Research Article

Analytical Solution for One-Dimensional Nonlinear Consolidation of Saturated Double-Layered Soil Subjected to Cyclic Loadings

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Cyclic loading-induced consolidation behavior of soft soil is of great interest for the analysis of offshore and onshore structures. In this study, an analytical solution for one-dimensional (1D) nonlinear consolidation of saturated double-layered soil under various types of cyclic loadings such as trapezoidal cyclic loading, rectangular cyclic loading, and triangular cyclic loading was derived. The proposed solution was subsequently degenerated into solutions for special cases and compared to the existing solutions. The degenerate solutions show good agreement with the existing results, which proves that the proposed solutions are more general ones for 1D nonlinear consolidation of saturated soils under time-dependent loading. Finally, a comprehensive parametric study was conducted to investigate the influences of different layer parameters, drainage conditions, and loading parameters on nonlinear consolidation of saturated double-layered soil under cyclic loadings.

1. Introduction

Soft soil beneath most of the marine structures is saturated and can be subjected to various types of cyclic loads. The sources of cyclic load include traffic loads on roads, highways, railways, and airports; silos or tanks filling and discharging; wave and wind actions for offshore structures; and so on. For those structures built on soft soil, predicting the cyclic loading-induced consolidation deformation is a very important design issue.

Up to the present, a number of methods have been proposed for consolidation analysis of saturated soil subjected to cyclic loadings, which can generally be divided into two categories according to the adopted main assumptions. Many scholars have presented different solutions for a consolidation of saturated soil under various cyclic loadings such as rectangular cyclic loading [1], haversine cyclic loading [2–4], and the other types of cyclic loadings [5, 6]. However, those solutions are based on the linear assumption that the compressibility and the permeability of soil are constant during the consolidation process, which is a well-known drawback of the linear consolidation theory. In order

to overcome this shortcoming, a number of studies have been performed to consider nonlinear consolidation characteristics of soft soil. Several investigators [7, 8] analysed 1D nonlinear consolidation of soil subjected to rectangular cyclic loading using superimposing rule. In addition, numerous researchers have employed the nonlinear assumption proposed by Davis and Raymond [9] and studied the nonlinear consolidation of soil under different types of cyclic loadings [10–13]. Furthermore, Geng et al. [14], Cai et al. [15], and Kim et al. [16] developed solutions for 1D nonlinear consolidation of saturated soil subjected to cyclic loading by considering the variable consolidation coefficient.

All the research works mentioned above only considered the single-layer soil and did not take the layered characteristics of soil into consideration. In view of this, a number of researchers made great contributions to the consolidation theories of saturated layered soil in recent years [17–24]. Among them, Hu et al. [24] applied the differential quadrature method to analyse cyclic loading-induced nonlinear consolidation behavior of saturated multi-layered soil. However, due to the complexity of the nonlinear

consolidation equation, none of the researchers have ever derived any analytical solutions for 1D nonlinear consolidation of soil taking both the layered characteristics of soil and the cyclic loading condition into account.

In this paper, analytical solutions are derived for 1D nonlinear consolidation of saturated double-layered soil under different cyclic loadings such as trapezoidal cyclic loading, rectangular cyclic loading, and triangular cyclic loading. The proposed solutions are degenerated into existing solutions in previous literature. A comprehensive parametric study is conducted to investigate the influences of different layer parameters, drainage conditions, and loading parameters on nonlinear consolidation behavior of saturated double-layered soil under cyclic loadings.

2. Mathematical Model

Figure 1 illustrates the schematic model for 1D nonlinear consolidation of double-layered soil subjected to time-dependent loading, where h_i , k_{v0}^i , m_{v0}^i , and c_v^i are the thickness, the initial coefficient of permeability, the initial coefficient of compressibility, and the consolidation coefficient of the i -th soil layer ($i = 1, 2$), respectively. H is the total thickness of soil that satisfies $H = h_1 + h_2$. $q(t)$ is the uniformly distributed time-dependent loading applied on the top surface of the soil.

Based on Davis and Raymond's nonlinear consolidation theory, the governing equation for 1D nonlinear consolidation of double-layered soil subjected to time-dependent loading can be written as follows:

$$c_v^i \left[\frac{\partial^2 u_i}{\partial z^2} + \frac{1}{\sigma'^i} \left(\frac{\partial u_i}{\partial z} \right)^2 \right] = \frac{\partial u_i}{\partial t} - \frac{dq}{dt}, \quad i = 1, 2, \quad (1)$$

where u_i and σ'^i are the excess pore water pressure and the effective vertical stress in the i -th soil layer, respectively; z is the spatial coordinate as shown in Figure 1; and t is time.

According to the assumption that the decrease in permeability is proportional to the decrease in compressibility during the consolidation process of soil, the consolidation coefficient c_v^i is given by $c_v^i = (k_{v0}^i/m_{v0}^i\gamma_w)$, in which γ_w is the unit weight of water; $m_{v0}^i = 0.434C_c^i/(1 + e_0^i)/\sigma_0^i$, where C_c^i and e_0^i are the compression index and the initial void ratio in the i -th layer corresponding to the initial effective vertical stress σ_0^i , respectively. By the assumption that the distribution of initial effective stress is constant with depth, $\sigma_0^i = \sigma_0'$.

FIGURE 1: Schematic model of double-layered soil.

Using Terzaghi's principle of effective stress, σ'^i can be written as follows:

$$\sigma'^i = \sigma_0' + q - u_i. \quad (2)$$

In order to simplify equation (1), a new variable can be defined as follows:

$$\omega_i = \ln \left(\frac{\sigma'^i}{\sigma_0' + q} \right). \quad (3)$$

Then, equation (1) can be simplified in terms of ω_i as follows:

$$\frac{\partial \omega_i}{\partial t} = c_v^i \frac{\partial^2 \omega_i}{\partial z^2} + R(t), \quad i = 1, 2, \quad (4)$$

where

$$R(t) = -\frac{1}{\sigma_0' + q} \frac{dq}{dt}. \quad (5)$$

The initial condition for equation (4) in terms of u_i and ω_i can be given by

$$u_i(z, 0) = q(0), \text{ or } \omega_i(z, 0) = \omega_0. \quad (6)$$

The boundary conditions in terms of u_i and ω_i can be written as follows:

