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Research Article

Unsteady MHD Mixed Convection Flow of Chemically Reacting Micropolar Fluid between Porous Parallel Plates with Soret and Dufour Effects

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The objective of the present study is to investigate the first-order chemical reaction and Soret and Dufour effects on an incompressible MHD combined free and forced convection heat and mass transfer of a micropolar fluid through a porous medium between two parallel plates. Assume that there are a periodic injection and suction at the lower and upper plates. The nonuniform temperature and concentration of the plates are assumed to be varying periodically with time. A suitable similarity transformation is used to reduce the governing partial differential equations into nonlinear ordinary differential equations and then solved numerically by the quasilinearization method. The fluid flow and heat and mass transfer characteristics for various parameters are analyzed in detail and shown in the form of graphs. It is observed that the concentration of the fluid decreases whereas the temperature of the fluid enhances with the increasing of chemical reaction and Soret and Dufour parameters.

1. Introduction

The flow through porous boundaries has many applications in science and technology such as water waves over a shallow beach, mechanics of the cochlea in the human ear, aerodynamic heating, flow of blood in the arteries, and petroleum industry. Several authors have studied theoretically the laminar flow in porous channels. Berman [1] considered the viscous fluid and analyzed the flow characteristics when it passed through the porous walls. Later the same problem for different permeability was studied by Terril and Shrestha [2]. The theory of micropolar fluids was introduced by Eringen [3] which are considered as an extension of generalized viscous fluids with microstructure. Examples for micropolar fluids include lubricants, colloidal suspensions, porous rocks, aerogels, polymer blends, and microemulsions. The same Berman problem with micropolar fluid was discussed by Sastry and Rama Mohan Rao [4]. The flow and heat transfer of micropolar fluid between two porous parallel plates was analyzed by Ojjela and Naresh Kumar [5]. Srinivasacharya et al. [6] obtained an analytical solution for the unsteady Stokes flow of micropolar fluid between two parallel plates. The effect of buoyancy parameter on flow and heat transfer of micropolar fluid between two vertical parallel plates was investigated by Maiti [7].

The study of MHD heat and mass transfer through porous boundaries has attracted many researchers in the recent past due to applications in engineering and science, such as oil exploration, boundary layer control, and MHD power generators. The steady incompressible free convection flow and heat transfer of an electrically conducting micropolar fluid in a vertical channel was studied by Bhargava et al. [8]. The laminar incompressible magnetohydrodynamic flow and heat transfer of micropolar fluid between porous disks was analyzed numerically by Ashraf and Wehgal [9]. Islam et al. [10] obtained a numerical solution for an incompressible unsteady magnetohydrodynamic flow through vertical porous medium. Nadeem et al. [11] discussed the unsteady MHD stagnation flow of a micropolar fluid through porous media. The effects of Hall and ion slip currents on micropolar fluid flow and heat and mass transfer in a porous medium between parallel plates with chemical reaction were considered by Ojjela and Naresh Kumar [12]. The MHD heat and mass transfer of micropolar fluid in a porous medium with chemical reaction and Hall and ion slip effects by considering variable viscosity and thermal diffusivity were investigated by Elgazery [13]. The mixed convection flow and heat transfer of an electrically conducting micropolar fluid over a vertical plate with Hall and ion slip effects was analyzed by Ayano [14].

When heat and mass transfer occurs simultaneously in a moving fluid, the energy flux caused by a concentration gradient is termed as diffusion thermoeffect, whereas mass fluxes can also be created by temperature gradients which is known as a thermal diffusion effect. These effects are studied as second-order phenomena and may have significant applications in areas like petrology, hydrology, and geosciences. The effect of thermophoresis on an unsteady natural convection flow and heat and mass transfer of micropolar fluid with Soret and Dufour effects was studied by Aurangzaib et al. [15]. Srinivasacharya and RamReddy [16] considered the problem of the steady MHD mixed convection heat and mass transfer of micropolar fluid through non-Darcy porous medium over a semi-infinite vertical plate with Soret and Dufour effects. Influence of the Soret and Dufour numbers on mixed convection flow and heat and mass transfer of non-Newtonian fluid in a porous medium over a vertical plate was analyzed by Mahdy [17]. Hayat and Nawaz [18] investigated analytically the effects of the Hall and ion slip on the mixed convection heat and mass transfer of second-grade fluid with Soret and Dufour effects. Rani and Kim [19] studied numerically the laminar flow of an incompressible viscous fluid past an isothermal vertical cylinder with Soret and Dufour effects. The effects of chemical reaction and Soret and Dufour on the mixed convection heat and mass transfer of viscous fluid over a stretching surface in the presence of thermal radiation were analyzed by Pal and Mondal [20]. Sharma et al. [21] studied the mixed convective flow, heat and mass transfer of viscous fluid in a porous medium past a radiative vertical plate with chemical reaction, and Soret and Dufour effects.

In the field of fluid mechanics many fluid flow problems are nonlinear boundary value problems. To solve these problems we can use a numerical technique, quasilinearization method which is a powerful technique having second-order convergence. Several authors (Lee and Fan [22], Hymavathi and Shanker [23], Huang [24], Motsa et al. [25], and Ojjela and Naresh Kumar [5, 12]) applied the quasilinearization method to solve the nonlinear boundary layer equations.

In the present study the effects of chemical reaction on two-dimensional mixed convection flow and heat transfer of an electrically conducting micropolar fluid in a porous medium between two parallel plates with Soret and Dufour have been considered. The reduced flow field equations are solved using the quasilinearization method. The effects of various parameters such as Hartmann number, inverse Darcy's parameter, Schmidt number, Prandtl number, chemical reaction rate, Soret and Dufour numbers on the velocity components, microrotation, temperature distribution, and concentration are studied in detail and presented in the form of graphs.

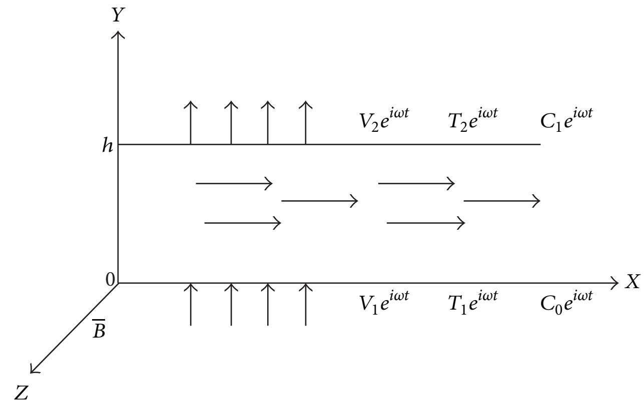


Figure 1: Schematic diagram of the fluid flow between parallel porous plates.

2. Formulation of the Problem

Consider a two-dimensional laminar incompressible micropolar fluid flow through an elongated rectangular channel, as shown in Figure 1. Assume that the fluid is injected and aspirated periodically through the plates with injection velocity $V_1e^{i\omega t}$ and suction velocity $V_2e^{i\omega t}$. Also the nonuniform temperature and concentration at the lower and upper plates are $T_1e^{i\omega t}$, $C_0e^{i\omega t}$ and $T_2e^{i\omega t}$, $C_1e^{i\omega t}$, respectively. The region inside the parallel plates is subjected to porous medium and a constant external magnetic field of strength B_0 perpendicular to the XY-plane is considered.

The governing equations of the micropolar fluid flow and heat and mass transfer in the presence of buoyancy forces, magnetic field and in the absence of body forces, body couples are given by

$$\nabla \cdot \overline{q} = 0, \tag{1}$$

$$\begin{aligned}\rho\left[\frac{\partial \overline{q}}{\partial t} + (\overline{q}\cdot\nabla)\overline{q}\right] &= -\operatorname{grad} p + k_1 \operatorname{curl} \overline{l} \\ &\quad - (\mu + k_1)\operatorname{curl}\operatorname{curl}(\overline{q}) \\ &\quad - \frac{\mu + k_1}{k_2}\overline{q} + \overline{J}\times\overline{B} + \overline{F_b},\end{aligned} \tag{2}$$

$$\begin{aligned}\rho j\left[\frac{\partial \overline{l}}{\partial t} + (\overline{q}\cdot\nabla)\overline{l}\right] &= -2k_1\overline{l} + k_1\operatorname{curl}\overline{q} \\ &\quad - \gamma\operatorname{curl}\operatorname{curl}(\overline{l}),\end{aligned} \tag{3}$$

$$\begin{aligned}\rho c\left[\frac{\partial T}{\partial t} + (\overline{q}\cdot\nabla)T\right] &= k\nabla^2 T + 2\mu D : D \\ &\quad + \frac{k_1}{2}\left(\operatorname{curl}(\overline{q}) - 2\overline{l}\right)^2 \\ &\quad + \gamma\nabla\overline{l} : \nabla\overline{l} + \frac{\mu + k_1}{k_2}\overline{q}^2 \\ &\quad + \frac{\left|\overline{J}\right|^2}{\sigma} + \frac{\rho D_1 k_T}{c_s}\nabla^2 C\end{aligned} \tag{4}$$

$$\left[\frac{\partial C}{\partial t} + (\bar{q}\cdot\nabla)C\right] = D_1\nabla^2 C - k_3\left(C - C_0 e^{i\omega t}\right) + \frac{D_1 k_T}{T_m}\nabla^2 T, \quad (5)$$

where $\overline{F_b}$ is the buoyancy force and it is defined as $(\rho g\beta_T(T - T_1 e^{i\omega t}) + \rho g\beta_C(C - C_0 e^{i\omega t}))\hat{i}$.

