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Research Article

Investigating Surface Effects on Thermomechanical Behavior of Embedded Circular Curved Nanosize Beams

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To investigate the surface effects on thermomechanical vibration and buckling of embedded circular curved nanosize beams, nonlocal elasticity model is used in combination with surface properties including surface elasticity, surface tension, and surface density for modeling the nanoscale effect. The governing equations are determined via the energy method. Analytically Navier method is utilized to solve the governing equations for simply supported nanobeam at both ends. Solving these equations enables us to estimate the natural frequency and critical buckling load for circular curved nanobeam including Winkler and Pasternak elastic foundations and under the effect of a uniform temperature change. The results determined are verified by comparing the results with available ones in literature. The effects of various parameters such as nonlocal parameter, surface properties, Winkler and Pasternak elastic foundations, temperature, and opening angle of circular curved nanobeam on the natural frequency and critical buckling load are successfully studied. The results reveal that the natural frequency and critical buckling load of circular curved nanobeam are significantly influenced by these effects.

1. Introduction

Nanomaterials are attracting many researchers over the recent years due to their improvement of the quality properties. Atomistic modeling and experimental researches show that the size effect gains important when the dimensions of structures become very small. Due to this fact, the size effect plays an important role on the mechanical behavior of micro- and nanostructures [1]. Among various nanostructures, nanobeams have more important applications [2, 3]. Reddy [4] has investigated nonlocal theories for bending, buckling, and vibration of beams. Moreover, Demir and Civalek [5] have surveyed torsional and longitudinal vibration and wave response of microtubules based on the nonlocal continuum and nonlocal discrete theories. Meanwhile a nonlocal beam theory is presented by Thai [6], in this research; bending, buckling, and vibration of nanobeams have been investigated. However, the nonlinear vibration of the piezoelectric nanobeams based on the Timoshenko beam theory and nonlocal modeling has been investigated by Ke et al. [7]. In addition Murmu and Adhikari [8] have investigated the nonlocal transverse vibration of double-nanobeam-system. In this research, an analytical method has been developed for determining the natural frequencies of the nonlocal double-nanobeam-system. Also Eltaher et al. [9] have presented free vibration analysis of functionally graded (FG) size-dependent nanobeams using finite element method.

Because the nanobeams has the high proportion of the surface to volume, the surface stress effects have important role in their mechanics behavior of these structures. The effect of the residual surface stress on the beam gives the distributed loading on the two surfaces. Hence Gurtin and Murdoch [10] have considered surface stress effects. In this theory the surface is considered as a part of (nonphysical) the two-dimensional body with zero thickness (mathematically) which has covered the total volume. This theory has been used in many researches about nanostructures [11–14]. Nonlinear free vibration of functionally graded nanobeams with surface effects has been investigated by Sharabiani and Yazdi [15]. Furthermore, the nonlinear free vibration of nanobeams with considering surface properties has been studied by Nazemnezhad et al. [16]. However, Hosseini-Hashemi and Nazemnezhad [17] have presented nonlinear free vibration of functionally graded nanoscale beams with surface properties. Also, Ansari et al. [18] have investigated nonlinear forced vibration characteristics of nanobeams including surface stress effect. In this study, a new formulation of the Timoshenko beam theory has been developed through the Gurtin-Murdoch elasticity theory in which the effect of surface stress has been incorporated. Moreover, the surface and nonlocal effects on the nonlinear flexural free vibrations of elastically supported nonuniform cross-section nanobeams have been investigated by Malekzadeh and Shojaee [19] simultaneously. In addition, Ebrahimi et al. [20] have presented surface and nonlocal effects on buckling and vibrational analysis of nanotubes with various boundary conditions.

In the field of elastic foundation there are linear and nonlinear ones which are called Winkler and Pasternak, respectively. Zhao et al. [21] have investigated the axial buckling of a nanowire resting on Winkler-Pasternak substrate medium with the Timoshenko beam theory. In addition, simple analytical expressions have been presented by Fallah and Aghdam [22] for large amplitude free vibration and postbuckling analysis of functionally graded beams rest on nonlinear elastic foundation. Furthermore, Jang et al. [23] have presented a new method of analyzing the nonlinear deflection behavior of an infinite beam on a nonlinear elastic foundation. Also, Niknam and Aghdam [24] have obtained a closed form solution for both natural frequency and buckling load of nonlocal functionally graded beams resting on nonlinear elastic foundation. Moreover, the static instability of a nanobeam with geometrical imperfections with elastic foundation has been investigated by Mohammadi et al. [25]. In this paper, size-dependent effect is included in the nonlinear model. Nevertheless, differential transformation method has been used to predict the buckling behavior of single walled carbon nanotube on Winkler foundation under various boundary conditions by Pradhan and Reddy [26].

In recent years vibrations of curved nanobeams and nanorings have been done in many empirical experiments and dynamic molecular simulations [27]. Hence some researchers are interested in studying of vibration curved nanobeams and nanorings. Malekzadeh et al. [28] have presented out-of-plane free vibration of functionally graded circular curved beams in thermal environment. However, Yan and Jiang [29] have investigated the electromechanical response of a curved piezoelectric nanobeam with the consideration of surface effects. In addition, a new numerical technique, the differential quadrature method, has been developed for dynamic analysis of the nanobeams in the polar coordinate system by Kananipour et al. [30]. Moreover, Khater et al. [31] have investigated the effect of surface energy and thermal loading on the static stability of nanowires. In this research, nanowires have been considered as curved fixed-fixed Euler-Bernoulli beams and have used Gurtin-Murdoch theory to represent surface effects. The model has taken into account both von Kármán strain and axial strain. Also, Wang and Duan [27] have surveyed the free vibration problem of nanorings/arches. In this research the problem was formulated on the framework of nonlocal elasticity theory. Furthermore, DQ thermal buckling analysis of embedded curved carbon nanotubes via nonlocal elasticity model has been presented by Setoodeh et al. [32]. Nevertheless, explicit solution has been shown for size- and geometry-dependent free vibration of curved nanobeams including surface effects by Assadi and Farshi [33].

