Hindawi
Journal of Engineering
Volume 2024, Article ID 5281868, 18 pages
https://doi.org/10.1155/2024/5281868

Research Article

Experimental and Numerical Studies on the Vibration of Hybrid Composite with an Edge Crack

Solomon Alemneh Adimass,[1] Ermias Gebrekidan Koricho,[2] Velmurugan Paramasivam ,[3] and Vignesh Krishnan[4]

[1]*Department of Mechanical Engineering, School of Mechanical and Chemical Engineering, Kombolcha Institute of Technology, Wollo University, P.O. Box 1145, Kombolcha, Ethiopia*
[2]*Department of Mechanical Engineering, Addis Ababa Science and Technology University, Addis Ababa, Ethiopia*
[3]*Faculty of Mechanical and Industrial Engineering, Bahir Dar Institute of Technology, Bahir Dar University (BiT), P.O. Box 26, Bahir Dar, Ethiopia*
[4]*Department of Mechanical Engineering, PSNA College of Engineering and Technology, Dindigul, Tamil Nadu, India*

Correspondence should be addressed to Velmurugan Paramasivam; drvelmuruganp@bdu.edu.et

Received 22 January 2024; Revised 18 March 2024; Accepted 8 April 2024; Published 26 April 2024

Academic Editor: Parviz Malekzadeh

The hybrid composite plate is increasingly used in marine, robotic arm, and other applications. Edge crack is the greatest common fault in the structural systems during its working time. In this research, the successful fabrications of a hybrid composite of epoxy-based composites reinforced with bamboo and glass fibers have been done to examine the vibration characteristics under free vibration. First-order shear deformation theory is used to study the fundamental natural frequencies of asymmetrically hybrid composite beam with edge cracks for cantilever beam boundary conditions. Both numerical (FEM) using ABAQUS software and experimental investigations are done for the vibration analysis of unidirectional bamboo/glass fiber epoxy hybrid composite (BGHC) beam with edge cracks using fracture mechanics theory. It is observed that implications edge crack with different crack depth and various fiber weight ratio parameters have a major effect on vibration analysis. Also, it is clear that the natural frequencies of the hybrid composite are significantly affected due to the fiber weight ratio and crack depth of the hybrid composite materials. So, crack lengths are playing a vital role in the dynamic or vibration characteristics of the hybrid composite materials. The results of natural frequencies have been shown that they are decreased with an increase in crack depth and an increase in the bamboo fiber weight. Thus, BGHC-2 made from 30% bamboo and 10% glass fiber can be a viable candidate for applications that will produce good vibration properties. Generally, the first-third natural frequency of the hybrid composite beam decreases within the increase of bamboo fiber and decreases in glass fiber. The composite materials having 60% of epoxy with 20% bamboo and 20% glass fiber, and 60% of epoxy with 10% glass and 30% bamboo fibers, are observed with a gradual decrease in the values of natural frequencies with respect to increasing in crack depth. A fine agreement was accomplished between the simulation and experimental results. Therefore, the present approach can be used to identify cracks for mechanical health monitoring by linking the variation in natural frequencies of the hybrid composite beams.

1. Introduction

The demand of composite materials for the type of structure have rapidly increased due to industrial stringency, especially in aerospace and automotive industries in which lightweights of a structure is essential. In recent years, the vibrational behavior of composite materials has received considerable attention as their usage is getting higher in primary and secondary structures of vehicles, airplanes, ships, and civil construction. However, the presence of defects, delamination, and cracks significantly reduces the stiffness and strength of the physical and may affect their mechanical properties, fracture toughness, and vibration characteristics (e.g., natural frequency and mode shape).

Therefore, the finding and localization of mutilation to composite structures at the initial stage of development can optimize system performance and safety.

Two plate theories are widely used in engineering problems, namely, the Kirchhoff plate theory, or classical plate theory, and the Mindlin–Reissner plate theory. Comprehensive background on plates has been provided by Timoshenko [1]. The composite reinforced materials could either use synthetic and natural reinforcement fibers [2]. The fiber-reinforced polymer composites are developed using synthetic fiber which are having many advantages such as high strength, high stiffness, long fatigue life, adaptability to the function of the structure, corrosion resistance, and environmental stability [3]. However, they are not biodegradable, and hence, motive a lot of pollution.

On the other hand, in recent years, the demand for natural fiber composites in automotive and aerospace industries as well as civil engineering fields is increasing. Compared with synthetic composites, natural fiber hybrid composites have distinct advantages such as low cost, high strength-weight ratio, recyclable, low energy consumption, high specific properties, biodegradability, and renewable. So, it is preferred over synthetic fibers [4]. Various natural fibers for example sisal, hemp, bamboo, cotton, jute, kenaf, bagasse, and so on have been used for years as a reinforcement of composites. As compared with other types of natural fibers, bamboo fiber has high specific stiffness and strength due to its low density and high mechanical strength with abandoned resources [5]. The ratio of strength to weight of bamboo fiber is comparatively higher than conventional metallic fibers. The bamboo plants are short life cycles to grow and a good amount of availability in places that qualify as tropical, subtropical, or temperate zones. By hybridizing natural fibers for example bamboo fiber, with another natural fiber/synthetic fiber in one matrix, the resulting composite is a unique product (hybrid composites) that displays better mechanical and thermal properties in comparison with individual fiber-reinforced polymer composites [6–8].

The performance of composite structures is detrimentally affected by the presence of damages such as delamination, edge cracks, voids, inclusion of undesired fillers. Determining the response hybrid composite subjected to vibration is more challenging due to the interaction among various types of fibers and the hosting matrix as compared with a monofiber composite structure. In fiber-reinforced composites, the vibration response depends on many factors for example fiber length, fiber/matrix interface, orientation, and content. So, it is a complex task to find the natural frequency for fiber-reinforced composites. The occurrence of vibration response in a structure is an undesirable phenomenon in order to guarantee structural stability, position control durability (particularly durability against fatigue), performance, and noise reduction. Considering the critical condition, the occurrence of vibration in the cracked plate or beam structure can reduce the reliability of a structure due to the expansion of crack opening.

Hence, it is very important to thoroughly understand the vibration response of the cracked hybrid composite plate structure and to prevent the accident by predicting the destruction of the structure [9]. Most of the physical and mechanical components are damaged due to the crack propagation of excessive vibrations' response in their service life since most people's activities involve vibration response in one form or in the other. And also, for engineering application, the structures of metal materials will undergo dangerously large oscillations which may prime to excessive deflection and failure and also, when the weight of the structure increases, the live performance of the structure decreases. The vibration of the structure also causes high stress concentrations at ascertain locations. In order to attain the right combination of material properties and service performance, the vibrational behavior is the main point to be considered in composite structure. To avoid the typical problems caused by vibration, it is significant to determine vibrational response of the structure cracked effects to reinforce the most flexible regions or to locate the right positions where weight should be reduced or the service time should be increased for bamboo glass fiber reinforcement epoxy hybrid composite.

Mohanty et al. [10] studied the cracks in carbon/epoxy composite beam which could reduce the structural stiffness. They mentioned that a better understating had been achieved on the impact of crack on frequencies when the experimental and FEM analysis were combined together. Sahu et al. [11] used ABAQUS platform for FEM analysis to study the crack effects on vibration frequencies of aluminum beam. They reported that higher crack depths observed with noticeable frequencies of vibration. In addition to that, ABAQUS is one of the best platforms for doing FE modelling they reported. Sahu et al. [12] modelled the frequencies of crack by dividing the beam into individual segments at cracks section using ABAQUS software. The C3D8R element was used for modelling and the beam is considered as laminated glass/epoxy composite beam with open cracks. Das et al. [13] carried out an experimental and numerical approach to find the effect of open cracks woven LCB (laminated composite beam). Fast fourier transformation system of analyzer was used for experimental modal analysis. Multi and single cracked LCB system were observed with very less frequencies at zero-degree ply orientation. Prusty et al. [14] analyzed the free vibration on sandwich plates using experimental and FEM approach. In this research, cut out model is used in the sand which plates instead of cracks. Sinha et al. [15] developed the laminated composite plate with and without and analyzed the free vibration using experimental and numerical methods. In this paper, Kelly et al. [16] introduces the utilization of the variational method for a Timoshenko beam model with a single-edge crack, reinforced by functionally graded graphene platelet (FG GPL). It is the first time this method has been applied in such a context. The variational expression allows for arbitrary and independent variations of displacement (Ui), total strain (γij), stress (σij), and momentum (mi) components. The model not only took into account the concentrations and properties of the matrix and GPL but also considered the orientation and aspect ratio of the GPL. Two modifications are made to the model, including a three-dimensional and a two-dimensional Halpin-Tsai model. The two-dimensional

Halpin-Tsai model, in particular, demonstrates a good ability to predict Young's modulus of randomly oriented GPL reinforced polymeric composites.

From the literature study, it is clear that the study of free vibration analysis is very much essential to identify the impact of natural frequencies when the composite beam is exposed to cracks and cutouts. Researchers focused on composite beam which mainly consists of epoxy, fiber, and laminated composites. According to the authors' knowledge, no study has been carried out to find the impact of natural frequencies on unidirectional bamboo/glass fiber epoxy hybrid composite (BGHC) beam with edge cracks when undergone into free vibration. Thus, keeping these aspects in consideration, the main objective of this paper is to study the dynamic behavior of three various hybrid bamboo/glass epoxy cantilever composite plates that consisted of various bamboo and glass fibers weight percentages using experimental and numerical approaches.

2. The Mathematical Formulation of Vibration Analysis

2.1. Uncracked Hybrid Composite Beam Modelling. The geometric configuration of the bamboo/glass fiber epoxy reinforced hybrid composite beam is shown in Figure 1: length L, width, w, and height, h, crack depths "a_i" ($i = 1, 3$), crack location at the arbitrary positions, Lc.