Neglecting the displacement currents, the Maxwell equations and the generalized Ohm's law are

$$\begin{aligned} \nabla\cdot\overline{B} &= 0,\\ \nabla\times\overline{B} &= \mu'\overline{J},\\ \nabla\times\overline{E} &= \frac{\partial\overline{B}}{\partial t},\\ \overline{J} &= \sigma\left(\overline{E} + \bar{q}\times\overline{B}\right), \end{aligned} \quad (6)$$

where $\overline{B} = B_0\hat{k} + \bar{b}$, $\bar{b}$ is induced magnetic field. Assume that the induced magnetic field is negligible compared to the applied magnetic field so that magnetic Reynolds number is small, the electric field is zero, and magnetic permeability is constant throughout the flow field.

The velocity and microrotation components are

$$\begin{aligned} \bar{q} &= u\hat{i} + v\hat{j},\\ \bar{l} &= N\hat{k}. \end{aligned} \quad (7)$$

Following Ojjela and Naresh Kumar [5, 12] the velocity and microrotation components are

$$\begin{aligned} u(x,\lambda,t) &= \left(\frac{U_0}{a} - \frac{V_2 x}{h}\right) f'(\lambda)\, e^{i\omega t},\\ v(x,\lambda,t) &= V_2 f(\lambda)\, e^{i\omega t},\\ N(x,\lambda,t) &= \left(\frac{U_0}{a} - \frac{V_2 x}{h}\right) A(\lambda)\, e^{i\omega t}. \end{aligned} \quad (8)$$

The temperature and concentration distributions can be taken as

$$\begin{aligned} &T(x,\lambda,t)\\ &\quad = \left(T_1 + \frac{\mu V_2}{\rho h c}\left[\phi_1(\lambda) + \left(\frac{U_0}{aV_2} - \frac{x}{h}\right)^2 \phi_2(\lambda)\right]\right) e^{i\omega t},\\ &C(x,\lambda,t)\\ &\quad = \left(C_0 + \frac{\dot{n_A}}{h\upsilon}\left[g_1(\lambda) + \left(\frac{U_0}{aV_2} - \frac{x}{h}\right)^2 g_2(\lambda)\right]\right) e^{i\omega t}, \end{aligned} \quad (9)$$

where $\lambda = y/h$ and $f(\lambda)$, $A(\lambda)$, $\phi_1(\lambda)$, $\phi_2(\lambda)$, $g_1(\lambda)$, and $g_2(\lambda)$ are to be determined.

The boundary conditions for the velocity, microrotation, temperature, and concentration are

$$\begin{aligned} u(x,\lambda,t) &= 0,\\ v(x,\lambda,t) &= V_1 e^{i\omega t},\\ N(x,\lambda,t) &= 0,\\ T(x,\lambda,t) &= T_1 e^{i\omega t},\\ C(x,\lambda,t) &= C_0 e^{i\omega t}\\ &\qquad \text{at } \lambda = 0\\ u(x,\lambda,t) &= 0,\\ v(x,\lambda,t) &= V_2 e^{i\omega t},\\ N(x,\lambda,t) &= 0,\\ T(x,\lambda,t) &= T_2 e^{i\omega t},\\ C(x,\lambda,t) &= C_1 e^{i\omega t}\\ &\qquad \text{at } \lambda = 1. \end{aligned} \quad (10)$$

Substituting (8) and (9) in (2), (3), (4), and (5) then we get

$$\begin{aligned} f^{IV} &= \frac{-R}{1+R}A'' + \frac{\mathrm{Re}}{1+R}\left(ff''' - f'f''\right)\cos\psi\\ &\quad + \frac{\mathrm{Ha}^2}{1+R}f'' + D^{-1}f'' - \frac{\mathrm{EcGr}}{(1+R)\zeta}\left(\phi_1' + \zeta^2\phi_2'\right)\\ &\quad - \frac{\mathrm{Sh\,Gm}}{(1+R)\zeta}\left(g_1' + \zeta^2 g_2'\right),\\ J_1\left(fA' - f'A\right)\cos\psi &= -s_1\left(2A + f''\right) + A'',\\ \phi_1'' &= -2\phi_2 - \mathrm{Re}s_2 A^2\cos\psi - \mathrm{Re\,Pr}\left((1+R)D^{-1}f^2\right.\\ &\quad \left. + \mathrm{Ha}^2 f^2 + 4f'^2 - f\phi_1'\right)\cos\psi - \mathrm{Du}\left(g_1'' + 2g_2\right),\\ \phi_2'' &= -\mathrm{Re\ Pr}\left(\frac{R}{2}\left(f'' + 2A\right)^2 + \frac{s_2}{\mathrm{Pr}}A'^2\right.\\ &\quad \left. + (1+R)D^{-1}f'^2 + \mathrm{Ha}^2 f'^2 + f''^2 + 2f'\phi_2 - f\phi_2'\right)\\ &\quad \cdot\cos\psi - \mathrm{Du}g_2'',\\ g_1'' &= -2g_2 + \mathrm{Kr}g_1 + \mathrm{Sc\,Re}fg_1'\cos\psi - \mathrm{Sc\,Sr}\left(\phi_1''\right.\\ &\quad \left. + 2\phi_2\right),\\ g_2'' &= \mathrm{Kr}g_2 + \mathrm{Sc\,Re}\left(fg_2' - 2f'g_2\right)\cos\psi - \mathrm{Sc\,Sr}\phi_2'', \end{aligned} \quad (11)$$

where the prime denotes the differentiation with respect to λ.

The dimensionless forms of temperature and concentration from (9) are

$$T^* = \frac{T - T_1 e^{i\omega t}}{(T_2 - T_1) e^{i\omega t}} = \mathrm{Ec}\left(\phi_1 + \zeta^2 \phi_2\right),$$
$$C^* = \frac{C - C_0 e^{i\omega t}}{(C_1 - C_0) e^{i\omega t}} = \mathrm{Sh}\left(g_1 + \zeta^2 g_2\right). \quad (12)$$

The boundary conditions (10) in terms of f, A, ϕ_1, ϕ_2, g_1, and g_2 are

$$\begin{aligned}
f(0) &= 1 - a,\\
f(1) &= 1,\\
f'(0) &= 0,\\
f'(1) &= 0,\\
A(0) &= 0,\\
A(1) &= 0\\
\phi_1(0) &= 0,\\
\phi_1(1) &= \frac{1}{\mathrm{Ec}}\\
\phi_2(0) &= 0,\\
\phi_2(1) &= 0,\\
g_1(0) &= 0,\\
g_1(1) &= \frac{1}{\mathrm{Sh}}\\
g_2(0) &= 0,\\
g_2(1) &= 0.
\end{aligned} \quad (13)$$

For micropolar fluids the shear stress τ_{kl} is given by

$$\tau_{kl} = \left(-p + \eta v_{r,r}\right)\delta_{kl} + 2\mu d_{kl} + 2k_1 e_{klr}\left(\omega_r - l_r\right). \quad (14)$$

Then the nondimensional shear stress at the lower and upper plates is

$$S_f = \frac{2\tau_{kl}}{\rho V_2^2} = \left[\frac{2}{\mathrm{Re}}\left(\frac{U_0}{aV_2} - \frac{x}{h}\right)(R-1) f''(\lambda)\cos\psi\right]_{\lambda=0,1}. \quad (15)$$