Recently, nanostructures have generated a great deal of interest from research. These distinguished properties make them apt for potential applications in nanoelectromechanical systems (NEMS) such as nanosensors [34, 35] and nanoactuators [36].

Ebrahimi and Salari [37] presented a semianalytical method for vibrational and buckling analysis of FG nanobeams considering the physical neutral axis position. Recently Ebrahimi and Barati [38–41] presented static and dynamic modeling of a thermopiezoelectrically actuated nanosize beam subjected to a magnetic field. Most recently small scale effects on hygrothermomechanical vibration of temperature-dependent nonhomogeneous nanoscale beams are investigated by Ebrahimi and Barati [42]. They [43] also proposed a nonlocal higher-order shear deformation beam theory for vibration analysis of size-dependent functionally graded nanobeams. The influences of various thermal environments on buckling and vibration of nonlocal temperature-dependent FG beams is analyzed by Ebrahimi and Salari [44] using Navier analytical solution. In another work, Ebrahimi and Salari [45] investigated thermomechanical vibration of FG nanobeams with arbitrary boundary conditions applying differential transform method. Also, Ebrahimi et al. [46] explored the effects of linear and nonlinear temperature distributions on vibration of FG nanobeams.

To the best of the author's knowledge, there is no study regarding the surface effects on thermomechanical vibration and buckling of embedded circular curved nanosize beams. Therefore, there is a strong scientific need to understand the thermomechanical vibration and buckling behavior of circular curved nanobeams with surface effects in considering the effect of elastic foundations. Curvature rather exists in all of the real beams and nanobeams. Moreover, in previous researches in order to streamline mathematical equations, straight beam models have been used, whilst curved beam models are more practicable than straight ones. Analyzing of the vibration treatment of nanostructures is an important topic in the design process of the nanodevices. The aim of this research is to survey the effects of temperature changes and Winkler and Pasternak elastic foundation on natural frequencies and critical buckling loads of curved nanobeams with and without surface properties. So the paper has investigated the effects of surface density, surface elasticity, and surface residual stress.

2. Theories and Relations

2.1. Displacement Fields. Consider a free vibration of curved nanobeam as is shown in Figure 1, with the radius curvature and thickness as R and h, respectively. In addition surface properties are assumed for surface elasticity, surface tension, and surface density.

Based on the Timoshenko beam model, the displacement fields of an arbitrary point of the circular curved nanobeam can be defined as follows:

$$\begin{aligned} U(\theta, z, t) &= u(\theta, t) + z\varphi_\theta(\theta, t), \\ W(\theta, z, t) &= w(\theta, t), \end{aligned} \tag{1}$$

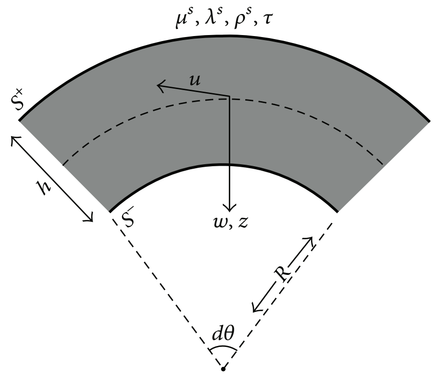

Figure 1: Geometry of an element of a circular curved nanobeam with surface layers.

where u and w give the middle displacements in θ and z directions, respectively. φ_θ is the rotation of transverse normal.

The relation between length of circular curved beam (α) and the angle of curvature of beam (θ) can be written as [32]

$$\alpha = R\theta. \tag{2}$$

The normal and shear strains at any point of the beam space can be denoted as follows:

$$\varepsilon_\theta = \frac{1}{(1+z/R)}\left(\varepsilon_\theta^0 + z\chi_\theta\right),$$
$$\lambda_{\theta z} = \frac{1}{(1+z/R)}\left(\gamma_{\theta z}^0\right), \tag{3}$$

where ε_θ^0 and $\gamma_{\theta z}^0$ are the normal and shear strains in the main surface. χ_θ is the curvature change. They are explained in terms of the middle surface displacements and rotation as [36]

$$\varepsilon_\theta^0 = \frac{\partial u}{R\partial\theta} + \frac{w}{R};$$
$$\chi_\theta = \frac{\partial\varphi_\theta}{R\partial\theta}; \tag{4}$$
$$\lambda_{\theta z}^0 = \frac{\partial w}{R\partial\theta} - \frac{u}{R} + \varphi_\theta.$$

By exerting the integration of the stresses with respect to the cross-section, the normal force and moment resultants can be determined as

$$\begin{bmatrix} N \\ Q \end{bmatrix} = b\int_{-h/2}^{h/2} \begin{bmatrix} \sigma_\theta \\ \tau_{\theta z} \end{bmatrix} dz;$$
$$M = b\int_{-h/2}^{h/2} \sigma_\theta z\, dz, \tag{5}$$

where Q deputes the transverse shear force resultant.

