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Research Article

Bit Rate Optimization with MMSE Detector for Multicast LP-OFDM Systems

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We propose a new resource allocation algorithm with minimum mean square error (MMSE) detector for multicast linear precoded orthogonal frequency division multiplexing (LP-OFDM) systems. To increase the total multicast bit rate, this algorithm jointly uses the LP-OFDM modulation technique and an adaptation of the OFDM-based multicast approaches to exploit the transmission link diversities of users. The LP technique applied to multicast OFDM systems with zero forcing (ZF) detector has already proved its ability to increase the unirate multicast system bit rate in a power line communication (PLC) context. The new MMSE detector and the new related bit-loading algorithm are developed to enhance the ZF detector results. To improve both the bit rate and the fairness among multicast users, the utilization of the LP component in multirate multicast systems is then investigated. Simulations are run over indoor PLC channels, and it is shown that the proposed LP-based methods outperform the OFDM-based methods in terms of total bit rate and fairness index for both unirate and multirate multicast systems. Additionally, it is shown that the proposed bit-loading algorithm with MMSE detector outperforms the ZF detector and the OFDM-based receiver in terms of total multicast bit rate and fairness among users.

1. Introduction

Multicasting is a network addressing method for the delivery of data to a group of users simultaneously. This technique offers a significant improvement compared to unicasting because it uses less network resources. Multicast routing is a well-investigated subject in the literature for both wired and wireless systems [1]. In multiuser communication systems, all users share the same downlink resources [2]. The allocation algorithms in multicast must adapt to the system parameters to satisfy all requirements of users. In this paper, the adaptation of the physical (PHY) layer parameters is addressed for multicast orthogonal frequency division multiplexing (OFDM) systems in indoor powerline communications (PLC) context. Since indoor PLC is being used to deliver triple play services, the multicast may be interesting in this case. Over the PHY layer, resources have to be allocated in order to satisfy requirements of each multicast user. However, the difference in link conditions of users makes it difficult to adapt the PHY layer (coding rate, modulation index, etc.) to the link conditions of each user. The conventional resource allocation method in multicast OFDM consists in adjusting the PHY parameters to serve the user who experiences the worst channel condition. Consequently, all users receive the same bit rate, and this final multicast bit rate is limited by the worst user channel conditions [3].

To increase the total multicast bit rate and to better fit the link conditions, the concept of heterogeneous multicast (also called multirate multicast) was brought in [4]. The conventional multicast system refers to unirate multicast system. In multirate multicast, users are grouped into subgroups, and the receivers of a multicast subgroup are offered services at different rates commensurate with their capabilities. Therefore, multirate schemes have a great advantage over unirate multicast in adapting to diverse receiver requirements and heterogeneous network conditions [5]. One way of attaining multirate multicast is by hierarchical encoding or layered streaming, which is particularly suitable for audio and video traffic. In this approach, the sender provides data in several layers organized in a hierarchy. Receivers subscribe to the layers cumulatively to provide progressive refinements [3, 6, 7]. Multicast users are separated into subgroups in frequency domain [3, 7], or in time domain [8]. In frequency domain and for OFDM systems, each subcarrier is assigned to a subgroup of users and carries the same data symbols. The number of loaded bits on each subcarrier is then determined considering the lowest one among the channel amplitudes of all the users sharing this subcarrier. It has been shown that this method significantly increases the total multicast bit rate compared to the conventional method in wireless communications, but degrade the fairness among users [7]. To enforce the fairness performance while minimizing throughput degradation, a subcarrier and bit allocation scheme for proportional fairness (PF) has been also proposed [7].

In [9–11], we proposed to exploit the transmission link diversities of users by jointly using the linear precoded OFDM (LP-OFDM) modulation technique and an adaptation of the conventional resource allocation scheme. The proposed resource allocation methods were applied only for unirate and time domain multirate multicast systems. LP-OFDM is a combination of multicarrier and spread spectrum techniques also known as MC-SS techniques in wireless applications. The proposed resource allocation algorithms are developed for an LP-OFDM system where the equalization is performed according to the zero forcing (ZF) criterion. The ZF detection technique consists in reversing the channel coefficients to fully correct the phase shift and the attenuation, and to completely cancel the interference between precoding sequences, but at the cost of increasing the noise level. Using this detector, we showed that the proposed algorithms offer a significant bit rate gain compared to the conventional resource allocation method in unirate multicast context. Furthermore, for multirate multicast systems, the proposed method is the most suitable method considering both the bit rate and the fairness index when users experience similar channels [11]. It has been shown that the minimum mean square error (MMSE) detector outperforms the ZF technique [12]. The MMSE detector corrects the phase shift and the attenuation of the channel fading taking into account the present signal-to-noise ratio (SNR) [13].

