Research Article

Command-Filtered Backstepping Control for a Class of Chaotic Systems

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The problem of command-filtered backstepping control for a class of chaotic systems was studied in this paper. At each step of the backstepping design, a command filtering backstepping method is introduced to use a new virtual control signal, in which the derivative of the virtual control was approximated by using the output of filter. Then, the asymptotic stability conditions for synchronization errors are obtained by using the Lyapunov stability criterion and this control method. The design technique has the advantages of the conventional command-filtered backstepping control. An example shows the proposed method was feasible.

1. Introduction

Chaos phenomenon is a seemingly irregular and inherently random complex phenomenon in a system, which widely exists in meteorology, water culture, communications, economy, and other fields. In recent years, chaotic systems have been studied deeply, especially in the field of chaos control. Chaos control mainly includes two purposes: to suppress chaos when it is not needed in practical application and to enhance chaos when it is needed. The idea of effectively controlling chaos is to control the chaotic phenomenon in the system to an ideal state by using small changes so that the chaotic phenomenon can be used effectively. Controlling chaos is the premise of making effective use of chaos. Therefore, controlling chaotic system to achieve the expected state has become one of the research focuses of scholars [1–5].

Since the development contribution of PC in synchronizing chaotic systems, synchronizing chaotic systems has aroused great interest of people, such as adaptive feedback control, backstepping control, impulse control, sliding mode control, fuzzy control, and other methods, which have been proposed [6–12]. In [13], the trajectory tracking control problem of mobile robot is studied. The trajectory tracking problem is transformed into the stabilization problem of error system through coordinate

transformation. Based on the idea of feedback linearization and variable structure control, a composite finite time variable structure linearization tracking control is proposed. Backstepping control is one of the most common methods to deal with chaotic systems [14]. Therefore, this paper mainly uses the same method to eliminate the nonlinear term in the error to study the stability and asymptotic synchronization of chaotic systems.

It is important to note as well that the backstepping control method of chaotic systems has caught scholars' much attentions in recent years [15, 16]. The control of pure feedback nonlinear systems is studied by using backstepping adaptive control and neural network adaptive control algorithms [17]. It is well known that for all kinds of control systems, there have been plenty of research achievements in different integer-order systems with the backstepping control method. Even so, there is a particular limitation of the traditional backstepping algorithm method. The limitation is that the backstepping design procedure has the problem of "explosion of complexity" [18–20]. This means that the derivative of the virtual control function increases with the increase of the system order, which makes the calculation prohibitive [21, 22]. Based on the backstepping control method, Li and Tong [23] studied the output regulation of strict feedback uncertain control system and finally obtained the global asymptotic stability.

In this paper, we will study the finite time tracking control problem of command-filtered backstepping method for nonlinear systems. When designing each step of the backstepping, the command filter is replaced by the filter with asymptotic convergence rate, which can ensure that the output of the filter is similar to the derivative of the virtual control. At the same time, choosing proper control parameters is a better method. Compared with the proposed virtual control signal, the finite time virtual control signal can make the closed-loop system have faster response speed and higher tracking accuracy in [21, 22]. Using a coordinate transformation and based on the idea of backstepping design, the stabilization problem of a class of nonlinear systems with triangular structure is studied. Through the designed virtual control law, the controller is obtained [24].

Many results on controller design have been used to study the stability of systems with nonlinear inputs. In [25], the adaptive fuzzy control problem of single input single output nonlinear stochastic systems in the form of nonstrict feedback was investigated, in which the fuzzy logic system is used to approximate the uncertain nonlinear function, and the adaptive fuzzy controller is constructed by backstepping technology. This shows that there is still a lot of room for research integer-order systems with nonlinear inputs in the future. In a word, the study of the stability of systems with nonlinear inputs has certain theoretical value. Sui et al. [26] studied the finite time control of uncertain MIMO non-triangular random nonlinear systems and proposed the switching control strategy by using the finite time backstepping algorithm and the general Lyapunov function method. In the simulation results, the integer-order gyro system is synchronized. In [27, 28], the quasisynchronization problems of delayed stochastic multiplex networks and coupled system of N heterogeneous harmonic oscillators are discussed, respectively.

Inspired by the above discussions, the finite time tracking control problem of command-filtered backstepping method for nonlinear systems. The main contributions can be summarized as follows: (1) a command filtering backstepping method is introduced to use a new virtual control signal, in which the derivative of the virtual control was approximated by using the output of filter. (2) Compared with the proposed virtual control signal, the finite time virtual control signal can make the closed-loop system have faster response speed and higher tracking accuracy.