2.2. Hamilton Principle. The strain energy (U_s) of the circular curved nanobeams for free vibration analysis can be expressed as

$$U_s = \frac{1}{2}\int_\theta \left\{N\varepsilon_\theta^0 + M\chi_\theta + Q\gamma_{\theta z}^0\right\} R\, d\theta. \tag{6}$$

Moreover, the kinetic energy (T) of the circular curved nanobeam can be defined as [36]

$$T = \frac{1}{2}\int_\theta \left\{\bar{I}_0\left(\frac{\partial u}{\partial t}\right)^2 + \bar{I}_0\left(\frac{\partial w}{\partial t}\right)^2\right\} R\, d\theta, \tag{7}$$

where the inertia components are given as

$$\bar{I}_0 = I_0 + \frac{I_1}{R},$$
$$[I_0, I_1] = b\int_{-h/2}^{h/2} \rho\,[1, z]\, dz, \tag{8}$$

where ρ is the density of the circular curved nanobeam. The work done corresponding to temperature changes and elastic foundations can be written in the following form [32]:

$$W_{\text{ext}} = \frac{1}{2}\int_\theta \left\{\frac{N^T}{R^2}\left(\frac{\partial w}{\partial\theta}\right)^2 + f\right\} R\, d\theta. \tag{9}$$

3. Mathematical Process

The Lagrangian functional (L) of the beams for free vibration analysis is demonstrated in terms of the energy expressions as

$$L = T - U_s - W_{\text{ext}}. \tag{10}$$

By substituting (6)–(9) into (10) and employing the Hamilton's principle as (11), governing equation can be determined:

$$\int_0^t \delta\left(T - U_s - W_{\text{ext}}\right) dt = 0. \tag{11}$$

Equation (11) can be satisfied only if the coefficients of the virtual displacements are zero. Hence, motion equations of circular curved nanobeams with considering surface properties and elastic foundations are obtained as follows:

$$\frac{\partial N}{\partial\theta} + Q = \left(\rho AR + 2bR\rho^s\right)\frac{\partial^2 u}{\partial t^2},$$
$$-N + \frac{\partial Q}{\partial\theta} - fR + \frac{N^T}{R}\frac{\partial^2 w}{\partial\theta^2} = \left(\rho AR + 2bR\rho^s\right)\frac{\partial^2 w}{\partial t^2}, \tag{12}$$
$$\frac{\partial M}{\partial\theta} - QR = 0,$$

where A, ρ, and ρ^s are the cross-sectional area, mass density, and surface density of the circular curved nanobeam, respectively. In (12) b is the width of circular curved nanobeam.

Omitting the normal stress resultant N from (12) obtains the relation between radial displacement and bending moment component. For this aim it is essential to differentiate the first of the two equations in (12) with respect to θ and then insert the obtained relation for $\partial N/\partial\theta$ into the other. Also by neglecting products of small quantities $\partial u/\partial\theta = -w$ [47], derivation of the obtained result with respect to θ, and streamline, this relation determines

$$\frac{1}{R}\left(\frac{\partial^4 M}{\partial\theta^4}+\frac{\partial^2 M}{\partial\theta^2}\right)+\frac{N^T}{R}\frac{\partial^4 w}{\partial\theta^4}-\frac{\partial^2 f}{\partial\theta^2}R = \left(\rho AR+2Rb\rho^s\right)\left(\frac{\partial^4 w}{\partial\theta^2\partial t^2}-\frac{\partial^2 w}{\partial t^2}\right). \quad (13)$$

Then again, the constitutive equation of elastic surface materials for the general case is defined as [10]

$$\tau^{\pm}_{\alpha\beta} = \tau\delta_{\alpha\beta}+\left(\mu^s-\tau\right)\left(u^{\pm}_{\alpha\beta}+u^{\pm}_{\beta\alpha}\right)+\left(\lambda^s+\tau\right)u^{\pm}_{\gamma\gamma}\delta_{\alpha\beta} + \tau u^{\pm}_{\alpha\beta}, \quad (14)$$

where τ is surface residual stress and λ^s and μ^s are the lame constants for the surface material. Plus and minus sign refer to S^+ and S^- surfaces, respectively. Other essential relation for derivation of the modified differential equation is the tangential strain in terms of planar displacement component, which can be given as [47]

$$\varepsilon_{\theta\theta} = \frac{1}{R}\left[-w+\frac{\partial u}{\partial\theta}-\frac{x}{R}\frac{\partial}{\partial\theta}\left(u+\frac{\partial w}{\partial\theta}\right)\right]. \quad (15)$$

Considering inextensible deformation of the curved element at $x = 0$, it can be concluded that employing the tangential strain statement by assuming the corresponding out-of-plane contractions for a curved beam at (14) and assuming the same surface properties for S^+ and S^-, the stress-strain relation for the surface material can be given as

$$\tau^{\pm}_{\theta\theta} = \tau \pm \frac{h\left[2\mu^s+\lambda^s(1-\nu)-\nu\tau\right]}{2R^2}\left(w+\frac{\partial^2 w}{\partial\theta^2}\right). \quad (16)$$

In addition, by integrating the strain components on the cross-section the resultant bending moment exerting on the cross-section of the circular curved beam can be determined as follows:

$$M = -b\int_{-h/2}^{h/2} E\varepsilon_{\theta\theta}x\,dx + \frac{bh}{2}\left(\tau^{+}_{\theta\theta}-\tau^{-}_{\theta\theta}\right). \quad (17)$$

By employing (15) and (16) in (17) the bending moment of the cross-section can be obtained with respect to the radial displacement as follows:

$$M = \left\{\frac{EI}{R^2}+\frac{bh^2\left[2\mu^s+\lambda^s(1-\nu)-\nu\tau\right]}{2R^2}\right\}\left(w+\frac{\partial^2 w}{\partial\theta^2}\right). \quad (18)$$

Inserting (18) into (13) yields the modified governing equation of motion for the circular curved beam as a pure function of the radial displacement components, which must be solved to obtain the natural frequencies.

