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Research Article

Finite-Time Control of Networked Control Systems with Time Delay and Packet Dropout

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This paper studies the finite-time stabilization and boundedness problem of a class of network control systems that are simultaneously affected by time delay and packet loss. Based on the Lyapunov function method, the sufficient conditions for the design of the state feedback controller in the form of linear matrix inequality are obtained. The state feedback controller makes the network control system stable for a finite time. Finally, a numerical example is given to illustrate the effectiveness and feasibility of the method. The research results of this paper will develop and enrich the control theory system of the network control system and provide advanced control theory methods and application technology reserves in order to promote the development process of the network control system application and improve the application level.

1. Introduction

With the development of high-speed network communication technology, numerous devices can easily form a distributed networked system through the network to realize remote data transmission and data exchange. Due to the use of the network, the complexity and cost of the distributed control system are fully reduced, and the maintenance of the system becomes simpler [1]. The network control system is used more and more in the industrial process. It is the product of the combination of the current control science and the rapidly developing computer network and communication technology. In the past ten years, the problem of network control has received more and more attention from the international control and computer science circles. At present, many complex control systems such as wireless network robots, transportation tools, remote teleoperation, Internet-based remote teaching and experimentation, telemedicine, manufacturing equipment, weapon systems, and Fieldbus and Industrial Ethernet Technology, in essence, can be attributed to network-based control systems. In addition, the network control system also has broad application prospects in the field of aerospace and complex and dangerous industrial control. Therefore, the analysis and design of network control systems have received more attention [2]. However, the application of the network in the control loop also brings some problems, such as time lag, data loss, and multipacket transmission of data generated by the network, which will affect the stability and performance of the control system. Since the network control system is a fusion of control technology and network communication technology, its analysis and design are often carried out from two perspectives of network and control. On the one hand, from the perspective of network, we design a communication protocol to reduce network time. The impact of these problems on the control system such as hysteresis and data loss, on the other hand, from the perspective of control, the existing network structure, protocol, etc., are regarded as established conditions, and the structure of the control system, control algorithm, etc., are designed on this basis. To compensate or reduce the adverse effects of network time lag, data loss and other issues on the control system, we mainly discuss the network control system from the perspective of control.

Usually, the system stability that people care about is mainly Lyapunov stability. The Lyapunov stability characterizes the steady-state performance of the system, and it does not reflect the transient performance of the system [3]. However, in actual engineering, in addition to the steady-state performance of the system, the transient performance is sometimes particularly important, such as missile systems, communication network systems, and robot control systems. These systems have short working hours, and people are interested in their asymptotic stability. In addition, they are more concerned about the system meeting certain transient performance requirements [4]. In fact, on the one hand, a system that is stable in the sense of Lyapunov may have very bad transient performance (such as excessive overshoot), and sometimes it is not even applicable in engineering. On the other hand, the system is not Lyapunov asymptotically stable, or even without equilibrium, but the system can maintain good performance within a limited time interval. Therefore, in actual engineering, people are often more concerned that the system should meet certain transient performance requirements. In order to study the transient performance of the system, the finite-time stability of the system must be discussed [5, 6].