2. Problem Formulation and Preliminaries

Suppose the master chaotic system as,

$$\begin{cases} \dot{x}_1(t) = x_2(t) + f_1(\tilde{x}_1(t)), \\ \dot{x}_2(t) = x_3(t) + f_2(\tilde{x}_2(t)), \\ \vdots \\ \dot{x}_{n-1}(t) = x_n(t) + f_{n-1}(\tilde{x}_{n-1}(t)), \\ \dot{x}_n(t) = f_n(x(t)), \end{cases} \quad (1)$$

where $x(t) = [x_1(t), \dots, x_n(t)]^T \in R^n$ is the measurable state variable vector, $\tilde{x}_i(t) =$

$[x_1(t), x_2(t), \dots, x_i(t)]^T \in R^i, i \in (1, \dots, n)$, and $\tilde{x}_n(t) = x(t)$. The smooth nonlinear functions $f_i(\tilde{x}_i(t)) \in R$ are assumed to be known, $i \in (1, \dots, n)$.

Let the slave system be,

$$\begin{cases} \dot{y}_1(t) = y_2(t) + g_1(\tilde{y}_1(t)), \\ \dot{y}_2(t) = y_3(t) + g_2(\tilde{y}_2(t)), \\ \vdots \\ \dot{y}_{n-1}(t) = y_n(t) + g_{n-1}(\tilde{y}_{n-1}(t)), \\ \dot{y}_n(t) = g_n(y(t)) + u(t), \end{cases} \quad (2)$$

where $y(t) = [y_1(t), \dots, y_n(t)]^T \in R^n$ is the measurable state variable vector, $\tilde{y}_i(t) = [y_1(t), y_2(t), \dots, y_i(t)]^T \in R^i, i \in (1, \dots, n)$, and $\tilde{y}_n(t) = y(t)$. The smooth nonlinear functions $g_i(\tilde{y}_i(t)) \in R$ are assumed to be known, $i \in (1, \dots, n)$. $u(t) \in R$ is the control input.

Let $e(t) = [y_1(t) - x_1(t), \dots, y_n(t) - x_n(t)]^T$, then the error dynamics is as follows:

$$\begin{cases} \dot{e}_1(t) = e_2(t) + l_1(\tilde{x}_1(t), \tilde{y}_1(t)), \\ \dot{e}_2(t) = e_3(t) + l_2(\tilde{x}_2(t), \tilde{y}_2(t)), \\ \vdots \\ \dot{e}_{n-1}(t) = e_n(t) + l_{n-1}(\tilde{x}_{n-1}(t), \tilde{y}_{n-1}(t)), \\ \dot{e}_n(t) = l_n(x(t), y(t)) + u(t), \end{cases} \quad (3)$$

where $l_i(\tilde{x}_i(t), \tilde{y}_i(t)) = g_i(\tilde{y}_i(t)) - f_i(\tilde{x}_i(t)), i \in (1, \dots, n)$ and $l_n(\tilde{x}_n(t), \tilde{y}_n(t)) = l_n(x(t), y(t)) = g_n(y(t)) - f_n(x(t))$. For convenience, let us replace $l_i(\tilde{x}_i(t), \tilde{y}_i(t))$ with $l_i(\tilde{w}_i(t))$, $i \in (1, \dots, n-1)$ and $l_n(\tilde{w}_n(t)) = l_n(\tilde{w}(t)) = l_n(x(t), y(t))$.

Remark 1 (see [29]). In reality, the strict feedback system is usually expressed in the form of compact function. For example, (1) and (2) are strict feedback systems. Therefore, system (3) is also a strict feedback system. There are a lot of chaotic systems that can be expressed in (3), such as Chen system, Chua's circuit, and Arneodo system. In fact, the chaotic systems mentioned above are not strict feedback forms. However, we can rewrite it into (3) form through simple variable transformation.

Definition 1 (see [30]). If $\varphi: [0, b) \rightarrow \Xi$ is a strictly increasing continuous function and $\varphi(0) = 0$, then φ belongs to class- $\mathcal{K}$.

Lemma 1. Let

$$\dot{\mu}(\tau) = \Phi(\tau, \mu(\tau)), \quad (4)$$

where $\Phi(\tau, \mu(\tau))$ is a Lipschitz continuous nonlinear function and 0 is an equilibrium point of (4). If there exists a Lyapunov function $V(\tau, \mu(\tau))$, $\mathcal{K}$ functions $\varphi_i (i = 1, 2, 3)$ such that

$$\varphi_1(\|\mu(\tau)\|) \leq V(\tau, \mu(\tau)) \leq \varphi_2(\|\mu(\tau)\|), \quad (5)$$

$$\dot{V}(\tau, \mu(\tau)) \leq -\varphi_3(\|\mu(\tau)\|), \quad (6)$$

then (4) is asymptotically stable.

