

Research Article

Prescribed Performance Synchronization Control of Chaotic Systems with Unknown Control Gain Signs

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For a class of uncertain nonlinear chaotic systems with unknown control gain signs and saturated input, by means of Nussbaum function, a scheme of finite-time prescribed performance synchronization control is proposed. Here, Nussbaum function is used to eliminate the influence of unknown control gain signs, and fuzzy logic systems are used to estimate unknown functions. Lyapunov theory is used to prove that all synchronization errors converge to a predefined small performance range under the designed control method. Finally, simulation results are provided to illustrate the feasibility of the proposed method.

1. Introduction

Since chaotic synchronization was first discovered in 1990, chaotic synchronization has always been a concern to scholars. Chaotic synchronization control is widely used in engineering, communication, and other fields. For different chaotic systems, many scholars have proposed some synchronization control schemes [1–10], such as fuzzy control, sliding mode control, backstepping control, adaptive control, and robust control. For example, based on the Takagi–Sugeno fuzzy model, Kumar and Khan [4] designed the adaptive synchronization of chaotic systems by utilizing linear matrix inequality. For the synchronization problems of fractional-order chaotic systems, Tabasi and Balochian [7] designed a fractional-order adaptive sliding mode controller. For chaotic systems, Dai et al. [10] designed a noise-suppression zeroing neural network, which has better synchronization control performance. As we all know, nonlinear systems will inevitably have uncertainties and external disturbances, if the systems also have unknown control gain signs and input saturation, then the controller design will become more challenging.

To eliminate the influence of unknown control gain signs, many scholars have proved the effectiveness of the

Nussbaum-type function method [11–18]. For example, Boulkroune [12] constructed a direct adaptive controller by using the Nussbaum function and the adaptive fuzzy system. Khettab et al. [13] introduced fractional Nussbaum gain to deal with unknown control directions and used the fuzzy logic set to identify the fractional-order chaotic system. Chen [16] introduced a heightened version of the Nussbaum function and discussed the necessity of using it. Using backstepping technique and Nussbaum gain method, Zhang et al. [18] proposed an adaptive control method. However, the Nussbaum gain function can only ensure the boundedness of the signal in the closed-loop system.

For the past few years, many literatures have studied prescribed performance control (PPC) schemes for nonlinear systems with unknown control directions [19–30]. A new finite time performance function is given for uncertain nonlinear systems in [20]. Wang and Yang [22] proposed a prescribed performance feedback tracking control scheme. Shi et al. [23] designed adaptive fuzzy predetermined performance controller by using symmetric matrix decomposition technique. Zhang and Yang constructed a state feedback fault-tolerant control method by introducing a new error transformation function in [25]. In [29], Xiang and Liu transformed the controlled system into an equivalent system

and constructed a fuzzy adaptive-prescribed performance control technique to make tracking error draw near the predefined neighborhood.

Inspired by the above works, for uncertain nonlinear chaotic systems with unknown control gain signs and input saturation, we complete the following works. (1) The synchronization of two uncertain chaotic systems is realized. (2) The synchronous controller is constructed by Nussbaum function, fuzzy logic systems, and finite time-prescribed performance function.

This brief is organized as follows. In Section 2, the description of problem and preparatory works are given, which is mainly to deduce the described system and give some assumptions and lemmas. In Section 3, a prescribed performance synchronization control scheme of chaotic systems is addressed. In Section 4, the simulation results are given. A short conclusion is presented in Section 5.

2. System Descriptions and Preparatory Works

The main system considered in this paper is

$$\begin{cases} \dot{x}_1(t) = f_1(x) \\ \dot{x}_2(t) = f_2(x) \\ \dots \\ \dot{x}_n(t) = f_n(x), \end{cases} \quad (1)$$

where $x_i(t)$ represents the state of master system, $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T$ is the state vector of the master system, and $f_i(x)$ is unknown smooth function, $i = 1, 2, \dots, n$.