In order to study the transient performance of the system, in 1961, Dorato proposed the concept of short-time stability, which analyzed and studied the problem of finite-time control of linear systems [7]. In 1965, Weiss et al. proposed the concept of finite-time convergent stability (contractive stability) [8], which studied the finite-time stability of nonlinear systems, and then extended it to nonlinear systems with disturbances. The concept of bounded input and bounded output stability (BIBO) [9] is usually called finite-time bounded (FTB). Compared with the research on the infinite time control of the network control system, the finite-time control of the network control system started relatively late, and the research results are relatively few. Research on finite-time control of packet loss network control systems: Mastellone [10–12] and others discussed the finite-time stability of model-based packet loss network control systems. However, literature studies [10–12] did not give the design method of the controller. Shang [13] and Sun [14], based on the linear matrix inequality method, respectively, gave the finite-time stability analysis and controller design method of random packet loss network control systems with all the transition probabilities and all known transition probabilities, but literature [13] did not give a finite-time bounded result. Sun [15] further studied the finite-time stability and finite-time boundedness of random packet loss network control systems on the basis of literature studies [13, 14]. Zhang et al. [16] discussed the finite-time control of random packet loss network control systems. Research on finite-time control of time-delay networked control systems: Shang et al. [17] and Sun and Xu [18] studied the finite-time control problems of constant-delay networked control systems based on linear transformation methods and discretization methods, respectively. Furthermore, Shang and Gao [19] and Li and Sun [20] discussed the finite-time control of constant-delay network control systems. Gao et al. [21] used the linear matrix inequality method to give the stability analysis and controller design method of the continuous uncertain time-delay network control system. Xue and Mao [22] used the time-delay system method to study the segmentation stability of the time-delay packet loss network control system.

In this paper, the finite-time control method for a class of network control systems with time-varying delay and random packet loss is proposed. The main contributions of this paper mainly include four aspects:

(1) Based on the analysis of network delay and packet loss, a network control system model that is simultaneously affected by time-varying delay and random packet loss is established
(2) By using the principle of deterministic equivalence, the stochastic time-varying network control system model is transformed into a time-invariant linear system
(3) Based on the Lyapunov function method and linear matrix inequality method, the finite-time control method for network control systems with time-varying delay and random packet loss is proposed
(4) The actual system is inevitably subject to external interference, so both finite-time stability and finite-time boundedness are discussed in our paper

The rest of this paper is outlined as follows. In Section 2, problem formulation and some preliminaries are presented. The finite-time stability analysis is given Section 3. In Section 4, based on the linear matrix inequality, a state feedback controller that makes the considered networked control systems finite-time stable is designed. A numerical example is presented in Section 5. Finally, Section 6 gives the conclusion of this work and the future research directions.

2. System Description and Some Preliminaries

Consider the time-invariant plant as follows:

$$\dot{x}(t) = \mathrm{Ax}(t) + \mathrm{Bu}(t) + \mathrm{Gw}(t), \tag{1}$$

where $x(t) \in R^n$ is system state, $u(t) \in R^m$ represents control input, and A and B are the system matrix and control matrix with appropriate dimension matrix, respectively.

Due to the existence of the network, system (1) can be modeled as a random system as follows:

$$\dot{x}(t) = \mathrm{Ax}(t) + \gamma(t)\mathrm{Bu}(t - \tau(t)) + \mathrm{Gw}(t), \tag{2}$$

where $\tau(t)$ is the time-varying network delay, $\gamma(t)$ is random matrix, when the control input data packet is lost, $\gamma(t)$ is zero matrix, and when the control input packet is received, $\gamma(t)$ is the identity matrix. The assumed sampling period is h. Network delay $\tau(t)$ satisfies $0 \leq \tau(t) \leq \kappa h$. Figure 1 shows the data packet transmission process.

