Research Article

Deep Federated Learning Based Convergence Analysis in Relaying-Aided MEC-IoT Networks

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Recently, deep federated learning has attracted much attention from researchers in the fields of wireless communications, where the relaying technique has been shown as a powerful technology to assist the wireless signals and enhance the transmission quality, which is very important to the development of mobile edge computing (MEC) based Internet of Things (IoT) networks. In a relaying-aided MEC-IoT system, it is of vital importance to deeply investigate the system signal-to-noise ratio (SNR) at the receiver side, as it mainly determines the system performance metrics, such as capacity (or achievable data rate), outage probability, and bit-error-rate (BER). To this end, we first investigate the instantaneous convergence error, by deeply studying the relationship between the instantaneous two-hop relaying channels. We then investigate the statistical convergence error, by performing the statistical expectation with respect to the two-hop relaying channels. We finally present some results to show that the analysis of the convergence error is effective. The work in this paper can provide some theoretical foundation for deep federated learning and computing networks.

1. Introduction

Relaying technology has attracted much attention from researchers [1–3], as it can help enhance the wireless transmission between the source and destination without adding additional energy consumption [4–6]. In this system, one or more relay nodes are added between the base station and the mobile station, which is responsible for one or more times of forwarding the wireless signal; that is, the wireless signal has to go through multiple hops to reach the mobile station [7, 8]. Take a simple two-hop relay as an example, that is, a base station terminal link is divided into two links: base station relay station and relay station terminal, so as to have the opportunity to replace a link with poor quality with two links with better quality and to obtain higher link capacity and better coverage [9].

The Internet of Things makes full use of the new generation of IT technology in all walks of life. Specifically, it embeds and equips sensors into various objects such as power grids, railways, bridges, tunnels, highways, buildings, water supply systems, dams, oil, and gas pipelines and then integrates the "Internet of Things" with the existing Internet to realize the integration of human society and physical systems. In this integrated network, there is a super powerful central computer group, which can implement real-time management and control of personnel, machines, equipment, and infrastructure in the integrated network. On this basis, human beings can manage production and life in a more refined and dynamic way, achieve a "smart" state, improve resource utilization and productivity, and improve the relationship between man and nature.

Due to the above advantages of relaying techniques, relaying has been widely applied to mobile edge computing (MEC)-aided Internet of Things (IoT) networks, which has become of the most hot topics in the area of wireless communication [10]. With the development trend of IoT and test cases in the next two years, this kind of situation will continue to exist. According to Gartner, a scientific research firm, up to 20 billion connected devices, will form billions of bytes of data for each customer by 2020 [11–13]. This equipment is not only an intelligent machine or notebook but also includes connected network cars, automatic vending machines, intelligent wearable equipment, surgical treatment, and medical robots. Many data formed by thousands of such devices must push messages to the centralized cloud for preservation (data management methods), analysis, and management decisions. Then, the analyzed data results are transmitted to the equipment. The back and forth of such data consume a lot of Internet infrastructure construction and cloud infrastructure construction resources, which increases the problems of delay time and network bandwidth consumption and then endangers the application of Internet of Things technology in important daily tasks. For example, in the driverless connected car, a lot of data are generated every hour. The data must be uploaded to the cloud, analyzed, and pushed back to the car. Low delay time or resource congestion is likely to delay the response to the vehicle, and it is likely to cause road traffic accidents when it is more serious. For the relaying-aided MEC-IoT system, it is of vital importance to deeply study the system signal-to-noise ratio (SNR) at the receiver side, as it mainly determines the

system performance evaluation, such as capacity (or achievable data rate), outage probability, and bit-error-rate (BER) [14, 15].

Hence, in this paper, we first investigate the instantaneous convergence error, by deeply studying the relationship between the instantaneous two-hop relaying channels. We then investigate the statistical convergence error, by performing the statistical expectation with respect to the two-hop relaying channels. We finally present some results to show that the analysis of the convergence error is effective. We believe that the work in this paper can provide some theoretical foundation for deep federated learning networks.