For micropolar fluids, the couple stress M_{ij} is given by

$$M_{ij} = \alpha l_{r,r}\delta_{ij} + \beta l_{i,j} + \gamma l_{j,i}. \quad (16)$$

Then the nondimensional couple stress at lower and upper plates is

$$m = \left[\left(\frac{U_0}{aV_2} - \frac{x}{h}\right) g'(\lambda)\cos\psi\right]_{\lambda=0,1}. \quad (17)$$

3. Solution of the Problem

The nonlinear equations (11) are converted into the following system of first-order differential equations by the substitution

$$\left(f, f', f'', f''', A, A', \phi_1, \phi_1', \phi_2, \phi_2', g_1, g_1', g_2, g_2'\right) = \left(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}\right)$$

$$\frac{dx_1}{d\lambda} = x_2,$$

$$\frac{dx_2}{d\lambda} = x_3,$$

$$\frac{dx_3}{d\lambda} = x_4,$$

$$\begin{aligned}
\frac{dx_4}{d\lambda} &= \frac{-R}{1+R}\left(s_1\left(x_3 + 2x_5\right) + J_1\left(x_1 x_6 - x_2 x_5\right)\right)\cos\psi\\
&\quad + \frac{\mathrm{Re}}{1+R}\left(x_2 x_3 - x_1 x_4\right)\cos\psi + D^{-1}x_3 + \frac{\mathrm{Ha}^2}{1+R}x_3\\
&\quad - \frac{\mathrm{Ec}\mathrm{Gr}}{(1+R)\zeta}\left(x_8 + \zeta^2 x_{10}\right) - \frac{\mathrm{Sh}\,\mathrm{Gm}}{(1+R)\zeta}\left(x_{12}\right.\\
&\quad \left. + \zeta^2 x_{14}\right),
\end{aligned}$$

$$\frac{dx_5}{d\lambda} = x_6,$$

$$\frac{dx_6}{d\lambda} = s_1\left(x_3 + 2x_5\right) + J_1\left(x_1 x_6 - x_2 x_5\right)\cos\psi,$$

$$\frac{dx_7}{d\lambda} = x_8,$$

$$\begin{aligned}
\frac{dx_8}{d\lambda} &= -2x_9 - \frac{\mathrm{Re}\,\mathrm{Pr}}{1 - \mathrm{Du}\,\mathrm{Sc}\,\mathrm{Sr}}\left(4x_2^2 + (1+R)D^{-1}x_1^2\right.\\
&\quad \left. + \mathrm{Ha}^2 x_1^2 - x_1 x_8 + \frac{s_2}{\mathrm{Pr}}x_5^2\right)\cos\psi - \frac{\mathrm{Kr}\,\mathrm{Du}}{1 - \mathrm{Du}\,\mathrm{Sc}\,\mathrm{Sr}}x_{11}\\
&\quad + \frac{\mathrm{Du}\,\mathrm{Sc}}{1 - \mathrm{Du}\,\mathrm{Sc}\,\mathrm{Sr}}\mathrm{Re}x_1 x_{12}\cos\psi,
\end{aligned}$$

$$\frac{dx_9}{d\lambda} = x_{10},$$

$$\begin{aligned}
\frac{dx_{10}}{d\lambda} &= -\frac{\mathrm{Re}\,\mathrm{Pr}}{1 - \mathrm{Du}\,\mathrm{Sc}\,\mathrm{Sr}}\left(\frac{R}{2}\left(x_3 + 2x_5\right)^2 + x_3^2 + (1+R)\right.\\
&\quad \left. \cdot D^{-1}x_2^2 + \mathrm{Ha}^2 x_2^2 + 2x_2 x_9 - x_1 x_{10}\right)\cos\psi\\
&\quad - \frac{\mathrm{Kr}\,\mathrm{Du}}{1 - \mathrm{Du}\,\mathrm{Sc}\,\mathrm{Sr}}x_{13} - \frac{\mathrm{Du}\,\mathrm{Sc}}{1 - \mathrm{Du}\,\mathrm{Sc}\,\mathrm{Sr}}\mathrm{Re}\left(x_1 x_{14}\right.\\
&\quad \left. - 2x_2 x_{13}\right)\cos\psi
\end{aligned}$$

$$\frac{dx_{11}}{d\lambda} = x_{12},$$

$$\begin{aligned}
\frac{dx_{12}}{d\lambda} &= -2x_{13} + \frac{\mathrm{Sc}\,\mathrm{Sr}\,\mathrm{Re}\,\mathrm{Pr}}{1 - \mathrm{Du}\,\mathrm{Sc}\,\mathrm{Sr}}\left(4x_2^2 + (1+R)D^{-1}x_1^2\right.\\
&\quad \left. + \mathrm{Ha}^2 x_1^2 - x_1 x_8 + \frac{s_2}{\mathrm{Pr}}x_5^2\right)\cos\psi + \frac{\mathrm{Kr}}{1 - \mathrm{Du}\,\mathrm{Sc}\,\mathrm{Sr}}x_{11}\\
&\quad + \frac{\mathrm{Sc}}{1 - \mathrm{Du}\,\mathrm{Sc}\,\mathrm{Sr}}\mathrm{Re}x_1 x_{12}\cos\psi,
\end{aligned}$$

$$\frac{dx_{13}}{d\lambda} = x_{14},$$
$$\frac{dx_{14}}{d\lambda} = \frac{\mathrm{Sc\,Sr\,Re\,Pr}}{1-\mathrm{Du\,Sc\,Sr}}\left(x_3^2 + (1+R)\,D^{-1}x_2^2 + \mathrm{Ha}^2x_2^2 + \frac{s_2}{\mathrm{Pr}}x_6^2 + \frac{R}{2}(x_3+2x_5)^2 + 2x_2x_9 - x_1x_{10}\right)\cos\psi + \frac{\mathrm{Kr}}{1-\mathrm{Du\,Sc\,Sr}}x_{13} + \frac{\mathrm{Sc}}{1-\mathrm{Du\,Sc\,Sr}}\mathrm{Re}\,(x_1x_{14} - 2x_2x_{13})\cos\psi. \tag{18}$$

The boundary conditions in terms of x_1, x_2, x_3, x_4x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14} are

$$\begin{aligned} x_1(0) &= 1-a, \\ x_2(0) &= 0, \\ x_5(0) &= 0, \\ x_7(0) &= 0, \\ x_9(0) &= 0, \\ x_{11}(0) &= 0, \\ x_{13}(0) &= 0, \\ x_1(1) &= 1, \\ x_2(1) &= 0, \\ x_5(1) &= 0, \\ x_7(1) &= \frac{1}{\mathrm{Ec}}, \\ x_9(1) &= 0, \\ x_{11}(1) &= \frac{1}{\mathrm{Sh}}, \\ x_{13}(1) &= 0. \end{aligned} \tag{19}$$

The system of (18) is solved numerically subject to boundary conditions (19) using the quasilinearization method given by Bellman and Kalaba [26].

Let (x_i^r, $i = 1,2,\ldots,14$) be an approximate current solution and let (x_i^{r+1}, $i = 1,2,\ldots,14$) be an improved solution of (18). Using Taylor's series expansion about the current solution by neglecting the second- and higher-order derivative terms, coupled first-order system (18) is linearized as