$$u_1(0, t) = 0, \text{ or } \omega_1(0, t) = 0$$

$$\frac{\partial u_2(H, t)}{\partial z} = 0, \text{ or } \frac{\partial \omega_2(H, t)}{\partial z} = 0, \quad (\text{for single drainage condition}), \quad (7)$$

$$u_2(H, t) = 0, \text{ or } \omega_2(H, t) = 0, \quad (\text{for double drainage condition}),$$

$$u_1(h_1, t) = u_2(h_1, t), \text{ or } \omega_1(h_1, t) = \omega_2(h_1, t), \quad (8)$$

$$k_v^1 \frac{\partial u_1(h_1, t)}{\partial z} = k_v^2 \frac{\partial u_2(h_1, t)}{\partial z}, \text{ or } \frac{\partial \omega_1(h_1, t)}{\partial z} = K \frac{\partial \omega_2(h_1, t)}{\partial z}, \quad (9)$$

where $\omega_0 = \ln\{\sigma'_0/[\sigma'_0 + q(0)]\}$ and $K = k_v^2/k_v^1 = c_v^2 m_{v0}^2 / c_v^1 m_{v0}^1 = k_{v0}^2 / k_{v0}^1$.

3. Derivation of Solutions for Cyclic Loadings

3.1. Solutions for Trapezoidal Cyclic Loading

3.1.1. *Excess Pore Water Pressure.* From equations (2) and (3), the excess pore water pressure for double-layered soil

subjected to time-dependent loading can be written as follows:

$$u_i = (\sigma'_0 + q)(1 - e^{\omega_i}), \quad i = 1, 2, \quad (10)$$

where ω_i can be obtained by equations (A.10)–(A.13) according to drainage conditions. The detailed derivation for ω_i is shown in Appendix A.

Trapezoidal cyclic loading shown in Figure 2 can be expressed as follows:

$$q(t) = \begin{cases} \frac{q_u}{\alpha t_0} [t - (N-1)\beta t_0], & (N-1)\beta t_0 \leq t \leq [(N-1)\beta + \alpha]t_0, \\ q_u, & [(N-1)\beta + \alpha]t_0 \leq t \leq [(N-1)\beta + (1-\alpha)]t_0, \\ -\frac{q_u}{\alpha t_0} [t - (N-1)\beta t_0 - t_0], & [(N-1)\beta + (1-\alpha)]t_0 \leq t \leq [(N-1)\beta + 1]t_0, \\ 0, & [(N-1)\beta + 1]t_0 \leq t \leq N\beta t_0, \end{cases} \quad (11)$$

where q_u is the maximum loading; βt_0 is the period of one loading cycle; α and β are the loading parameters corresponding to the rate of loading increment or decrement and the rest period of loading, respectively; and N is the cycle number.

By solving the integrals in equations (A.10)–(A.13) for the trapezoidal cyclic loading function, complete solutions for the excess pore water pressure of saturated double-layered soil under trapezoidal cyclic loading can be expressed as follows:

$$u_i = \begin{cases} \frac{q_u T_1}{N_\sigma - 1} (1 - e^{\omega_{i1}}), & T_{vb1} \leq T_{v1} \leq T_{vb1} + T_{vc1}, \\ \frac{q_u N_\sigma}{N_\sigma - 1} (1 - e^{\omega_{i2}}), & T_{vb1} + T_{vc1} \leq T_{v1} \leq T_{vf1} - T_{vc1}, \\ \frac{q_u T_2}{N_\sigma - 1} (1 - e^{\omega_{i3}}), & T_{vf1} - T_{vc1} \leq T_{v1} \leq T_{vf1}, \\ \frac{q_u}{N_\sigma - 1} (1 - e^{\omega_{i4}}), & T_{vf1} \leq T_{v1} \leq T_{vN1}, \end{cases} \quad i = 1, 2, \quad (12)$$

where

FIGURE 2: Trapezoidal cyclic loading.

$$\begin{aligned}
 T_1 &= \frac{T_{vc1} + (T_{v1} - T_{vb1})(N_\sigma - 1)}{T_{vc1}}, \quad T_2 = \frac{T_{vc1} + (T_{vf1} - T_{v1})(N_\sigma - 1)}{T_{vc1}}, \\
 N_\sigma &= \frac{\sigma'_0 + q_u}{\sigma'_0}, \quad T_{v1} = \frac{c_{v1}t}{h_1^2}, \quad T_{vc1} = \frac{c_{v1}\alpha t_0}{h_1^2}, \\
 T_{vb1} &= \frac{c_{v1}(N-1)\beta t_0}{h_1^2}, \quad T_{vf1} = \frac{c_{v1}[(N-1)\beta + 1]t_0}{h_1^2}, \quad T_{vN1} = \frac{c_{v1}N\beta t_0}{h_1^2}.
 \end{aligned} \tag{13}$$

For a single drainage condition,

$$\omega_{1j} = - \sum_{m=1}^{\infty} \sin\left(\lambda_m \frac{z}{h_1}\right) e^{-\lambda_m^2 T_{v1}} \frac{2C_j}{\lambda_m(1 + \mu^2 KcA_m^2)} \quad j = 1, 2, 3, 4, \tag{14a}$$

$$\omega_{2j} = - \sum_{m=1}^{\infty} A_m \cos\left(\mu \lambda_m \frac{H-z}{h_1}\right) e^{-\lambda_m^2 T_{v1}} \frac{2C_j}{\lambda_m(1 + \mu^2 KcA_m^2)} \quad j = 1, 2, 3, 4. \tag{14b}$$

For a double drainage condition,

$$\omega_{1j} = - \sum_{m=1}^{\infty} \sin\left(\lambda_m \frac{z}{h_1}\right) e^{-\lambda_m^2 T_{v1}} \frac{2(1 + \mu K A_m)C_j}{\lambda_m(1 + \mu^2 KcA_m^2)} \quad j = 1, 2, 3, 4, \tag{14c}$$

$$\omega_{2j} = - \sum_{m=1}^{\infty} A_m \sin\left(\mu \lambda_m \frac{H-z}{h_1}\right) e^{-\lambda_m^2 T_{v1}} \frac{2(1 + \mu K A_m)C_j}{\lambda_m(1 + \mu^2 KcA_m^2)} \quad j = 1, 2, 3, 4, \tag{14d}$$

where A_m and λ_m can be given by equations (A.2), (A.6), (A.14), and (A.15) according to drainage conditions. c and μ can be obtained by equations (A.3) and (A.5), respectively.