3.1. The Nonlocal Circular Curved Nanobeam Model. Eringen's nonlocal model states that the stress at any physical point of a body is assumed to be a function of the strain field for the whole body. The differential form of this theory is as follows [48]:

$$\left(1-\mu\nabla^2\right)\sigma_{kl} = t_{kl}, \quad (19)$$

where ∇^2 stands for gradient operator and μ is the nonlocality. The size effect on the response of nanostructure is taken into account by the scale length e_0a. However, the relation between nonlocality (μ) and scale length (e_0a) can be written as follows:

$$\mu = \left(e_0a\right)^2. \quad (20)$$

In one-dimensional problems, the nonlocal constitutive relations of an elastic material can be written as follows:

$$\sigma_{xx}-\mu\frac{\partial^2\sigma_{xx}}{\partial x^2} = E\varepsilon_{xx}, \quad (21)$$

where σ and ε depute the nonlocal stress and strain at any point, respectively. Also, E represents Young's modulus. By integrating (19) and (20) over cross-sectional area of the beam, force-strain and moment-strain of nonlocal circular beam model will be given as

$$M-\frac{\mu}{R^2}\frac{\partial^2 M}{\partial\theta^2} = -\frac{D}{R^2}\left(w+\frac{\partial^2 w}{\partial\theta^2}\right), \quad (22)$$

where D is defined as follows:

$$D = EI+\frac{bh^2\left(2\mu s+\lambda s(1-\nu)-\nu\tau\right)}{2}. \quad (23)$$

By inserting (22) in (13), the governing equation is obtained as follows:

$$\frac{D}{R^3}\left(\frac{\partial^6 w}{\partial\theta^6}+2\frac{\partial^4 w}{\partial\theta^4}+\frac{\partial^2 w}{\partial\theta^2}\right)+\frac{N^T}{R}\frac{\partial^4 w}{\partial\theta^4}-\frac{\mu}{R^3}N^T\frac{\partial^6 w}{\partial\theta^6} - \frac{\partial^2 f}{\partial\theta^2}R+\frac{\mu}{R}\frac{\partial^4 f}{\partial\theta^4} = -\left(\rho AR+2bR\rho^s\right)\left(\frac{\partial^4 w}{\partial\theta^2\partial t^2}-\frac{\partial^2 w}{\partial t^2}\right) + \frac{\mu}{R^2}\left(\frac{\partial^6 w}{\partial\theta^4\partial t^2}-\frac{\partial^4 w}{\partial\theta^2\partial t^2}\right), \quad (24)$$

where f is related to Winkler and Pasternak elastic foundations which is defined as

$$f = -K_w w+\frac{K_p}{R^2}\frac{\partial^2 w}{\partial\theta^2}. \quad (25)$$

(i) Vibration Equation. By employing (25) into (24), the governing equation is determined as follows:

$$\frac{D}{R^3}\left(\frac{\partial^6 w}{\partial\theta^6}+2\frac{\partial^4 w}{\partial\theta^4}+\frac{\partial^2 w}{\partial\theta^2}\right)+\frac{N^T}{R}\frac{\partial^4 w}{\partial\theta^4}-\frac{\mu}{R^3}N^T\frac{\partial^6 w}{\partial\theta^6}$$
$$+K_w R\frac{\partial^2 w}{\partial\theta^2}-\frac{K_p}{R}\frac{\partial^4 w}{\partial\theta^4}-\mu\frac{K_w}{R}\frac{\partial^4 w}{\partial\theta^4}+\mu\frac{K_p}{R^3}\frac{\partial^6 w}{\partial\theta^6}$$
$$=-\left(\rho AR+2bR\rho^s\right)\left(\frac{\partial^4 w}{\partial\theta^2\partial t^2}-\frac{\partial^2 w}{\partial t^2}\right)$$
$$+\frac{\mu}{R^2}\left(\frac{\partial^6 w}{\partial\theta^4\partial t^2}-\frac{\partial^4 w}{\partial\theta^2\partial t^2}\right). \tag{26}$$

(ii) Critical Buckling Load Equation

$$-\frac{D}{R^3}\left(\frac{\partial^5 w}{\partial\theta^5}+\frac{\partial^3 w}{\partial\theta^3}+\frac{\partial w}{\partial\theta}\right)-\frac{\partial f}{\partial\theta}+\frac{\mu}{R^2}\frac{\partial^3 f}{\partial\theta^3}$$
$$+\frac{N^T}{R}\frac{\partial^3 w}{\partial\theta^3}-\frac{\mu}{R^3}N^T\frac{\partial^5 w}{\partial\theta^5}+\frac{N_0}{R}\frac{\partial^3 w}{\partial\theta^3}$$
$$-\frac{\mu}{R^3}N_0\frac{\partial^5 w}{\partial\theta^5}=0, \tag{27}$$

where f is defined in (24).

4. Solution Method

The nanoring or nanoarch with total central angle α is considered. The analytical solution for free vibration of nanoarches can be written as

$$w(\theta,t)=W(\theta)\,e^{i\omega_n t} \tag{28}$$

in which ω_n is the natural frequency of the nanoring. Navier solution is employed for simply-simply supported circular curved nanobeam. Therefore (27) can be rewritten as follows:

$$w(\theta,t)=\sin\left(\frac{n\pi}{\alpha}\theta\right)e^{i\omega_n t}, \tag{29}$$

where $\sin((n\pi/\alpha)\theta)$ is the corresponding deformation shape of the circular curved nanobeam and nanoring, and i is the conventional imaginary number $\sqrt{-1}$. Substituting (29) into (26) and (27), natural frequencies and critical buckling load of the circular curved nanobeam and nanoring with surface properties and elastic foundation can be obtained.

5. Numerical Results

In this section, numerical results are presented for the vibration and buckling of circular curved nanobeam embedded in an elastic medium with surface properties. In the first step, the accuracy study of the present analytically method is illustrated. The bulk elastic properties are $E = 177.3$ Gpa, $\rho = 7000\,\mathrm{Kg/m^3}$, and $\nu = 0.27$, and the surface elastic properties are $\lambda^s = -8$ N/m, $\mu^s = 2.5$ N/m, $\tau = 1.7$ N/m, and $\rho^s = 7\times10^{-6}\,\mathrm{Kg/m^2}$ [8].