$$\frac{dx_1^{r+1}}{d\lambda} = x_2^{r+1},$$
$$\frac{dx_2^{r+1}}{d\lambda} = x_3^{r+1},$$
$$\frac{dx_3^{r+1}}{d\lambda} = x_4^{r+1},$$
$$\frac{dx_4^{r+1}}{d\lambda} = -\frac{\mathrm{Re}}{1+R}\left(x_1^{r+1}x_4^r + x_4^{r+1}x_1^r - x_2^{r+1}x_3^r - x_2^rx_3^{r+1}\right)\cos\psi + D^{-1}x_3^{r+1} + \frac{\mathrm{Ha}^2}{1+R}x_3^{r+1} - \frac{R}{1+R}\left(s_1\left(x_3^{r+1}+2x_5^{r+1}\right) + J_1\left(x_1^{r+1}x_6^r + x_1^rx_6^{r+1} - x_2^{r+1}x_5^r - x_5^{r+1}x_2^r\right)\cos\psi\right) - \frac{\mathrm{Ec\,Gr}}{(1+R)\,\zeta}\left(x_8^{r+1} + \zeta^2x_{10}^{r+1}\right) - \frac{\mathrm{Sh\,Gm}}{(1+R)\,\zeta}\left(x_{12}^{r+1} + \zeta^2x_{14}^{r+1}\right) + \frac{\mathrm{Re}}{1+R}\left(-x_2^rx_3^r + x_1^rx_4^r\right)\cos\psi + \frac{RJ_1}{1+R}\left(x_1^rx_6^r - x_2^rx_5^r\right)\cos\psi,$$
$$\frac{dx_5^{r+1}}{d\lambda} = x_6^{r+1},$$
$$\frac{dx_6^{r+1}}{d\lambda} = s_1\left(x_3^{r+1}+2x_5^{r+1}\right) + J_1\left(x_1^{r+1}x_6^r + x_1^rx_6^{r+1} - x_2^{r+1}x_5^r - x_5^{r+1}x_2^r\right)\cos\psi - J_1\left(x_1^rx_6^r - x_2^rx_5^r\right)\cos\psi,$$
$$\frac{dx_7^{r+1}}{d\lambda} = x_8^{r+1},$$
$$\frac{dx_8^{r+1}}{d\lambda} = -2x_9^{r+1} - \frac{\mathrm{Re\,Pr}}{1-\mathrm{Du\,Sc\,Sr}}\left(8x_2^rx_2^{r+1} + 2(1+R)\cdot D^{-1}x_1^rx_1^{r+1} + 2\mathrm{Ha}^2x_1^rx_1^{r+1} - x_1^rx_8^{r+1} - x_8^rx_1^{r+1} + \frac{2s_2}{\mathrm{Pr}}x_5^rx_5^{r+1}\right)\cos\psi - \frac{\mathrm{Kr\,Du}}{1-\mathrm{Du\,Sc\,Sr}}x_{11}^{r+1} + \frac{\mathrm{Du\,Sc}}{1-\mathrm{Du\,Sc\,Sr}}\mathrm{Re}\left(x_1^rx_{12}^{r+1} + x_{12}^rx_1^{r+1} - x_1^rx_{12}^r\right)\cdot\cos\psi + \frac{\mathrm{Re\,Pr}}{1-\mathrm{Du\,Sc\,Sr}}\left(4x_2^rx_2^r + (1+R)\,D^{-1}x_1^rx_1^r + \mathrm{Ha}^2x_1^rx_1^r - x_1^rx_8^r + \frac{s_2}{\mathrm{Pr}}x_5^rx_5^r\right)\cos\psi,$$
$$\frac{dx_9^{r+1}}{d\lambda} = x_{10}^{r+1},$$
$$\frac{dx_{10}^{r+1}}{d\lambda} = -\frac{\mathrm{Re\,Pr}}{1-\mathrm{Du\,Sc\,Sr}}\left(\frac{R}{2}\left(2x_3^rx_3^{r+1} + 8x_5^rx_5^{r+1} + 4x_3^rx_5^{r+1} + 4x_5^rx_3^{r+1}\right) + 2x_3^rx_3^{r+1} + 2(1+R)\cdot D^{-1}x_2^rx_2^{r+1} + 2\mathrm{Ha}^2x_2^rx_2^{r+1} + 2x_2^rx_9^{r+1} + 2x_2^{r+1}x_9^r - x_1^rx_{10}^{r+1} - x_1^{r+1}x_{10}^r\right)\cos\psi - \frac{\mathrm{Du\,Sc\,Re}}{1-\mathrm{Du\,Sc\,Sr}}\left(x_1^rx_{14}^{r+1} + x_1^{r+1}x_{14}^r - 2x_2^rx_{13}^{r+1} - 2x_2^{r+1}x_{13}^r - x_1^rx_{14}^r + 2x_2^rx_{13}^r\right)\cos\psi + \frac{\mathrm{Re\,Pr}}{1-\mathrm{Du\,Sc\,Sr}}\left(\frac{R}{2}\left(x_3^rx_3^r + 4x_5^rx_5^r + 4x_3^rx_5^r\right) + x_3^rx_3^r + (1+R)\,D^{-1}x_2^rx_2^r + \mathrm{Ha}^2x_2^rx_2^r + 2x_2^rx_9^r - x_1^rx_{10}^r\right)\cos\psi - \frac{\mathrm{Kr\,Du}}{1-\mathrm{Du\,Sc\,Sr}}x_{13}^{r+1},$$

$$\frac{dx_{11}^{r+1}}{d\lambda} = x_{12}^{r+1},$$

$$\begin{aligned}\frac{dx_{12}^{r+1}}{d\lambda} &= -2x_{13}^{r+1} + \frac{\text{Sc Sr Re Pr}}{1 - \text{Du Sc Sr}}\Big(8x_2^r x_2^{r+1} + 2(1+R) \\ &\quad \cdot D^{-1}x_1^r x_1^{r+1} + 2\text{Ha}^2 x_1^r x_1^{r+1} - x_1^r x_8^{r+1} - x_1^{r+1} x_8^r \\ &\quad + \frac{2s_2}{\text{Pr}} x_5^r x_5^{r+1}\Big)\cos\psi + \frac{\text{Kr}}{1 - \text{Du Sc Sr}} x_{11}^{r+1} \\ &\quad + \frac{\text{Sc Re}}{1 - \text{Du Sc Sr}}\left(x_1^r x_{12}^{r+1} + x_1^{r+1} x_{12}^r - x_1^r x_{12}^r\right)\cos\psi \\ &\quad - \frac{\text{Sc Sr Re Pr}}{1 - \text{Du Sc Sr}}\Big(4x_2^r x_2^r + (1+R) D^{-1} x_1^r x_1^r \\ &\quad + \text{Ha}^2 x_1^r x_1^r - x_1^r x_8^r + \frac{s_2}{\text{Pr}} x_5^r x_5^r\Big)\cos\psi,\end{aligned}$$

$$\frac{dx_{13}^{r+1}}{d\lambda} = x_{14}^{r+1},$$

$$\begin{aligned}\frac{dx_{14}^{r+1}}{d\lambda} &= \frac{\text{Sc Sr Re Pr}}{1 - \text{Du Sc Sr}}\Big(2x_3^{r+1} x_3^r + 2(1+R) \\ &\quad \cdot D^{-1} x_2^r x_2^{r+1} + 2\text{Ha}^2 x_2^r x_2^{r+1} + \frac{2s_2}{\text{Pr}} x_6^r x_6^{r+1} \\ &\quad + \frac{R}{2}\left(2x_3^r x_3^{r+1} + 8x_5^r x_5^{r+1} + 4x_3^r x_5^{r+1} + 4x_5^r x_3^{r+1}\right) \\ &\quad + 2x_2^r x_9^{r+1} + 2x_2^{r+1} x_9^r - x_1^r x_{10}^{r+1} - x_1^{r+1} x_{10}^r\Big)\cos\psi \\ &\quad + \frac{\text{Kr}}{1 - \text{Du Sc Sr}} x_{13}^{r+1} + \frac{\text{Sc Re}}{1 - \text{Du Sc Sr}}\Big(x_1^r x_{14}^{r+1} \\ &\quad + x_1^{r+1} x_{14}^r - 2x_2^r x_{13}^{r+1} - 2x_2^{r+1} x_{13}^r - x_1^r x_{14}^r \\ &\quad + 2x_2^r x_{13}^r\Big)\cos\psi - \frac{\text{Sc Sr Re Pr}}{1 - \text{Du Sc Sr}}\Big(x_3^r x_3^r + (1+R) \\ &\quad \cdot D^{-1} x_2^r x_2^r + \text{Ha}^2 x_2^r x_2^r + \frac{s_2}{\text{Pr}} x_6^r x_6^r + \frac{R}{2}\big(x_3^r x_3^r \\ &\quad + 4x_5^r x_5^r + 4x_3^r x_5^r\big) + 2x_2^r x_9^r - x_1^r x_{10}^r\Big)\cos\psi.\end{aligned} \tag{20}$$