The expressions of C_j (i.e., C_1 , C_2 , C_3 , and C_4) are as follows:

$$\begin{aligned}
 C_1 &= D(N-1) + D_1, \quad C_2 = D(N-1) + D_2, \quad C_3 = D(N-1) + D_2 + D_3, \\
 C_4 &= D(N), \quad D(n) = \sum_{N=1}^n (D_2 + D_4),
 \end{aligned} \tag{14e}$$

where

$$\begin{aligned}
 D_1 &= e^{\frac{\lambda_m^2 [(N_\sigma - 1)T_{vb1} - T_{vc1}]}{N_\sigma - 1}} \left[\ln T_1 + \sum_{k=1}^{\infty} \frac{(\lambda_m^2 T_{vc1})^k (T_1^k - 1)}{k!k(N_\sigma - 1)^k} \right], \\
 D_2 &= e^{\frac{\lambda_m^2 [(N_\sigma - 1)T_{vb1} - T_{vc1}]}{N_\sigma - 1}} \left[\ln N_\sigma + \sum_{k=1}^{\infty} \frac{(\lambda_m^2 T_{vc1})^k (N_\sigma^k - 1)}{k!k(N_\sigma - 1)^k} \right], \\
 D_3 &= e^{\frac{\lambda_m^2 [(N_\sigma - 1)T_{vf1} + T_{vc1}]}{N_\sigma - 1}} \left[\ln \frac{T_2}{N_\sigma} + \sum_{k=1}^{\infty} \frac{(-\lambda_m^2 T_{vc1})^k (T_2^k - N_\sigma^k)}{k!k(N_\sigma - 1)^k} \right], \\
 D_4 &= e^{\frac{\lambda_m^2 [(N_\sigma - 1)T_{vf1} + T_{vc1}]}{N_\sigma - 1}} \left[\ln \frac{1}{N_\sigma} + \sum_{k=1}^{\infty} \frac{(-\lambda_m^2 T_{vc1})^k (1 - N_\sigma^k)}{k!k(N_\sigma - 1)^k} \right].
 \end{aligned} \tag{14f}$$

3.1.2. Average Degree of Consolidation. The average degree of consolidation can be expressed either by settlement or by effective stress. The average degrees of consolidation for double-layered soil subjected to time-dependent loading can be written as follows [22]:

$$\begin{aligned}
 U_s &= \frac{U_{s1} + \mu^2 Kc U_{s2}}{1 + \mu^2 Kc}, \\
 U_p &= \frac{U_{p1} + c U_{p2}}{1 + c},
 \end{aligned} \tag{15}$$

where U_{s1} , U_{s2} , U_{p1} , and U_{p2} are average degrees of consolidation in terms of settlement and effective stress for each layer, respectively, defined as follows:

$$\begin{aligned}
 U_{s1} &= \frac{\ln[(\sigma'_0 + q)/\sigma'_0] + 1/h_1 \int_0^{h_1} \omega_1 dz}{\ln N_\sigma}, \quad U_{s2} = \frac{\ln[(\sigma'_0 + q)/\sigma'_0] + 1/h_2 \int_{h_1}^H \omega_2 dz}{\ln N_\sigma}, \\
 U_{p1} &= \frac{q - 1/h_1 \int_0^{h_1} u_1 dz}{q_u}, \quad U_{p2} = \frac{q - 1/h_2 \int_{h_1}^H u_2 dz}{q_u}.
 \end{aligned} \tag{16}$$

For trapezoidal cyclic loading, complete solutions for the average degrees of consolidation can be written as follows:

$$U_s = \begin{cases} \frac{1}{\ln N_\sigma} \left(\ln T_1 - \frac{1}{1 + \mu^2 Kc} \sum_{m=1}^{\infty} \frac{2aC_1}{\lambda_m^2 (1 + \mu^2 KcA_m^2)} e^{-\lambda_m^2 T_{v1}} \right), & T_{vb1} \leq T_{v1} \leq T_{vb1} + T_{vc1}, \\ \frac{1}{\ln N_\sigma} \left(\ln N_\sigma - \frac{1}{1 + \mu^2 Kc} \sum_{m=1}^{\infty} \frac{2aC_2}{\lambda_m^2 (1 + \mu^2 KcA_m^2)} e^{-\lambda_m^2 T_{v1}} \right), & T_{vb1} + T_{vc1} \leq T_{v1} \leq T_{vf1} - T_{vc1}, \\ \frac{1}{\ln N_\sigma} \left(\ln T_2 - \frac{1}{1 + \mu^2 Kc} \sum_{m=1}^{\infty} \frac{2aC_3}{\lambda_m^2 (1 + \mu^2 KcA_m^2)} e^{-\lambda_m^2 T_{v1}} \right), & T_{vf1} - T_{vc1} \leq T_{v1} \leq T_{vf1}, \\ \frac{1}{\ln N_\sigma (1 + \mu^2 Kc)} \sum_{m=1}^{\infty} \frac{2aC_4}{\lambda_m^2 (1 + \mu^2 KcA_m^2)} e^{-\lambda_m^2 T_{v1}}, & T_{vf1} \leq T_{v1} \leq T_{vN1}, \end{cases} \quad (17)$$

$$U_p = \begin{cases} \frac{T_1}{N_\sigma - 1} \left[\frac{1}{H} \left(\int_0^{h_1} e^{\omega_{11}} dz + \int_{h_1}^H e^{\omega_{21}} dz \right) - \frac{1}{T_1} \right], & T_{vb1} \leq T_{v1} \leq T_{vb1} + T_{vc1}, \\ \frac{N_\sigma}{N_\sigma - 1} \left[\frac{1}{H} \left(\int_0^{h_1} e^{\omega_{12}} dz + \int_{h_1}^H e^{\omega_{22}} dz \right) - \frac{1}{N_\sigma} \right], & T_{vb1} + T_{vc1} \leq T_{v1} \leq T_{vf1} - T_{vc1}, \\ \frac{T_2}{N_\sigma - 1} \left[\frac{1}{H} \left(\int_0^{h_1} e^{\omega_{13}} dz + \int_{h_1}^H e^{\omega_{23}} dz \right) - \frac{1}{T_2} \right], & T_{vf1} - T_{vc1} \leq T_{v1} \leq T_{vf1}, \\ \frac{1}{N_\sigma - 1} \left[\frac{1}{H} \left(\int_0^{h_1} e^{\omega_{14}} dz + \int_{h_1}^H e^{\omega_{24}} dz \right) - 1 \right], & T_{vf1} \leq T_{v1} \leq T_{vN1}, \end{cases} \quad (18)$$

where $a = 1$ for a single drainage condition and $a = (1 + \mu K A_m)^2$ for a double drainage condition.