3. Control Law Design

Define the tracking errors as follows:

$$\begin{cases} z_1 = e_1, \\ z_2 = e_2 - \alpha_1^c, \\ z_3 = e_3 - \alpha_2^c, \\ \vdots \\ z_n = e_n - \alpha_{n-1}^c. \end{cases} \quad (7)$$

The finite time command filter is designed as follows [31]:

$$\begin{cases} \dot{\varphi}_{i,1} = v_{i,1}, \\ v_{i,1} = -r_{i,1} |\varphi_{i,1} - \alpha_i|^{1/2} \text{sign}(\varphi_{i,1} - \alpha_i) + \varphi_{i,2}, \\ \dot{\varphi}_{i,2} = -r_{i,2} \text{sign}(\varphi_{i,2} - v_{i,1}), \quad i = 1, \dots, n-1, \end{cases} \quad (8)$$

in which α_i is the input and $x_{i+1,c}(t) = \varphi_{i,1}(t)$ and $\dot{x}_{i+1,c}(t) = v_{i,1}(t)$ are the outputs.

Remark 2 (see [32]). Note that parameters $r_{i,1}$ and $r_{i,2}$ should be large enough; $r_{i,2}$ is selected first.

Construct the virtual signals α_i during the procedure of the finite time command-filtered backstepping control design as follows:

$$\begin{cases} \alpha_1 = -k_1 z_1 - l_1 (\tilde{w}_1(t)), \\ \alpha_2 = -k_2 z_2 - l_2 (\tilde{w}_2(t)) - z_1 + \dot{\alpha}_1^c, \\ \vdots \\ \alpha_i = -k_i z_i - l_i (\tilde{w}_i(t)) - z_{i-1} + \dot{\alpha}_{i-1}^c, \\ \alpha_n = -k_n z_n - l_n (\tilde{w}_n(t)) - z_{n-1} + \dot{\alpha}_{n-1}^c, \end{cases} \quad (9)$$

where k_i are positive constants.

Proof. Step 1: Define the Lyapunov function as

$$V_1 = \frac{1}{2} z_1^2. \quad (10)$$

Taking the derivative of V_1 yields,

$$\begin{aligned} \dot{V}_1 &= z_1 \dot{z}_1 = z_1 \dot{e}_1, \\ &= z_1 (e_2 + l_1 (\tilde{x}_1(t), \tilde{y}_1(t))) \\ &= z_1 (e_2 + l_1 (\tilde{w}_1(t)) - \alpha_1 + \alpha_1 - \alpha_1^c + \dot{\alpha}_1^c) \\ &= z_1 (z_2 - k_1 z_1 + \alpha_1^c - \alpha_1) \\ &= -k_1 z_1^2 + z_1 z_2 + z_1 (\alpha_1^c - \alpha_1). \end{aligned} \quad (11)$$

Step 2: Define the Lyapunov function as

$$V_2 = V_1 + \frac{1}{2} z_2^2. \quad (12)$$

Taking the derivative of V_2 yields,

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + z_2 \dot{z}_2 = \dot{V}_1 + z_2 (\dot{e}_2 - \dot{\alpha}_1^c), \\ &= \dot{V}_1 + z_2 (e_3 + l_2 (\tilde{w}_2(t)) - \alpha_2 + \alpha_2 - \alpha_2^c + \dot{\alpha}_2^c - \dot{\alpha}_1^c) \\ &= \dot{V}_1 + z_2 (z_3 + \alpha_2^c - \alpha_2 - k_2 z_2 - z_1) \\ &= -k_1 z_1^2 + z_1 z_2 + z_1 (\alpha_1^c - \alpha_1) - k_2 z_2^2 + z_2 z_3 \\ &\quad - z_1 z_2 + z_2 (\alpha_2^c - \alpha_2) \\ &= -k_1 z_1^2 - k_2 z_2^2 + z_2 z_3 + z_1 (\alpha_1^c - \alpha_1) + z_2 (\alpha_2^c - \alpha_2). \end{aligned} \quad (13)$$