In this paper, we propose a new resource allocation scheme with MMSE detection technique for multicast LP-OFDM systems. In addition to the studies done in [10, 11], the linear precoding component is also applied in frequency domain multirate multicast systems to improve both the bit rate and the fairness performances. Numerical results show that the proposed LP-based methods outperform the OFDM-based methods both in unirate and multirate multicast systems. Additionally, it is shown that the new bit-loading algorithm with MMSE detector offers the best performances in terms of total multicast bit rate and fairness among users.

This paper is organized as follows. Section 2 describes the linear precoded multicarrier systems and the achievable bit rate for LP-OFDM systems using ZF and MMSE detectors. Section 3 describes the multicast systems. Section 4 presents the resource allocation algorithms in unirate multicast systems. The study of the multirate multicast systems is done in Section 5. The performance comparisons of all algorithms are given in Section 6 over PLC channels. Finally, Section 7 concludes the paper.

2. Linearly Precoded Multicarrier Systems

2.1. System Description. Multicarrier modulation techniques like OFDM for wireless and discrete multitone (DMT) for wireline have been selected as major modulation schemes, which are able to ensure high data rates in frequency-selective channels. The utilization of adaptive multicarrier systems allows to dynamically distribute the information over the subcarriers of the signal based on the value of the SNR of each subcarrier. However, when the signal power spectral density (PSD) is limited, as in PLC systems, there is a loss of quantification due to discrete modulation orders. This loss is minimized by transmitting the information on subsets of subcarriers instead of individual subcarriers [14]. The linear precoding technique consists in connecting a subset of subcarriers with precoding sequences to mutually exploit their capabilities [15]. If judiciously done, each resulting subset holds an equivalent SNR such that the total supported throughput is greater than the sum of the individual throughputs supported by each subcarrier taken separately. In the following, the subset of subcarriers is known as block and the subcarriers in one block are not necessarily adjacent. The number of blocks B is the ratio of the total number of subcarriers N to the length L of the precoding sequences. The length L is assumed to be the same for all blocks.

Figure 1 gives the LP-OFDM transmitter-receiver model. This figure shows the various operations involved when setting up an LP-OFDM signal. From the classical OFDM systems, the linear precoding matrix which is a Hadamard matrix is added. The resource allocator dynamically distributes the bits and the powers based on the channel conditions and the quality of service (QoS) requirements. In this figure, r_u and E_u are respectively the number of bits and the power allocated to the precoding sequence u. U is the number of precoding sequence, N_0 the background noise level, h_n represents the channel coefficient on subcarrier n. The PSD constraint in this context is written as

$$\sum_{u=1}^{U} E_u \leq E_{\mathrm{PSD}}, \tag{1}$$

where E_{PSD} is the power per symbol imposed by the PSD limit.

2.2. Mutual Information of the LP-OFDM System. The frequency characteristics of the LP-OFDM signal are the same as those of the OFDM signal. The multicarrier OFDM signal is assumed to be adapted to the channel. The guard interval, the number N of subcarriers, and the carrier spacing are selected to perfectly absorb the multipaths caused by the channel

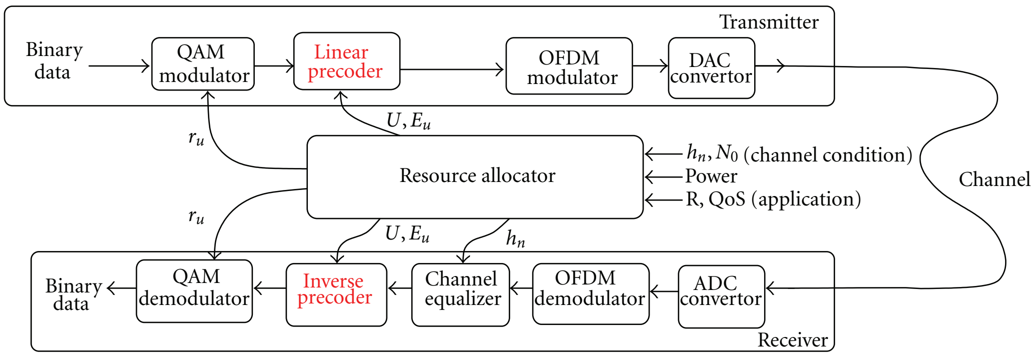

Figure 1: Transmitter-receiver model.