3.2. Solutions for Rectangular and Triangular Cyclic Loadings. Trapezoidal cyclic loading can be degenerated into rectangular cyclic loading and triangular cyclic loading. When $\alpha = 0$, trapezoidal cyclic loading reduces into rectangular cyclic loading (Figure 3(a)). On the other hand, trapezoidal cyclic loading reduces to triangular cyclic loading (Figure 3(b)) when $\alpha = 0.5$.

Analytical solutions for the excess pore water pressure under rectangular cyclic loading and triangular cyclic loading can be obtained substituting $\alpha = 0$ and $\alpha = 0.5$ into equation (12), respectively. Similarly, the average degrees of consolidation in terms of settlement and effective stress can also be given by equations (17) and (18).

4. Degeneration to Special Cases

The proposed solutions can be degenerated into existing solutions for 1D nonlinear consolidation of saturated double-layered soil under constant and ramp loadings and saturated single-layer soil under trapezoidal cyclic loading.

4.1. Solutions for Double-Layered Soil under Constant Loading. When $\alpha = 0$, $\beta = 1$, and $N = 1$, trapezoidal cyclic loading reduces to constant loading (Figure 4(a)) expressed by $q(t) = q_u$. Then, the solution can be obtained as follows:

$$u_i = \frac{q_u N_\sigma}{N_\sigma - 1} (1 - e^{\omega_i}), \quad i = 1, 2, \quad (19)$$

$$U_s = 1 - \frac{1}{1 + \mu^2 Kc} \sum_{m=1}^{\infty} \frac{2a}{\lambda_m^2 (1 + \mu^2 KcA_m^2)} e^{-\lambda_m^2 T_{v1}}, \quad (20)$$

$$U_p = \frac{N_\sigma}{N_\sigma - 1} \left[\frac{1}{H} \left(\int_0^{h_1} e^{\omega_1} dz - \int_{h_1}^H e^{\omega_2} dz \right) - \frac{1}{N_\sigma} \right], \quad (21)$$

where ω_i can be written as follows.

For a single drainage condition,

$$\omega_1 = -\ln N_\sigma \sum_{m=1}^{\infty} \sin\left(\lambda_m \frac{z}{h_1}\right) e^{-\lambda_m^2 T_{v1}} \frac{2}{\lambda_m (1 + \mu^2 KcA_m^2)}, \quad (22a)$$

$$\omega_2 = -\ln N_\sigma \sum_{m=1}^{\infty} A_m \cos\left(\mu \lambda_m \frac{H - z}{h_1}\right) e^{-\lambda_m^2 T_{v1}} \frac{2}{\lambda_m (1 + \mu^2 KcA_m^2)}. \quad (22b)$$

FIGURE 3: Rectangular cyclic loading (a) and triangular cyclic loading (b).

For a double drainage condition,

$$\omega_1 = -\ln N_\sigma \sum_{m=1}^{\infty} \sin\left(\lambda_m \frac{z}{h_1}\right) e^{-\lambda_m^2 T_{v1}} \frac{2(1 + \mu K A_m)}{\lambda_m(1 + \mu^2 K c A_m^2)}, \quad (22c)$$

$$\omega_2 = -\ln N_\sigma \sum_{m=1}^{\infty} A_m \sin\left(\mu \lambda_m \frac{H-z}{h_1}\right) e^{-\lambda_m^2 T_{v1}} \frac{2(1 + \mu K A_m)}{\lambda_m(1 + \mu^2 K c A_m^2)}. \quad (22d)$$

Equations (19)–(21) are the solutions for double-layered soil subjected to constant loading proposed by Xie et al. [22].

4.2. Solutions for Double-Layered Soil under Ramp Loading. When \$N = 1\$ and \$t_0\$ tend to infinity, trapezoidal cyclic loading reduces to ramp loading (Figure 4(b)) given by

$$q(t) = \begin{cases} \frac{t}{t_c} q_u, & t \leq t_c, \\ q_u, & t \geq t_c, \end{cases} \quad (23)$$

where \$t_c = \alpha t_0\$.

The solutions for double-layered soil under ramp loading can be obtained by

$$u_i = \begin{cases} \frac{q_u T}{N_\sigma - 1} (1 - e^{\omega_{i1}}), & T_{v1} \leq T_{vc1}, \\ \frac{q_u N_\sigma}{N_\sigma - 1} (1 - e^{\omega_{i2}}), & T_{v1} \geq T_{vc1}, \end{cases}, \quad i = 1, 2, \quad (24)$$

$$U_s = \begin{cases} \frac{1}{\ln N_\sigma} \left(\ln T - \frac{1}{1 + \mu^2 K c} \sum_{m=1}^{\infty} \frac{2aC_1}{\lambda_m^2 (1 + \mu^2 K c A_m^2)} e^{-\lambda_m^2 T_{v1}} \right), & T_{v1} \leq T_{vc1}, \\ \frac{1}{\ln N_\sigma} \left(\ln N_\sigma - \frac{1}{1 + \mu^2 K c} \sum_{m=1}^{\infty} \frac{2aC_2}{\lambda_m^2 (1 + \mu^2 K c A_m^2)} e^{-\lambda_m^2 T_{v1}} \right), & T_{v1} \geq T_{vc1}, \end{cases} \quad (25)$$

$$U_p = \begin{cases} \frac{T}{N_\sigma - 1} \left[\frac{1}{H} \left(\int_0^{h_1} e^{\omega_{11}} dz - \int_{h_1}^H e^{\omega_{21}} dz \right) - \frac{1}{T} \right], & T_{v1} \leq T_{vc1}, \\ \frac{N_\sigma}{N_\sigma - 1} \left[\frac{1}{H} \left(\int_0^{h_1} e^{\omega_{12}} dz - \int_{h_1}^H e^{\omega_{22}} dz \right) - \frac{1}{N_\sigma} \right], & T_{v1} \geq T_{vc1}, \end{cases} \quad (26)$$