The assumptions made in the analysis are as follows:

(i) The deformation of the beam takes place in the x-z plane

(ii) There is no warping effect

(iii) The stress-strain distribution is linear and small displacement theory is applied

(iv) Damping effects are neglected

According to FSDT for laminated composite beam, the displacement field can be as follows:

$$\begin{aligned} U(x,z,t) &= u(x,t) + z\varphi(x,t) \\ W(x,z,t) &= w(x,t), \end{aligned} \tag{1}$$

where "u" and "w" represent the centerline displacement along x and z-direction of the plate and φ is an unknown function that represents a shear strain on the centerline of the plate.

The strain displacement relations are given by

$$\begin{aligned} \varepsilon x &= \varepsilon xx^0 + kxx \\ \gamma xz &= \gamma xz^0, \end{aligned} \tag{2}$$

where

$$\begin{aligned} \varepsilon xx^0 &= \frac{\partial u}{\partial x}, \\ kxx &= \frac{\partial \varphi}{\partial x}, \\ \gamma xz^0 &= \varphi + \frac{\partial w}{\partial x}. \end{aligned} \tag{3}$$

2.2. Stress Intensity Factor of Cracked Composite Plate. The methods for calculating stress intensity factors of cracked composite plates corresponding to generalized loading conditions are described in equations (4)–(16). The geometric dimension cracked hybrid composite plates and cracked plate under generalized loading are shown in Figures 2 and 3, respectively. A cracked hybrid composite plate of rectangular cross-section has an edge crack with a tip line parallel to the z-axis, i.e., with a uniform depth, and the generalized loading is indicated by six general forces, that is, P_1 is an axial force, P_2 and P_3 are shear force, P_4 and P_5 are bending moments, and P_6 a torsional. The stress intensity factors of the three modes of fracture that are opening, sliding and tearing types correspond to the generalized loading P_n. In general, the stress intensity factors K_{jn} ($j = \text{I}$, II, and III) of the composite plate cannot be taken in the same form as the stress intensity factors of anisotropic material in the same loading and geometry conditions. The stress intensity factors K_{jn} ($j = \text{I}$, II, and III) of the fiber-reinforced composite beam can be expressed in the following equation [17–19]:

$$\mathrm{K_{jn}} = \sigma_{\mathrm{n}} \sqrt{\pi a}\, \mathrm{F_{jn}}\left(\frac{a}{\mathrm{h}}, \lambda^{1/4}\frac{\mathrm{L}}{\mathrm{h}}, \delta\right), \tag{4}$$

where σ_n is the stress, a is the crack depth, F_{jn} is the correction function according to the geometric shape of the plate, and $-\lambda$ and δ are dimensionless parameters taking into account the in-plane orthotropic and defined as functions of the elastic constants by using the following equation [17–19]:

$$\begin{aligned} \lambda &= \frac{\mathrm{E}_{22}}{\mathrm{E}_{11}}, \\ \delta &= \sqrt{\frac{\mathrm{E}_{22}\mathrm{E}_{11}}{2\mathrm{G}12}} - \sqrt{\mu_{12}\mu_{21}}, \end{aligned} \tag{5}$$

where $\lambda^{1/4}$ L/h ≥ 2, the term related to $\lambda^{1/4}$ L/h is ignored. For a fiber-reinforced composite beam whose aspect (L/4) ratio is greater than 4, it satisfies this condition, the stress intensity factor K_{jn} becomes as shown in the following equation [17–19]:

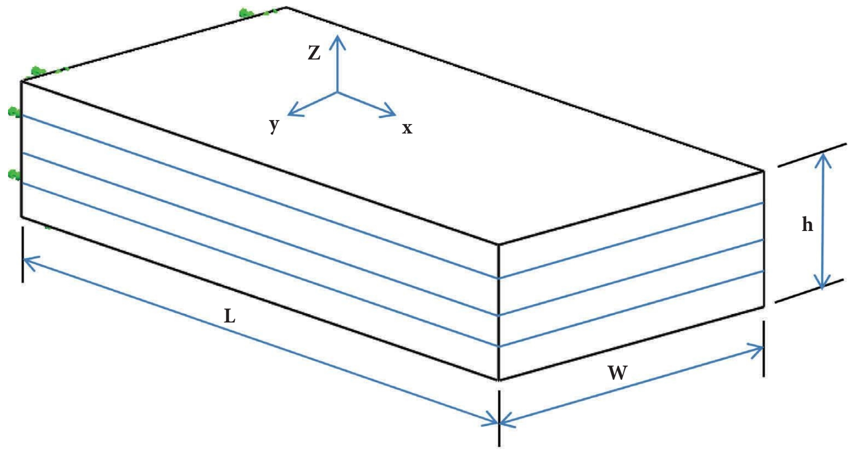

Figure 1: Cantilever hybrid composite plate.

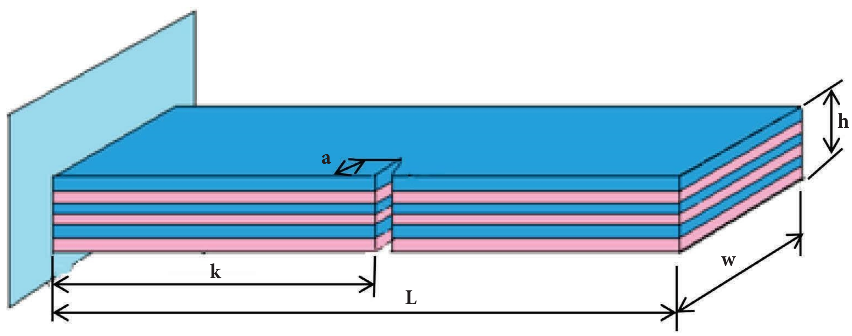

Figure 2: The geometric dimension of cracked hybrid composite plate.

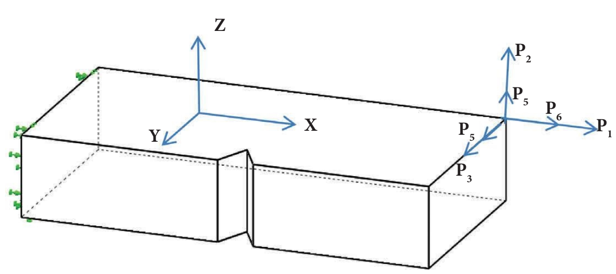

Figure 3: The cracked plate under generalized loading conditions.

$$\text{Kjn} = \sigma_{n}\sqrt{\pi \text{a}}\text{F}_{\text{jn}}\left(\frac{\text{a}}{\text{h}}\right)Y_{j}(\delta), \tag{6}$$

where $Y_j(\delta)$ is the correction function for orthotropic material and $F_{jn}(a/b)$ takes the same form as correction function in orthotropic material selected according to geometry shape of the plate and loading acting on the beam. The stress intensity factors of the unidirectional bamboo/glass fiber-reinforced hybrid composite beam are determined by using equations (7)–(10) [17–19].

$$
\begin{aligned}
&\text{K}_{\text{I1}} = \sigma_1\sqrt{\pi a}\,\text{F}_1\left(\frac{a}{\text{b}}\right)\text{Y}_1(\delta), \sigma_1 = \frac{P1}{bh}\\
&\text{K}_{\text{I4}} = \sigma_4\sqrt{\pi a}\,\text{Y}_1(\delta)\text{F}_2\left(\frac{a}{\text{b}}\right), \sigma_4 = \frac{12P_4}{bh^3}z\\
&\text{K}_{\text{I5}} = \sigma_5\sqrt{\pi a}\,\text{Y}_1(\delta)\text{F}_2\left(\frac{a}{\text{b}}\right), \sigma_4 = \frac{6P_4}{bh^2}\\
&\text{K}_{\text{I2}} = \text{K}_{\text{I3}} = \text{K}_{\text{I6}} = 0\\
&\text{K}_{\text{II3}} = \sigma_1\sqrt{\pi a}\,\text{Y}_{\text{II}}(\delta)\text{F}_{\text{II}}\left(\frac{a}{\text{b}}\right), \sigma_1 = \frac{P_3}{bh}\\
&\text{K}_{\text{II1}}\,\text{K}_{\text{II2}} = \text{K}_{\text{II4}} = \text{K}_{\text{II5}} = \text{K}_{\text{I6}} = 0\\
&\text{K}_{\text{III2}} = \sigma_2\sqrt{\pi a}\,\text{Y}_{\text{III}}(\delta)\text{F}_{\text{III}}\left(\frac{a}{\text{b}}\right), \sigma_2 = \frac{P_2}{bh}\\
&\text{K}_{\text{III6}} = \sigma 6\sqrt{\pi \text{a}}\,\text{Y}_{\text{III}}(\delta)\,\text{F}_{\text{III}}\left(\frac{a}{\text{b}}\right), \sigma 6 = \frac{24\text{P}_6\pi^3}{\pi^5 bh^2 - 192h^3}\cos\left(\frac{\pi}{h}\text{z}\right)\\
&\text{K}_{\text{III1}} = \text{K}_{\text{III3}} = \text{K}_{\text{III5}} = 0,
\end{aligned}
\tag{7}
$$

where

$$
\begin{aligned}
&\text{F}_1\left(\frac{a}{\text{b}}\right) = \sqrt{\frac{2h}{\pi\mathfrak{I}}\tan\frac{\pi a}{2b}}\,\frac{0.752 + 2.02(a/h) + 0.37(1 - \sin \pi a/2b)^3}{\cos \pi a/2b}\\
&\text{F}_2\left(\frac{a}{\text{b}}\right) = \sqrt{\frac{2h}{\pi\mathfrak{I}}\tan\frac{\pi a}{2b}}\,\frac{0.923 + 0.199(1 - \sin \pi a/2b)^4}{\cos \pi a/2b}\\
&\text{F}_{\text{II}}\left(\frac{a}{\text{b}}\right) = \frac{1.122 - 0.561(a/b) + 0.085(a/b)^2 + 0.18(a/b)^3}{\sqrt{1-(a/b)}}\\
&\text{FIII}\left(\frac{a}{\text{b}}\right) = \sqrt{\frac{2h}{\pi a}\tan\frac{\pi a}{2b}}
\end{aligned}
\tag{8}
$$