Discretizing the system (2), we obtain

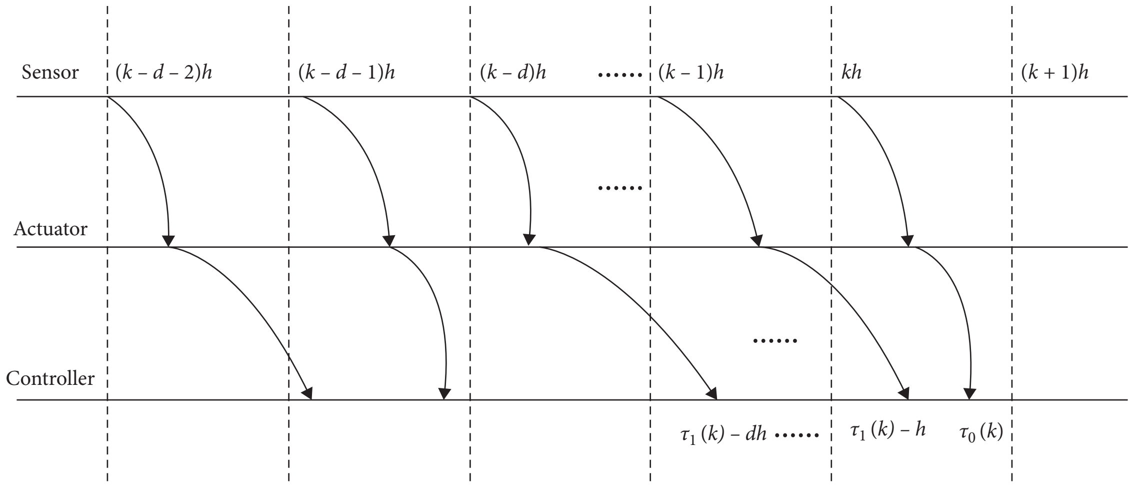

Figure 1: Schematic diagram of data packet transmission.

$$
\begin{aligned}
x(k+1) = {} & A_s x(k) + B_0(k)\tilde{u}(k) \\
& + B_1(k)\tilde{u}(k-1) + \cdots + B_d(k)\tilde{u}(k-\kappa) + G_s w(k),
\end{aligned}
\tag{3}
$$

where

$$
\begin{aligned}
A_s &= e^{\mathrm{Ah}}, \\
G_s &= \int_0^h e^{\mathrm{As}} G \mathrm{d}s, \\
\tilde{u}(k-i) &= \gamma(k-i)u(k-i) \\
B_i(k) &= \int_{\tau_i(k)-ih}^{\tau_{i-1}(k)-(i-1)h} e^{A(h-s)} B\varphi \\
&\quad (h-\tau_{i-1}(k)-\tau_i(k))\varphi(\tau_i(k)-ih)\mathrm{d}s, \\
B_0(j) &= \int_{\tau_0(k)}^{h} e^{A(h-s)} B\varphi((k+1)h-\tau_0(k))\mathrm{d}s, \\
\varphi(x) &= \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}.
\end{aligned}
\tag{4}
$$

Define augmentation vector

$$
\tilde{x}(k) = \left[x^T(k), u^T(k-1), \ldots, u^T(k-\kappa)\right]^T. \tag{5}
$$

Then, the equivalent model of (3) can be obtained as follows:

$$
\tilde{x}(k+1) = A(k)\tilde{x}(k) + B(k)\tilde{u}(k) + \tilde{G}w(k), \tag{6}
$$

where

$$
A(k) = \begin{bmatrix}
A & \gamma(k-1)B_1(k) & \cdots & \gamma(k-1)B_1(k) & \cdots & \gamma(k-1)B_1(k) \\
0 & 0 & \cdots & 0 & \cdots & 0 \\
0 & I & \cdots & 0 & \cdots & 0 \\
\vdots & 0 & I & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & I & 0
\end{bmatrix},
$$

$$
B(k) = \begin{bmatrix} \gamma(k)B_0(k) \\ I \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},
\tilde{G} = \begin{bmatrix} G_s \\ I \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{7}
$$

Considering that system (2) obtained through the discretization combined with the augmented vector method (6) is a time-varying system, the complexity of the calculation is greatly increased in the aspects of analysis and synthesis. This section will model the time-delay packet loss network control system as a linear time-invariant system based on deterministic equivalence and stochastic process theory and then discuss its finite-time control problem. Based on deterministic equivalence and stochastic process theory, the time-varying system (6) can be approximately expressed as follows:

$$
\tilde{x}(k+1) = \tilde{A}\tilde{x}(k) + \tilde{B}\tilde{u}(k) + \tilde{G}w(k), \tag{8}
$$