The corresponding slave system is described by

$$\begin{cases} \dot{y}_1(t) = g_1(y) + d_1(t) + \lambda_1 \text{sat}(u_1(t)) \\ \dot{y}_2(t) = g_2(y) + d_2(t) + \lambda_2 \text{sat}(u_2(t)) \\ \dots \\ \dot{y}_n(t) = g_n(y) + d_n(t) + \lambda_n \text{sat}(u_n(t)), \end{cases} \quad (2)$$

where $y_i(t)$ represents the state of slave system, $d_i(t)$ is unknown external disturbance, $g_i(y)$ is unknown smooth function, λ_i is control gain with unknown sign, and $\text{sat}(u_i(t))$ is the control input affected by saturation type nonlinearity, $i = 1, 2, \dots, n$. Here, $\text{sat}(u_i(t))$ is described as

$$\text{sat}(u_i(t)) = \begin{cases} M \text{sign}(u_i(t)), & |u_i(t)| \geq M, \\ u_i(t), & |u_i(t)| < M. \end{cases} \quad (3)$$

A fuzzy adaptive PPC scheme is designed as the objective of this paper to meet the following requirements:

P1) The synchronization error $e_i = y_i - x_i$ can be limited within predefined range

P2) The designed controller can ensure the boundedness of all signals of the closed-loop system

We make the following assumptions to meet our objective.

Assumption 1. $g_i(y)$ and $f_i(x)$ are unknown but bounded.

Assumption 2. $d_i(t)$ is a bounded external disturbance.

Assumption 3. The input coefficient λ_i is unknown.

Let $e = [e_1, e_2, \dots, e_n]^T$, and one gets the synchronization error dynamic system:

$$\begin{cases} \dot{e}_1 = g_1(y) + d_1(t) + \lambda_1 \text{sat}(u_1(t)) - f_1(x) \\ \dot{e}_2 = g_2(y) + d_2(t) + \lambda_2 \text{sat}(u_2(t)) - f_2(x) \\ \dots \\ \dot{e}_n = g_n(y) + d_n(t) + \lambda_n \text{sat}(u_n(t)) - f_n(x). \end{cases} \quad (4)$$

The Nussbaum function $N(\cdot)$ is given, which has the following properties:

$$\begin{cases} \lim_{t \rightarrow +\infty} \sup \frac{1}{t} \int_0^t N(\xi) d\xi = +\infty, \\ \lim_{t \rightarrow +\infty} \inf \frac{1}{t} \int_0^t N(\xi) d\xi = -\infty, \end{cases} \quad (5)$$

where $N(\cdot)$ is used to handle unknown control gain signs, and the following lemma will be used in conjunction with Nussbaum function.

Lemma 1 (see [29]). *Let $N(\cdot)$ be an even Nussbaum function and $V(t)$ and $\zeta(t)$ be smooth function with $V(t) \geq 0$, $t \in [0, t_f]$. The following inequality holds:*

$$0 \leq V(t) \leq a_0 + e^{-a_1 t} \int_0^t [\omega(x(\tau))N(\zeta) + 1] \dot{\zeta} e^{a_1 \tau} d\tau, \quad (6)$$

and then, $V(t)$, $\int_0^t \omega(x(\tau))N(\zeta)\dot{\zeta} d\tau$, and $\zeta(t)$ must be bounded on $t \in [0, t_f]$. $a_1 > 0$, and a_0 and a_1 are suitable constants, and $\omega(x(t))$ is defined on $D = [h^-, h^+]$ with $0 \notin D$.

3. Control Design and Stability Analysis

The finite-time performance constraints are set as follows to maintain the transient and steady-state performance of e_i :

$$-\Upsilon_i(t) < e_i < \Upsilon_i(t), \quad i = 1, 2, \dots, n, \quad (7)$$

where $\Upsilon_i(t)$ is defined as

$$\Upsilon_i(t) = \begin{cases} \left(\gamma_{i,0} - \frac{t}{t_f} \right) e^{(1 - (t_f/(t_f-t)))} + \gamma_{i,1}, & 0 \leq t < t_f, \\ \gamma_{i,1}, & t_f \leq t < +\infty, \end{cases} \quad (8)$$

where $\gamma_{i,0}$ and $\gamma_{i,1}$ are positive design parameters.

By introducing the error transformation function $\Omega_i(\rho_i)$, we transform the constrained synchronization error system (4) into an unrestraint equivalent transformation system.