And $\text{YI}(\delta), = 1 + 0.1(\delta - 1) - 0.016(\delta - 1)^2 + 0.002(\delta - 1)^3, \text{YII}(\delta) = \text{YIII}(\delta) = 1.$

The equation of motion using first-order shear deformation beam theory is

$$\frac{\partial N}{\partial x} = I_1\frac{\partial^2 u}{\partial t^2} + I_2\frac{\partial^2 \varnothing}{\partial t^2} - p_x$$
$$\frac{\partial M}{\partial x} = I_2\frac{\partial^2 u}{\partial t^2} + I_3\frac{\partial^2 \varnothing}{\partial t^2} + Q \quad (9)$$
$$\frac{\partial Q}{\partial x} = I_1\frac{\partial^2 \varnothing}{\partial t^2} - p_z.$$

The solution of the governing equation assumes the displacements are in the following equations:

$$\mathrm{u_0, w_0}, \varphi_0 = \sum_{\mathrm{m=1}}^{\mathrm{M}} \left[\mathrm{A_m} \cos(\alpha_{\mathrm{m}}\mathrm{x}), \mathrm{C_m} \sin(\alpha_{\mathrm{m}}\mathrm{x}), \mathrm{B_m} \cos(\alpha_{\mathrm{m}}\mathrm{x})\right] \sin(\omega t). \quad (10)$$

Substituting equation (10) into the equation of motion, the obtained characteristics equation for a hybrid composite beam without crack is

$$\begin{bmatrix} c11 & c12 & c13 \\ c21 & c22 & c23 \\ c31 & c32 & c33 \end{bmatrix}\begin{bmatrix} Am \\ Cm \\ Bm \end{bmatrix} + \omega^2\begin{bmatrix} I1 & 0 & I2 \\ 0 & -I1 & 0 \\ I2 & 0 & I3 \end{bmatrix}\begin{bmatrix} Am \\ Cm \\ Bm \end{bmatrix} + \begin{bmatrix} Pxm \\ -Pzm \\ 0 \end{bmatrix} = 0, \quad (11)$$

where

$$c_{11} = -\alpha_m{}^2 A_{11}, c_{22} = \alpha_m{}^2 A_{55}, c_{33} = -\alpha_m{}^2 D_{11} - A_{55}$$
$$c_{31} = c_{13} = -\alpha_m{}^2 B_{11}, c_{23} = c_{32} = \alpha_m{}^2 A_{55}, c_{12} = c_{21} = 0. \quad (12)$$

And the stiffen matrixes given are as follows:

$$A_{ij} = \sum_{k=1}^{Nk} \left[Q_{ij}{}^{-k}\right]_k (Z_{K+1} - Z_k)$$
$$B_{ij} = \frac{1}{2}\sum_{k=1}^{Nk} \left[Q_{ij}{}^{-k}\right]_k \left(Z_{K+1}{}^2 - Z_k{}^2\right) \quad (13)$$
$$C_{ij} = \frac{1}{3}\sum_{k=1}^{Nk} \left[Q_{ij}{}^{-k}\right]_k \left(Z_{K+1}{}^3 - Z_k{}^3\right).$$

A_{ij}, B_{ij}, and D_{ij} are the extensional, bending-stretching coupling, and bending stiffnesses, respectively. The transformed reduced stiffness constants $\bar{Q}_{11}^{k}$ are as follows:

$$\begin{aligned}
\bar{Q}_{11}^{k} &= Q_{11}^{k}m^4 + 2\left(Q_{12}^{k} + 2Q_{66}^{k}\right)m^2n^2 + Q_{22}^{k}n^2\\
\bar{Q}_{12}^{k} &= \left(Q_{11}^{k} + Q_{22}^{k} - 4Q_{66}^{k}\right)m^2n^2 + Q_{12}^{k}\left(m^2 + n^2\right)\\
\bar{Q}_{22}^{k} &= Q_{11}^{k}n^4 + 2\left(Q_{12}^{k} + 2Q_{66}^{k}\right)m^2n^2 + Q_{22}^{k}m^4\\
\bar{Q}_{16}^{k} &= \left(Q_{11}^{k} - Q_{12}^{k} - 2Q_{66}^{k}\right)m^3n + \left(Q_{12}^{k} - Q_{22}^{k} + 2Q_{66}^{k}\right)mn^3\\
\bar{Q}_{26}^{k} &= \left(Q_{11}^{k} - Q_{12}^{k} - 2Q_{66}^{k}\right)n^3m + \left(Q_{12}^{k} - Q_{22}^{k} + 2Q_{66}^{k}\right)nm^3\\
\bar{Q}_{66}^{k} &= \left(Q_{11}^{k} + Q_{22}^{k} - 2Q_{12}^{k} - 2Q_{66}^{k}\right)m^2n^2 + Q_{66}^{k}\left(m^4 + n^4\right)\\
\bar{Q}_{44}^{k} &= Q_{44}^{k}m^2 + Q_{55}^{k}n^2\\
\bar{Q}_{45}^{k} &= \left(Q_{55}^{k} - Q_{44}^{k}\right)mn, \bar{Q}_{55}^{k} = Q_{55}^{k}m^2 + Q_{44}^{k}n^2,
\end{aligned} \tag{14}$$

where

$$m = cos\alpha_{\text{fiber}}^{k}, n = sin\alpha_{\text{fiber}}^{k}. \tag{15}$$

The lamina elastic coefficients $Q_{ij}{}^{k}$ in the above equation can be obtained from the material properties of the k^{th} orthotropic lamina layer.

$$\left.\begin{aligned}
&Q_{11}^{k} = \frac{E1^k}{1-\mu_{12}^{k}\mu_{21}^{k}}, Q_{12}^{k} = \frac{\mu_{12}^{k}E2^k}{1-\mu_{12}^{k}\mu_{21}^{k}}, Q_{22}^{k} = \frac{E2^k}{1-\mu_{12}^{k}\mu_{21}^{k}}\\
&Q_{44}^{k} = G_{23}^{k}, Q_{55}^{k} = G_{13}^{k}, Q_{66}^{k} = G_{12}^{k}.
\end{aligned}\right\} \tag{16}$$

2.3. Cracked Hybrid Composite Beam Modelling. At the crack section, the cracked beam is separated into two parts and a continuous condition at the connecting face is modelled by the inverse of the flexibility coefficient. The additional strain energy due to the existence of the crack can be expressed as follows:

$$U_C = \int J(\xi)dA. \tag{17}$$

The flexibility coefficients which are functions of stress intensity factor and the crack shape can be expressed as follows:

$$c_{ij} = \frac{\partial w i}{\partial pi} = \frac{\partial^2 Uc}{\partial pipj} = \frac{\partial^2}{\partial pipj}\int J(\xi)dA. \tag{18}$$

For the existing stress intensity factor K_{jn}, the local flexibility matrix C_{jn} is expressed as follows:

$$c_{ij} = \frac{\partial^2}{\partial pipj}\int_{-h/2}^{h/2}\int_0^a \left[D1(KI1 + KI4 + KI5)^2 + D2KII3^2 + D12(KI1 + KI4 + KI5)KII3 + D3(KIII2 + KIII6)^2\right]d\xi\, dz. \tag{19}$$

For the composite under this study, there exist three independent variables which are the axial displacement, the vertical displacement, and the rotational displacement of the cross-section. Also, the corresponding flexibility matrix is expressed as follows:

$$c_{ij} = \frac{\partial^2}{\partial pipj}\int_{-h/2}^{h/2}\int_0^a \left[D1(KI1 + KI4)^2 + D3(KIII2)^2\right]d\xi\, dz. \tag{20}$$

If the orientation angle of the fiber in every layer is different, the coefficients D will be changed, so the flexibility matrix corresponding to the entire height of the cracked laminated beam can be written as follows:

$$c_{11} = \frac{2\pi Y\mathrm{I}(\delta)^2}{b^2h^2}\sum_{k=1}^{n} D1k\int_{h1}^{hu}\int_0^a \xi F1^2\left(\frac{\xi}{b}\right)d\xi\, dz$$
$$c_{22} = \frac{2\pi Y\mathrm{III}(\delta)^2}{b^2h^2}\sum_{k=1}^{n} D3k\int_{h1}^{hu}\int_0^a \xi F\mathrm{III}^2\left(\frac{\xi}{b}\right)d\xi\, dz \quad (21)$$
$$c_{44} = \frac{288\pi Y\mathrm{I}(\delta)^2}{b^2h^6}\sum_{k=1}^{n} D1k\int_{h1}^{hu}\int_0^a \xi F2^2\left(\frac{\xi}{b}\right)d\xi\, dz.$$

The connecting spring can be expressed as follows:

$$k_{uc} = c_{11}^{-}, k_{wc} = c_{22}^{-}, k_{\phi c} = c_{44}^{-}. \quad (22)$$

The spring force stored in the spring is expressed as follows:

$$Kt = k_{uc} * u + k_{wc} * w + k_{\phi c} * \phi, \quad (23)$$

$$\begin{bmatrix} Kuc & 0 & 0 \\ 0 & Kwc & 0 \\ 0 & 0 & K\phi c \end{bmatrix}\begin{bmatrix} Am \\ Cm \\ Bm \end{bmatrix}. \quad (24)$$