To solve for (x_i^{r+1}, $i = 1, 2, \ldots, 14$), the solutions to seven separate initial value problems, denoted by $x_i^{h1}(\lambda)$, $x_i^{h2}(\lambda)$, $x_i^{h3}(\lambda)$, $x_i^{h4}(\lambda)$, $x_i^{h5}(\lambda)$, $x_i^{h6}(\lambda)$, and $x_i^{h7}(\lambda)$ (which are the solutions of the homogeneous system corresponding to (20)) and $x_i^{p1}(\lambda)$ (which is the particular solution of (20)), with the following initial conditions are obtained by using the 4th-order Runge-Kutta method:

$$\begin{aligned}&x_3^{h1}(0) = 1, \\ &x_i^{h1}(0) = 0 \quad \text{for } i \neq 3, \\ &x_4^{h2}(0) = 1, \\ &x_i^{h2}(0) = 0 \quad \text{for } i \neq 4, \\ &x_6^{h3}(0) = 1, \\ &x_i^{h3}(0) = 0 \quad \text{for } i \neq 6, \\ &x_8^{h4}(0) = 1, \\ &x_i^{h4}(0) = 0 \quad \text{for } i \neq 8 \\ &x_{10}^{h5}(0) = 1, \\ &x_i^{h5}(0) = 0 \quad \text{for } i \neq 10 \\ &x_{12}^{h6}(0) = 1, \\ &x_i^{h6}(0) = 0 \quad \text{for } i \neq 12 \\ &x_{14}^{h7}(0) = 1, \\ &x_i^{h7}(0) = 0 \quad \text{for } i \neq 14 \\ &x_1^{p1}(0) = 1 - a, \\ &x_2^{p1}(0) = x_3^{p1}(0) = x_4^{p1}(0) = x_5^{p1}(0) = 0 \\ &x_6^{p1}(0) = x_7^{p1}(0) = x_8^{p1}(0) = x_9^{p1}(0) = 0 \\ &x_{10}^{p1}(0) = x_{11}^{p1}(0) = x_{12}^{p1}(0) = 0 \\ &x_{13}^{p1}(0) = x_{14}^{p1}(0) = 0.\end{aligned} \tag{21}$$

By using the principle of superposition, the general solution can be written as

$$\begin{aligned}x_i^{n+1}(\lambda) &= C_1 x_i^{h1}(\lambda) + C_2 x_i^{h2}(\lambda) + C_3 x_i^{h3}(\lambda) \\ &\quad + C_4 x_i^{h4}(\lambda) + C_5 x_i^{h5}(\lambda) + C_6 x_i^{h6}(\lambda) \\ &\quad + C_7 x_i^{h7}(\lambda) + x_i^{p1}(\lambda),\end{aligned} \tag{22}$$

where C_1, C_2, C_3, C_4, C_5, C_6, and C_7 are the unknown constants and are determined by considering the boundary conditions at $\lambda = 1$. This solution (x_i^{r+1}, $i = 1, 2, \ldots, 14$) is then compared with solution at the previous step (x_i^r, $i = 1, 2, \ldots, 14$) and further iteration is performed if the convergence has not been achieved.

4. Results and Discussion

The system of nonlinear differential equations (18) subject to boundary conditions (19) is solved numerically by the quasi-linearization method. The influences of various fluid and geometric parameters such as Soret number Sr, Dufour number Du, Hartmann number Ha, chemical reaction parameter Kr, Schmidt number Sc, Eckert number Ec, and Prandtl number Pr on nondimensional velocity components, microrotation, temperature distribution, and concentration are analyzed through graphs in the domain [0, 1].

Figures 2 and 3 show the influence of Sr and Du on temperature and concentration. From these figures, it is

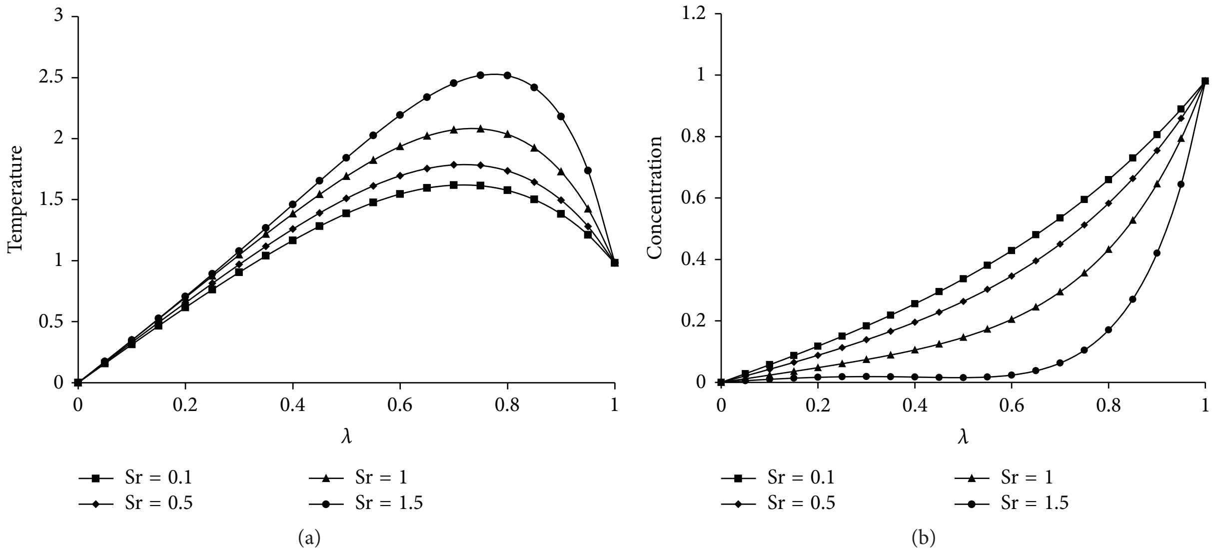


Figure 2: Effect of Sr on (a) temperature and (b) concentration for Kr = 2, Gr = 4, Gm = 4, Re = 2, Du = 2, Sc = 0.22, Pr = 1, $a = 0.2$, $\psi = 0.2$, $R = 0.2$, $J_1 = 0.2$, $s_1 = 2$, $s_2 = 10$, Ha = 1, $D^{-1} = 2$, and Ec = 1.

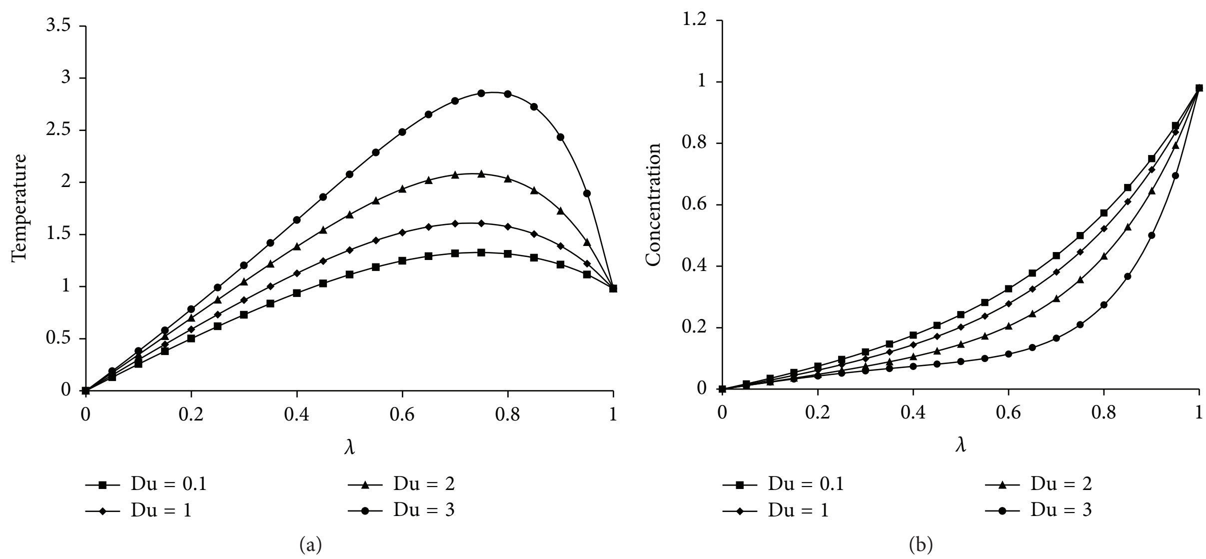
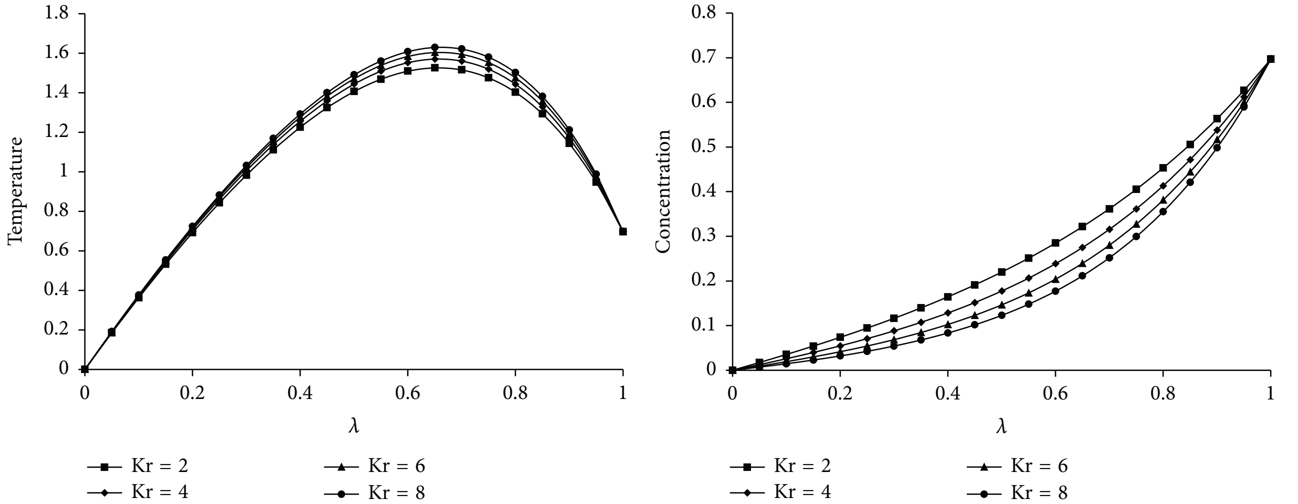