2. Problem Formulation on the Instantaneous Error

We can formulate the problem as

$$P1: Z = \left| \frac{XY/(X+Y) - \min\{X,Y\}}{XY/(X+Y)} \right|, \quad (1)$$

where the random variables X and Y are subject to the exponential distribution. Both X and Y are independent and identically distributed. Therefore, the jointly joint probability density function of X and Y is presented by

$$f(x,y) = \begin{cases} \frac{1}{\beta_1} \frac{1}{\beta_2} e^{-1/\beta_1 x} e^{-1/\beta_2 y}, & x > 0, y > 0, \\ 0, & other. \end{cases} \quad (2)$$

Without generality, we assume $X \geq Y$. Then, $P1$ can be simplified as

$$P2: Z = \begin{cases} \frac{Y}{X}, & X \geq Y > 0, \\ \frac{X}{Y}, & Y > X > 0. \end{cases} \quad (3)$$

The problem $P2$ can be rewritten as

$$P3: Z = \frac{Y}{X}, \quad (4a)$$

$$C_1: X \geq Y. \quad (4b)$$

Then, we take the expectation for the variable Z

$$\mathbb{E}(Z) = \int_0^{+\infty} \int_0^x \frac{y}{x} \frac{1}{\beta_1} \frac{1}{\beta_2} e^{(-1/\beta_1)x} e^{(-1/\beta_2)y} dy\, dx$$

$$= \frac{\beta_2}{\beta_1} \int_0^\infty \left(\frac{e^{(-1/\beta_1)x}}{x} - \frac{e^{(-1/\beta_1)x - 1/\beta_2 x}}{x} \right) dx$$

$$- \frac{1}{\beta_1} \int_0^\infty e^{(-1/\beta_1)x - 1/\beta_2 x} dx$$

$$= A - B$$

$$= \frac{\beta_2}{\beta_1} \ln \frac{\beta_1 + \beta_2}{\beta_2} - \frac{\beta_2}{\beta_1 + \beta_2}. \quad (5)$$

For convenience, A and B are used to denote

$$A = \frac{\beta_2}{\beta_1} \int_0^\infty \left(\frac{e^{(-1/\beta_1)x}}{x} - \frac{e^{-(1/\beta_1)x - (1/\beta_2)x}}{x} \right) dx$$

$$= \frac{\beta_2}{\beta_1} \int_0^1 \left(\frac{e^{(-1/\beta_1)x}}{x} - \frac{e^{-(1/\beta_1)x(-1/\beta_2)x}}{x} \right) dx + \frac{\beta_2}{\beta_1} \int_1^{+\infty} \left(\frac{e^{(-1/\beta_1)x}}{x} - \frac{e^{(-1/\beta_1)x(-1/\beta_2)x}}{x} \right) dx$$

$$= \frac{\beta_2}{\beta_1} \left\{ \mathrm{Ei}\left(-\frac{1}{\beta_1} \right) - \mathrm{Ei}\left(-\frac{1}{\beta_1} - \frac{1}{\beta_2} \right) - \ln \frac{1/\beta_1}{1/\beta_1 + 1/\beta_1 1\beta_2} \right\} + \frac{\beta_2}{\beta_1} \left\{ -\mathrm{Ei}\left(-\frac{1}{\beta_1} \right) + \mathrm{Ei}\left(\frac{1}{\beta_1} - \frac{1}{\beta_2} \right) \right\} \tag{6}$$

$$= \frac{\beta_2}{\beta_1} \ln \frac{\beta_1 + \beta_2}{\beta_2},$$

$$B = \frac{1}{\beta_1} \int_0^\infty e^{(-1/\beta_1)x - (1/\beta_2)x} dx = \frac{\beta_2}{\beta_1 + \beta_2},$$

where $\mathrm{Ei}(x) = -\int_{-x}^\infty (e^{-t}/t) dt$ denotes the exponential integral function [15].