Subtract equations (24) from (11) in the flexibility matrix part of the characteristic's equation for a hybrid composite beam without crack, then the characteristics equation for a hybrid composite beam with crack is as follows:

$$\begin{bmatrix} c11 & c12 & c13 \\ c21 & c22 & c23 \\ c31 & c32 & c33 \end{bmatrix}\begin{bmatrix} Am \\ Cm \\ Bm \end{bmatrix} - \begin{bmatrix} Kuc & 0 & 0 \\ 0 & Kwc & 0 \\ 0 & 0 & K\phi c \end{bmatrix}\begin{bmatrix} Am \\ Cm \\ Bm \end{bmatrix} + \omega^2\begin{bmatrix} I1 & 0 & I2 \\ 0 & -I1 & 0 \\ I2 & 0 & I3 \end{bmatrix}\begin{bmatrix} Am \\ Cm \\ Bm \end{bmatrix} + \begin{bmatrix} Pxm \\ -Pzm \\ 0 \end{bmatrix} = 0. \quad (25)$$

3. Experimentation

3.1. Materials. The application of lightweight and low cost, natural fibers offer the possibility to replace a large section of the glass and other synthetic fibers in various automotive and construction applications. Natural fibers such as hemp, kenaf, bamboo, jute, flax, and sisal are providing reinforcement due to reductions in weight and cost, less CO_2 consumption for manufacturing, recyclability, and the added benefit that these fiber sources are "green" or eco-friendly. Among the all-natural fibers, bamboo fiber has good mechanical properties. This study utilizes bamboo fiber as a natural reinforcement for manufacturing hybrid composite plate and glass fibers as synthetic reinforcement and glass and bamboo fibers as hybrid reinforcement for manufacturing composite laminates for analysis purposes. Bamboo was purchased from Gish Abay, the place where the source of the Blue Nile river. And glass fiber was purchased from the Dejen Aviation industry. Epoxy was used as the matrix for manufacturing composite laminates for both bamboo and glass fibers. Epoxy 501 and its hardener were purchased from the Company Kadisco color factory, Addis Ababa.

3.2. Extraction of Bamboo Fiber. To extract fibers from bamboo culm firstly, the diaphragm and node have been removed, and then the hollow portions have been used for processing. Nodes of raw bamboo (3.5 four years old) were first removed and the remaining parts were cleaved in the longitudinal direction to thin slabs with 20–30 cm in length and 2-3 mm in thickness by the slicer. Then, the strips were bundled and kept in water for 3 days in order to soften them. Then, the strips were further sliced by a slicer to get desired thin strips and immersed in NaOH solution at 27 C for 8 hours [20]. Three NaOH concentrations, namely 1%, 2%, and 3% were used initially to identify the best extraction process and mechanical properties of the resulting bamboo fibers. Based on preliminary results, the 1% concentration of NaOH was selected in this study due to ease of extraction and the higher mechanical properties of fiber as compared with the 2% and 3% of NaOH. Following fiber extraction, the bamboo fibers were subjected to gently apply repeated impact loads using a rubber hammer in order to loosen and separate the fibers.

Finally, they were washed with fresh water or distilled water to neutralize, and also to separate the long and the short fiber, the fiber was dried in the sun for 24 hours at 27°C before the treatment of bamboo fiber [21].

3.3. Chemical Treatment of Bamboo Fiber. Alkaline treatment is the extreme widely used chemical treatment for natural fiber applications. The main goal of this treatment is to recover the surface roughness. Bamboo in the natural form contains wax, lignin, and oils on its outer surface, which reduces the bonding between resin and fiber [22]. This treatment removes some amount of wax, lignin, and oils [23]. To use the bamboo fiber in making composite, they should be dry and chemically treated. After drying bamboo fiber, they were dipped into the 1% NaOH solution for 8 hours [24, 25], and the fiber was washed with freshwater or distilled water to neutralize the fiber, and the fibers were dried in the sun for 8 days at room temperature before it made the fiber mat.

3.4. Fabrication of Hybrid Composite Laminates. The form of reinforcement and some preliminary conditions for this bamboo and glass fiber-reinforced epoxy hybrid composite material fabrication process has been presented here. The sequence of lamina hybrid composite structure is shown in Figure 4. Plies in composite materials are substructures of the hybrid composite plates and most of the time they are considered in prepreg form. So, hybrid composite plates are formed by stacking different plies with different angles of orientations and stacking plies with fiber arrangements by forming different laminated layers [26]. The different lamina sequences of the sisal and hemp fiber mats which are used for the manufacturing of the hybrid laminates and symbols for the designated plying sequences were presented, that is, [hemp-hemp-hemp-hemp], [sisal-sisal-sisal-sisal], [hemp-sisal-sisal-hemp], [sisal-hemp-hemp-sisal], [hemp-hemp-sisal-sisal], and [hemp-sisal-hemp-sisal]. From the investigation, the highest tensile strength recorded from [hemp-sisal-hemp-sisal] composite was 31.762 MPa and the least was recorded from the green epoxy matrix was 25.66 MPa [27].

The effect of stacking sequence on tensile, flexural, and interlinear shear properties of untreated woven glass and jute fabric-reinforced epoxy hybrid composite has been evaluated experimentally. The experimental values showed that the characteristics of E-glass/jute epoxy and its hybrid laminates can be considerably 30 enhanced by the incorporation of glass fiber as extreme glass piles. The stacking sequence of the composite was [glass-jute-glass-jute], [jute-jute-jute-jute], [glass-jute-jute-glass], and [jute-glass-glass-jute]. The tensile strength of [glass-jute-glass-jute] was the highest [28]. The composite laminate, [0 90 0] s [19, 26, 28–30], bamboo/glass epoxy (BGFRE) hybrid, was manufactured by a hand layup-assisted vacuum bagging method. The hybrid composite laminate has 6 layers, and the fiber arrangement of the composite laminate was [bamboo-glass-bamboo-glass-bamboo-glass] fibers, as shown in Figure 4. To manufacture the hybrid composite materials, the mold surface was cleaned using the cleaning agent, acetone. A release agent, wax, was applied on the mold surface for easy removal of hybrid composite laminates. The plies (unidirectional mat) were cut to the required shape and size and were placed on the mold surface. Table 1 shows the details of the proportions of bamboo fiber, glass, and epoxy resin used for fabrication. The peel ply was placed over the fibers and followed by the distribution medium. The entire layup was then bagged and placed under a vacuum. The resin used in this fabrication process was an epoxy 501 mixed with a hardener (2 : 1 ratio). The impregnation process was taken place under the vacuum for 8 hrs. Then, the bag was removed, and the laminate was exposed to an atmosphere venting in the hood to remove the styrene gas produced from the curing of the hybrid composite.

Finally, the hybrid laminates were cut to the required shape and dimensions, $200 \times 30 \times 3\,\text{mm}^3$, by a composite circular cutting machine for vibrational tests according to ASTM E756 [31]. After cutting into specimen an edge crack has been created at the length of 75 mm from the fixed end as shown in Figure 5 and different crack depths, 3 mm, 7 mm, and 11 mm using a hacksaw as shown in Figure 6. For each test configuration, five specimens were prepared for statistical significance.

3.5. Experimental Setup for Vibration Test. Vibration tests of hybrid composite made from bamboo/glass fibers were performed by considering the plates as a cantilever beam, as shown in Figure 7. A vibration test was performed using the ACC-103 accelerometer with DAQ USB NI-6009 were connected to the computer by integrating the LABVIEW 2018 software. The DAQ was used to convert the analog data to the digital data to determine the amplitude versus frequency responses. The DAQ NI-6009 is the central unit to control various input/output signals, collects data and analysis by the means of lab view block diagram, as shown in Figure 8. The accelerometer was attached at the free end of the cantilever hybrid composite laminate (dimensions: $200 \times 30 \times 3\,\text{mm}^3$) with wax (Figure 7) and connected to DAQ USB NI-6009 data acquisition.

In this work, a data sampling rate of 1000 readings per second was chosen to effectively capture the free vibration response of the cantilever beam subjected to the initial exiting load. To convert the out-voltage signal, shown in Figure 9(a), to a natural frequency domain, the fast Fourier transform (FFT) method was used. FFT was performed in MATLAB which gives the required results as shown in Figure 9(b). A representative graphical output of hybrid bamboo/glass epoxy plate containing 30% bamboo fiber and is 10% glass fiber is shown in Figure 9.

4. Finite Element Modelling

The vibration response of uncracked and cracked hybrid composite cantilever beams was investigated numerically using commercially available FEM software ABAQUS 2017, as shown in Figures 10 and 11. Properties required for modelling of the modal analysis of the hybrid composite are shown in Table 2. Geometric data for the hybrid composite were adopted from the experimental data, $200\,\text{mm} \times 30\,\text{mm} \times 3\,\text{mm}$, as per ASTM E756. The mechanical properties of the hybrid composite of BGHCs (bamboo/glass fiber epoxy hybrid composite) were calculated by classical laminated theory, as shown in Table 2. The fixed boundary condition and the exciting load were applied at the fixed-end and free-end, respectively. Figures 10 and 11 show the representative mode shapes results of uncracked and cracked specimens, respectively. In this study, the results of BGHC at a different weight ratio of bamboo and glass fiber are discussed and interpreted from different perspectives in the following section.

5. Results and Discussion

The experimental and numerical results of natural frequencies of free vibration for 6 layers of unidirectional bamboo/glass fiber-reinforced epoxy hybrid composite beam without and with crack are presented for various parameters.