Figure 3: Effect of Du on (a) temperature and (b) concentration for Kr = 2, Gr = 4, Gm = 4, Re = 2, Sr = 1, Sc = 0.22, Pr = 1, $a = 0.2$, $\psi = 0.2$, $R = 0.2$, $J_1 = 0.2$, $s_1 = 2$, $s_2 = 10$, Ha = 1, $D^{-1} = 2$, and Ec = 1.

evident that the temperature of the fluid increases whereas the concentration decreases with the increasing of Sr and Du. This is because of the difference between the temperatures of the fluid and surface as well as the difference between concentrations of the fluid and surface concentrations are increased with Sr and Du. Figure 4 describes the behavior of the temperature distribution and concentration for the various values of Kr. As Kr increases the temperature distribution of the fluid also increases, whereas the concentration decreases from the lower plate to the upper plate. It is clear that the increase in the Kr produces a decrease in the species concentration. This causes the concentration buoyancy effects to decrease as Kr increases. The effect of Ec on velocity components, microrotation, and temperature is presented in Figure 5. As Ec increases the radial velocity, microrotation and temperature are decreasing towards the upper plate. However, the axial velocity decreases towards the center of the channel and then increases. Since the Eckert number is the relation between the kinetic energy and enthalpy, as enthalpy increases, the temperature distribution decreases. Figure 6 elucidates the change in velocity components, microrotation, temperature distribution, and concentration for different values of Pr. It is observed that the axial velocity reaches highest value near the hot plate and the radial velocity, microrotation, and concentration decrease whereas the temperature increases with the increasing value of Pr. Physically, if Pr increases the thermal diffusivity decreases and this leads to the decrease in the heat transfer ability at the thermal boundary layer.



(a) (b)

Figure 4: Effect of Du on (a) temperature and (b) concentration for Du = 1, Gr = 4, Gm = 4, Re = 2, Sr = 0.2, Sc = 0.22, Pr = 1, $a = 0.2$, $\psi = 0.8$, $R = 2$, $J_1 = 0.2$, $s_1 = 2$, $s_2 = 10$, Ha = 2, $D^{-1} = 2$, and Ec = 1.

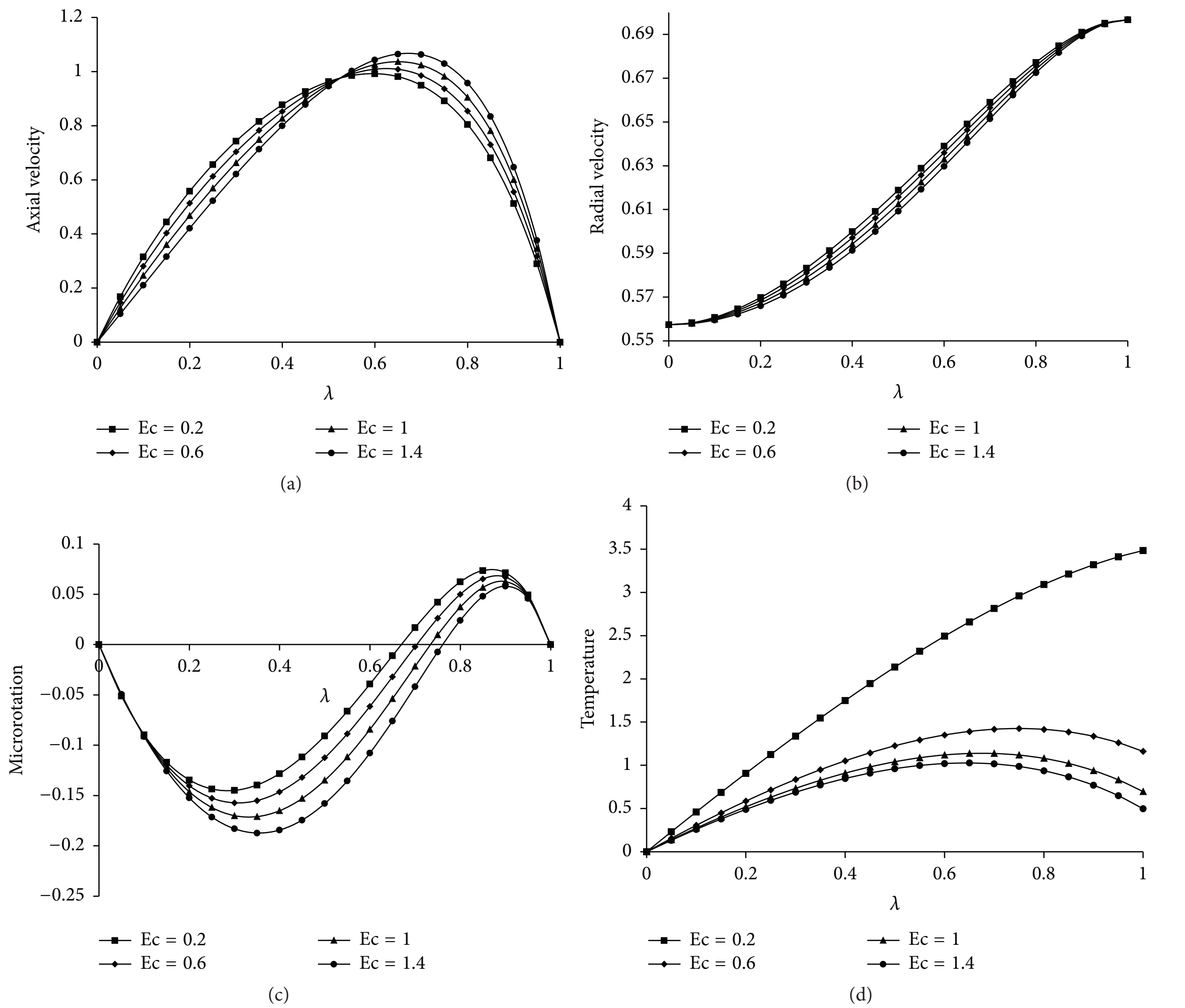


Figure 5: Effect of Ec on (a) axial velocity, (b) radial velocity, (c) microrotation, and (d) temperature for $D^{-1} = 2$, Du = 0.2, Gr = 4, Re = 2, Sr = 0.2, Ha = 2, Sc = 0.22, $a = 0.2$, $\psi = 0.8$, $R = 2$, $J_1 = 2$, $s_1 = 2$, $s_2 = 1$, Gm = 4, Kr = 2, and Pr = 0.71.

