Research Article

Prandtl Number and Viscosity Correlations of Titanium Oxide Nanofluids

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Many features of nanofuids, such as the Prandtl number and viscosity, are researched as the number of studies conducted in the field of nanofuids increases. Observations on the Prandtl number and viscosity of titanium oxide nanofuids are made in this study. These observations are made at low concentrations of titanium oxide nanoparticles and temperatures ranging from 30.4°C to 70.4°C. Novel correlations for viscosity and Prandtl number as functions of temperature have been developed and compared to the previously published models for Prandtl number and viscosity. The results indicate that titanium oxide-ethylene glycol nanofluid has a greater viscosity and Prandtl number than all other titanium oxide nanofuids observed in the study at 0.01 nanoparticle concentration. The results on viscosity and Prandtl number for the new correlations fall within the same range as those found in the literature, indicating that the new correlations introduced as functions of temperature in this study can be used in future research to establish viscosity and Prandtl number calculations for the different types of nanofuids at specific temperatures.

1. Introduction

As nanoparticles are added to base fluids such as water, glycols, oils, and refrigerants to generate nanofuids, the thermophysical properties of the resulting fluids improve. These nanofuids are utilized in several industrial applications. The research topic on nanofuids has advanced significantly as scientists have been interested in the manufacture of these specialized fluids that contain nanoparticles. As it was discovered that nanofuids possess increased thermophysical characteristics, curiosity emerged. Various nanoparticle systems have been studied to demonstrate the validity of the tested hypotheses. In this article, the nanofluid viscosity and Prandtl number of titanium oxide-ethylene glycol (40%)/water (60%), titanium oxide-ethylene glycol nanofluid viscosity and Prandtl number, and titanium oxide-water nanofluid viscosity and Prandtl number are analysed. Observations are also made about the viscosity and Prandtl numbers on their respective base fluids, specifically the enhancement of viscosity and Prandtl number at varied nanoparticle concentrations of 0.004, 0.006, 0.008, and 0.01. It is vital to evaluate how high or low the Prandtl number can be, as a greater Prandtl number indicates a higher viscosity of the nanofluid, while a lower Prandtl number indicates a lower viscosity of the nanofluid. The titanium oxide nanofluids, at the measured nanoparticle concentration, have the potential for usage in numerous applications due to a minor increase in viscosity for the selected nanoparticle concentrations. The Prandtl number depends on the nanofluid’s specific heat, viscosity, and thermal conductivity. In addition to detecting the Prandtl number and viscosity of nanofluids using the aforementioned published models, we examine new correlations for Prandtl number and viscosity and compare them to previously published models. It is
important to evaluate the effects of temperature on the thermophysical properties of nanofluids, as determined by a thorough analysis of the relevant literature. The correct analysis of the thermophysical and rheological properties that nanofluids should possess when utilized in a variety of industrial applications necessitates the development of novel correlations that involve temperature. Temperature plays a significant role in the thermophysical characteristics of nanofluids. Nanofluids are analysed at a variety of temperatures and nanoparticle concentrations to identify the optimal outcomes. Numerous experiments have been conducted in which researchers have been able to observe these findings measured at specific temperatures using measuring devices or theoretical models; however, there is a gap in the literature where more correlations, particularly correlations as functions of temperature, are required to analyse the nanofluids at specific temperatures using correlations. Hence, theoretical models and experimental investigations may be predicted with more ease, and more precise results can be acquired. This study provides more recent relationships between the Prandtl number and viscosity.

2. Literature Review

Viscosity and the Prandtl number were examined by several experts in their respective fields, and their work and findings are discussed in Table 1.