Step k: Define the Lyapunov function as

$$V_k = V_{k-1} + \frac{1}{2} z_k^2. \quad (14)$$

Then, taking the derivative of V_k yields,

$$\begin{aligned} \dot{V}_k &= \dot{V}_{k-1} + z_k \dot{z}_k = \dot{V}_{k-1} + z_k (\dot{e}_k - \dot{\alpha}_{k-1}^c), \\ &= \dot{V}_{k-1} + z_k (e_{k+1} + l_k (\tilde{w}_k(t)) - \alpha_k + \alpha_k - \alpha_k^c + \dot{\alpha}_k^c - \dot{\alpha}_{k-1}^c) \\ &= \dot{V}_{k-1} + z_k (e_{k+1} + l_k (\tilde{w}_k(t)) - \alpha_k + \alpha_k - \alpha_k^c + \dot{\alpha}_k^c - \dot{\alpha}_{k-1}^c) \\ &= \dot{V}_{k-1} + z_k (-k_k z_k + z_{k+1} - z_{k-1} + \alpha_k^c - \alpha_k) \\ &= \dot{V}_{k-1} - k_k z_k^2 + z_k z_{k+1} - z_{k-1} z_k + z_k (\alpha_k^c - \alpha_k) \\ &= -\sum_{i=1}^k k_i z_i^2 + z_k z_{k+1} + \sum_{i=1}^k z_i (\alpha_i^c - \alpha_i). \end{aligned} \quad (15)$$

Step n: For the n th subsystem, define the Lyapunov function as

$$V_n = V_{n-1} + \frac{1}{2} z_n^2. \quad (16)$$

Then, it follows that,

$$\begin{aligned} \dot{V}_n &= \dot{V}_{n-1} + z_n \dot{z}_n = \dot{V}_{n-1} + z_n (\dot{e}_n - \dot{\alpha}_{n-1}^c), \\ &= \dot{V}_{n-1} + z_n (l_n (\tilde{w}_n(t)) + u - \dot{\alpha}_{n-1}^c) \\ &= \dot{V}_{n-1} - k_n z_n^2 - z_{n-1} z_n \\ &= -\sum_{i=1}^n k_i z_i^2 + \sum_{i=1}^{n-1} z_i (\alpha_i^c - \alpha_i). \end{aligned} \quad (17)$$

Step $n+1$: Finally,

$$\forall \mu > 0, \quad \exists t_0, \quad (18)$$

where $r_{i,2}$ is large enough; we have

$$t > t_0, \quad |\alpha_i^c - \alpha_i| \leq \mu, \quad (19)$$

that means $\alpha_i^c - \alpha_i$ is bounded.

$$\dot{V}_n \leq -\sum_{i=1}^n k_i z_i^2 + \mu. \quad (20)$$

FIGURE 1: Chaotic phenomenon of integer-order gyro system (21).

FIGURE 2: Simulation results: (a) state synchronization; (b) virtual control input; (c) control input.

From (5), it satisfies the stability characteristic. Therefore, we can get the following theorem. $\square$

Theorem 1. Consider system (3). The finite time command filter and the virtual signals α_i are given in (8) and (9), respectively. If the choice of μ in (18) is large enough, then the system is asymptotically stable so that system (1) and system (2) are synchronized.

$$\begin{cases} \dot{x}_1(\tau) = x_2(\tau), \\ \dot{x}_2(\tau) = -a^2(1 - \cos x_1(\tau))^2 \sin^{-3} x_1(\tau) - c_1 x_2(\tau) - c_2 x_2^3(\tau) + b \sin x_1(\tau) + c \sin \omega \tau \sin x_1(\tau). \end{cases} \quad (21)$$

4.2. Slave System.

$$\begin{cases} \dot{y}_1(\tau) = y_2(\tau), \\ \dot{y}_2(\tau) = u(\tau) - a^2(1 - \cos y_1(\tau))^2 \sin^{-3} y_1(\tau) - c_1 y_2(\tau) - c_2 y_2^3(\tau) + b \sin y_1(\tau) + c \sin \omega \tau \sin y_1(\tau). \end{cases} \quad (22)$$

In system (21), we select the parameter as $a^2 = 100, b = 1, c_1 = 0.5, c_2 = 0.05, c = 35.5, \omega = 25$ and the initial condition $x(0) = [-1, 1]^T$, which shows chaotic phenomenon in Figure 1.

In the simulation, the simulation results are shown in Figure 2.

5. Conclusion

In this paper, a command filter backstepping controller is designed, which gives full play to the advantages of backstepping control in dealing with nonlinear systems. Firstly, the system backstepping controller is designed according to Lyapunov stability theory, and then the constrained command filter is introduced to solve the control saturation problem existing in the application of traditional backstepping control engineering, and a controller is constructed so that the synchronization error can asymptotically approach zero in a short time. Simulation results show that the algorithm has better power tracking performance and dynamic response ability and is more suitable for engineering applications.

Data Availability

The data used to support the findings of this study are available from the author upon request.

Conflicts of Interest

The author declares no conflicts of interest.

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4. Simulation Results

A simulation example is given to declare the availability of the propound controller in this section. We selected the master and slave systems as follows [33, 34].

4.1. Master System.

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