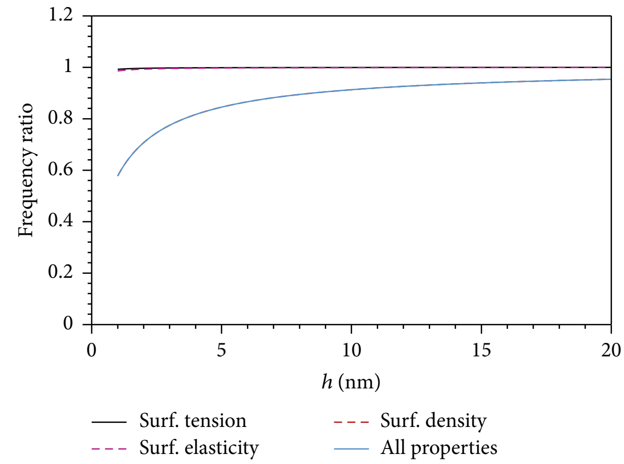

FIGURE 2: Comparison between the effect of various surface properties on natural frequency with respect to thickness h (nm).

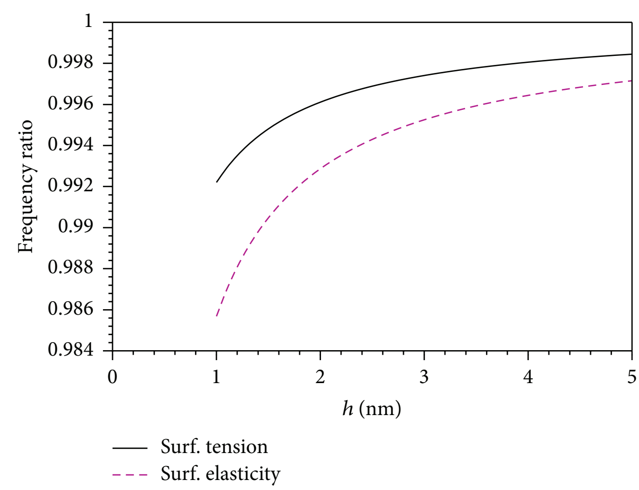

FIGURE 3: Comparison between the effect of surface tension and surface elasticity on natural frequency with respect to thickness h (nm).

5.1. Validation of the Results. To validate results, elastic foundations, thermal effect, and nonlocal coefficient are eliminated and simply-simply supported boundary conditions are considered. Therefore, the plot of frequency ratio versus thickness of curved nanobeam is illustrated in Figures 2 and 3. The results by comparison with [33] have been reached to reasonable results. In this survey the validity of our research can be represented as is shown in Figures 2 and 3. However, to validate results with nonlocal coefficient, the accuracy of the nonlocal natural frequencies of straight nanotube is investigated [4, 6]. The nondimensional fundamental frequencies and critical buckling loads of the nonlocal nanotube without consideration of the surface properties are compared to the results presented by Reddy [4], Thai, [6], and Ebrahimi et al. [20] and are listed in Tables 1 and 2, respectively. It is observed that the present results agree very well with those given by [4, 6, 20] and that increasing the nonlocality parameter tends to decrease the natural frequency and critical buckling

Table 1: Comparison of the dimensionless fundamental natural frequencies of simply supported nanobeam.

L/h	μ	Thai [6]	Reddy [4]	Ebrahimi et al. [20]	Present study
10	0	9.8293	9.8696	9.8696	9.8696
	1	9.3774	9.4159	9.4159	9.4159
	2	8.9826	9.0195	9.0195	9.0195
	3	8.6338	8.6693	8.6693	8.6693
	4	8.3228	8.3569	8.3569	8.3569
20	0	9.8293	9.8696	9.8696	9.8696
	1	9.3774	9.4159	9.4159	9.4159
	2	8.9826	9.0195	9.0195	9.0195
	3	8.6338	8.6693	8.6693	8.6693
	4	8.3228	8.3569	8.3569	8.3569
100	0	9.8293	9.8696	9.8696	9.8696
	1	9.3774	9.4159	9.4159	9.4159
	2	8.9826	9.0195	9.0195	9.0195
	3	8.6338	8.6693	8.6693	8.6693
	4	8.3228	8.3569	8.3569	8.3569

Table 2: Comparison of the dimensionless critical buckling load of simply supported nanobeam.

L/h	μ	Thai [6]	Reddy [4]	Ebrahimi et al. [20]	Present study
10	0	9.8696	9.8696	9.8696	9.8696
	1	8.9830	8.9830	8.9830	8.9830
	2	8.2426	8.2426	8.2426	8.2426
	3	7.6149	7.6149	7.6149	7.6149
	4	7.0761	7.0761	7.0761	7.0761
20	0	9.8696	9.8696	9.8696	9.8696
	1	8.9830	8.9830	8.9830	8.9830
	2	8.2426	8.2426	8.2426	8.2426
	3	7.6149	7.6149	7.6149	7.6149
	4	7.0761	7.0761	7.0761	7.0761
100	0	9.8696	9.8696	9.8696	9.8696
	1	8.9830	8.9830	8.9830	8.9830
	2	8.2426	8.2426	8.2426	8.2426
	3	7.6149	7.6149	7.6149	7.6149
	4	7.0761	7.0761	7.0761	7.0761

load. The reason is that the presence of the nonlocal effect tends to decrease the stiffness of the nanostructures and hence decrease the values of natural frequencies and critical buckling loads.