3. Data Reduction

The Prandtl number depends on the nanofluid’s thermal conductivity, specific heat, and viscosity. Equation (1a) depicts the formula used by Tiandho et al. [21] and other researchers to analyse the nanofluid Prandtl number in scientific literature.

\[ Pr = \frac{\mu C_p}{k}, \]  

(1a)

For nanofluid computations, equation (1a) can be rewritten as shown in the following equation:

\[ Pr_{nf} = \frac{\mu_{nf} C_{p,nf}}{k_{nf}}, \]  

(1b)

where \( Pr_{nf}, \mu_{nf}, C_{p,nf}, \) and \( k_{nf} \) are the nanofluid Prandtl number, nanofluid viscosity, and nanofluid specific heat, respectively. In this study, the Prandtl number and viscosity of three nanofluids are measured at temperatures ranging from 30.4°C to 70.4°C. The titanium oxide nanoparticles are combined with ethylene glycol (40%)/water (60%), ethylene glycol, and water as the basis of fluids. Equation (2) of the Einstein model is the most popular model for calculating nanofluid viscosity. Manikandan and Baskar [16] were among the researchers who utilised this model.

\[ \mu_{nf} = \mu_{bf}(1 + 2.5\varnothing), \]  

(2)

where \( \mu_{nf}, \mu_{bf}, \) and \( \varnothing \) indicate the viscosity of the nanofluid and the base fluid, respectively, and \( \varnothing \) represents the nanoparticle concentration. The thermophysical properties are observed using ASHRAE (2017) Handbook-Fundamental (SI).

3.1. Viscosity and Prandtl Number’s Base Fluid Properties and Correlations. Prior to introducing the new nanofluid correlations for Prandtl number and viscosity, it is crucial to have examined the base fluid properties for the nanofluid analysis, as base fluids form the basis of the new correlations in this study. The base fluids are used to compare the findings obtained using both the theoretical models and the novel correlations to determine the difference between utilizing nanoparticles to enhance thermophysical properties and not using them.

Table 2 provides an examination of the base fluids. The correlations of base fluids are explored at temperatures ranging from 30.4°C to 70.4°C.

4. New Correlations for Viscosity and Prandtl Number for Nanofluids as Functions of Temperature

This section’s study is based on the latest correlations between the viscosity and the Prandtl number. The correlations are used for calculating the viscosity and Prandtl number of nanofluids. The correlations are also reported for the range of temperatures between 30.4 and 70.4 degrees Celsius. In addition to its usage in titanium oxide nanofluid investigation, the novel correlations can be utilized to compute the viscosity and Prandtl number of different nanofluids at various temperatures.

4.1. New Viscosity Correlation as a Function of Temperature. The equation in 4.1 represents the new link between viscosity and temperature, where \( \mu_{nf}, \mu_{bf}, \varnothing, \) and \( T \) represent the viscosity, base fluid, nanoparticle concentration, and temperature of the nanofluid.

\[ \mu_{nf} = \mu_{bf}(1 + 3.5\varnothing) - 0.035\mu_{bf}T\varnothing. \]  

(3)