FIGURE 4: Constant loading (a) and ramp loading (b).

where ω_{ij} can be written as follows:

For a single drainage condition,

$$\omega_{1j} = - \sum_{m=1}^{\infty} \sin\left(\lambda_m \frac{z}{h_1}\right) e^{-\lambda_m^2 T_{v1}} \frac{2C_j}{\lambda_m (1 + \mu^2 K c A_m^2)}, \quad j = 1, 2, \quad (27a)$$

$$\omega_{2j} = - \sum_{m=1}^{\infty} A_m \cos\left(\mu \lambda_m \frac{H-z}{h_1}\right) e^{-\lambda_m^2 T_{v1}} \frac{2C_j}{\lambda_m (1 + \mu^2 K c A_m^2)}, \quad j = 1, 2. \quad (27b)$$

For a double drainage condition,

$$\omega_{1j} = - \sum_{m=1}^{\infty} \sin\left(\lambda_m \frac{z}{h_1}\right) e^{-\lambda_m^2 T_{v1}} \frac{2(1 + \mu K A_m) C_j}{\lambda_m (1 + \mu^2 K c A_m^2)}, \quad j = 1, 2, \quad (27c)$$

$$\omega_{2j} = - \sum_{m=1}^{\infty} A_m \sin\left(\mu \lambda_m \frac{H-z}{h_1}\right) e^{-\lambda_m^2 T_{v1}} \frac{2(1 + \mu K A_m) C_j}{\lambda_m (1 + \mu^2 K c A_m^2)}, \quad j = 1, 2,$$

$$C_1 = e^{-\lambda_m^2 T_{vc1} / (N_\sigma - 1)} \left[\ln T + \sum_{k=1}^{\infty} \frac{(\lambda_m^2 T_{vc1})^k (T^k - 1)}{k! k (N_\sigma - 1)^k} \right],$$

$$C_2 = e^{-\lambda_m^2 T_{vc1} / (N_\sigma - 1)} \left[\ln N_\sigma + \sum_{k=1}^{\infty} \frac{(\lambda_m^2 T_{vc1})^k (N_\sigma^k - 1)}{k! k (N_\sigma - 1)^k} \right], \quad (27d)$$

$$T = \frac{T_{vc1} + T_{v1} (N_\sigma - 1)}{T_{vc1}},$$

$$T_{vc1} = \frac{c_{v1} t_c}{h_1^2}.$$

Equations (24)–(26) are the solutions for double-layered soil under ramp loading proposed by Xie et al. [22].

4.3. Solutions for Single-Layer Soil under Trapezoidal Cyclic Loading. When $\mu = K = 1$, the solutions for double-layered soil reduce to the solutions for single-layer soil. Therefore,

the proposed solutions can be reduced to the solutions for 1D nonlinear consolidation of saturated single-layer soil subjected to trapezoidal cyclic loading as follows:

$$u = \begin{cases} \frac{q_u T_1}{N_\sigma - 1} (1 - e^{-B_1}), & T_{vb} \leq T_v \leq T_{vb} + T_{vc}, \\ \frac{q_u N_\sigma}{N_\sigma - 1} (1 - e^{-B_2}), & T_{vb} + T_{vc} \leq T_v \leq T_{vf} - T_{vc}, \\ \frac{q_u T_2}{N_\sigma - 1} (1 - e^{-B_3}), & T_{vf} - T_{vc} \leq T_v \leq T_{vf}, \\ \frac{q_u}{N_\sigma - 1} (1 - e^{-B_4}), & T_{vf} \leq T_v \leq T_{vN}, \end{cases} \quad (28)$$

$$U_s = \begin{cases} \frac{1}{\ln N_\sigma} \left(\ln T_1 - \sum_{m=1}^{\infty} \frac{2E_1}{M^2} e^{-M^2 T_v} \right), & T_{vb} \leq T_v \leq T_{vb} + T_{vc}, \\ \frac{1}{\ln N_\sigma} \left(\ln N_\sigma - \sum_{m=1}^{\infty} \frac{2E_2}{M^2} e^{-M^2 T_v} \right), & T_{vb} + T_{vc} \leq T_v \leq T_{vf} - T_{vc}, \\ \frac{1}{\ln N_\sigma} \left(\ln T_2 - \sum_{m=1}^{\infty} \frac{2E_3}{M^2} e^{-M^2 T_v} \right), & T_{vf} - T_{vc} \leq T_v \leq T_{vf}, \\ -\frac{1}{\ln N_\sigma} \sum_{m=1}^{\infty} \frac{2E_4}{M^2} e^{-M^2 T_v}, & T_{vf} \leq T_v \leq T_{vN}, \end{cases} \quad (29)$$

$$U_p = \begin{cases} \frac{T_1}{N_\sigma - 1} \left(\frac{1}{H} \int_0^H e^{-B_1} dz - \frac{1}{T_1} \right), & T_{vb} \leq T_v \leq T_{vb} + T_{vc}, \\ \frac{N_\sigma}{N_\sigma - 1} \left(\frac{1}{H} \int_0^H e^{-B_2} dz - \frac{1}{N_\sigma} \right), & T_{vb} + T_{vc} \leq T_v \leq T_{vf} - T_{vc}, \\ \frac{T_2}{N_\sigma - 1} \left(\frac{1}{H} \int_0^H e^{-B_3} dz - \frac{1}{T_2} \right), & T_{vf} - T_{vc} \leq T_v \leq T_{vf}, \\ \frac{1}{N_\sigma - 1} \left(\frac{1}{H} \int_0^H e^{-B_4} dz - 1 \right), & T_{vf} \leq T_v \leq T_{vN}, \end{cases} \quad (30)$$