Select the error transformation function as $\Omega_i(\rho_i) = (e^{\rho_i} - e^{-\rho_i}) / (e^{\rho_i} + e^{-\rho_i})$; let

$$e_i = \Upsilon_i(t)\Omega_i(\rho_i), \quad (9)$$

where ρ_i is transformation error, $i = 1, 2, \dots, n$.

Lemma 2 (see [29]). e_i can be limited to (7) if ρ_i is bounded.

According to (9), one has

$$\dot{\rho}_i = r_i(\dot{e}_i + \Xi_i), \quad (10)$$

where $r_i = 1/(\Upsilon_i(t)(\partial\Omega_i(\rho_i)/\partial\rho_i))$ and $\Xi_i = -\Upsilon_i(t)\Omega_i(\rho_i)$, $i = 1, 2, \dots, n$.

Using fuzzy logic systems (FLSs), let

$$\begin{aligned} g_i(y) &= \eta_{g_i}^{*T} \phi_{g_i}(y) + \rho_{g_i}(y), \\ f_i(x) &= \eta_{f_i}^{*T} \phi_{f_i}(x) + \rho_{f_i}(x), \end{aligned} \quad (11)$$

where $\eta_{g_i}^*$ and $\eta_{f_i}^*$ are the optimal approximation vectors, $\phi_{g_i}(y)$ and $\phi_{f_i}(x)$ are the basis function vectors, and $\rho_{g_i}(y)$ and $\rho_{f_i}(x)$ are fuzzy estimation errors which are bounded.

Substituting (4) and (11) into (10), then we have

$$\begin{aligned} \dot{\rho}_i &= r_i [g_i(y) - f_i(x) + d_i(t) + \lambda_i \text{sat}(u_i) + \Xi_i] \\ &= r_i [\eta_{g_i}^{*T} \phi_{g_i}(y) + \rho_{g_i}(y) - \eta_{f_i}^{*T} \phi_{f_i}(x) - \rho_{f_i}(x) + d_i(t) + (\lambda_i \text{sat}(u_i) - \lambda_i u_i) + \lambda_i u_i + \Xi_i] \\ &= r_i [\eta_{g_i}^{*T} \phi_{g_i}(y) - \eta_{f_i}^{*T} \phi_{f_i}(x) + \bar{d}_i(t) + \lambda_i u_i + \Xi_i], \end{aligned} \quad (12)$$

where $\bar{d}_i(t) = d_i(t) + \rho_{g_i}(y) - \rho_{f_i}(x) + \lambda_i [\text{sat}(u_i) - u_i]$ and $|\bar{d}_i(t)| \leq \bar{d}_i^*$, $\bar{d}_i^* > 0$ is an unknown constant.

Then, the error transformation dynamic system can be given as

$$\begin{cases} \dot{\rho}_1(t) = r_1 [\eta_{g_1}^{*T} \phi_{g_1}(y) - \eta_{f_1}^{*T} \phi_{f_1}(x) + \bar{d}_1(t) + \lambda_1 u_1 + \Xi_1] \\ \dot{\rho}_2(t) = r_2 [\eta_{g_2}^{*T} \phi_{g_2}(y) - \eta_{f_2}^{*T} \phi_{f_2}(x) + \bar{d}_2(t) + \lambda_2 u_2 + \Xi_2] \\ \dots \\ \dot{\rho}_n(t) = r_n [\eta_{g_n}^{*T} \phi_{g_n}(y) - \eta_{f_n}^{*T} \phi_{f_n}(x) + \bar{d}_n(t) + \lambda_n u_n + \Xi_n]. \end{cases} \quad (13)$$