5.2. The Effect of Temperature Change. To illustrate effects of temperature on vibration and buckling of circular curved nanobeams, buckling η_{Thermal} and vibration ξ_{Thermal} nondimensional thermal ratio parameters are defined as follows:

$$\eta_{\text{Thermal}} = \frac{N_{\text{Thermal}}}{N_{\text{Non-thermal}}}, \quad \xi_{\text{Thermal}} = \frac{\Omega_{\text{Thermal}}}{\Omega_{\text{Non-thermal}}}, \tag{30}$$

where N_{Thermal} and $N_{\text{Non-thermal}}$ represent the dimensionless critical buckling load when the influence of temperature change is assumed or eliminated, respectively. Also Ω_{Thermal}

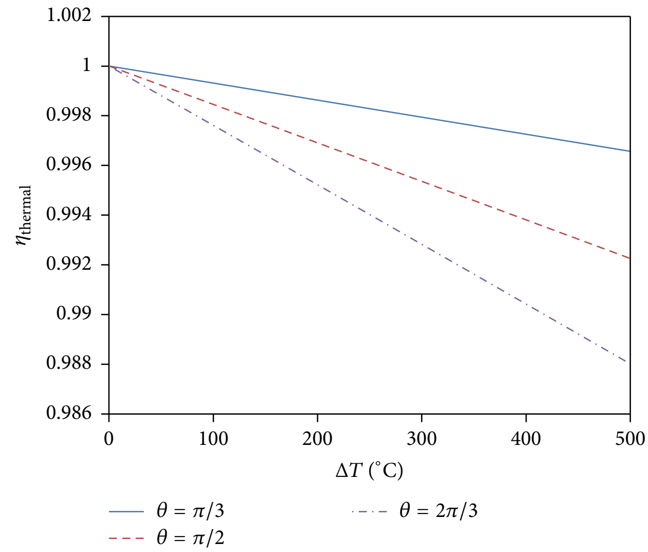

Figure 4: Buckling thermal ratio parameter of circular curved nanobeam with respect to temperature change ($\mu = 1$, $K_w = 10^{10}$, $K_p = 10^{-6}$, $R = 10$ nm).

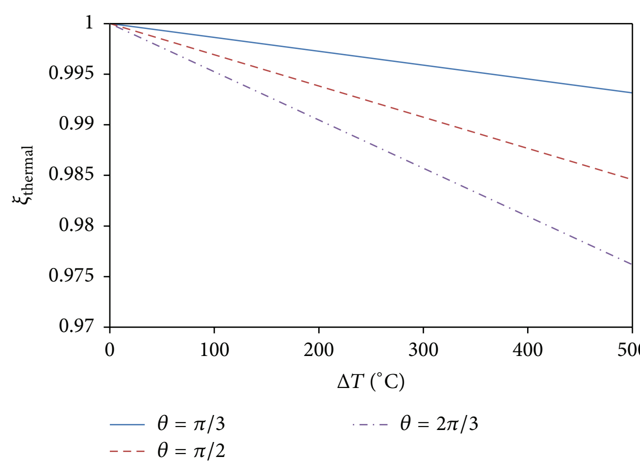

Figure 5: Vibration thermal ratio parameter of circular curved nanobeam with respect to temperature change ($\mu = 1$, $K_w = 10^{10}$, $K_p = 10^{-6}$, $R = 10$ nm).

and $\Omega_{\text{Non-thermal}}$ represent the dimensionless natural frequency when the effect of temperature change is considered or neglected, respectively.

As it is clear in Figures 4 and 5, for all types of opening angles considered, the vibration and critical buckling load thermal ratios η_{Thermal} and ξ_{Thermal} reduce with increasing the temperature change. In other words, with increasing of the temperature, the nondimensional natural frequency and critical buckling load reduce. Furthermore, the slope of vibration and buckling thermal ratio versus temperature change increases as the opening angles become greater.

5.3. Influence of the Nonlocal Parameter. Next, In order to illustrate the effect of the nonlocal parameter on the circular curved nanobeam in thermal environment, the

Table 3: Comparison of the dimensionless fundamental natural frequencies of circular curved nanobeam with and without surface effects with various temperature changes and nonlocality $R = 20$ nm, $K_w = 10^{10}$, and $K_p = 10^{-6}$.

ΔT	μ	$\theta = \pi/3$ With surface effects	$\theta = \pi/3$ Without surface effects	$\theta = \pi/2$ With surface effects	$\theta = \pi/2$ Without surface effects	$\theta = 2\pi/3$ With surface effects	$\theta = 2\pi/3$ Without surface effects
0	0	9.6217	11.4112	12.3122	14.5812	16.5025	19.5307
	1	9.3899	11.1352	12.1984	14.4455	16.4623	19.4828
	2	9.1919	10.8994	12.1026	14.3312	16.4287	19.4426
	3	9.0207	10.6954	12.0208	14.2337	16.4002	19.4086
	4	8.8710	10.5171	11.9501	14.1493	16.3756	19.3793
250	0	9.5838	11.3666	12.2531	14.5113	16.4347	19.4504
	1	9.3512	11.0865	12.1388	14.3750	16.3943	19.4023
	2	9.1523	10.8527	12.0425	14.2601	16.3606	19.3620
	3	8.9803	10.6478	11.9602	14.1621	16.3319	19.3278
	4	8.8300	10.4687	11.8892	14.0773	16.3073	19.2984
500	0	9.5459	11.3218	12.1937	14.4411	16.3665	19.3699
	1	9.3123	11.0436	12.0788	14.3041	16.3261	19.3216
	2	9.1126	10.8057	11.9820	14.1887	16.2922	19.2811
	3	8.9398	10.5999	11.8994	14.0901	16.2634	19.2468
	4	8.7888	10.4200	11.8280	14.0049	16.2386	19.2172

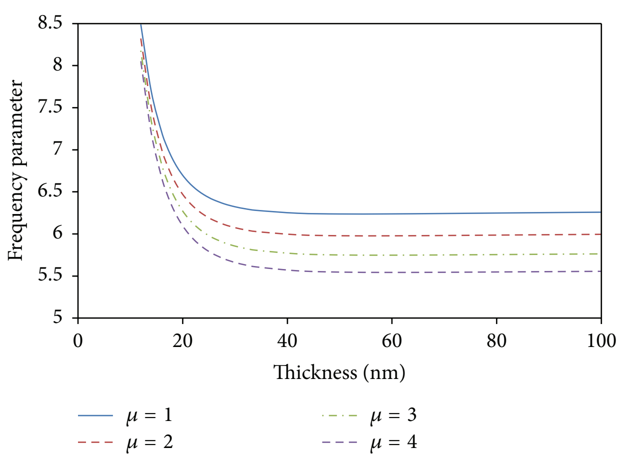

Figure 6: Dimensionless natural frequency with respect to thickness h for various nonlocalities ($K_w = 10^{10}$, $K_p = 10^{-6}$, $R = 30$ nm, $\Delta T = 100$).