4.2. New Prandtl Number Correlation as a Function of Temperature. The equation in 4.2. represents the new Prandtl number correlation, where \( Pr_{nf}, \mu_{nf}, k_{nf}, \varnothing, P_{r,nf}, k_{bf}, C_{p,bf}, C_{p,nf}, k_{p,nf}, \) and \( T \) represent the nanofluid Prandtl number, base fluid viscosity, nanoparticle density, nanoparticle thermal conductivity, base fluid specific heat, base fluid thermal conductivity, nanoparticle concentration, nanoparticle specific heat, base fluid density, and temperature, respectively. This new association between the Prandtl number and viscosity is applicable to the research of numerous nanofluids that require examination.
<table>
<thead>
<tr>
<th>Authors</th>
<th>Base fluid</th>
<th>Nanoparticle</th>
<th>Nanoparticle concentration</th>
<th>Results obtained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rudyak et al. [1]</td>
<td>Distilled</td>
<td>Zirconia, aluminum, silicon, and</td>
<td>1% and 8% by volume</td>
<td>The Prandtl number increased with increasing nanoparticle concentration and dropped with decreasing nanoparticle size [1]</td>
</tr>
<tr>
<td>Ganesh et al. [2]</td>
<td>Water and ethylene glycol</td>
<td>Gamma aluminum oxide</td>
<td>0.05, 0.1, 0.15, and 0.2 by volume</td>
<td>A velocity profile rise was noticed [2]</td>
</tr>
<tr>
<td>Raza et al. [3]</td>
<td>Water</td>
<td>Copper</td>
<td></td>
<td>Nanoparticle concentrations and Prandtl number influenced the velocity profile [3]</td>
</tr>
<tr>
<td>Kim et al. [4]</td>
<td>Distilled water</td>
<td>Aluminum oxide</td>
<td>0.25%, 0.5% and 1% wt%</td>
<td>Found a decline in Prandtl number when nanoparticle concentrations rose [4]</td>
</tr>
<tr>
<td>Özdemir and Öğüt [5]</td>
<td>Water/ethylene glycol</td>
<td>Ethylene glycol</td>
<td></td>
<td>The Prandtl number grew as the concentration of ethylene glycol likewise increased [5]</td>
</tr>
<tr>
<td>Al-Amir et al. [6]</td>
<td>Water</td>
<td>Silver</td>
<td>0 &lt; φ &lt; 0.2 by volume</td>
<td>Conclusion: the Nusselt number increased as the Prandtl number increased [6]</td>
</tr>
<tr>
<td>Mikkola et al. [7]</td>
<td>Water</td>
<td>Aluminum oxide, micelles, polystyrene, and silicon oxide</td>
<td>0.1% to 1.8% volume percentage</td>
<td>In comparing nanofuids, the significance of the Prandtl number varies. The Nusselt number remained negligible since the Prandtl number was considered [7]</td>
</tr>
<tr>
<td>Sundar et al. [8]</td>
<td>Oil magnetic</td>
<td>Ferrous oxide</td>
<td>0.05% to 0.5%</td>
<td>The Prandtl number of nanofuid is 1.52 times greater at 30°C and 1.6 times greater at 60°C compared to the base fluid. With increasing temperature, the Prandtl number decreased [8]</td>
</tr>
<tr>
<td>Veera Krishna [9]</td>
<td>Water</td>
<td>Aluminum copper</td>
<td>0.05%, 0.1%, 0.15%</td>
<td>When the Prandtl number increased, a reduction in the thickness of the thermal boundary layer was noticed [9]</td>
</tr>
<tr>
<td>Zargartalebi et al. [10]</td>
<td>Water/ethylene glycol</td>