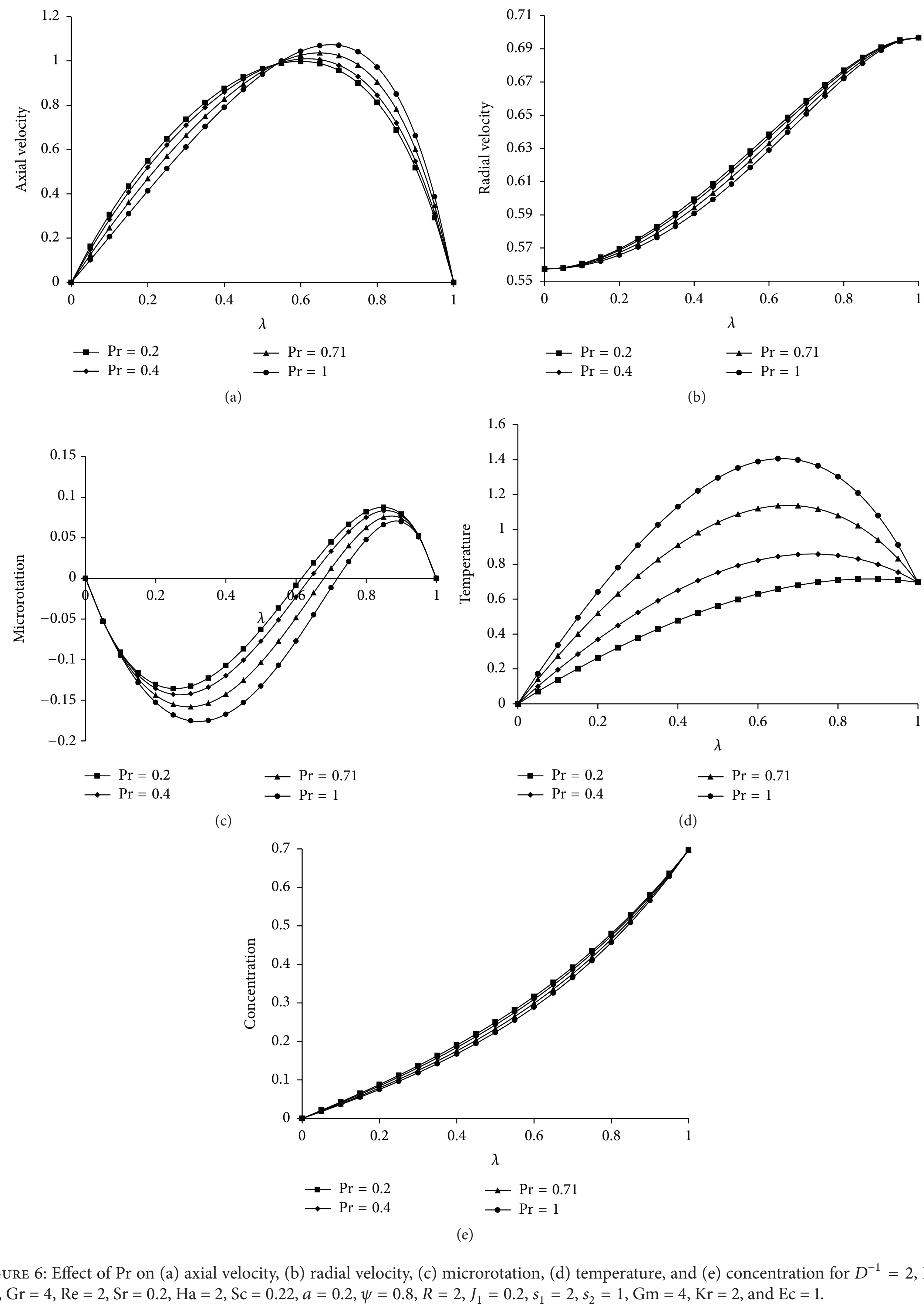


Figure 6: Effect of Pr on (a) axial velocity, (b) radial velocity, (c) microrotation, (d) temperature, and (e) concentration for $D^{-1} = 2$, Du = 0.2, Gr = 4, Re = 2, Sr = 0.2, Ha = 2, Sc = 0.22, $a = 0.2$, $\psi = 0.8$, $R = 2$, $J_1 = 0.2$, $s_1 = 2$, $s_2 = 1$, Gm = 4, Kr = 2, and Ec = 1.

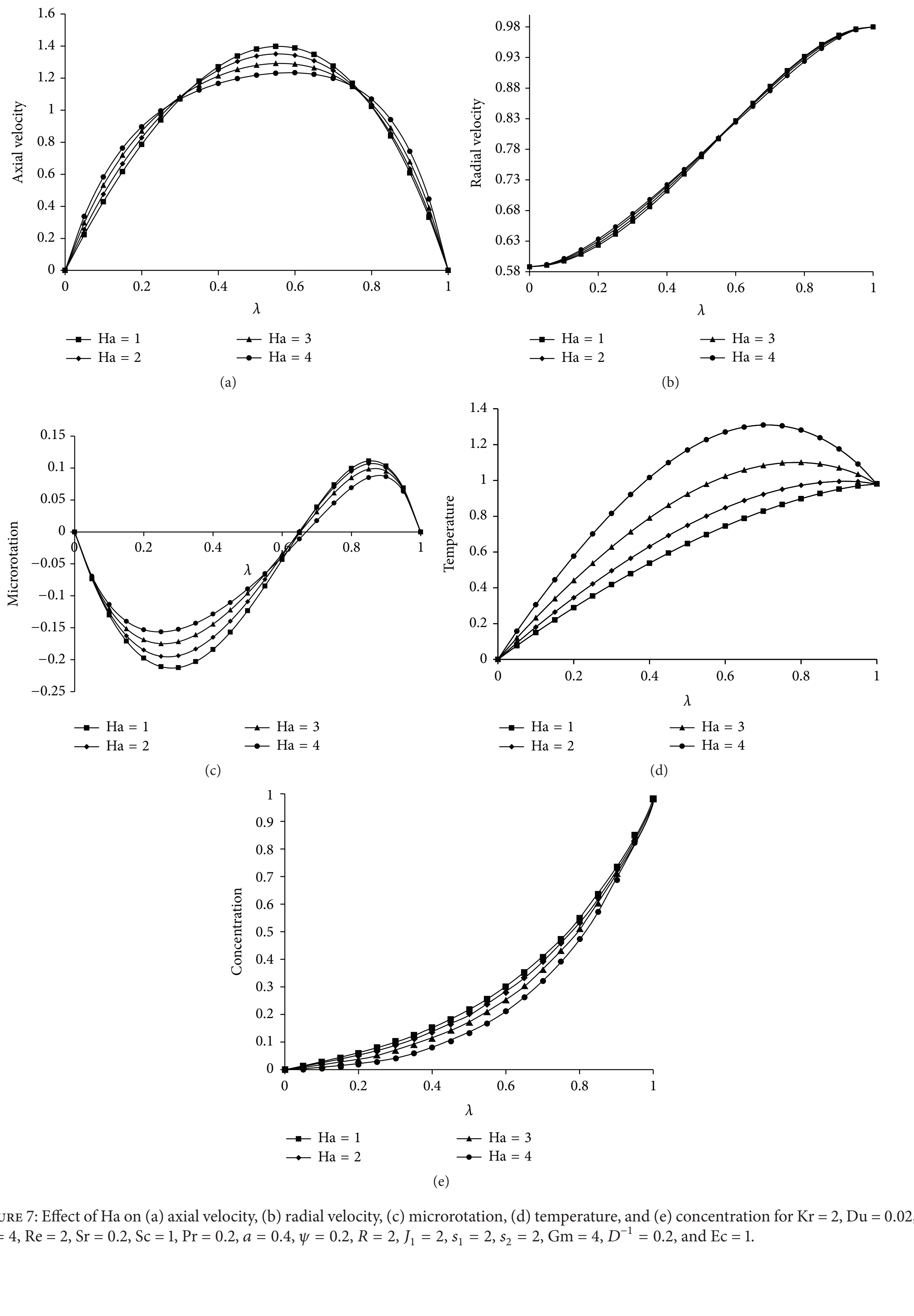


Figure 7: Effect of Ha on (a) axial velocity, (b) radial velocity, (c) microrotation, (d) temperature, and (e) concentration for Kr = 2, Du = 0.02, Gr = 4, Re = 2, Sr = 0.2, Sc = 1, Pr = 0.2, $a = 0.4$, $\psi = 0.2$, $R = 2$, $J_1 = 2$, $s_1 = 2$, $s_2 = 2$, Gm = 4, $D^{-1} = 0.2$, and Ec = 1.

Figure 7 displays the change in the velocity components, microrotation, temperature distribution, and concentration for several values of Ha. From this it is observed that when Ha increases, the temperature distribution also increases whereas the concentration decreases from the lower plate to upper plate and the axial velocity attains maximum value at the center of the plates. However, the radial velocity and microrotation increase towards the center of the plates and then decrease. This is due to the fact that the magnetic force retards the flow in both axial and radial directions. The variations in the velocity components, microrotation, temperature distribution, and concentration for different values of D^{-1} are shown in Figure 8. From these one can deduce that the temperature distribution is increasing with D^{-1} whereas the radial velocity, microrotation, and concentration are decreasing towards the upper plate and the axial velocity decreases towards the center of the plates and then increases because the resistance offered by the porosity of the medium is more than the resistance due to the magnetic lines of force.