where

$$A = \begin{bmatrix} A & \mu_1 & \cdots & \mu_{d-1} & \mu_d \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix}, B = \begin{bmatrix} \mu_0 \\ I \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\begin{aligned}
\mu_0 &= \lim_{k\longrightarrow\infty} \frac{1}{k} \sum_{j=1}^{k} \gamma_{j-i} B_0(j), \\
&= \lim_{k\longrightarrow\infty} \frac{1}{k} \sum_{j=1}^{k} \gamma_{j-i} \int_{\tau_0(j)}^{h} e^{A(h-s)} B\varphi((k+1)h - \tau_0(j))\mathrm{d}s, \\
\mu_i &= \lim_{k\longrightarrow\infty} \frac{1}{k} \sum_{j=1}^{k} \gamma_{j-i} B_i(j), \\
&= \lim_{k\longrightarrow\infty} \frac{1}{k} \sum_{j=1}^{k} \gamma_{j-i} \int_{\tau_i(j)-ih}^{\tau_{i-1}(j)-(i-1)h} e^{A(h-s)} B\varphi(h - \tau_{i-1}(j) \\
&\quad -\tau_i(j))\varphi(\tau_i(j) - \mathrm{ih})\mathrm{d}s.
\end{aligned} \tag{9}$$

The following form controller is proposed:

$$\tilde{u}(k) = K\tilde{x}(k), \tag{10}$$

where K is the control gain matrix which will be designed later. Therefore, we can derive the time-invariant closed-loop system as follows:

$$\tilde{x}(k+1) = (\tilde{A} + \tilde{B}K)\tilde{x}(k) + \tilde{G}w(k). \tag{11}$$

Definition 1. For the time-invariant closed-loop system (11), when external interference $w(k) = 0$, then system (11) is finite-time stability about (α, β, R, N), if

$$\tilde{x}^T(k)R\tilde{x}(k) \le \alpha^2 \Rightarrow \tilde{x}^T(k)R\tilde{x}(k) \le \beta^2, \quad k \in \{1, \ldots, N\}, \tag{12}$$

where R is the energy matrix, N is a natural number, and $0 < \alpha < \beta$.

Definition 2. For the time-invariant closed-loop system (11), when external interference $w(k) \ne 0$ and $w(k)$ satisfy

$$\sum_{k=1}^{N} w^T(k)w(k) \le d^2, \tag{13}$$

then system (11) is finite-time boundedness about (α, d, β, R, N), if

$$\tilde{x}^T(k)R\tilde{x}(k) \le \alpha^2 \Rightarrow \tilde{x}^T(k)R\tilde{x}(k) \le \beta^2, \quad k \in \{1, \ldots, N\}, \tag{14}$$

where R is the energy matrix, N is a natural number, and $0 < \alpha < \beta$.

Lemma 1 (see [23]). *Schur complement theorem). For the given symmetric matrix* $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, *the following three conditions are equivalent:*

(1) $S < 0$
(2) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$
(3) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$

3. Stability Analysis

This section mainly discusses the finite-time stability about (α, β, R, N) and finite-time boundedness about (α, d, β, R, N) of system (11) when the control gain matrix K is known.

Theorem 1. *Given the state feedback control gain matrix K, if there are positive definite matrices* P_1 *and* P_2, *scalar* $\gamma \ge 1$, *such that the following linear matrix inequality holds*

$$\begin{bmatrix} (\tilde{A} + \tilde{B}K)^T P_1 (\tilde{A} + \tilde{B}K) - \gamma P_1 & (\tilde{A} + \tilde{B}K)^T P_1 \tilde{G} \\ \tilde{G}^T P_1 (\tilde{A} + \tilde{B}K) & \tilde{G}^T P_1 \tilde{G} - \gamma P_2 \end{bmatrix} < 0, \tag{15}$$

$$\frac{\lambda_2}{\lambda_1}\gamma^N \alpha^2 + \frac{\lambda_3}{\lambda_1}\gamma^N d^2 < \beta^2, \tag{16}$$

where

$$\lambda_1 = \lambda_{\min}(\tilde{P}_1), \lambda_2 = \lambda_{\max}(\tilde{P}_1), \lambda_3 = \lambda_{\max}(P_2), \tilde{P}_1 = R^{-(1/2)} P_1 R^{-(1/2)}, \tag{17}$$

then system (11) is finite-time bounded with respect to (α, d, β, R, N).