<td>Titanium oxide-silicon dioxide (50:50)</td>
<td>0.5% to 3.0%</td>
<td>The rise in the Prandtl number enhanced the heat transfer rate, but it decreased the collector's efficiency [10]</td>
</tr>
<tr>
<td>Nasrin [11]</td>
<td>Water</td>
<td>Aluminum oxide</td>
<td>2%</td>
<td>With an increase in temperature, viscosity was shown to diminish [12]</td>
</tr>
<tr>
<td>Nabil et al. [12]</td>
<td>Water/ethylene glycol (60:40)</td>
<td>Titanium oxide-silicon dioxide (50:50)</td>
<td>0.0047%, 0.023%, 0.0571%, 0.1428%, and 0.2381%</td>
<td>Results indicated a small increase in viscosity when temperature rose over the critical threshold [13]</td>
</tr>
<tr>
<td>Yu et al. [13]</td>
<td>Water</td>
<td>MWCNT</td>
<td></td>
<td>Reduced graphene oxide/ethylene glycol nanofuid viscosity was lowered by 22% [14]</td>
</tr>
<tr>
<td>Shah et al. [14]</td>
<td>Ethylene glycol</td>
<td>Reduced graphene oxide</td>
<td>0.02%, 0.04%, and 0.05%</td>
<td>Viscosity rose with increasing nanoparticle concentrations [15]</td>
</tr>
<tr>
<td>Iqbal et al. [15]</td>
<td>Water</td>
<td>Titanium oxide</td>
<td>0.1%, 0.25%, 0.5%, and 0.75%</td>
<td>Found an increase in viscosity after adding nanoparticles to the basic fluids [16]</td>
</tr>
<tr>
<td>Manikandan and Baskar [16]</td>
<td>Water/ethylene glycol at 40:60, 50:50, and 30:70</td>
<td>Titanium oxide zinc oxide</td>
<td>0.2 to 1.0 by volume</td>
<td></td>
</tr>
<tr>
<td>Authors</td>
<td>Base fluid</td>
<td>Nanoparticle</td>
<td>Nanoparticle concentration</td>
<td>Results obtained</td>
</tr>
<tr>
<td>------------------</td>
<td>-----------------------------------</td>
<td>-------------------------------</td>
<td>----------------------------</td>
<td>---------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Samylingam et al. [17]</td>
<td>Ethylene glycol (40%)/water (60%)</td>
<td>Cellulose nanocrystal CNC</td>
<td>0.1%, 0.3%, 0.5%, 0.7%, 0.9, 1.1, and 1.5%</td>
<td>Measurements revealed a rise in viscosity as nanoparticle concentration rose, with the lowest viscosity measured at 70°C for a concentration of 0.1% by volume [17]. Temperature led to an increase in viscosity. The ceric oxide nanofluid displayed a greater rise in viscosity compared to the zinc oxide nanofluid [18].</td>
</tr>
<tr>
<td>Kumar et al. [18]</td>
<td>Water</td>
<td>Ceric oxide zinc oxide</td>
<td>0.5% to 2.0% by volume</td>
<td>It was noticed that viscosity increased whenever nanoparticle concentrations were raised [19].</td>
</tr>
<tr>
<td>Żyła et al. [19]</td>
<td>Ethylene glycol</td>
<td>Nanodiamond</td>
<td>0.01 to 0.1 by volume</td>
<td>An observation was made of the rise in viscosity [20].</td>
</tr>
<tr>
<td>Zheng et al. [20]</td>
<td>Water</td>
<td>Copper oxide, iron oxide black, aluminum oxide, and silicon carbide</td>
<td>0.05%, 0.1%, 0.5%, and 1.0%</td>
<td></td>
</tr>
</tbody>
</table>
new correlation results. At 30.4°C, the Prandtl number for value increases by a small proportion in both theoretical and