and to limit the loss of spectral efficiency due to the guard interval. Under these assumptions, the signal received after the OFDM demodulator can be expressed as

$$\mathbf{Y} = \mathbf{HCX} + \mathbf{Z}, \tag{2}$$

where $\mathbf{X}$ is the vector of size N of transmitted symbols before OFDM modulation and carrying the information, $\mathbf{C}$ is the precoding matrix composed of orthogonal Hadamard matrices, $\mathbf{H} = \mathrm{diag}(h_1, \ldots, h_N)$ the diagonal matrix of the channel transfer function, $\mathbf{Z}$ is the noise vector such that $\mathbb{E}[\mathbf{ZZ}^H] = N_0\mathbf{I}_N$, and $\mathbf{Y}$ is the demodulated OFDM signal. In the general case, the estimated symbols, after the equalization and the inverse linear precoding processes, are written as

$$\hat{\mathbf{X}} = \mathbf{W}^H\mathbf{Y}, \tag{3}$$

where $\mathbf{W}$ represents the matrix of the complex equalization coefficients and the operator $(\cdot)^H$ the Hermitian conjugate. In general case, $\mathbf{W}$ can be written [16] as

$$\mathbf{W} = \mathbf{CG}^H, \tag{4}$$

where $\mathbf{G}$ is the equalization matrix.

As previously stated, we consider in this paper, the ZF and MMSE detection technique for the equalization process. Using linear precoding sequences and for full-loaded system, that is, $U = L$, the equalization matrix $\mathbf{G}$ is then a diagonal matrix [12]. The equalization coefficients g_n based on ZF and MMSE criteria are then written as [13]:

$$\begin{aligned} \text{ZF:} \quad & g_n = \frac{1}{h_n}, \\ \text{MMSE:} \quad & g_n = \frac{h_n^*}{|h_n|^2 + N_0/E_{\mathrm{PSD}}}. \end{aligned} \tag{5}$$

The inverse precoding process is performed block per block without dependence between block. In one block of L subcarriers $l \in [1; L]$, the received symbol for the precoding sequence v, after the inverse linear precoding process, is written as:

$$\hat{x}_v = \underbrace{\sum_{l=1}^{L} c_{v,l} g_l h_l c_{l,v} x_v}_{A_1} + \underbrace{\sum_{l=1}^{L} \sum_{\substack{u=1 \\ u \neq v}}^{U} c_{v,l} g_l h_l c_{l,u} x_u}_{A_2} + \underbrace{\sum_{l=1}^{L} c_{v,l} g_l z_l}_{A_3}. \tag{6}$$

In this expression, there are, from left to right, the term A_1 of the useful signal, an interference term A_2 and a noise term A_3.

Under the assumption of simple linear receiver with independent sequence demodulation, the system capacity is expressed as the sum of the capacities provided by each precoding sequences. It is then sufficient to calculate the mutual information $\mathcal{I}_v$ between processes $\hat{x}_v$ and x_v. Notice that without independent sequence demodulation, the channel capacity is given by the maximum of the mutual information between the input and output of the channel, where the maximization is with respect to the input distribution, and the capacity is the capacity of the Gaussian interference channel [17]. In the case of ZF receiver, the assumption of independent sequence demodulation is not needed since the transmission is orthogonal, $A_2 = 0$. The mutual information $\mathcal{I}_v$ is then as follows:

$$\mathcal{I}_v = \log_2\left(1 + \frac{\mathbb{E}\left[A_1 A_1^H\right]}{\mathbb{E}\left[A_3 A_3^H\right]}\right). \tag{7}$$

Using (5), the development of the mathematical expectation terms in the mutual information leads to

$$\mathcal{I} = \sum_{v=1}^{U} \log_2\left(1 + \frac{L^2}{\sum_{l=1}^{L}\left(1/|h_l|^2\right)} \frac{E_u}{N_0}\right). \tag{8}$$

In the case of MMSE, $A_2 \neq 0$ and the mutual information under independent sequence demodulation is

$$\mathcal{I} = \sum_{v=1}^{U} \log_2\left(1 + \frac{\phi E_v}{\sum_{\substack{u=1 \\ u \neq v}}^{U} \varphi_{u,v} E_u + \lambda N_0}\right). \tag{9}$$

with

$$\begin{aligned} \phi &= \left| \sum_{l=1}^{L} \frac{|h_l|^2}{|h_l|^2 + N_0/E_{\mathrm{PSD}}} \right|^2, \\ \varphi_{u,v} &= \left| \sum_{l=1}^{L} \frac{|h_l|^2}{|h_l|^2 + N_0/E_{\mathrm{PSD}}} c_{v,l} c_{u,l} \right|^2, \\ \lambda &= \sum_{l=1}^{L} \frac{|h_l|^2}{\left(|h_l|^2 + N_0/E_{\mathrm{PSD}}\right)^2}. \end{aligned} \tag{10}$$

Optimization procedures for the maximisation of the system bit rate can then be applied on these mutual information expressions.