According to (13), the controller is designed as

$$u_i = u_{i1} + u_{i0},$$

$$u_{i1} = \frac{1}{\lambda_i} (\tilde{\eta}_{f_i}^T \phi_{f_i}(x) - \tilde{\eta}_{g_i}^T \phi_{g_i}(y) - k_i \rho_i - \Xi_i),$$

$$u_{i0} = N(\zeta_i) \bar{u}_{i0}, \quad (14)$$

$$\bar{u}_{i0} = \hat{\bar{d}}_i,$$

$$\dot{\zeta}_i = |\rho_i| \cdot \bar{u}_{i0}.$$

The parameter adaptation laws $\dot{\tilde{\eta}}_{g_i}$, $\dot{\tilde{\eta}}_{f_i}$, and $\dot{\hat{\bar{d}}}_i$ are given as

$$\begin{cases} \dot{\tilde{\eta}}_{g_i} = k_{g_i} (\rho_i \phi_{g_i}(y) - b_{g_i} \tilde{\eta}_{g_i}), \\ \dot{\tilde{\eta}}_{f_i} = k_{f_i} (-\rho_i \phi_{f_i}(x) - b_{f_i} \tilde{\eta}_{f_i}), \\ \dot{\hat{\bar{d}}}_i = k_{d_i} (|\rho_i| - b_{d_i} \hat{\bar{d}}_i), \end{cases} \quad (15)$$

where k_{g_i} , k_{f_i} , k_{d_i} , b_{g_i} , b_{f_i} , and b_{d_i} are all positive constants.

We construct Lyapunov function as

$$V = \frac{1}{2r_i} \rho_i^2 + \frac{1}{2} \tilde{\eta}_{g_i}^T k_{g_i}^{-1} \tilde{\eta}_{g_i} + \frac{1}{2} \tilde{\eta}_{f_i}^T k_{f_i}^{-1} \tilde{\eta}_{f_i} + \frac{1}{2k_{d_i}} \hat{\bar{d}}_i^2, \quad (16)$$

where $\tilde{\eta}_{g_i} = \eta_{g_i}^* - \hat{\eta}_{g_i}$, $\tilde{\eta}_{f_i} = \eta_{f_i}^* - \hat{\eta}_{f_i}$, and $\tilde{\bar{d}}_i = \bar{d}_i^* - \hat{\bar{d}}_i$.

Since

$$\begin{aligned} \tilde{\eta}_{g_i}^T \tilde{\eta}_{g_i} &= \tilde{\eta}_{g_i}^T (\eta_{g_i}^* - \tilde{\eta}_{g_i}) = \tilde{\eta}_{g_i}^T \eta_{g_i}^* - \tilde{\eta}_{g_i}^T \tilde{\eta}_{g_i} \\ &\leq -\frac{1}{2} \tilde{\eta}_{g_i}^T \tilde{\eta}_{g_i} + \frac{1}{2} \|\eta_{g_i}^*\|^2, \end{aligned} \quad (17)$$

similarly, we have

$$\begin{aligned} \tilde{\eta}_{f_i}^T \tilde{\eta}_{f_i} &= \tilde{\eta}_{f_i}^T (\eta_{f_i}^* - \tilde{\eta}_{f_i}) = \tilde{\eta}_{f_i}^T \eta_{f_i}^* - \tilde{\eta}_{f_i}^T \tilde{\eta}_{f_i} \\ &\leq -\frac{1}{2} \tilde{\eta}_{f_i}^T \tilde{\eta}_{f_i} + \frac{1}{2} \|\eta_{f_i}^*\|^2, \\ \tilde{\bar{d}}_i \hat{\bar{d}}_i &= \tilde{\bar{d}}_i (\bar{d}_i^* - \tilde{\bar{d}}_i) = -\tilde{\bar{d}}_i^2 + \tilde{\bar{d}}_i \bar{d}_i^* \\ &\leq -\frac{1}{2} \tilde{\bar{d}}_i^2 + \frac{1}{2} \bar{d}_i^{*2}. \end{aligned} \quad (18)$$