Proof. Select the Lyapunov function as follows:

$$V(\tilde{x}(k)) = \tilde{x}^T(k) P_1 x(k). \tag{18}$$

Thus, we can obtain

$$\begin{aligned}
V(\tilde{x}(k+1)) &= \tilde{x}^T(k+1) P_1 \tilde{x}(k+1) \\
&= ((\tilde{A} + \tilde{B}K)\tilde{x}(k) + \tilde{G}\tilde{w}(k))^T P_1 \\
&\quad ((\tilde{A} + \tilde{B}K)\tilde{x}(k) + \tilde{G}\tilde{w}(k)) \\
&= \begin{bmatrix} \tilde{x}(k) \\ \tilde{w}(k) \end{bmatrix}^T \begin{bmatrix} (\tilde{A} + \tilde{B}K)^T P_1 (\tilde{A} + \tilde{B}K) & (\tilde{A} + \tilde{B}K)^T P_1 \tilde{G} \\ \tilde{G}^T P_1 (\tilde{A} + \tilde{B}K) & \tilde{G}^T P_1 \tilde{G} \end{bmatrix} \\
&\quad \cdot \begin{bmatrix} \tilde{x}(k) \\ \tilde{w}(k) \end{bmatrix}.
\end{aligned} \tag{19}$$

It can be obtained from condition (15) that

$$V(\tilde{x}(k+1)) \le \gamma V(\tilde{x}(k)) + \gamma \tilde{w}^T(k) P_2 \tilde{w}(k). \tag{20}$$

Repeated use of condition (20) can be obtained:

$$V(\tilde{x}(k)) \le \gamma^k V(\tilde{x}(0)) + \sum_{j=1}^{k} \gamma^j \tilde{\omega}^T(k-j)P_2\tilde{\omega}(k-j)$$
$$= \gamma^k\left(V(\tilde{x}(0)) + \sum_{j=1}^{k}\gamma^{j-k}\tilde{\omega}^T(k-j)P_2\tilde{\omega}(k-j)\right)$$
$$\le \gamma^k\left(V(\tilde{x}(0)) + \lambda_3\sum_{j=1}^{k}\gamma^{j-k}\tilde{\omega}^T(k-j)\tilde{\omega}(k-j)\right). \tag{21}$$

Due to the fact that$\gamma \ge 1$, we have

$$V(\tilde{x}(k)) \le \gamma^k\left(V(\tilde{x}(0)) + \lambda_3\sum_{j=1}^{k}\gamma^{j-k}\tilde{\omega}^T(k-j)\tilde{\omega}(k-j)\right)$$
$$\le \gamma^N\left(\lambda_2\alpha^2 + \lambda_3 d^2\right). \tag{22}$$

Note that

$$V(\tilde{x}(k)) = \tilde{x}^T(k)P_1\tilde{x}(k) \ge \lambda_1\tilde{x}^T(k)R\tilde{x}(k). \tag{23}$$

By using (22) and (23), we can derive that

$$\tilde{x}^T(k)R\tilde{x}(k) \le \frac{\lambda_2}{\lambda_1}\gamma^N\alpha^2 + \frac{\lambda_3}{\lambda_1}\gamma^N d^2 < \beta^2. \tag{24}$$

It can be seen from Definition 2 that system (11) is finite-time bounded about (α, d, β, R, N).

4. Controller Design

Based on the stability analysis results obtained in Section 3, this section mainly discusses how to design the a suitable controller to make the closed-loop delay packet loss network control system (11) finite-time stability about (α, β, R, N) and finite-time boundedness about (α, d, β, R, N). The design of the controller is given by the following theorem.

