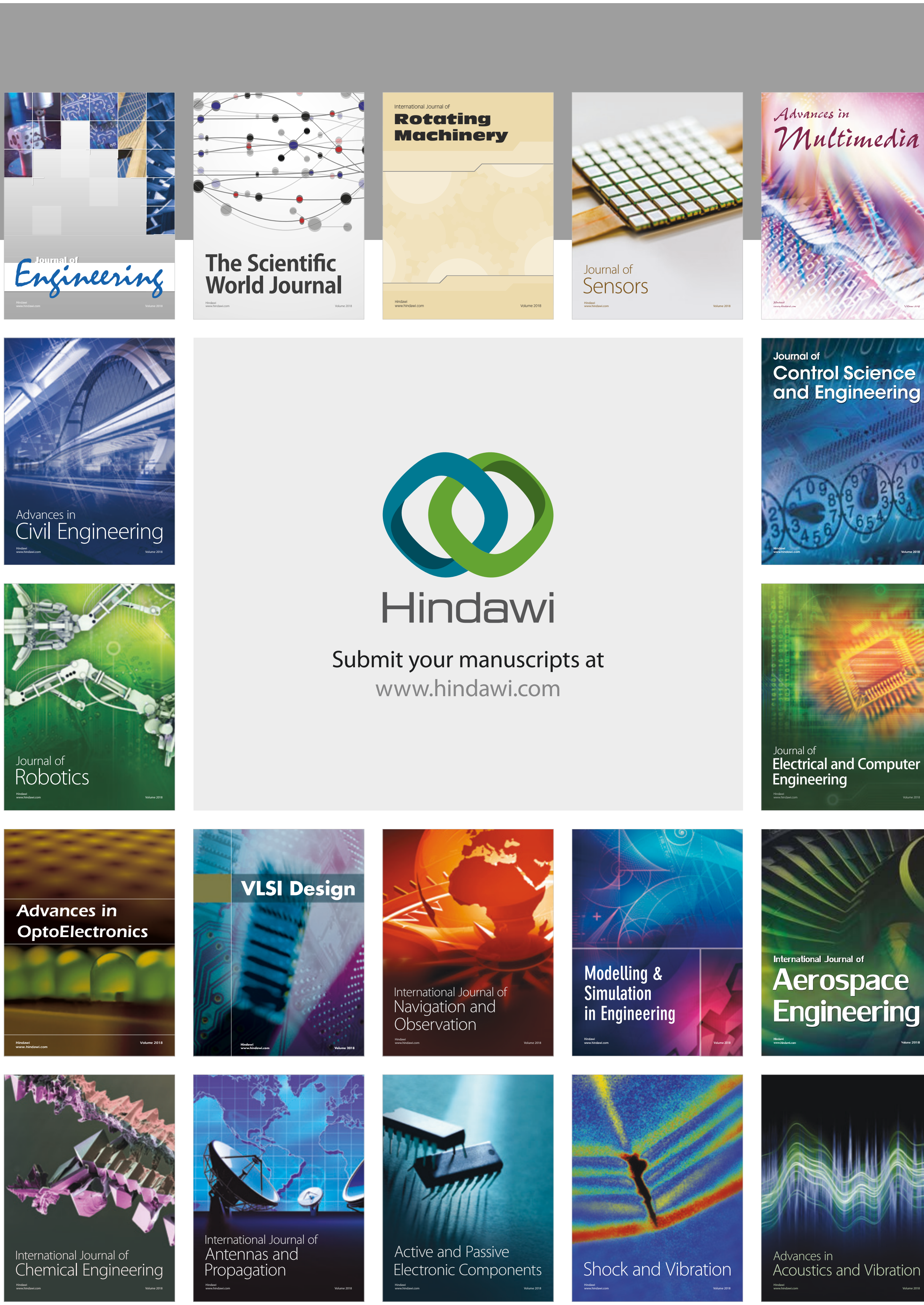

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