5. Discussion of Results

Shown in Figure 6

5.2.2. Titanium Oxide-Ethylene Glycol Nanofuid Prandtl

Number Shown in Figure 5

5.2.1. Titanium Oxide-Ethylene Glycol Nanofuid Viscosity

Shown in Figure 1

5.1.3. Titanium Oxide-Water Nanofuid Viscosity Shown in

Figure 3

5.1.1. Titanium Oxide-Ethylene Glycol (40%)/Water (60%)

Nanofuid Viscosity Shown in Figure 5

5.1.2. Titanium Oxide-Ethylene Glycol Nanofuid Viscosity

Shown in Figure 2

5.1. Viscosity Comparison Results

5.1.1. Titanium Oxide-Ethylene Glycol (40%)/Water (60%)

Nanofuid Viscosity Shown in Figure 1

5.1.2. Titanium Oxide-Ethylene Glycol Nanofuid Viscosity Shown in Figure 2

5.1.3. Titanium Oxide-Water Nanofuid Viscosity Shown in Figure 3

5.2. Prandtl Number Comparison Results

5.2.1. Titanium Oxide-Ethylene Glycol (40%)/Water (60%)

Nanofuid Prandtl Number Shown in Figure 4

5.2.2. Titanium Oxide-Ethylene Glycol Nanofuid Prandtl Number Shown in Figure 5

5.2.3. Titanium Oxide-Water Nanofuid Prandtl Number Shown in Figure 6

6. Discussion of Results

6.1. Prandtl Number. In accordance with the observations made in Sections 3–5, the Prandtl number for the titanium oxide-ethylene glycol (40%)/water (60%) mixture at various temperatures is 19 at 30.4°C, as shown in Figure 4. The Prandtl value increases by a small proportion in both theoretical and new correlation results. At 30.4°C, the Prandtl number for titanium oxide-ethylene glycol is shown to range from 137 to 147 in Figure 5. In Figure 6, the Prandtl number for titanium oxide-water nanofuid ranges from 5.50 to 5.79 at 30.40 degrees Celsius. The Prandtl number of pure ethylene glycol is greater than that of water and ethylene glycol/water mixtures. Observing the various nanofuids, the Prandtl number increases as 0.004, 0.006, 0.008, and 0.01 nanoparticle concentrations are added to the basic fluids. Yet, as the temperature increases, the Prandtl number similarly falls. In both the theoretical formula for the Prandtl number and the new correlation model, it can be observed that as the viscosity increases, the Prandtl number also increases. As the temperature of the nanofuid increases, the viscosity of the nanofuid lowers, causing the Prandtl number to fall as well. The basic fluids have a lower measured Prandtl number than the nanofuids. Compared to ethylene glycol (40%)/water (60%) base fluid and water base fluid, ethylene glycol has a high Prandtl number. The Prandtl number increases ranging from 1.4% to above 3.45% for the titanium oxide-ethylene glycol (40%)/water (60%) nanofuid, from 2.4% to 5.8% for the titanium oxide-ethylene glycol nanofuid, and from 1.1% to 2.6% for the titanium oxide-water nanofuid, with reference to the base fluids. Moreover, the Prandtl number determines the thickness of the boundary layer. The greater the Prandtl number, the greater the nanofuid's high viscosity, which causes nanofluid flow constraints when the nanofuid thickens due to its high viscosity. Kho et al. [22] came to the conclusion that when the Prandtl number increased, the temperature profile continued to decrease.

6.2. Viscosity. When the concentration of nanoparticles increases, the viscosity of titanium oxide nanofuids increases. Considering the previous Sections 3–5, the viscosity increases for all the titanium oxide nanofuids are minimal. In the fluid analysis, it is essential that the viscosity is kept low. Adding nanoparticle concentrations to nanofuids increases their viscosity. Yet, as demonstrated in Figures 1–3, the viscosity increases slightly when nanoparticle concentrations of 0.004, 0.006, 0.008, and 0.01 are added to a solution. Viscosity increases slightly by fractions of percentage points in all the detected nanofuids, indicating that the necessary results in nanofuid analysis can be obtained. Among the thermophysical parameters of nanofuids, such as thermal conductivity, specific heat, and density, viscosity increases less than thermal conductivity, specific heat, and density do at the same nanoparticle concentrations. Maintaining a low viscosity is a desirable outcome in the fluid analysis and makes nanofuids even more suitable for

<table>
<thead>
<tr>
<th>Base fluid properties</th>
<th>Water</th>
<th>Ethylene glycol</th>
<th>Ethylene glycol (40%)/water (60%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity (mPa.s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_{bf}$</td>
<td>1.0461 – 0.0095T</td>
<td>19.681 – 0.2293T</td>
<td>3.0648 – 0.0311T</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9762</td>
<td>0.9529</td>
<td>0.9750</td>
</tr>
<tr>
<td>Prandtl number (–)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_{r,bf}$</td>
<td>7.3418 – 0.0707T</td>
<td>187.99 – 2.161T</td>
<td>26.048 – 0.2669T</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9723</td>
<td>0.9529</td>
<td>0.9719</td>
</tr>
</tbody>
</table>

$$P_{r,\text{nbf}} = \frac{\mu_{bf}P_{k}}{\rho_{bf}} \left[ \frac{C_{p,bf} (1 + 2\varnothing) + \left( \frac{C_{p} \varnothing}{\rho_{bf}} \right)}{(1 - 4\rho_{p} \varnothing + 4\rho_{p} \varnothing T) - \left( 0.035 \rho_{p} \varnothing^2 T / \rho_{bf} \right)} \right].$$ (4)
industrial applications. This is supported by the viscosity graphs and results presented in the study. Viscosity plays a significant role in fluid movement, and it is always preferable for the fluid to have a low viscosity. All the base fluids containing Titanium oxide nanoparticle concentrations of 0.004 have the lowest viscosity as seen in the study. Considering the selected titanium oxide nanoparticle concentrations and how the viscosity increases slightly, titanium oxide nanofluids at lower nanoparticle concentrations could be considered for use in a variety of applications, and careful consideration should also be given to titanium oxide water-based nanofluid, as the addition of higher nanoparticle