Therefore, \dot{V} can be obtained as

$$\begin{aligned}
\dot{V} &= -\frac{\dot{r}_i}{2r_i^2}\rho_i^2 + \frac{1}{r_i}\rho_i\dot{\rho}_i + \tilde{\eta}_{g_i}^T k_{g_i}^{-1}\dot{\tilde{\eta}}_{g_i} + \tilde{\eta}_{f_i}^T k_{f_i}^{-1}\dot{\tilde{\eta}}_{f_i} + \frac{1}{k_{d_i}}\tilde{d}_i\dot{\tilde{d}}_i \\
&= -\frac{\dot{r}_i}{2r_i^2}\rho_i^2 + \rho_i[\eta_{g_i}^{*T}\phi_{g_i}(y) - \eta_{f_i}^{*T}\phi_{f_i}(x) + \bar{d}_i(t) + \lambda_i u_i + \Xi_i] \\
&\quad + \tilde{\eta}_{g_i}^T k_{g_i}^{-1}\dot{\tilde{\eta}}_{g_i} + \tilde{\eta}_{f_i}^T k_{f_i}^{-1}\dot{\tilde{\eta}}_{f_i} + \frac{1}{k_{d_i}}\tilde{d}_i\dot{\tilde{d}}_i \\
&\leq -\frac{\dot{r}_i}{2r_i^2}\rho_i^2 - k_i\rho_i^2 + \rho_i\tilde{\eta}_{g_i}^T\phi_{g_i}(y) - \rho_i\tilde{\eta}_{f_i}^T\phi_{f_i}(x) + \lambda_i\rho_i N(\zeta_i)\bar{u}_{i0} + |\rho_i|\bar{d}_i^* \\
&\quad + \tilde{\eta}_{g_i}^T k_{g_i}^{-1}\dot{\tilde{\eta}}_{g_i} + \tilde{\eta}_{f_i}^T k_{f_i}^{-1}\dot{\tilde{\eta}}_{f_i} + \frac{1}{k_{d_i}}\tilde{d}_i\dot{\tilde{d}}_i \\
&\leq -\frac{\dot{r}_i}{2r_i^2}\rho_i^2 - k_i\rho_i^2 + \rho_i\tilde{\eta}_{g_i}^T\phi_{g_i}(y) - \rho_i\tilde{\eta}_{f_i}^T\phi_{f_i}(x) + \lambda_i N(\zeta_i)\dot{\zeta}_i + |\rho_i|\widehat{\bar{d}}_i + |\rho_i|\widetilde{\bar{d}}_i \\
&\quad - \tilde{\eta}_{g_i}^T(\rho_i\phi_{g_i}(y) - b_{g_i}\widehat{\eta}_{g_i}) - \tilde{\eta}_{f_i}^T(-\rho_i\phi_{f_i}(x) - b_{f_i}\widehat{\eta}_{f_i}) - \widetilde{\bar{d}}_i(|\rho_i| - b_{d_i}\widehat{\bar{d}}_i) \\
&= -\frac{\dot{r}_i}{2r_i^2}\rho_i^2 - k_i\rho_i^2 + [\lambda_i N(\zeta_i) + 1]\dot{\zeta}_i + \tilde{\eta}_{g_i}^T b_{g_i}\widehat{\eta}_{g_i} + \tilde{\eta}_{f_i}^T b_{f_i}\widehat{\eta}_{f_i} + b_{d_i}\widetilde{\bar{d}}_i\widehat{\bar{d}}_i \\
&\leq -\left(\frac{\dot{r}_i}{2r_i^2} + k_i\right)\rho_i^2 + [\lambda_i N(\zeta_i) + 1]\dot{\zeta}_i - \frac{1}{2}b_{g_i}\tilde{\eta}_{g_i}^T\tilde{\eta}_{g_i} - \frac{1}{2}b_{f_i}\tilde{\eta}_{f_i}^T\tilde{\eta}_{f_i} \\
&\quad + \frac{1}{2}b_{g_i}\|\eta_{g_i}^*\|^2 + \frac{1}{2}b_{f_i}\|\eta_{f_i}^*\|^2 - \frac{1}{2}b_{d_i}\widetilde{\bar{d}}_i^2 + \frac{1}{2}b_{d_i}\bar{d}_i^{*2},
\end{aligned} \tag{19}$$

where $b = \min\{2r_i((\dot{r}_i/2r_i^2) + k_i), b_{g_i} \cdot \lambda_{\min}(k_{g_i}), b_{f_i} \cdot \lambda_{\min}(k_{f_i}), b_{d_i}k_{d_i}\}$, and $R_1 = (1/2)b_{g_i}\|\eta_{g_i}^*\|^2 + (1/2)b_{f_i}\|\eta_{f_i}^*\|^2 + (1/2)b_{d_i}\bar{d}_i^{*2}$. Then, we can obtain

$$\dot{V} \leq -bV + R_1 + [\lambda_i N(\zeta_i) + 1]\dot{\zeta}_i. \tag{20}$$

The two sides of (20) are multiplied by e^{bt} , and the result is

$$\frac{d(Ve^{bt})}{dt} \leq R_1 e^{bt} + e^{bt}[\lambda_i N(\zeta_i) + 1]\dot{\zeta}_i. \tag{21}$$