3. Problem Formulation on the Statistical Error

As to the statistical error, we can write the error expression as

$$P1: \; Z = \left| \frac{XY}{X+Y} - \min\{X, Y\} \right|. \tag{7}$$

Note that P1 can be specified as

$$P2: \; Z = \begin{cases} \dfrac{Y^2}{X+Y}, & X \geq Y > 0, \\[2mm] \dfrac{X^2}{X+Y}, & Y > X > 0. \end{cases} \tag{8}$$

Without loss of generality, we assume that $X \geq Y$. Then, the expectation of Z is expressed as

$$\mathbb{E}(Z) = \frac{1}{\beta_1 \beta_2} \int_0^{+\infty} \int_0^x \frac{y^2}{x+y} e^{-(x/\beta_1) - (y/\beta_2)} dy \, dx,$$

$$= \frac{1}{\beta_1 \beta_2} \int_0^{(\pi/4)} \int_0^{+\infty} \frac{r^2 (\sin \theta)^2}{\sin \theta + \cos \theta} e^{-r(\cos \theta/\beta_1) + (\sin \theta/\beta_2))} dr \, d\theta, \tag{9}$$

where $x = r \cos \theta$ and $y = r \sin\theta$. We can further write $\mathbb{E}(Z)$ as

$$\mathbb{E}(Z) = \frac{\Gamma(3)}{\beta_1 \beta_2} \int_0^{(\pi/4)} \frac{(\sin \theta)^2}{(\sin \theta + \cos \theta)(\cos \theta \beta_1 + \sin \theta \beta_2)^3} d\theta$$

$$= \frac{2!}{\beta_1 \beta_2} \int_0^{(\pi/4)} \frac{(\csc \theta)^2}{(1 + \cot \theta)(1\beta_2 + \cot \theta \beta_1)^3} d\theta,$$

$$= -\frac{2!}{\beta_1 \beta_2} \int_0^{(\pi/4)} \frac{1}{(1 + \cot \theta)(1\beta_2 + \cot \theta \beta_1)^3} dc \, ot\theta, \tag{10}$$

where $\Gamma(x)$ is the gamma function, given by,

$$\Gamma(x) = \int_0^{+\infty} t^{x-1} e^{-t} dt \quad x > 0. \tag{11}$$

Then, the result of $\mathbb{E}(Z)$ is

$$P2: \; \mathbb{E}(Z) = \begin{cases} \dfrac{\beta_1}{12}, \beta_1 == \beta_2, \\[4mm] \dfrac{2}{\beta_1 \beta_2} \left(\dfrac{(\beta_1 \beta_2)^3}{(\beta_2 - \beta_1)^3} \ln \dfrac{2\beta_2}{\beta_1 + \beta_2} - \dfrac{(\beta_1 \beta_2)^3 (\beta_2 + 3\beta_1)}{2((\beta_2)^2 - (\beta_1)^2)^2} \right), \beta_1 \neq \beta_2. \end{cases} \tag{12}$$

4. Simulation of the Instantaneous Error

In this part, simulation results are presented to verify the difference between the analytic results and simulation results.

Figure 1 and Table 1 demonstrate the instantaneous and simulated error results versus β_1 with $\beta_2 = 1$. According to Figure 1 and Table 1, we can perceive that the instantaneous error achieves the highest value when β_1 approximates to one, while the instantaneous error drops gradually when β_1 increases or decreases from one. Moreover, the analytical curve matches the simulation result very well, which verifies the effectiveness on theoretical analysis.

Figure 2 and Table 2 present the instantaneous and simulated error results versus β_2, with $\beta_1 = 1$. From Figure 1 and Table 2, we can observe that the curve of the instantaneous error goes up and then downs, and it achieves the peak when β_2 approximates to one. Moreover, the analytical curve matches the simulation result very well, which also verified the effectiveness of the theoretical analysis.

Figure 3 and Table 3 demonstrate the instantaneous and simulated error results versus x with $y = 1$. From Figure 3 and Table 3, we can see that the instantaneous error achieves its peak when x and y are both equal to one. When $x > 1$, the
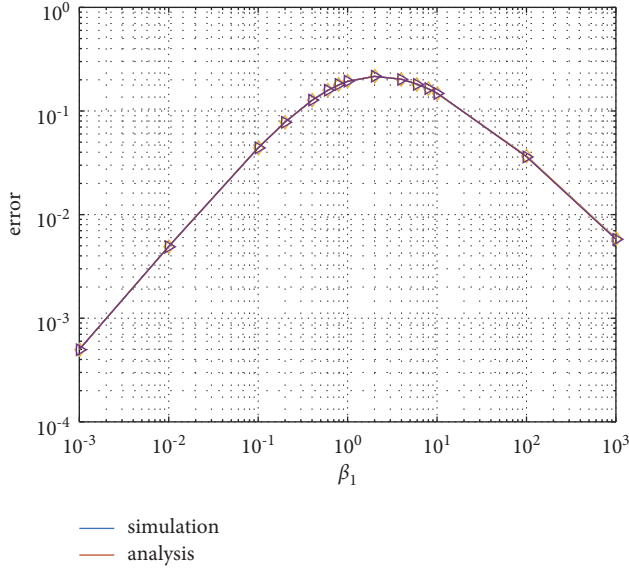
FIGURE 1: Statistical error versus β_1 with $\beta_2 = 1$.