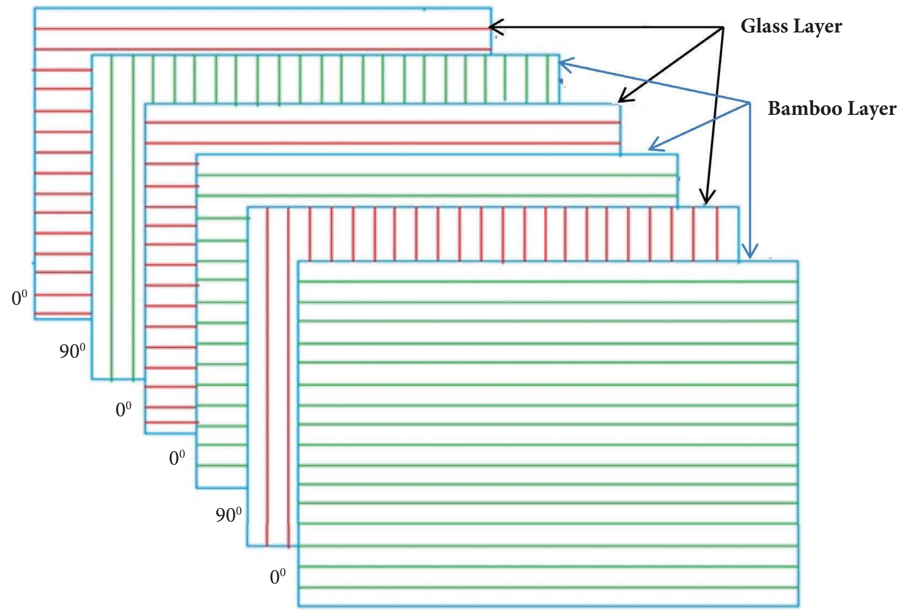

Figure 4: The sequence of lamina hybrid composite structure with [00 900 00 00 900 00].

5.1. Free Vibration Analysis of Hybrid Composite Beam without Crack. The variation of the first-third natural frequency of 6 layered cantilever hybrid composite beam has been obtained by both numerically and experimentally without crack.

Figure 12 shows both the experimental and FEM results of uncrack hybrid composite specimens. Results showed that the first three natural frequencies of the hybrid composite with a ratio of 60% of epoxy with 20% bamboo and 20% glass fibers 39.999 Hz, 250 Hz, and 698.080 Hz, respectively. The hybrid composite with the weight ratio of 60% of epoxy with 10% glass and 30% bamboo fiber, exhibited a reduction of the first three natural frequencies, 38.455 Hz, 240.530 Hz, and 671.870 Hz, respectively, as compared with the 20% bamboo hybrid specimen. The highest value of the first three natural frequencies was obtained by the hybrid specimen with a weight ratio of 60% of epoxy with 10% bamboo and 30% glass, that is, 41.817 Hz, 261.340 Hz, and 729.020 Hz, respectively, as compared with the remaining uncracked hybrid material configurations. Similar trends had been shown in the numerical results for three samples of hybrid composites, however, in all cases, the numerical results exhibited slightly reduced values, as compared with the experimental results. This could be explained by either by the uncertainty of materials homogeneity during composite. As can be seen in Figure 12, the variation in all cases could be considered negligible, for example, from mode-3 of the hybrid specimen with a weight ratio of 60% of epoxy with 10% bamboo and 30% glass fiber, the variation was 0.95%. Generally, the first-third natural frequency of the hybrid composite beam decreases within the increase of bamboo fiber and decreases in glass fiber.

5.2. Free Vibration Analysis of Hybrid Composite Beam with an Edge Crack. Figure 13 represents the graph of natural frequency for the first mode according to crack depth of 3 mm, 7 mm, and 11 mm, the natural frequencies of composite for the first mode of natural frequency obtained from experimental and numerical methods are plotted. The decrease in natural frequency and increase in depth of the crack for the first mode is observed. In general, the vibrational behaviors of composite materials were affected by the crack depth.

The graph Figure 13 shows that the first natural frequency of BGHC materials at the different crack depths, 3, 7, and 11 mm for crack location Lc/L = 0.375 were considered. It was observed that from the crack depth of 3, 7, and 11 mm, at the relative crack location of Lc/L = 0.375, the fundamental natural frequencies of BGHC-1 reduced by 8.874%, 15.473%, and 20.332%, respectively, as compared with the uncracked BGHC materials. For the BGHC- 2, the fundamental natural frequency reduced by 3.792%, 11.532%, and 14.524%, as compared with the uncrack BGHC materials. Similarly, for the BGHC-3, the fundamental natural frequency reduced by 8.692%, 19.888%, and 26.341%, as compared with the uncrack BGHC materials. An observed in BGHC-1, BGHC-2, and BGHC-3, generally, the fundamental frequency decreased as the crack depth increased in all the three methods for a relative crack location Lc/L = 0.375.

Figure 14 represents the graph of fundamental natural frequency for the second mode according to crack depth of 3 mm, 7 mm, and 11 mm, the natural frequencies of composite, 60% of epoxy with 20% bamboo and 20% glass fiber, composite 60% of epoxy with 10% glass and 30% bamboo

Table 1: The percentage weight of fibers and matrix.

Plate no	Epoxy in grams	Percentage of epoxy (%)	Bamboo fiber in grams	Percentage of bamboo fiber (%)	Glass fiber in grams	Percentage of glass fiber (%)	Total weight in grams
1	217.158	60	72.386	20	72.386	20	361.930
2	204.416	60	102.209	30	34.070	10	340.694
3	231.424	60	38.572	10	115.715	30	385.714

Figure 5: Sample composite plates for testing.

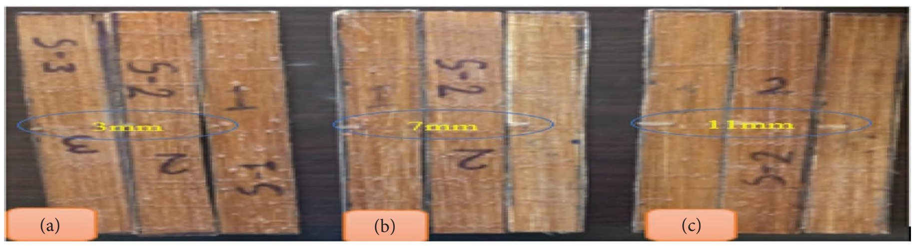

Figure 6: Composite plate specimen with crack depth: (a) 3 mm, (b) 7 mm, and (c) 11 mm in each sample.

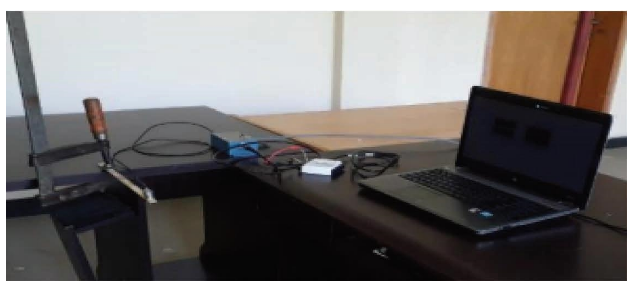

Figure 7: Vibration test setup with accelerometer attachment.

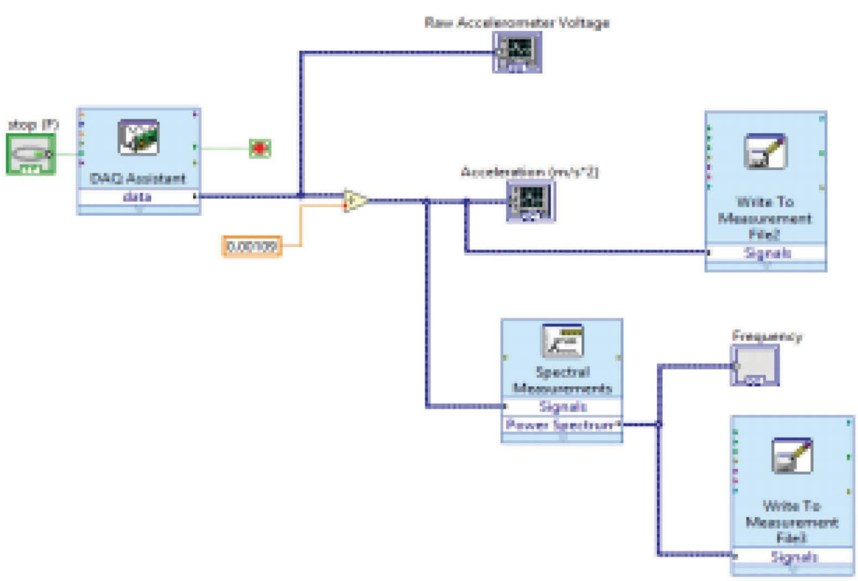

Figure 8: Block diagram used for acceleration velocity and displacement data.

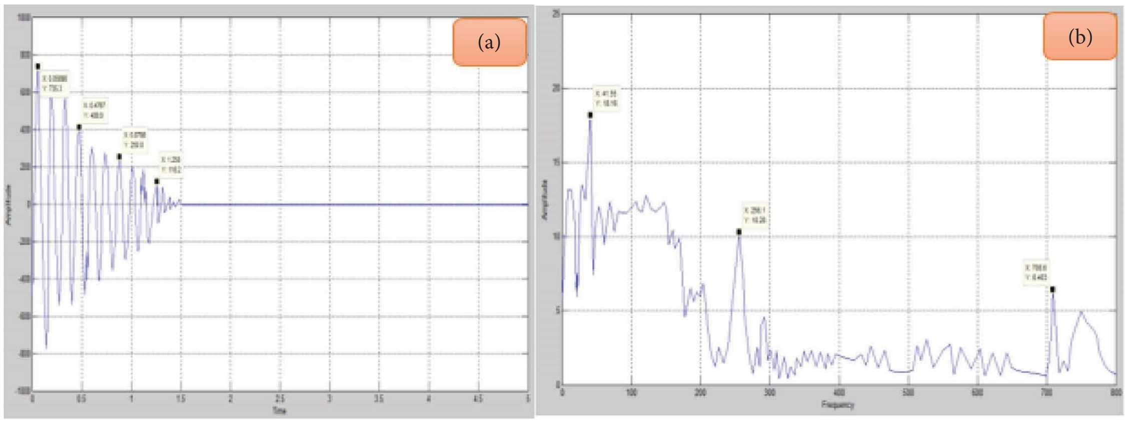

Figure 9: Sample 1 (lab view software and MATLAB (FFT result for 30% bamboo fiber and 10% glass fiber)).