where

$$\begin{aligned} T_1 &= \frac{T_{vc} + (T_v - T_{vb})(N_\sigma - 1)}{T_{vc}}, \quad T_2 = \frac{T_{vc} + (T_{vf} - T_v)(N_\sigma - 1)}{T_{vc}}, \\ N_\sigma &= \frac{\sigma'_0 + q_u}{\sigma'_0}, \quad T_v = \frac{c_v t}{h^2}, \quad T_{vc} = \frac{c_v \alpha t_0}{h^2}, \quad M = \frac{(2m-1)\pi}{2}, \\ T_{vb} &= \frac{c_v (N-1)\beta t_0}{h^2}, \quad T_{vf} = \frac{c_v [(N-1)\beta + 1]t_0}{h^2}, \quad T_{vN} = \frac{c_v N\beta t_0}{h^2}, \\ B_j &= \sum_{m=1}^{\infty} \frac{2E_j}{M} \sin\left(\frac{Mz}{h}\right) e^{-M^2 T_v}, \quad j = 1, 2, 3, 4, \end{aligned}$$

$$\begin{aligned}
E_1 &= F(N-1) + F_1, \quad E_2 = F(N-1) + F_2, \quad E_3 = F(N-1) + F_2 + F_3, \\
E_4 &= F(N), \quad F(n) = \sum_{N=1}^n (F_2 + F_4), \\
F_1 &= e^{M^2 [T_{vb}(N_\sigma-1)-T_{vc}]/(N_\sigma-1)} \left[\ln T_1 + \sum_{k=1}^{\infty} \frac{(M^2 T_{vc})^k (T_1^k - 1)}{k!k(N_\sigma-1)^k} \right], \\
F_2 &= e^{M^2 [T_{vb}(N_\sigma-1)-T_{vc}]/(N_\sigma-1)} \left[\ln N_\sigma + \sum_{k=1}^{\infty} \frac{(M^2 T_{vc})^k (N_\sigma^k - 1)}{k!k(N_\sigma-1)^k} \right], \\
F_3 &= e^{M^2 [T_{vf}(N_\sigma-1)+T_{vc}]/(N_\sigma-1)} \left[\ln \frac{T_2}{N_\sigma} + \sum_{k=1}^{\infty} \frac{(-M^2 T_{vc})^k (T_2^k - N_\sigma^k)}{k!k(N_\sigma-1)^k} \right], \\
F_4 &= e^{M^2 [T_{vf}(N_\sigma-1)+T_{vc}]/(N_\sigma-1)} \left[\ln \frac{1}{N_\sigma} + \sum_{k=1}^{\infty} \frac{(-M^2 T_{vc})^k (1 - N_\sigma^k)}{k!k(N_\sigma-1)^k} \right].
\end{aligned} \tag{31}$$

$h = H$ (for single drainage condition) and $h = H/2$ (for a double drainage condition).

Equations (28)–(30) are the solutions for 1D nonlinear consolidation of single-layer soil under trapezoidal cyclic loading presented by Xie et al. [10].

Figure 5 shows the comparison between the degenerate solutions and the existing solutions for double-layered soil subjected to constant loading (Figures 5(a) and 5(b)) and ramp loading (Figure 5(c)) and single-layer soil subjected to trapezoidal cyclic loading (Figure 5(d)). It can be found that the degenerate solutions from this paper show good agreement with the existing results, which proves that the proposed solutions are more general ones for 1D nonlinear consolidation of saturated soils under time-dependent loading.

5. Parametric Study

In this section, a comprehensive parametric study is conducted to investigate the influences of different layer parameters, drainage conditions, and loading parameters on nonlinear consolidation behavior of saturated double-layered soil under various cyclic loadings.

5.1. Influence of Layer Parameters. In order to investigate the influences of layer parameters such as the permeability ratio K (i.e., $K = k_v^2/k_v^1$), the compressibility ratio b (i.e., $b = m_v^2/m_v^1$), and the thickness ratio c (i.e., $c = h_2/h_1$), the saturated double-layered soil of total thickness $H = 10$ m with single drainage condition under various cyclic loadings is considered. The cyclic loadings are trapezoidal ($\alpha = 0.2$) and rectangular and triangular cyclic loadings with $\beta = 1.5$, $T_0 = 0.04$, and $N_\sigma = 2.5$, where T_0 is time factor defined by $T_0 = c_{v1}t_0/H^2$.

5.1.1. The Permeability Ratio K . For the effect of K , it is assumed that $b = 1$ and $c = 1$. Figure 6 presents the excess pore pressure isochrones at the end of loading and

unloading stages of the 10th cycle for trapezoidal cyclic loading. Cyclic loadings include loading stage and unloading stage in each cycle unlike static loading, which results in squeezing out of pore water during loading stage and then absorbing during unloading stage. Thus, the excess pore pressures have both positive values for the loading stage and negative values for the unloading stage as shown in Figure 6. It can be shown that the excess pore pressure is smaller for the bigger value of K in the lower layer, while K has no significant effects in the upper layer. It implies that the bigger permeability of the lower layer leads to the faster dissipation rate of pore pressure when the compressibilities of the two layers are the same.

Figure 7 shows the variations of average degrees of consolidation in terms of settlement and effective stress with time factor T_v under different values of K for trapezoidal cyclic loading (Figure 7(a)), rectangular cyclic loading (Figure 7(b)), and triangular cyclic loading (Figure 7(c)). The time factor T_v is defined by $T_v = c_{v1}t/H^2$. It can be seen that there are almost no differences in U_s and U_p between different values of K at the early stages of consolidation, but the greater K increases the rates of settlement and dissipation of excess pore pressure with fluctuation as time grows. Moreover, the effects are obvious for the case of $K < 1$ but not clear for the opposite case. It can be explained that the effects of K is mainly related to the permeability of the lower layer, indicating that for a single drainage condition, the smaller permeability of the lower layer decreases the rates of settlement and dissipation of excess pore pressure.

5.1.2. The Compressibility Ratio b . The effect of the compressibility ratio b is investigated for double-layered soil with $K = 1$ and $c = 1$. Figure 8 illustrates the excess pore pressure isochrones at the end of loading and unloading stages of the 10th cycle for trapezoidal cyclic loading. It can be found that when the permeabilities of the two layers are the same, the greater the compressibility ratio b is, the greater the excess

FIGURE 5: Comparison between the degenerate solutions and the existing solutions.

FIGURE 6: The excess pore pressure isochrones under different values of K .

FIGURE 7: The variations of average degrees of consolidation U_s and U_p with time factor T_v under different values of K .

FIGURE 8: The excess pore pressure isochrones under different values of b .

pore pressure is, indicating that an increase in the compressibility of the lower layer will decrease the rate of consolidation.