TABLE 1: Data of instantaneous error versus β_1 with $\beta_2 = 1$.

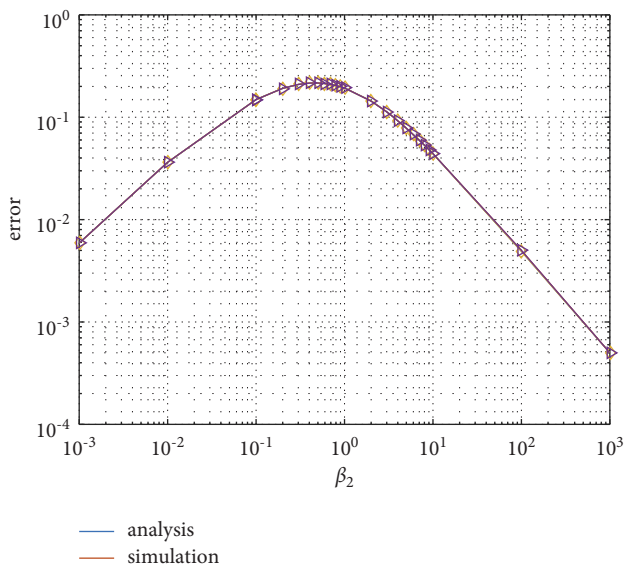| β_1	Simulation	Analysis
0.001	0.0005	0.0005
0.01	0.0049	0.0049
0.1	0.0442	0.044
1	0.1923	0.1931
10	0.1491	0.1489
100	0.0364	0.0363
1000	0.0060	0.0059

FIGURE 2: Instantaneous error versus β_2 with $\beta_1 = 1$.

TABLE 2: Data of instantaneous error versus β_2 with $\beta_1 = 1$.

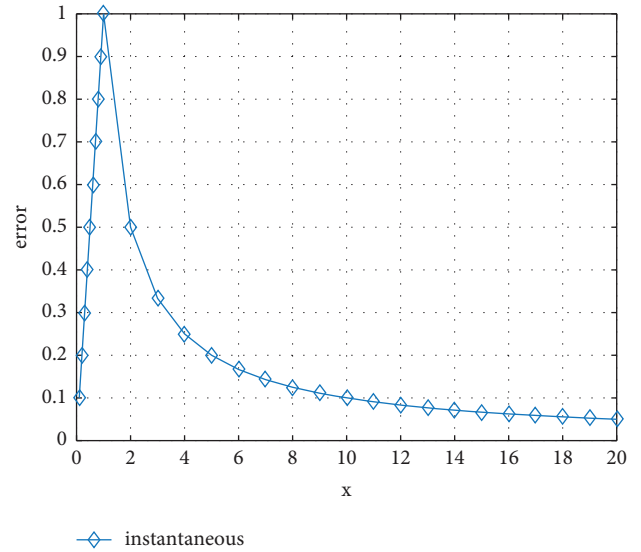| β_2	Simulation	Analysis
0.001	0.0059	0.0059
0.01	0.0363	0.0363
0.1	0.1489	0.1489
1	0.1931	0.1931
10	0.0441	0.044
100	0.005	0.0049
1000	0.0005	0.0005

FIGURE 3: Instantaneous error versus β_1 with $\beta_2 = 1$.

TABLE 3: Data of instantaneous error versus β_2 with $\beta_1 = 1$.

x	Error
0.1	0.1
0.5	0.5
0.9	0.9
1	1
5	0.2
9	0.1111
11	0.0909
15	0.0667
19	0.0526
20	0.05

instantaneous error decreases swiftly with the increase of x. Similarly, the instantaneous error increases linearly when y grows from 0.1 to 1.