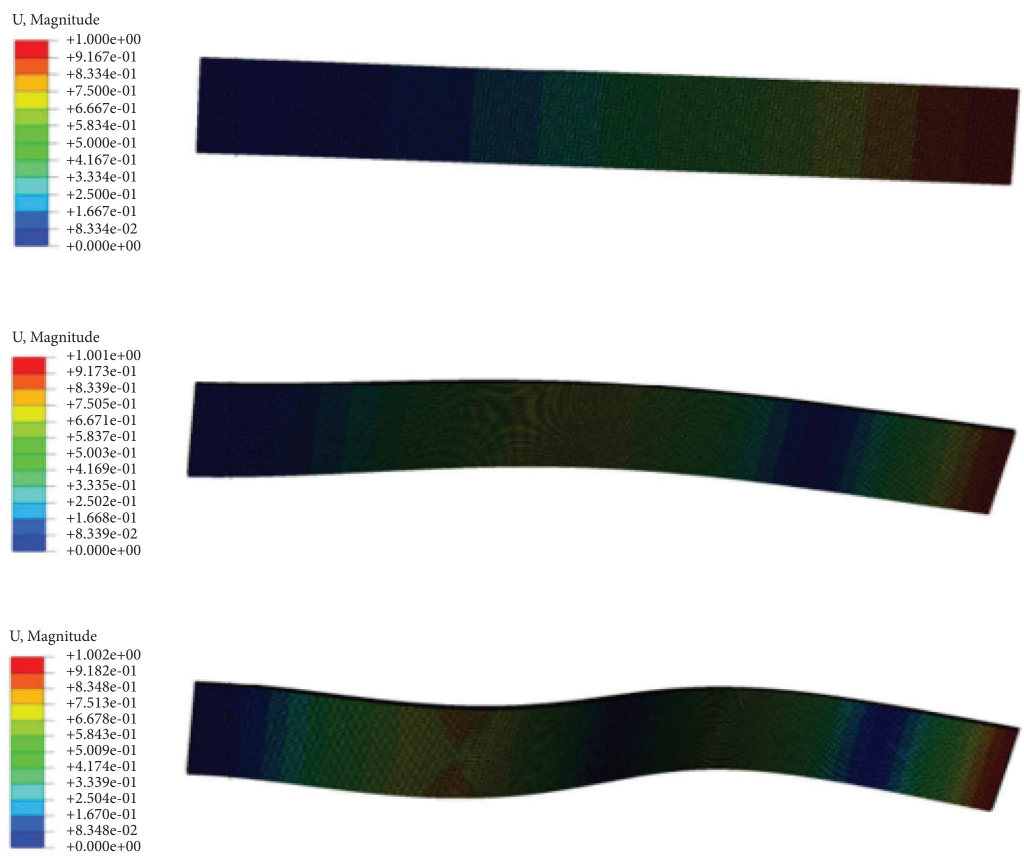

Figure 10: Natural frequency without a crack composite plate of 60% epoxy with 20% bamboo and 20% glass fiber.

fiber and composite 60% of epoxy with 10% bamboo and 30% glass fiber for the second mode of natural frequency obtained from experimental and numerical methods are plotted. It is noticed that the natural frequencies of composite 60% of epoxy with 10% bamboo and 30% glass fiber are varying slightly with an increase in crack depth. From a composite material, 60% of epoxy with 20% bamboo and 20% glass fiber, and the composite material 60% of epoxy with 10% glass and 30% bamboo fibers, there is a gradual decrease in the values of natural frequencies with respect to increase in crack depth.

In addition, Figure 14 shows that the second natural frequency of the BGHC materials, considered as the cantilever beam with different crack depth for BGHC-1, BGHC2, and BGHC-3. The second natural frequency is higher than the fundamental frequency of the cracked BGHC materials. It is observed that the second natural frequency of BGHC-1 for a relative crack location of Lc/L = 0.375 reduced by 6.710%, 12.539%, and 19.177%, respectively, as compared with the corresponding uncracked BGHC materials for the BGHC-2, the second natural frequency reduced by 5.820%, 13.205%, and 20.238% for crack

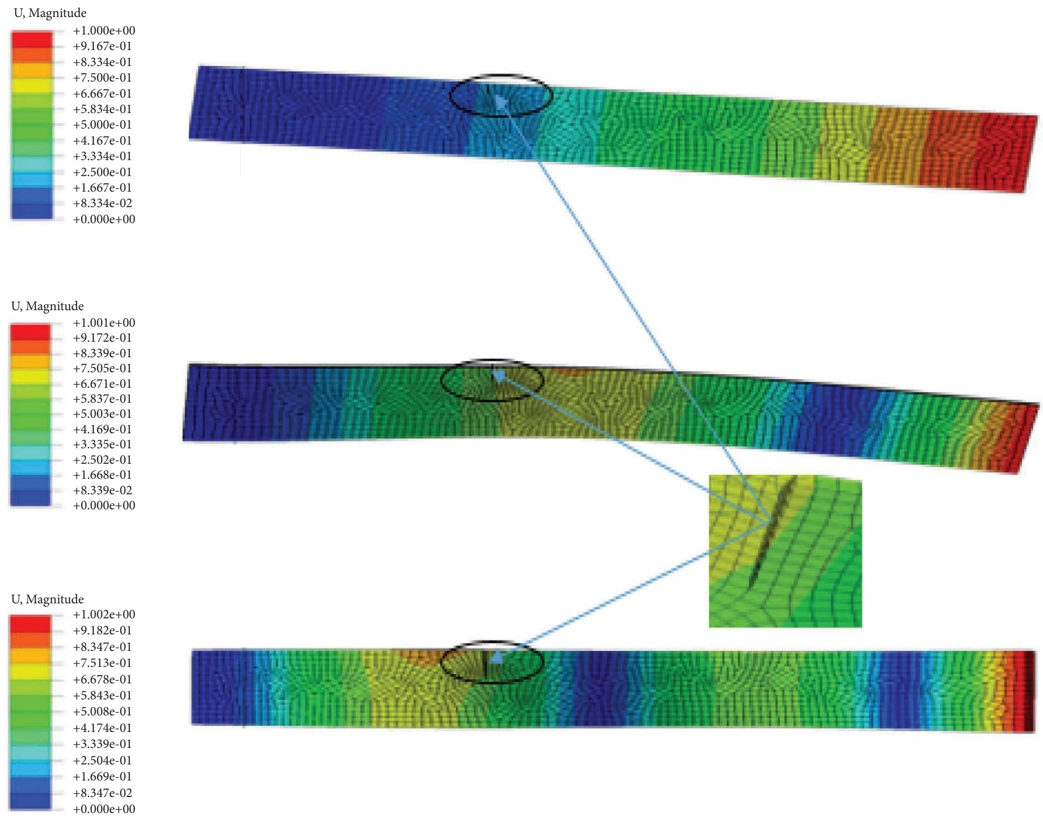

Figure 11: Natural frequency with crack depth composite plate of 60% epoxy with 20% bamboo and 20% glass fiber.

Table 2: Mechanical properties of hybrid composite plate.

Mechanical properties of BGHC plate	60% of epoxy with 20% bamboo and 20% glass fiber (BGHC-1)	60% of epoxy with 30% bamboo and 10% glass fiber (BGHC-2)	60% of epoxy with 10% bamboo and 30% glass fiber (BGHC-3)
ρ (kg/m^2)	1340	1263	1429
E_{11} (Gpa)	14.829	12.811	17.180
E_{22} (Gpa)	7.111	7.321	6.899
E_{33} (Gpa)	7.111	7.321	6.899
V_{12}	0.32	0.345	0.334
V_{21}	0.163	0.197	0.134
V_{23}	0.163	0.197	0.134
G_{12} (Gpa)	2.411	2.497	2.324
G_{13} (Gpa)	2.411	2.497	2.324
G_{23} (Gpa)	3.057	3.058	3.042

depths of 3, 7, and 11 mm, respectively, as compared with the corresponding uncracked BGHC materials. Similarly, for the BGHC-3, the second natural frequency reduced by 6.147%, 13.384%, and 20.986% for crack depths of 3, 7, and 11 mm, respectively, as compared with the corresponding uncracked beam.

Figure 15 represents the graph of natural frequency for third mode according to crack depth of 3 mm, 7 mm, and 11 mm, the natural frequencies of composite material, 60% of epoxy with 20% bamboo and 20% glass fiber, composite material, 60% of epoxy with 10% glass and 30% bamboo fibers and composite material, 60% of epoxy with 10% bamboo and 30% glass fibers of the third mode obtained from numerical experimental methods were plotted. The decrease in natural frequency and increase in depth of the crack for the first mode is observed. Generally, during the change in vibration properties of the composite, the resonance will occur and that causes catastrophic failure of the composite.

On the other hand, the introduction of cracks in BGHC-1, BGHC-2, and BGHC-3 materials showed in Figure 15 a slightly reduced effect on the third natural frequency, but the change in magnitude was less significant in the unidirectional composite beam. For the BGHC-1 materials, the third natural frequency reduced by 3.384%, 12.254%, and 18.951% for crack depths of 3, 7, and 11 mm, respectively.