By directly integrating the above inequalities, we obtain

$$\begin{aligned}
V(t) &\leq V(0)e^{-bt} + \frac{R_1}{b}(1 - e^{-bt}) + e^{-bt} \int_0^t [\lambda_i N(\zeta_i) + 1] \\
&\quad \dot{\zeta}_i \cdot e^{b\tau} d\tau \\
&\leq V(0)e^{-bt} + \frac{R_1}{b} + e^{-bt} \int_0^t [\lambda_i N(\zeta_i) + 1]\dot{\zeta}_i \cdot e^{b\tau} d\tau.
\end{aligned} \tag{22}$$

From Lemma 1, we can obtain that $V(t)$, $\int_0^t [\lambda_i N(\zeta_i) + 1]\dot{\zeta}_i d\tau$, and $\zeta(t)$ are bounded on $[0, t_f]$. Similar to the discussion in [31], the boundedness of

ρ_i , $\widehat{\eta}_{g_i}$, $\widehat{\eta}_{f_i}$, and $\widehat{\bar{d}}_i$ on $[0, +\infty)$ can be obtained. Using Lemma 2, because ρ_i is bounded, so $-\Upsilon_i(t) < e_i < \Upsilon_i(t)$ can be guaranteed.

Theorem 1. Consider the master chaotic system (1) and slave system (2) with unknown control gain signs, and under the condition that assumptions 1–3 hold, the synchronization controller (14) with parameter adaptive laws (15) can guarantee the realization of objectives P1 and P2.

Remark 1. If the control gain λ_i in this paper is replaced by the function $\chi(y)$, similar conclusions can also be obtained. However, to avoid singularity, we need to replace $(1/\lambda_i)$ in u_{i1} with $(\widehat{\chi}(y)/(\varrho + \widehat{\chi}^2(y)))$, where ϱ is a small positive constant.

Remark 2. In [5], the feedback control method was employed to realize the state synchronization of two different chaotic systems, but this synchronization method can only satisfy the steady-state performance. In the actual synchronous control, it is expected that the steady-state performance and instantaneous performance can be satisfied at the same time. Therefore, the finite-time PPC method

used in this paper can achieve this aim. It can be said that the work of this paper is a continuation of [5].

Remark 3. In [19], the initial value of the traditional performance function $Y_i(t)$ needs to be satisfied:

$$-Y_i(0) < e_i(0) < \varepsilon_i Y_i(0), \quad \text{if } e_i(0) < 0, \quad (23)$$

or

$$-\varepsilon_i Y_i(0) < e_i(0) < Y_i(0), \quad \text{if } e_i(0) > 0, \quad (24)$$

where $0 \leq \varepsilon_i \leq 1$. This means that it is necessary to design the controller repeatedly to adapt to the change of the sign of initial synchronization error.

Remark 4. Theorem 1 shows that, for $t > t_f$, the synchronization error $e_i(t)$ is limited within a specific range $[-\gamma_{i,1}, \gamma_{i,1}]$. Meanwhile, the presetting time t_f can be changed according to the actual needs.

4. Numerical Simulations

Consider the classical financial system as the master system:

$$\begin{cases} \dot{x}_1 = x_3 + (x_2 - 0.6)x_1, \\ \dot{x}_2 = 1 - 0.6x_2 - x_1^2, \\ \dot{x}_3 = -x_1 - 0.9x_3. \end{cases} \quad (25)$$

The corresponding slave system is as follows:

$$\begin{cases} \dot{y}_1 = y_3 + (y_2 - 6)y_1 + d_1(t) + \lambda_1 \text{sat}(u_1), \\ \dot{y}_2 = 1 - 0.1y_2 - y_1^2 + d_2(t) + \lambda_2 \text{sat}(u_2), \\ \dot{y}_3 = -y_1 - y_3 + d_3(t) + \lambda_3 \text{sat}(u_3), \end{cases} \quad (26)$$

where

$$\lambda_1 = 1, \lambda_2 = 1, \lambda_3 = -1, d_1(t) = \sin(t), d_2(t) = 2 \sin(t), \text{ and } d_3(t) = 3 \sin(t).$$

The synchronization error is $e_i = y_i - x_i, i = 1, 2, 3$.