5. Simulation of the Statistical Error

In this part, simulation results are presented to verify the difference between the statistical results and simulation results.

Figure 4 and Table 4 demonstrate the statistical and simulation error results versus β_1, with $\beta_2 = 1$. According to Figure 4 and Table 4, we can perceive that the statistical error achieves the highest value when β_1 approximates to one,
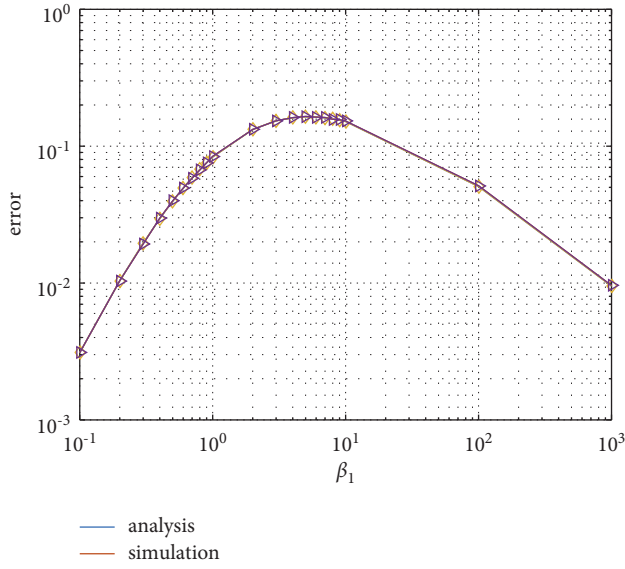
FIGURE 4: Statistical error versus β_1 with $\beta_2 = 1$.

TABLE 4: Data of statistical error versus β_1 with $\beta_2 = 1$.

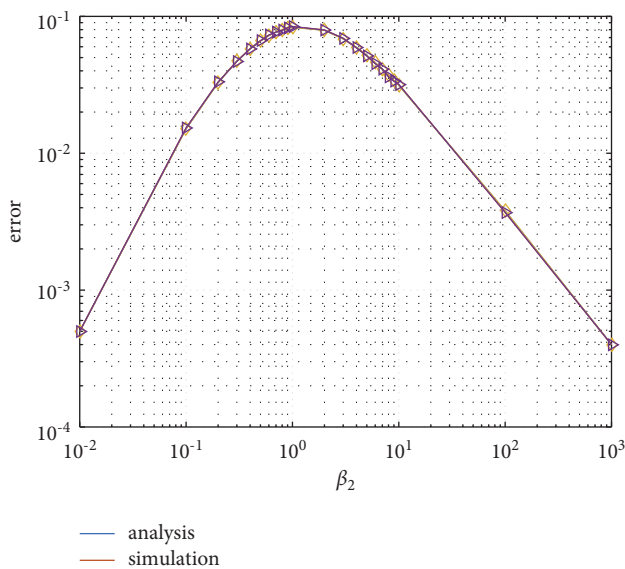| β_1	Simulation	Analysis
0.1000	0.0031	0.0031
0.5000	0.0396	0.0398
1.0000	0.0833	0.0843
5.0000	0.1638	0.1650
10.0000	0.1514	0.1527
100.0000	0.0507	0.0514
1000.0000	0.0095	0.0096

FIGURE 5: Statistical error versus β_2 with $\beta_1 = 1$.

TABLE 5: Data of statistical error versus β_2 with $\beta_1 = 1$.

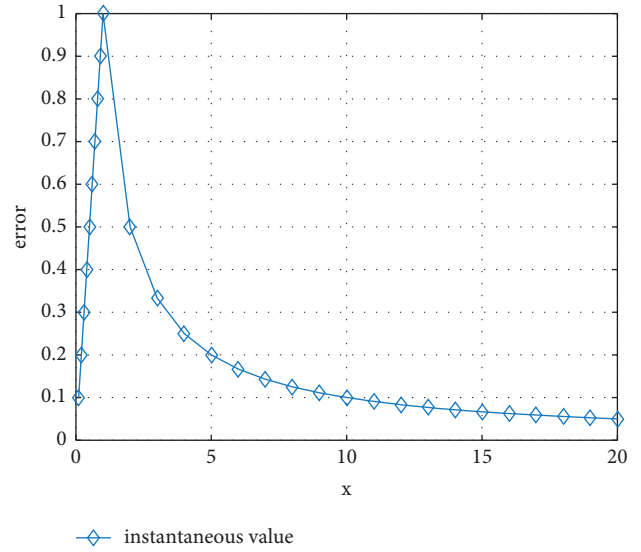| β_2	Simulation	Analysis
0.0100	0.0005	0.0005
0.1000	0.0151	0.0153
0.5000	0.0663	0.0663
1.0000	0.0833	0.0843
5.0000	0.0519	0.0515
10.0000	0.0314	0.0318
100.0000	0.0038	0.0037
1000.0000	0.0004	0.0004