The effect of the compressibility ratio b can also be found in Figure 9. Figure 9 represents the variations of average degree of consolidation in terms of effective stress U_p with time factor T_v under different values of b for three different cyclic loadings, such as trapezoidal cyclic loading (Figure 9(a)), rectangular cyclic loading (Figure 9(b)), and triangular cyclic loading (Figure 9(c)). Similar to the above-mentioned result, it can be seen that the increase in the compressibility ratio b decreases the dissipation rate of excess pore pressure with fluctuation when the permeabilities of the two layers are the same.

5.1.3. The Thickness Ratio c . The effect of the thickness ratio c is studied for two cases of double-layered soil: Case 1 is for $K=2$ and $b=1$ and Case 2 is for $K=1$ and $b=2$. Figure 10 depicts the excess pore pressure isochrones at the end of loading and unloading stages of the 10th cycle for trapezoidal cyclic loading. It can be found that excess pore pressure decreases when the thickness ratio c increases for Case 1, but the opposite occurs for Case 2. It implies that in the lower layer with higher permeability, the greater thickness results in the smaller excess pore pressure, whereas the increase in thickness of the lower layer with higher compressibility leads to the bigger excess pore pressure.

Figure 11 demonstrates the variations of average degree of consolidation in terms of settlement U_s with time factor T_v under different values of the thickness ratio c for trapezoidal cyclic loading (Figures 11(a) and 11(b)), rectangular cyclic loading (Figures 11(c) and 11(d)), and triangular

cyclic loading (Figures 11(e) and 11(f)). It can be seen that the effect of the thickness ratio c is similar to the above described in Figure 10. For Case 1, the rate of settlement increases a little with fluctuation when the thickness ratio c increases. As for Case 2, the effect of the thickness ratio c is opposite. In addition, the rate of settlement increases obviously when the thickness of the lower layer with higher compressibility decreases.

5.2. Influence of Drainage Conditions. The influence of drainage conditions is investigated for double-layered soil with $K=2$, $b=1$, and $c=1$. Figure 12 shows the variations of average degrees of consolidation in terms of settlement and effective stress with time factor T_v under different drainage conditions for trapezoidal cyclic loading (Figure 12(a)), rectangular cyclic loading (Figure 12(b)), and triangular cyclic loading (Figure 12(c)). It can be found that the rates of settlement and dissipation of excess pore pressure for a double drainage condition are bigger than those for a single drainage condition. Moreover, the amplitudes of fluctuation in the rates of settlement and dissipation of excess pore pressure under a double drainage condition are also greater than those under a single drainage condition.

5.3. Influence of Loading Parameters. The influence of loading parameters such as α , β , and T_0 is analysed for double-layered soil with $K=2$, $b=2$, and $c=1$.

5.3.1. The Loading Parameter α . Figure 13 presents the variations of average degrees of consolidation U_s and U_p with time factor T_v under cyclic loadings with different values of

FIGURE 9: The variations of average degree of consolidation U_p with time factor T_v under different values of b .

α . It can be seen that both the settlement rate and the dissipation rate of excess pore pressure decrease with the increase of the value of α . The loading parameter α reflects the rate of increment or decrement of loading, and thus, it can be concluded that the greater the rate of increment or decrement of loading, the greater the rate of consolidation. In this context, the rate of consolidation for rectangular cyclic loading is the biggest and the rate of consolidation for triangular cyclic loading is the smallest.

5.3.2. The Loading Parameter β . Figure 14 depicts the variations of average degrees of consolidation U_s and U_p with time factor T_v under trapezoidal cyclic loading (Figure 14(a)), rectangular cyclic loading (Figure 14(b)), and triangular cyclic loading (Figure 14(c)) with different values of β . It can be found that the increase of the value of β results

in the smaller rates of settlement and dissipation of excess pore pressure and less cycles and bigger amplitude of fluctuation in the rates of settlement and dissipation of excess pore pressure. The loading parameter β reflects the rest period of cyclic loadings. Therefore, it can be concluded that the rate of consolidation under cyclic loadings increases with the decrease of the rest period of cyclic loading. As a result, the rate of consolidation under cyclic loadings without rest period (i.e., $\beta = 1$) is the biggest; especially, the rectangular cyclic loading without rest period reduces the constant loading, and the rate of consolidation under constant loading is the highest.

5.3.3. The Loading Parameter T_0 . Figure 15 represents the variations of average degrees of consolidation U_s and U_p with time factor T_v under trapezoidal cyclic loading

FIGURE 10: The excess pore pressure isochrones under different values of c .

FIGURE 11: Continued.

FIGURE 11: The variations of average degree of consolidation U_s with time factor T_v under different values of c .

(Figure 15(a)), rectangular cyclic loading (Figure 15(b)), and triangular cyclic loading (Figure 15(c)) with different values of T_0 . It can be seen that a smaller value of T_0 induces more cycles and smaller amplitude of fluctuation in the rates of settlement and dissipation of excess pore pressure, but the loading parameter T_0 has no effects on the average values of

the rates of settlement and dissipation of excess pore pressure. Since the loading parameter T_0 reflects the period of loading, it can be found that the longer the period of loading, the bigger amplitude and less cycles of fluctuation in the settlement rate and the dissipation rate of excess pore pressure.

FIGURE 12: The variations of average degrees of consolidation U_s and U_p with time factor T_v under different drainage conditions.

FIGURE 13: The variations of average degrees of consolidation U_s and U_p with time factor T_v under different values of α .

FIGURE 14: Continued.

FIGURE 14: The variations of average degrees of consolidation U_s and U_p with time factor T_v under different values of β .

FIGURE 15: Continued.

FIGURE 15: The variations of average degrees of consolidation U_s and U_p with time factor T_v under different values of T_0 .