Let $Y_1(t) = Y_2(t) = Y_3(t) = p(t)$, where

$$p(t) = \begin{cases} \left(2.95 - \frac{t}{t_f}\right) e^{(1-(t_f/(t_f-t)))} + 0.05, & 0 \leq t < t_f, \\ 0.05, & t_f \leq t < +\infty. \end{cases} \quad (27)$$

The control inputs are designed as

$$\text{sat}(u_i) = \begin{cases} 15 \text{sign}(u_i), & |u_i| \geq 15, \\ u_i, & |u_i| < 15, i = 1, 2, 3. \end{cases} \quad (28)$$

Gaussian membership function is taken as

$$\phi(\kappa) = \exp\left[-\frac{1}{2}\left(\frac{\kappa + 7.5 - 2.5i}{1.2}\right)^2\right], \quad (29)$$

$$\kappa = x_1, x_2, x_3, y_1, y_2, y_3; i = 1, 2, 3, 4, 5, 6, 7,$$

where the fuzzy set is defined on the interval $[-5, 5]$.

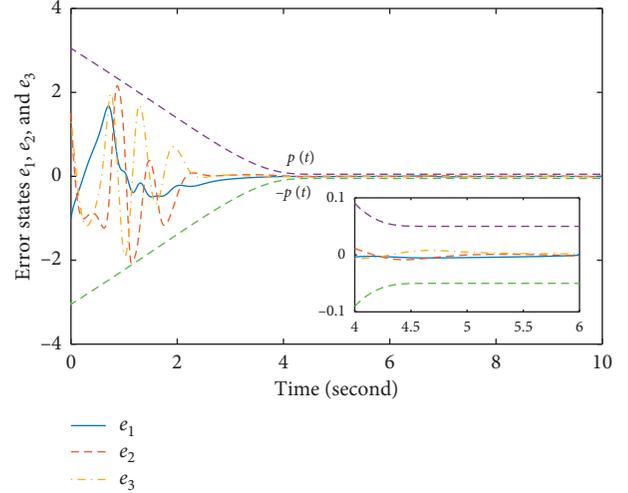


FIGURE 1: Trajectories of synchronization errors e_1, e_2 , and e_3 at $t_f = 5$.

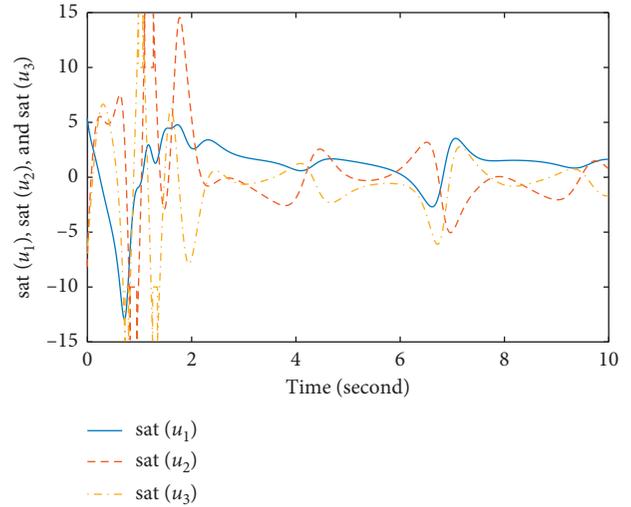


FIGURE 2: Trajectories of $\text{sat}(u_1), \text{sat}(u_2)$, and $\text{sat}(u_3)$ at $t_f = 5$.