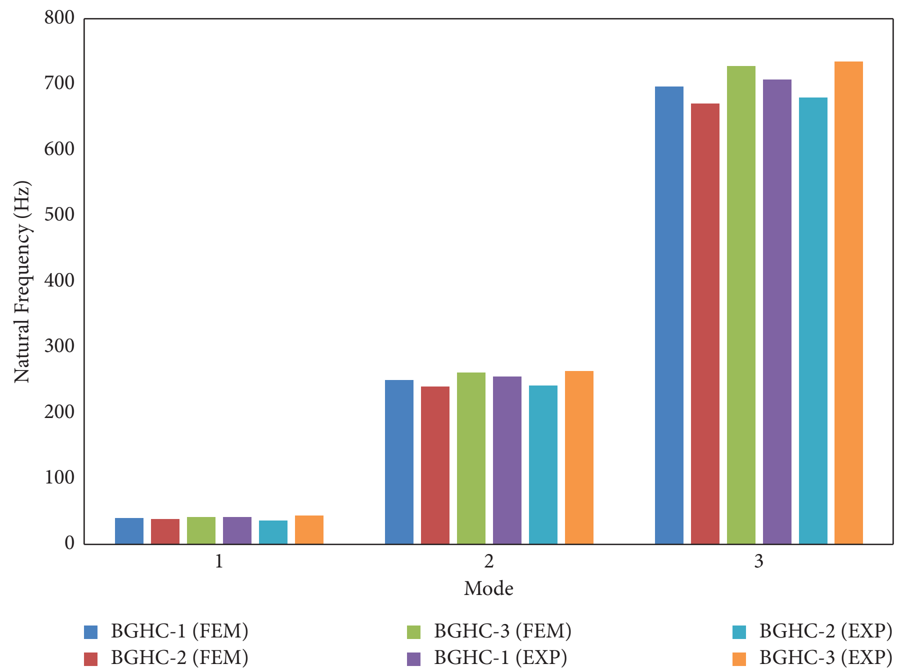

Figure 12: Natural frequency of hybrid composite plate by experimental and numerical methods.

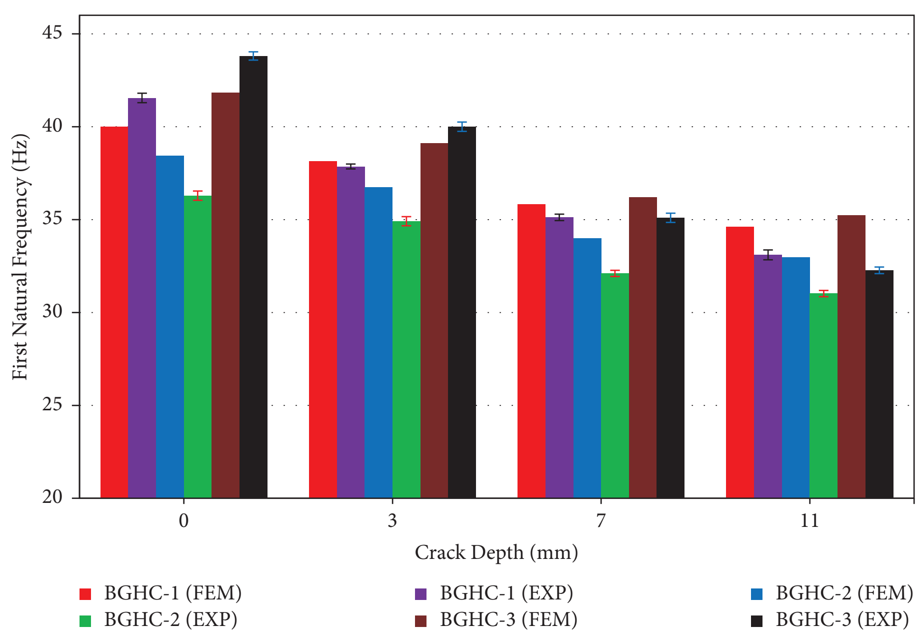

Figure 13: Natural frequency for first mode according to crack depth.

And for the BGHC-2, the third natural frequency reduced by 6.040%, 12.784%, and 21.784% for the given crack depths of 3, 7, and 11 mm, respectively. Similarly, for the BGHC-3, the third natural frequency reduced by 5.511%, 12.856%, and 20.854% for the given crack depths of 3, 7, and 11 mm, respectively. From Figures 13–15, the natural frequencies obtained experimentally at the first, second, and third mode of vibration of all the three samples of composites, BGHC-1, BGHC-2, and BGHC-3 have been compared with the theoretical and numerical values. The result shows that there is a better agreement between the experimental, analytical, and numerical results of natural frequency. The discrepancy may be due to a manufacturing defect of hybrid composite materials and loading effects during experimentation. Generally, good agreement was observed between experimental, analytical, and numerical natural frequencies.

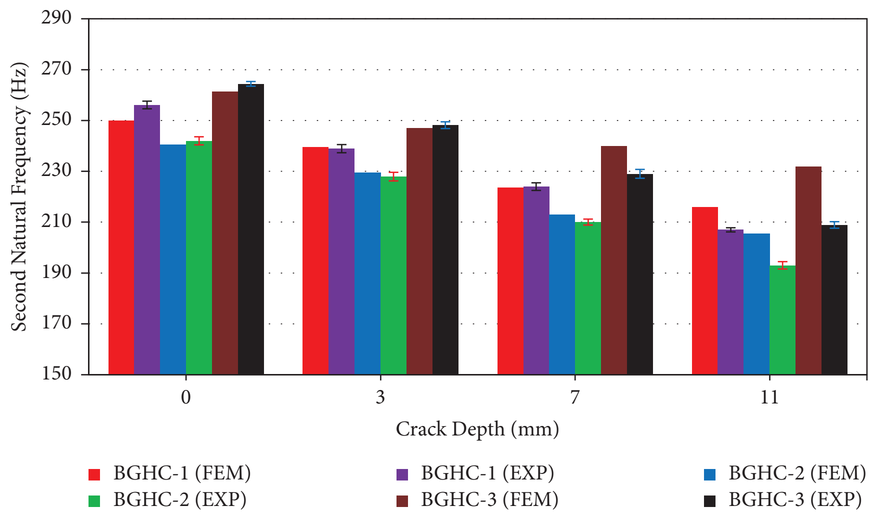

Figure 14: Natural frequency for second mode according to crack depth.

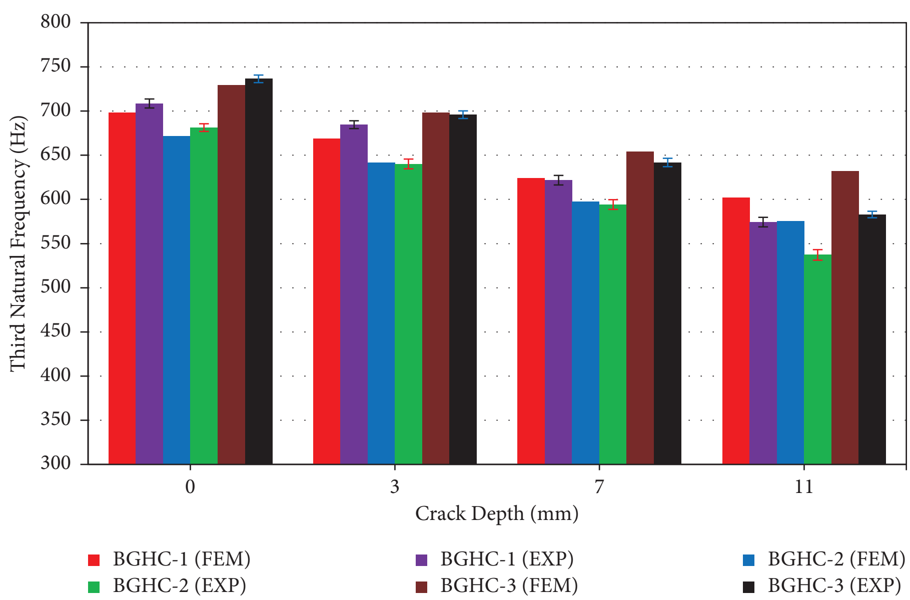

Figure 15: Natural frequency for the third mode according to crack depth.

6. Conclusions

In this research, the successful fabrications of a hybrid composite of epoxy-based composites reinforced with bamboo and glass fibers have been done. And also, first-order shear deformation theory is used to study the fundamental natural frequencies of asymmetrically hybrid composite beam with edge cracks for cantilever beam boundary conditions. Both numerical (FEM) using ABAQUS software and experimental investigations are done for the vibration analysis of unidirectional bamboo/glass fiber epoxy hybrid composite beam with edge cracks using fracture mechanics theory. From the above results, it is concluded that implications edge crack with different crack depth and different fiber weight ratio parameters have a major effect on vibration analysis.

The following decisions are drawn from the present investigation as follows:

(i) It is observed that the results gained from the present finite element analysis are in good promise with the experimental values.

(ii) The cracked hybrid composite beam with 60% of epoxy with 10% glass and 30% bamboo fiber shows a relative reduction in frequencies.

(iii) As the natural fiber weight ratio increases, the changes in frequencies reduce for all modes. In the reverse, the material performance was increased.

(iv) Variation of the first fundamental frequency with different fiber weight ratio and crack depth for a cantilever condition with constant crack location (ccl) = 75 mm are investigated.

(v) The crack depth 3, 7, and 11 mm has an important effect on the natural frequencies of the crack hybrid composite beam for all modes.

(vi) There is a decrease of frequencies happens with the increase in crack depth on the hybrid composite plate.

(vii) By using natural fiber-based hybrid composite structures, it is likely to create a hybrid composite laminate with superior vibrational performance without disadvantages in stiffness-to-weight ratios.

(viii) The use of bamboo fiber-reinforced composites has better benefits to the environment because the recycling process is short as compared to other natural fibers. It is also biodegradable.