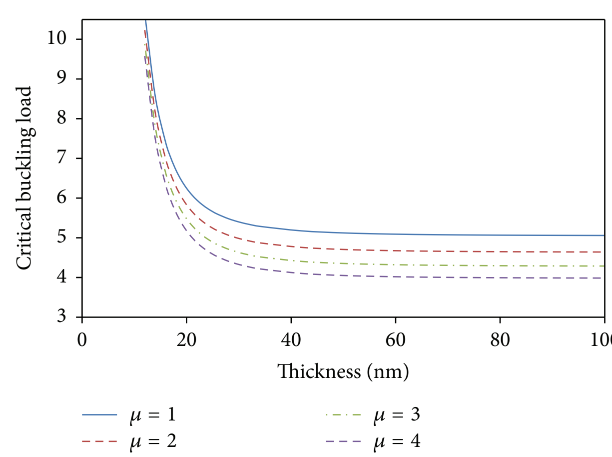

Figure 7: Dimensionless critical buckling load with respect to thickness h for various nonlocalities ($K_w = 10^{10}$, $K_p = 10^{-6}$, $R = 30$ nm, $\Delta T = 100$).

nondimensional fundamental natural frequency and critical buckling load for various values of nonlocal parameters are illustrated in Figures 6 and 7. It is obviously seen from Figures 6 and 7 that the critical buckling load and frequency parameter decrease with the increase of values of the nonlocal parameter.

The reason is that the presence of the nonlocal effect tends to decrease the stiffness of the nanostructures and furthermore decrease the values of dimensionless natural frequencies and critical buckling loads.

5.4. The Effect of Surface Properties. To illustrate surface effects in thermal environment for vibration and buckling of circular curved nanobeams, Tables 3 and 4 are presented. According to the tables, the dimensionless frequency and critical buckling loads are investigated for different values of temperature changes and nonlocality with various opening angles, with and without surface properties.

According to Tables 3 and 4, it is obviously seen that the dimensionless frequency and critical buckling load decrease with considering surface properties and also decrease when opening angle increases. It is interesting to say that natural frequencies and critical buckling load also decrease with increase in temperature changes. However, to compare Table 3 with Table 4 it can be seen that surface properties are more sensitive in vibration analysis than buckling analysis.

TABLE 4: Comparison of the dimensionless critical buckling loads of circular curved nanobeam with and without surface effects with various temperature changes and nonlocality $R = 20$ nm, $K_w = 10^{10}$, and $K_p = 10^{-6}$.

ΔT	μ	$\theta = \pi/3$ With surface effects	$\theta = \pi/3$ Without surface effects	$\theta = \pi/2$ With surface effects	$\theta = \pi/2$ Without surface effects	$\theta = 2\pi/3$ With surface effects	$\theta = 2\pi/3$ Without surface effects
0	0	14.5910	14.6596	26.8786	26.9274	55.7991	55.8259
	1	13.8967	13.9591	26.3843	26.4287	55.5279	55.5523
	2	13.3168	13.3740	25.9715	26.0122	55.3014	55.3237
	3	12.8252	12.8781	25.6215	25.6592	55.1094	55.1300
	4	12.4032	12.4523	25.3211	25.3561	54.9445	54.9637
250	0	14.4766	14.5451	26.6212	26.6700	55.3414	55.3682
	1	13.7822	13.8446	26.1268	26.1713	55.0702	55.0946
	2	13.2023	13.2596	25.7140	25.7548	54.8437	54.8660
	3	12.7108	12.7637	25.3641	25.4017	54.6516	54.6723
	4	12.2888	12.3379	25.0636	25.0986	54.4868	54.5060
500	0	14.3622	14.4307	26.3637	26.4125	54.8837	54.9105
	1	13.6678	13.7302	25.8694	25.9138	54.6125	54.6368
	2	13.0879	13.1452	25.4565	25.4973	54.3859	54.4083
	3	12.5963	12.6492	25.1066	25.1442	54.1939	54.2146
	4	12.1743	12.2235	24.8062	24.8412	54.0291	54.0483

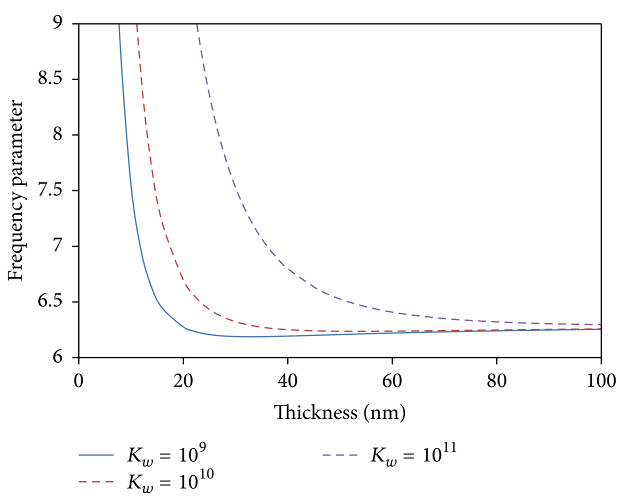

FIGURE 8: Dimensionless natural frequency with respect to thickness h for various Winkler elastic foundations ($K_p = 10^{-6}$, $R = 30$ nm, $\Delta T = 100$, $\mu = 1$, $b = 50$ nm, $\theta = \pi/2$).