Simulation is executed with $x(0) \equiv (2, -1, -2.5)^T, y(0) = (1, 0.5, -1.2)^T, \hat{\eta}_{g_i}(0) = \hat{\eta}_{f_i}(0) = 0, \hat{d}_i(0) = 0.1, k_{g_i} = k_{f_i} = 5, b_{g_i} = b_{f_i} = 0.1$, and $N(\zeta) = \zeta^2 \cos \zeta, \zeta(0) = 0.02$.

In order to demonstrate the effectiveness of the presented control scheme (14), we first take $t_f = 5$, and simulation results are exhibited in Figures 1 and 2.

Figure 1 gives the trajectories of synchronization errors when $t_f = 5$, and we can find that synchronization errors converge to $(-0.05, 0.05)$ within 5 s and remain in this region after 5 s. The saturation inputs are illustrated in Figure 2. We can also find that $\text{sat}(u_i)$ tends to stabilize after 3 s.

Sequentially, we take $t_f = 2$, and the comparison results are illustrated in Figures 3 and 4.

Figure 3 gives the trajectories of synchronization errors when $t_f = 2$, and we can find synchronization errors converge to $(-0.05, 0.05)$ within 2 s and remain in this region after 2 s. From Figure 4, obviously, because the limited time

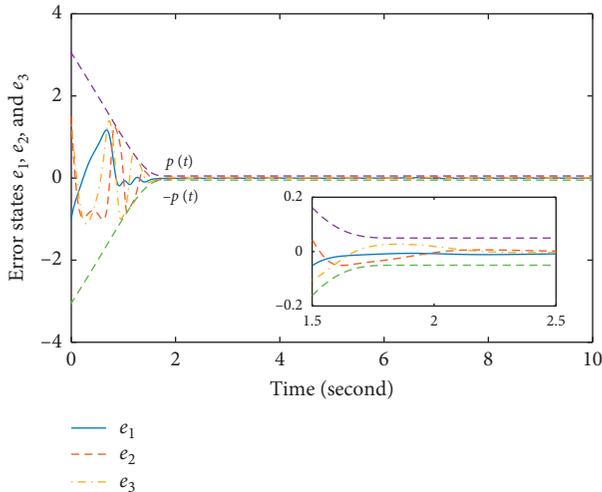


FIGURE 3: Trajectories of synchronization errors e_1 , e_2 , and e_3 at $t_f = 2$.

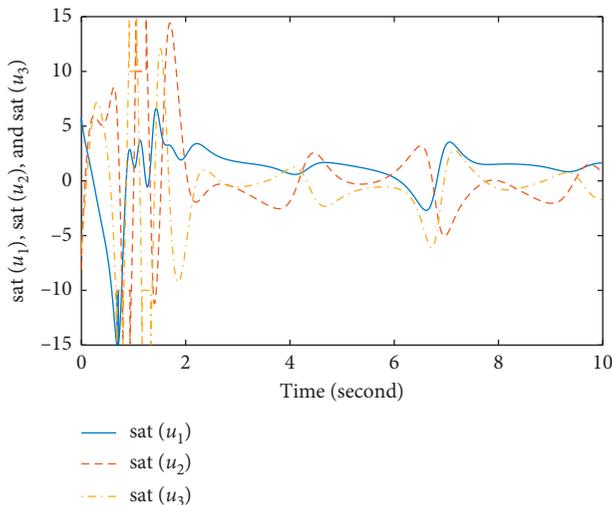


FIGURE 4: Trajectories of $\text{sat}(u_1)$, $\text{sat}(u_2)$, and $\text{sat}(u_3)$ at $t_f = 2$.

is shortened, the fluctuation of saturation input is larger than that of the previous control effect, but it still tends to be stable after 3 s.

To sum up, through the simulation results, we can find that our control time can be set according to the actual situation, and the control effect is better.

5. Conclusion

A finite-time synchronization control problem for a class of uncertain chaotic systems with unknown control direction is investigated. Based on finite-time performance function, PPC, Nussbaum function, and FLSs algorithm, an adaptive fuzzy synchronization control method is designed, which ensures that the synchronization errors satisfy the given performance and converge to the predefined region in finite time, and all the signals in the closed-loop system remain bounded. Finally, numerical simulations verify the effectiveness of the proposed control method.

Data Availability

All datasets generated for this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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