**Figure 1:** Comparison of titanium oxide-ethylene glycol (40%)/water (60%) nanofluid viscosity theoretical model and new correlation.

**Figure 2:** Comparison of titanium oxide-ethylene glycol nanofluid viscosity theoretical model and new correlation.
Figure 3: Comparison of titanium oxide-water nanofluid viscosity theoretical model and new correlation.

Figure 4: Comparison of titanium oxide-ethylene glycol (40%)/water (60%) nanofluid Prandtl number theoretical model and new correlation.
concentrations would affect the decrease percentage of viscosity as shown in this paper. Despite the favourable viscosity of water nanofluid, the addition of nanoparticle concentrations increases the decreasing percentage of the nanofluid relative to the base fluid, as shown in this study. This gives ethylene glycol/water combinations so much promise for usage in industrial applications that the drop in viscosity with increasing temperature for ethylene glycol (40%)/water (60%) nanofluid is identical to the fall in viscosity percentage for ethylene glycol (40%)/water (60%) base
fluid. With the introduction of the new correlation for viscosity, it is confirmed that other studies have seen an increase in viscosity when nanoparticles are added and that the percentage increase in viscosity throughout the nanofluids tested is consistent. It has been demonstrated that a novel correlation as a function of temperature contributes to the research models found in the literature since it produces the desired results. With the results of the new correlation shown in Figures 1–3, we observe a decrease in viscosity, making the usage of this new correlation a viable alternative for analysing viscosity at various temperature ranges. Einstein’s model does not include temperature, but with this new correlation, temperature is included, and it has been shown to produce lower viscosity rise findings when nanoparticles are introduced, making the new correlation preferable for viscosity analysis.

7. Conclusion

It is always desired that any type of fluid flows freely; therefore, in the observation of the researched study, titanium oxide-ethylene glycol (40%)/water (60%) nanofluid and titanium oxide water nanofluid had the lowest Prandtl number and were preferable to the titanium oxide ethylene glycol nanofluid. In all nanofluids observed, the highest Prandtl number and viscosity were observed at 0.01 nanoparticle concentration and 30.4 degree Celsius. Using the new correlations for Prandtl number and viscosity, this study demonstrated an increase in the Prandtl number and correlated with the previously published data. The nanofluid research study requires more precise analysis methods, which will aid in the creation of more precise designs for industrial equipment and the rescaling of existing industrial designs. The additional correlations as functions of temperature augment the literature-based research models that have demonstrated their ability to achieve the required results. With the results of the new correlations shown in Figures 1–6, we observe a decrease in viscosity and Prandtl number with an increase in temperature, making the application of the new correlations a viable alternative for analysing the viscosity at various temperature ranges. Einstein’s viscosity model for nanofluid viscosity analysis does not include temperature; with this new correlation for viscosity, temperature is included, as is the case for Prandtl number. With the new correlations introduced in this study, we note that the accuracy of results remains within the same range as when using theoretical models and that the new correlations for Prandtl number and viscosity contribute to the analysis of nanofluids.

Nomenclature

- $P_f$: Prandtl number
- $\mu$: Viscosity
- $k$: Thermal conductivity
- $C_p$: Specific heat
- $P_{nf}$: Nanofluid Prandtl number
- $P_{bf}$: Base fluid Prandtl number
- $\rho_{nf}$: Nanofluid density
- $\rho_{bf}$: Base fluid density
- $\phi$: Nanoparticle concentration
- $T$: Temperature
- $TiO_2$: Titanium oxide.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this article.

Acknowledgments

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References