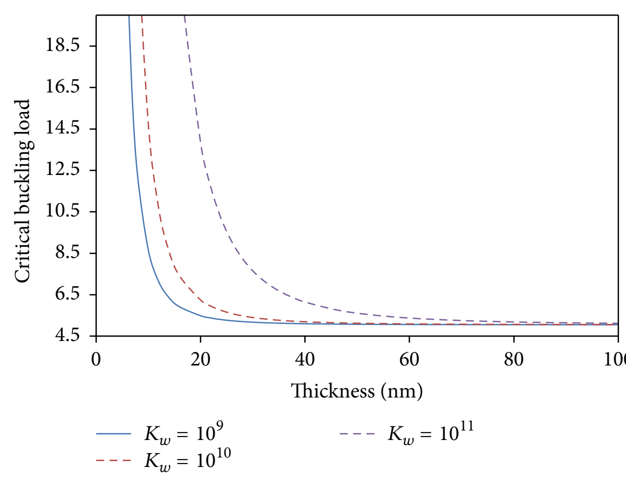

FIGURE 9: Dimensionless critical buckling load with respect to thickness h for various Winkler elastic foundations ($K_p = 10^{-6}$, $R = 30$ nm, $\Delta T = 100$, $\mu = 1$, $b = 50$ nm, $\theta = \pi/2$).

The results in Tables 3 and 4 can be used for design of circular curved nanobeams and nanorings in future.

6. Influence of the Winkler and Pasternak Elastic Foundations

In order to investigate the influence of the elastic foundation surrounding the circular curved nanobeam in thermal environment, the nondimensional fundamental natural frequency and critical buckling load for various values of Winkler and Pasternak parameters are illustrated in Figures 8, 9, 10, and 11. It can be observed that the nondimensional natural frequency and critical buckling load increase when the Winkler elastic foundation becomes stiffer. It is also seen that the influence of thickness on the nondimensional natural frequency and critical buckling load is higher than the Winkle parameter.

The nondimensional natural frequency and critical buckling load as a function of thickness, for three different Pasternak elastic foundation values, are illustrated in Figures 9 and 10, respectively. It can be observed that the nondimensional natural frequency and critical buckling load increase when the Pasternak elastic foundation becomes greater. It is also seen that the effect of thickness on the nondimensional

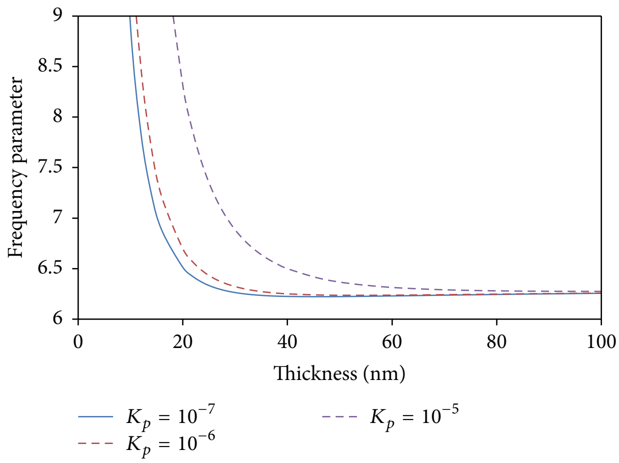

FIGURE 10: Dimensionless natural frequency with respect to thickness h for various Pasternak elastic foundations ($K_w = 10^{10}$, $R = 30\,\text{nm}$, $\Delta T = 100$, $\mu = 1$, $b = 50\,\text{nm}$, $\theta = \pi/2$).

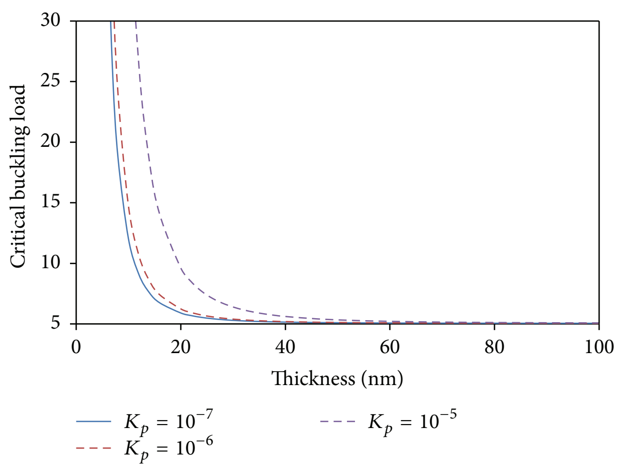

FIGURE 11: Dimensionless critical buckling load with respect to thickness h for various Pasternak elastic foundations ($K_w = 10^{10}$, $R = 30\,\text{nm}$, $\Delta T = 100$, $\mu = 1$, $b = 50\,\text{nm}$, $\theta = \pi/2$).

natural frequency and critical buckling load is higher than the Pasternak parameter. Moreover, by comparing Figures 7 and 8 with Figures 9 and 10, it should be noted that the influence of the Pasternak elastic foundation on the nondimensional natural frequency and critical buckling load is higher than the effect of the Winkler elastic foundation.

7. Conclusion

In this research, the surface properties on dimensionless natural frequency and critical buckling load of circular curved nanobeam embedded in an elastic medium were studied with various opening angles in the thermal environment, using the nonlocal elasticity model. Hamilton's principle was employed to derive the governing equations. Next, the analytically exact solution was employed to solve the governing equations for simply supported curved nanobeam. The effects of the surface properties, thickness of circular curved nanobeam, Winkler and Pasternak elastic foundations, opening angle, temperature changes, and nonlocal parameter were investigated on the frequency and critical buckling load parameters of the circular curved nanobeams. It is clearly observed that, by increasing thickness h, the surface effects tend to vanish. Furthermore it is shown that natural frequencies and critical buckling load also decrease with increase in temperature changes. In addition, results revealed that the elastic foundations and surface properties play an important role in vibration and buckling behavior of circular curved nanobeams. However it is interesting to say that the surface properties are more sensitive in vibration analysis than buckling analysis.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this article.

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