FIGURE 6: Statistical error versus x with $y = 1$.

TABLE 6: Data of statistical error versus x with $y = 1$.

x	Error
0.1000	0.0091
0.5000	0.1667
1.0000	0.5000
5.0000	0.1667
10.0000	0.0909
15.0000	0.0625
20.0000	0.0476

while the statistical error drops gradually when β_1 increases or decreases from one. Moreover, the analytical curve matches the simulation result very well, which verifies the effectiveness of theoretical analysis.

Figure 5 and Table 5 present the statistical and simulation error results versus β_2, with $\beta_1 = 1$. From Figure 5 and Table 5, we can observe that the curve of the statistical error goes up and then downs and achieves its peak when β_2 approximates one. Moreover, the analytical curve matches the simulation result very well, which verifies the effectiveness of theoretical analysis.

Figure 6 and Table 6 demonstrate the statistical and simulated error results versus x with $y = 1$. From this figure and table, we can find that the statistical error achieves its

peak when x and y are both equal to one. When $x > 1$, the statistical error decreases swiftly with the increase of x. Similarly, the statistical error increases linearly when y grows from 0.1 to 1.

6. Conclusions

In the relaying-aided MEC-IoT system, it was of vital importance to deeply investigate the system signal-to-noise ratio (SNR) at the receiver side, as it mainly determined the system performance evaluation, such as capacity (or achievable data rate), outage probability, and bit-error-rate (BER). For this issue, we first investigated the instantaneous convergence error, by deeply studying the relationship between the instantaneous two-hop relaying channels. We then investigated the statistical convergence error, by performing the statistical expectation with respect to the two-hop relaying channels. We finally presented some results to show that the analysis of the convergence error is effective. We believe that the work in this paper can provide some theoretical foundation for deep-federated learning networks. In future works, we will apply the results in this paper and try to analyze the convergence of the local and global training in the federated learning networks.

Data Availability

The data of this paper can be obtained through e-mail to the authors. Specifically, the data of Section 2 can be obtained through the email to Jun Liu (junliu.thu@ieee.org), Tao Cui (taocui@ieee.org), Lin Zhang (lzhang.ee@ieee.org), Yuwei Zhang (yzhang.thu@ieee.org), Jing Wang (jingwang.thu@ieee.org), Chao Li (chaoli.eecs@ieee.org), and Kai Chen (kchen.huawei@ieee.org). The data of Section 3 can be obtained through the email to Huang Huang (hhuang.huawei@ieee.org), Xuan Zhou (xzhou.huawei@ieee.org), Wei Zhou (wzhou.huawei@ieee.org), Sun Li (lisun@ieee.org), Suili Feng (slfeng@ieee.org), Dongqing Xie (dqxie@ieee.org), and Yun Li (yunli.ericsson@ieee.org). The data of Section 4 can be obtained through the email to Haige Xiang (haigexiang@ieee.org), Kaimeno Dube (Kaimeno.Dube@ieee.org), Abbarbas Muazu (Abbarbas.Muazu@ieee.org), Nakilavai Rono (Nakilavai.Rono@ieee.org), and Wen Zhou (wenzhou.nfu@gmail.com). The data of Section 5 can be obtained through the email to Fusheng Zhu (fushengzhu.gdcni@hotmail.com), Liming Chen (lmchen_CSPG@hotmail.com), Dan Deng (dengdan.ustc@hotmail.com), Zhao Wang (zhaowang.ericsson@ieee.org), and Yajuan Tang (yjtang@ieee.org).

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this work.

Acknowledgments

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