Theorem 2. *If there exist positive definite matrices Q_1 and Q_2, matrix L, scalar $\varepsilon > 0$ and $\gamma \ge 1$, such that the following linear matrix inequalities hold*

$$\begin{bmatrix} -\gamma Q_1 & 0 & \left(\tilde{A}Q_1 + \tilde{B}L\right)^T \\ 0 & -\gamma Q_2 & \tilde{G}^T \\ \tilde{A}Q_1 + \tilde{B}L & \tilde{G} & -Q_1 \end{bmatrix} < 0, \tag{25}$$

$$\frac{\lambda_5}{\lambda_4}\gamma^N\alpha^2 + \lambda_5\lambda_6\gamma^N d^2 < \beta^2, \tag{26}$$

where

$$\lambda_4 = \lambda_{\min}\left(\tilde{Q}_1\right),\ \lambda_5 = \lambda_{\max}\left(\tilde{Q}_1\right),\ \lambda_6 = \lambda_{\max}(Q_2),\ \tilde{Q}_1 = R^{1/2}Q_1R^{1/2}, \tag{27}$$

then the control gain matrix $\tilde{K} = LQ_1^{-1}$ can make the closed-loop system (11) finite-time boundedness with respect to (α, d, β, R, N).

Proof. On the one hand, in Theorem 1, let $Q_1 = P_1^{-1}$ and $Q_2 = P_2$, then it is not difficult to conclude that condition (16) and condition (26) are equivalent. On the other hand, let $\hat{A} = \tilde{A} + \tilde{B}K$, combined with $Q_1 = P_1^{-1}$ and $Q_2 = P_2$, condition (15) can be rewritten as the following form:

$$\begin{bmatrix} \hat{A}^T Q_1^{-1}\hat{A} - \gamma Q_1^{-1} & \hat{A}^T Q_1^{-1}\tilde{G} \\ \tilde{G}^T Q_1^{-1}\hat{A} & \tilde{G}^T Q_1^{-1}\tilde{G} - \gamma Q_2 \end{bmatrix} < 0. \tag{28}$$

Multiplying the left and right matrices by $\begin{bmatrix} Q_1 & 0 \\ 0 & I \end{bmatrix}$ and $\begin{bmatrix} Q_1 & 0 \\ 0 & I \end{bmatrix}^T$ at both ends of equation (28), respectively, we have

$$\begin{bmatrix} Q_1\hat{A}^T Q_1^{-1}\hat{A}Q_1 - \gamma Q_1 & Q_1\hat{A}^T Q_1^{-1}\tilde{G} \\ \tilde{G}^T Q_1^{-1}\hat{A}Q_1 & \tilde{G}^T Q_1^{-1}\tilde{G} - \gamma Q_2 \end{bmatrix} < 0. \tag{29}$$

From Lemma 1, it can be seen that (29) is equivalent to

$$\begin{bmatrix} Q_1\hat{A}^T Q_1^{-1}\hat{A}Q_1 - \gamma Q_1 & Q_1\hat{A}^T Q_1^{-1}\tilde{G} & 0 \\ \tilde{G}^T Q_1^{-1}\hat{A}Q_1 & \tilde{G}^T Q_1^{-1}\tilde{G} - \gamma Q_2 & \tilde{G}^T \\ 0 & \tilde{G} & -Q_1 \end{bmatrix} < 0. \tag{30}$$

Multiplying the left and right matrices by $\begin{bmatrix} I & 0 & -Q_1\hat{A}^T Q_1^{-1} \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$ and $\begin{bmatrix} I & 0 & -Q_1\hat{A}^T Q_1^{-1} \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}^T$ at both ends of equation (30), respectively, we can obtain

$$\begin{bmatrix} -\gamma Q_1 & 0 & Q_1\hat{A}^T \\ 0 & -\gamma Q_2 & \tilde{G}^T \\ \hat{A}Q_1 & \tilde{G} & -Q_1 \end{bmatrix} < 0. \tag{31}$$

Let $KQ_1 = L$, that is, $K = LQ_1^{-1}$, it can be obtained that condition (26) and the condition (31) are equivalent.