6. Conclusion

In this paper, analytical solutions have been presented for 1D nonlinear consolidation of saturated double-layered soil subjected to cyclic loadings such as trapezoidal cyclic loading, rectangular cyclic loading, and triangular cyclic loading. It shows through the degeneration of the proposed solutions into special cases that the solutions presented in this paper are more general for the nonlinear consolidation of saturated soils under time-dependent loading. Based on the proposed solutions, a comprehensive parametric study is conducted, and the following conclusions can be drawn:

- (1) The nonlinear consolidation of saturated double-layered soil under cyclic loadings is affected by layer parameters such as the permeability ratio, the compressibility ratio, and the thickness ratio. The smaller permeability of the lower layer decreases the rates of settlement and dissipation of excess pore pressure. An increase in the compressibility of the lower layer will decrease the rate of consolidation. Furthermore, the rate of settlement increases when the thickness of the lower layer with higher compressibility decreases.
- (2) The drainage conditions have great effects on the nonlinear consolidation behavior of saturated double-layered soil under cyclic loadings. The rates of settlement and dissipation of excess pore pressure and the amplitudes of fluctuation in the rates for a double drainage condition are bigger than those for a single drainage condition.
- (3) The loading parameters have significant influences on the nonlinear consolidation behavior of saturated double-layered soil subjected to various cyclic loadings. The greater the rate of increment or decrement of loading, the greater the rate of

consolidation. The rate of consolidation under cyclic loadings increases with the decrease of the rest period of cyclic loadings. The longer the period of loading, the bigger amplitude and less cycles of fluctuation in the settlement rate and the dissipation rate of excess pore pressure.

Appendix

A. The Derivation of General Solutions for ω_i

The general solutions of equation (4) for a single drainage condition can be assumed as follows:

$$\omega_i = \sum_{m=1}^{\infty} g_{mi}(z) e^{-\beta_m t} [B_m + C_m T_m(t)], \quad (i = 1, 2), \quad (\text{A.1})$$

where $g_{m1}(z) = \sin(\lambda_m z/h_1)$, $g_{m2}(z) = A_m \cos(\mu \lambda_m (H - z/h_1))$; β_m , μ , λ_m , A_m , B_m , and C_m are unknown coefficients to be found; and $T_m(t)$ is a function of t , resulting from $R(t)$.

Substituting equation (10) into the boundary condition equation (8), we can get A_m as follows:

$$A_m = \frac{\sin(\lambda_m)}{\cos(\mu c \lambda_m)}, \quad (\text{A.2})$$

where

$$c = \frac{h_2}{h_1}. \quad (\text{A.3})$$

Substituting equation (10) into (4) and simplifying, β_m and μ can be obtained as follows:

$$\beta_m = \frac{c_v^1 \lambda_m^2}{h_1^2}, \quad (\text{A.4})$$

$$\mu = \sqrt{\frac{c_v^1}{c_v^2}}. \quad (\text{A.5})$$

In order to determine λ_m , the eigen equation can be obtained by substituting equation (10) into the boundary condition equation (9) as follows:

$$\mu K \tan(\lambda_m) \tan(\mu \lambda_m) = 1. \quad (\text{A.6})$$

Furthermore, using the coefficients obtained above, we can get the following relations:

$$\begin{aligned} T_m(t) &= \int_0^t e^{\beta_m \tau} R(\tau) d\tau, \\ \sum_{m=1}^{\infty} C_m g_{mi}(z) &= 1, \quad (i = 1, 2), \\ \sum_{m=1}^{\infty} B_m g_{mi}(z) &= \omega_0, \quad (i = 1, 2), \end{aligned} \quad (\text{A.7})$$

Using the following orthogonal relation,

$$\int_0^{h_1} g_{m1}(z) g_{n1}(z) dz + \mu^2 K \int_{h_1}^H g_{m2}(z) g_{n2}(z) dz = \begin{cases} 0, & m \neq n, \\ \frac{1}{2}, & h_1(1 + \mu^2 K c A_m^2), m = n, \end{cases} \quad (\text{A.8})$$

B_m and C_m can be written as follows:

$$\begin{aligned} B_m &= \frac{\int_0^{h_1} g_{m1}(z) dz + \mu^2 K \int_{h_1}^H g_{m2}(z) dz}{\int_0^{h_1} g_{m1}^2(z) dz + \mu^2 K \int_{h_1}^H g_{m2}^2(z) dz} \omega_0 = \frac{2\omega_0}{\lambda_m(1 + \mu^2 K c A_m^2)}, \\ C_m &= \frac{\int_0^{h_1} g_{m1}(z) dz + \mu^2 K \int_{h_1}^H g_{m2}(z) dz}{\int_0^{h_1} g_{m1}^2(z) dz + \mu^2 K \int_{h_1}^H g_{m2}^2(z) dz} = \frac{2}{\lambda_m(1 + \mu^2 K c A_m^2)}. \end{aligned} \quad (\text{A.9})$$

Now, substituting all those coefficients into equation (10), the general solutions of equation (4) for a single drainage condition can be written as follows:

$$\omega_1 = \sum_{m=1}^{\infty} \sin\left(\lambda_m \frac{z}{h_1}\right) e^{-\beta_m t} \left[B_m + C_m \int_0^t e^{\beta_m \tau} R(\tau) d\tau \right], \quad (\text{A.10})$$

$$\omega_2 = \sum_{m=1}^{\infty} A_m \cos\left(\mu \lambda_m \frac{H-z}{h_1}\right) e^{-\beta_m t} \left[B_m + C_m \int_0^t e^{\beta_m \tau} R(\tau) d\tau \right]. \quad (\text{A.11})$$

Similarly, the general solutions of equation (4) for a double drainage condition can be obtained as follows:

$$\omega_1 = \sum_{m=1}^{\infty} \sin\left(\lambda_m \frac{z}{h_1}\right) e^{-\beta_m t} \left[B_m + C_m \int_0^t e^{\beta_m \tau} R(\tau) d\tau \right], \quad (\text{A.12})$$

$$\omega_2 = \sum_{m=1}^{\infty} A_m \sin\left(\mu \lambda_m \frac{H-z}{h_1}\right) e^{-\beta_m t} \left[B_m + C_m \int_0^t e^{\beta_m \tau} R(\tau) d\tau \right], \quad (\text{A.13})$$

where

$$A_m = \frac{\sin(\lambda_m)}{\sin(\mu\lambda_m)}. \quad (\text{A.14})$$

The eigen equation for λ_m can be obtained as follows:

$$-\mu K \tan(\lambda_m) \cot(\mu\lambda_m) = 1, \quad (\text{A.15})$$

$$B_m = \frac{2(1 + \mu K A_m) \omega_0}{\lambda_m (1 + \mu^2 K c A_m^2)},$$

$$C_m = \frac{2(1 + \mu K A_m)}{\lambda_m (1 + \mu^2 K c A_m^2)}. \quad (\text{A.16})$$

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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