5. Conclusions

The thermal diffusion and diffusion thermoeffects on combined free and forced convection magnetohydrodynamic flow of micropolar fluid in a porous medium between two parallel plates with chemical reaction are considered. The numerical solution of the transformed governing equations is obtained by the method of quasilinearization and the results are analyzed for various fluid and geometric parameters through graphs. From the results the following is concluded:

(i) The influences of Sr and Du on temperature and concentration are similar.

(ii) The temperature of the fluid is enhanced whereas the concentration of the fluid is decreased with the increasing of Ha and D^{-1}.

(iii) Kr reduces the concentration and enhances the temperature of the fluid.

(iv) Ec and Pr exhibit similar effects on the velocity components and microrotation whereas it is opposite in the case of temperature.

Nomenclature

a: Injection suction ratio, $1 - V_1/V_2$
t: Time
h: Distance between two parallel plates
$V_1 e^{i\omega t}$: Injection velocity
$V_2 e^{i\omega t}$: Suction velocity
p: Fluid pressure
$\bar{q}$: Velocity vector
c: Specific heat at constant temperature
$\bar{l}$: Microrotation vector
N: Microrotation component
Ec: Eckert number, $\mu V_2/\rho hc(T_2 - T_1)$
k: Thermal conductivity
k_1: Viscosity parameter
k_2: Permeability of the medium
k_3: Chemical reaction rate
u: Velocity component in x-direction
v: Velocity component in y-direction
Pr: Prandtl number, $\mu c/k$
Re: Suction Reynolds number, $\rho V_2 h/\mu$
j: Gyration parameter
$\bar{J}$: Current density
J_1: Nondimensional gyration parameter, $\rho j h V_2/\gamma$
$\bar{B}$: Total magnetic field
$\bar{b}$: Induced magnetic field
B_0: Magnetic flux density
D: Rate of deformation tensor
$\bar{E}$: Electric field
Ha: Hartmann number, $B_0 h\sqrt{\sigma/\mu}$
D^{-1}: Inverse Darcy parameter, h^2/k_2
R: Nondimensional viscosity parameter, k_1/μ
s_1: Nondimensional micropolar parameter, $k_1 h^2/\gamma$
s_2: Nondimensional micropolar parameter, $\gamma c/h^2 k$
T: Temperature
$T_1 e^{i\omega t}$: Temperature of the lower plate
$T_2 e^{i\omega t}$: Temperature of the upper plate
T^*: Dimensionless temperature, $(T - T_1 e^{i\omega t})/(T_2 - T_1)e^{i\omega t}$
C: Concentration
C^*: Nondimensional concentration, $(C - C_0 e^{i\omega t})/(C_1 - C_0)e^{i\omega t}$
$C_0 e^{i\omega t}$: Concentration of the lower plate
$C_1 e^{i\omega t}$: Concentration of the upper plate
D_1: Mass diffusivity
Kr: Nondimensional chemical reaction parameter, $k_3 h^2/D_1$
Sc: Schmidt number, υ/D_1
Gr: Thermal Grashof number, $\rho g\beta_T(T_2 - T_1)h^2/\mu V_2$
Gm: Solutal Grashof number, $\rho g\beta_C(C_1 - C_0)h^2/\mu V_2$
Sh: Sherwood number, $\dot{n}_A/h\upsilon(C_1 - C_0)$
$\dot{n}_A$: Mass transfer rate
Sr: Soret number, $D_1 k_T \upsilon V_2/cT_m \dot{n}_A$
Du: Dufour number, $D_1 k_T \dot{n}_A \rho c/\upsilon^2 V_2 c_s k$
T_m: Mean temperature
k_T: Thermal diffusion ratio
c_s: Concentration susceptibility.

Greek Letters

λ: Dimensionless y coordinate, y/h
α, β, γ: Gyro viscosity parameters
ζ: Dimensionless axial variable, $(U_0/aV_2 - x/h)$
υ: Kinematic viscosity
ρ: Fluid density
μ: Fluid viscosity

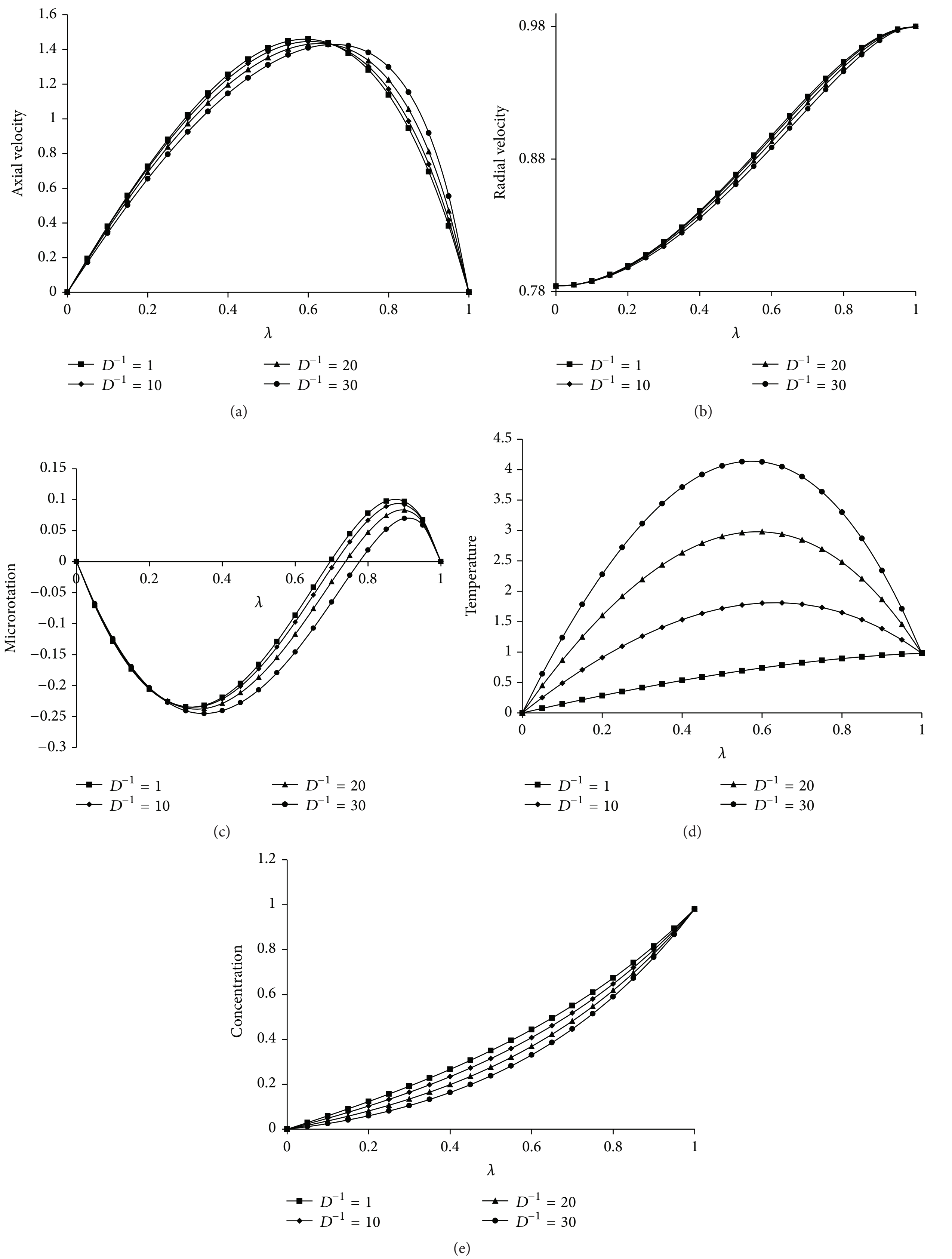


Figure 8: Effect of D^{-1} on (a) axial velocity, (b) radial velocity, (c) microrotation, (d) temperature, and (e) concentration for Kr = 2, Du = 0.02, Gr = 4, Re = 2, Sr = 0.2, Ha = 1, Sc = 0.2, Pr = 0.2, $a = 0.2$, $\psi = 0.2$, $R = 2$, $J_1 = 2$, $s_1 = 2$, $s_2 = 2$, Gm = 4, and Ec = 1.

μ': Magnetic permeability
σ: Electric conductivity
ψ: Nondimensional frequency parameter, ωt.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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