Remark 1. A system that is stable in the sense of Lyapunov may have very bad transient performance (such as excessive overshoot), and sometimes it is not even applicable in engineering. On the other hand, the system is not Lyapunov asymptotically stable, or even without equilibrium, but the system can maintain good performance within a limited time interval.

Remark 2. For the finite-time control of the network control system, on the one hand, due to the lack of effective tools to test the finite-time stability, people's interest is mainly focused on the classic Lyapunov stability. On the other hand, the limited time control research on networked control systems is still in its infancy, and there are still many problems to be solved.

Remark 3. In reality, the controlled object is inevitably subject to many disturbances from the outside world, so this article not only discusses the limited time control problem of the network control system free from external disturbance but also studies the limited time control problem of the network control system subject to external disturbance. On the other hand, in the modeling process of the network control system, we can regard the network delay and network packet loss as the external disturbance of the system. Therefore, the control method proposed in this paper is very effective for solving the limited time control of the network control system.

5. Numerical Example

Consider the following network control system with time delay and packet loss:

$$\dot{x}(t) = \begin{bmatrix} 1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{bmatrix} u(t). \tag{32}$$

Assuming the upper bound of the delay $\kappa = 2$, the average value of the delay from the sensor to the controller $E\{\tau_{sc}\} = 0.08$ s, and the average delay from the controller to the actuator $E\{\tau_{ca}\} = 0.15$ s, the data packet loss obeys the Bernoulli distribution $p = 0.3$, and the sampling period of the sensor is $h = 0.1$ s, according to the principle of determining equivalence, the coefficient matrix of the discrete system is obtained as follows:

$$\begin{aligned} \widetilde{A} &= \begin{bmatrix} 1.48 & 0.04 & 1.47 & -1.23 & 0.02 & -0.19 & 0.01 & -0.1 \\ -0.10 & 0.45 & -0.13 & 0.20 & 0.46 & 0.01 & 0.15 & 0 \\ 0.07 & 0.54 & -0.18 & 1.09 & 0.36 & -0.22 & 0.16 & -0.07 \\ -0.18 & 0.54 & -1.33 & 2.21 & 0.35 & -0.03 & 0.16 & -0.02 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \widetilde{B} &= \begin{bmatrix} 0.0056 & 0.2361 & 0.1338 & 0.1337 & 1 & 0 & 0 & 0 \\ -0.0636 & 0.0014 & -0.1170 & -0.0110 & 0 & 1 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{33}$$

Select $\alpha = 1$, $\beta = 2.8$, $R = I$, $N = 10$, and $\gamma = 1$, using Theorem 2, we can get the following control gain matrix:

$$K = \begin{bmatrix} -0.0749 & -0.4731 & 0.5115 & -1.0385 & -0.4314 & 0.0808 & -0.1493 & 0.0317 \\ 0.8835 & 0.1122 & 1.1682 & -0.9149 & 0.1210 & -0.2687 & 0.0322 & -0.0744 \end{bmatrix}. \tag{34}$$

6. Conclusions

This paper uses the linear matrix inequality method and the principle of deterministic equivalence to study the finite-time stabilization problem of a class of network systems with time-varying delay and random packet loss. Fully considered the impact of delay and packet loss on system performance, explored the finite-time stabilization condition of the system, and transformed the condition and controller design condition into a linear matrix inequality form, which can be solved easily by using MATLAB toolbox. Therefore, the research results can be easily applied to the actual network system to ensure the stability of the system within a limited time and have relatively important application value. The obtained method is an effective method to deal with the control problem of the time-delay network system. Finally, numerical examples are used to verify the validity and feasibility of the obtained results. In the future, we will study finite-time control of fractional nonlinear networked control systems.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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