From the above investigations, it is clear that the natural frequencies of the hybrid composite are significantly affected due to the fiber weight ratio and crack depth of the hybrid composite materials. So, crack lengths play a vital role in the dynamic or vibration characteristics of the hybrid composite materials. The outcomes of this research can be used to identify cracks for mechanical health monitoring by linking the variation in natural frequencies of the hybrid composite beams.

7. Future Works

(i) Experimental investigation of natural frequencies of bamboo/glass fiber-reinforced hybrid epoxy matrix composite has some limitations related to fabrication methods such as injection molding, and other techniques. The external force such as the wind force is not controlled during the experimental testing. From current work, the following future scopes are outlined.

(ii) Effect of crack length on the natural frequencies.

(iii) Studying the forced vibration of the hybrid composites.

(iv) It is worth noting that further research is required to optimize the composition and manufacturing process of the bamboo/glass fiber-reinforced hybrid composite, as well as to assess its long-term durability and performance under different environmental conditions. Addressing these aspects will unlock the full potential of bamboo fiber-reinforced composites and contribute to advancements in vibration properties.

(v) Experimental and numerical analysis on unidirectional bamboo/glass fiber epoxy hybrid composite (BGHC) beam with multiedge cracks can be done to identify the impact of natural frequencies under free vibration.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Solomon Alemneh Adimass contributed to writing the original draft, data curation, methodology, experiment investigation, writing review, and editing. Ermias Gebrekidan Koricho was involved in conceptualization, editing, advising, and visualization. Velmurugan Paramasivam provided conceptualization and editing. K. Vignesh[d] was involved in conceptualization. All authors have read and approved the final version of the manuscript for publication.

Acknowledgments

The authors would like to express their gratitude for the support received from Bahir Dar University and Bahir Dar Institute of Technology, Ethiopia, in setting up the vibration experiments. It is important to note that this research work was self-financed.

References

[1] S. Timoshenko, *Woinowsky- Kriege, Theory of Plates and Shells*, Mcgraw- Hill book company, New York, NY, USA, 2nd edition, 1989.

[2] N. V. Rachchh and D. N. Trivedi, "Mechanical characterization and vibration analysis of hybrid E-glass/bagasse fiber polyester composites," *Materials Today: Proceedings*, vol. 5, no. 2, pp. 7692–7700, 2018.

[3] R. S. Rana, R. Purohit, and R. K. Dwivedi, "Effect of variation of low cost reinforcement on natural frequency of polyester resin glass fibre hybrid composites," *Materials Today: Proceedings*, vol. 4, no. 2, pp. 3451–3457, 2017.

[4] D. Xu, C. Cerbu, H. Wang, and I. C. Rosca, "Analysis of the hybrid composite materials reinforced with natural fibers considering digital image correlation (DIC) measurements," *Mechanics of Materials*, vol. 135, pp. 46–56, 2019.

[5] P. C. Jena, "Free vibration analysis of short bamboo fiber based polymer composite beam structure," *Materials Today: Proceedings*, vol. 5, no. 2, pp. 5870–5875, 2018.

[6] M. Kurhe Nikhil, B. Shedbale Indrajeet, D. Charapale Utkarsh, and R. Manoharan, "Modal analysis of hybrid laminated composite sandwich plate," *Materials Today: Proceedings*, vol. 5, no. 5, pp. 12453–12466, 2018.

[7] A. Draoui, M. Zidour, A. Tounsi, and B. Adim, "Static and dynamic behavior of nanotubes-reinforced sandwich plates using (FSDT)," *Journal of Nano Research*, vol. 57, pp. 117–135, 2019.

[8] K. Kim, S. Kim, K. Sok, C. Pak, and K. Han, "A modeling method for vibration analysis of cracked beam with arbitrary boundary condition," *Journal of Ocean Engineering and Science*, vol. 3, no. 4, pp. 367–381, 2018.

[9] P. Das and S. Kr. Sahu, "Experimental and numerical study on free vibration of cracked woven fiber glass/epoxy composite beam," *Materials Today: Proceedings*, vol. 33, pp. 5505–5510, 2020.

[10] A. Mohanty, P. Das, S. Choudhury, and M. Muni, "Crack effect on the natural frequency of composite beam: finite element analysis with experimental study," *Suranaree Journal of Science and Technology*, vol. 30, no. 6, Article ID 010264, pp. 1–10, 2024.

[11] O. Sahu, P. Das, M. U. N. I. Manoj Kumar, N. Pradhan, B. Basa, and S. Kumar Sahu, "Frequency-based crack effect study on beams under free vibration using finite element analysis," *Engineering Transactions*, vol. 72, no. 1, pp. 95–114, 2024.

[12] O. Sahu, P. Das, S. Choudhury, N. Pradhan, B. Basa, and B. Jena, "On transverse crack effects in beams under free vibration using finite element analysis," *Materials Today: Proceedings*, 2024.

[13] P. Das, M. K. Muni, S. Choudhury, N. Pradhan, B. Basa, and S. K. Sahu, "A concerted experimental and numerical approach for frequency based non-destructive analysis of Bi-directional cracked laminated composite beams," *Nondestructive Testing and Evaluation*, pp. 1–26, 2024.

[14] J. K. Prusty, G. Papazafeiropoulos, and S. C. Mohanty, "Free vibration analysis of sandwich plates with cut-outs: an experimental and numerical study with artificial neural network modelling," *Composite Structures*, vol. 321, no. 1, Article ID 117328, 2023.

[15] L. Sinha, D. Das, A. N. Nayak, and S. K. Sahu, "Experimental and numerical study on free vibration characteristics of laminated composite plate with/without cut-out," *Composite Structures*, vol. 256, no. 15, Article ID 113051, 2021.

[16] Y. Kelly, H. B. Khaniki, and M. H. Ghayesh, "Ching-Tai Ng, Free vibrations of cracked functionally graded graphene platelets reinforced Timoshenko beams based on Hu-Washizu-Barr variational method," *Engineering Structures*, vol. 293, no. 15, Article ID 116587, 2023.

[17] C. T. Sun and Z. H. Jin, *Fracture Mechanics*, 2012.

[18] K. Kim, K. Choe, S. Kim, and Q. Wang, "A modeling method for vibration analysis of cracked laminated composite beam of uniform rectangular cross-section with arbitrary boundary condition," *Composite Structures*, vol. 208, no. 15, pp. 127–140, 2019.

[19] S. K. Sahu and P. Das, "Experimental and numerical studies on vibration of laminated composite beam with transverse multiple cracks," *Mechanical Systems and Signal Processing*, vol. 135, Article ID 106398, 2020.

[20] F. Gu, Y. Zheng, W. Zhang et al., "Can bamboo fibres be an alternative to flax fibres as materials for plastic reinforcement? A comparative life cycle study on polypropylene/flax/bamboo laminates," *Industrial Crops and Products*, vol. 121, pp. 372–387, 2018.

[21] C. Huang, W. Lin, C. Lai, X. Li, Y. Jin, and Q. Yong, "Coupling the post-extraction process to remove residual lignin and alter the recalcitrant structures for improving the enzymatic digestibility of acid-pretreated bamboo residues," *Bioresource Technology*, vol. 285, Article ID 121355, 2019.

[22] V. Fiore, T. Scalici, F. Nicoletti, G. Vitale, M. Prestipino, and A. Valenza, "A new eco-friendly chemical treatment of natural fibres: effect of sodium bicarbonate on properties of sisal fibre and its epoxy composites," *Composites Part B: Engineering*, vol. 85, pp. 150–160, 2016.

[23] Y. Yu, X. Huang, and W. Yu, "A novel process to improve yield and mechanical performance of bamboo fiber reinforced composite via mechanical treatments," *Composites Part B: Engineering*, vol. 56, pp. 48–53, 2014.

[24] M. M. E. Costa, S. L. S. Melo, J. V. M. Santos et al., "Influence of physical and chemical treatments on the mechanical properties of bamboo fibers," *Procedia Engineering*, vol. 200, pp. 457–464, 2017.

[25] G. Pramudi, W. W. Raharjo, D. Ariawan, Z. A. Ubaidillah, and Z. Arifin, "Utilization of bamboo fiber in the development of environmentally friendly composite – a review," *IOP Conference Series: Materials Science and Engineering*, vol. 1096, no. 1, Article ID 012038, 2021.

[26] S. Shams, A. R. Torabi, M. F. Narab, and M. A. Atashgah, "Free vibration analysis of a laminated beam using dynamic stiffness matrix method considering delamination," *Thin-Walled Structures*, vol. 166, Article ID 107952, 2021.

[27] S. M. K. Thiagamani, S. Krishnasamy, C. Muthukumar et al., "Investigation into mechanical, absorption and swelling behaviour of hemp/sisal fibre reinforced bioepoxy hybrid composites: effects of stacking sequences," *International Journal of Biological Macromolecules*, vol. 140, pp. 637–646, 2019.

[28] M. R. Sanjay and B. Yogesha, "Studies on mechanical properties of jute/E-glass fiber reinforced epoxy hybrid composites," *Journal of Minerals and Materials Characterization and Engineering*, vol. 04, no. 01, pp. 15–25, 2016.

[29] N. Nayak, S. R. Meher, and S. Kumar Sahu, "Experimental and numerical study on vibration and buckling characteristics of glass-carbon/epoxy hybrid composite plates," 2014, https://api.semanticscholar.org/CorpusID:36379597.

[30] Mulatu Achenef, "Free vibration study on edge cracked glass fiber reinforced polymer laminate," Master of Engineering Thesis, 2019, Addis Ababa Institute of Technology, Ethiopia, 2019.

[31] ASTM, *Standard Test Method for Measuring Vibration-Damping Properties of Materials*, ASTM, West Conshohocken, PA, USA, 2017.