2.3. LP-OFDM System Bit Rate. The mathematical expressions obtained with MMSE detector (9) have a form such that the studied optimization problems can be shown to be almost intractable or leads to a prohibitive complexity. The proposed bit-loading algorithm with MMSE detector uses as an input the bit distribution obtained with ZF detector. This choice will be justified later. The optimization of the LP-OFDM systems using the ZF detection technique has been studied in many works [15, 18]. Here, we provide the main results on the bit rate optimization procedures. The optimum achieved bit rate of the LP-OFDM system, under assumption of perfect synchronization, perfect channel estimation, PSD constraint and unconstrained modulations, is [15]

$$\mathcal{R} = \sum_{b=1}^{B} \mathcal{R}_b = \sum_{b=1}^{B} L \log_2\left(1 + \frac{1}{\Gamma}\frac{L}{\sum_{n\in S_b}\left(1/|h_n|^2\right)}\frac{E_{\text{PSD}}}{N_0}\right), \tag{11}$$

where $|h_n|$ is the channel amplitude of subcarrier n, Γ is the SNR gap, and S_b is the subset of subcarriers within the bth block of size L. The SNR gap Γ defines the gap between a practical coding and modulation scheme and the channel capacity. This SNR gap Γ depends on the used coding and modulation scheme, and also on the target probability of error. Following the conventional SNR gap analysis [19], Γ has a constant value for all the modulation orders of uncoded quadrature amplitude modulation (QAM) and for a fixed target symbol error rate (SER). Γ is approximated by

$$\Gamma = \frac{1}{3}\left(Q^{-1}\left(\frac{\text{SER}}{4}\right)\right). \tag{12}$$

To maximize the bit rate $\mathcal{R}$ given by (11), it is sufficient to minimize the sum $\sum_{n\in S_b}(1/|h_n|^2)$ for each block b. This minimization corresponds to the selection of available subcarriers with the best channel amplitudes $|h_n|$ for each block. A simple solution is then to sort subcarriers in descending order and to choose the first L subcarriers for the first block, the following L subcarriers for the second block, and so forth [15].

For real systems, the achieved bit rate using discrete modulation is maximized if, on block S_b, r_u bits are allocated to precoding sequence u, and r_u is expressed as [15]

$$r_u = \begin{cases} \left\lfloor \dfrac{\mathcal{R}_b}{L} \right\rfloor + 1 & \forall u \in [1; n_u] \\[2ex] \left\lfloor \dfrac{\mathcal{R}_b}{L} \right\rfloor & \forall u \in (n_u; L], \end{cases} \tag{13}$$

where

$$n_u = \left\lfloor L\left(2^{\mathcal{R}_b/L - \lfloor \mathcal{R}_b/L \rfloor} - 1\right)\right\rfloor. \tag{14}$$

In this paper, we propose a new bit-loading algorithm with MMSE detector for LP-OFDM systems. According to (9), to determine the optimal distribution of the power on each subcarrier, an initial power allocation is needed. Knowing that the MMSE detector outperforms the ZF technique [12] and considering the bit allocation provided in (13) as the optimal solution for ZF detector, we define this ZF result as the initialisation point. Hence, using (9), the allocated powers $\{E_u\}_{u\in[1;U]}$ to precoding sequences for a given bit distribution $\{r_u\}_{u\in[1;U]}$ in a block of subcarriers, satisfy

$$\frac{1}{\Gamma}\phi\frac{E_u}{E_{\text{PSD}}} - (2^{r_u} - 1)\sum_{\substack{v=1\\ v\neq u}}^{U} \varphi_{u,v}\frac{E_v}{E_{\text{PSD}}} = (2^{r_u} - 1)\lambda\frac{N_0}{E_{\text{PSD}}} \tag{15}$$

for all $u \in [1; U]$.

Let Φ, Λ, and Ψ the matrices defined by

$$\begin{aligned} \Phi_{u,u} &= \frac{\phi}{\Gamma}; \\ \forall u \neq v, \quad \Phi_{u,v} &= -(2^{r_u} - 1)\varphi_{u,v}; \\ \Lambda_u &= (2^{r_u} - 1)\lambda\frac{N_0}{E_{\text{PSD}}}; \\ \Psi_u &= \frac{E_u}{E_{\text{PSD}}}. \end{aligned} \tag{16}$$

Then, the distribution of relative energies $\{\Psi_u\}_{u\in[1;U]}$ satisfies

$$\Psi = \Phi^{-1}\Lambda. \tag{17}$$

In practice, the matrix Φ is diagonally dominant and then invertible. In fact, the used distribution strategy of subcarriers into blocks leads to low intersequence interference. Notice that this strategy consists in selecting subcarriers with similar amplitudes in each block in order to reduce the distortion into the blocks. As a result, the intersequence interference terms are minimized, and then Φ is diagonally dominant. The proposed algorithm consists in iteratively updating the bit distribution, while the PSD constraint is satisfied. Namely, while the sum of the relative energies $\{\Psi_u\}_{u\in[1;U]}$ is lower than one, the algorithm adds one bit more to the bit distribution $\{r_u\}_{u\in[1;U]}$, while minimizing the dispersion of values in the block. Algorithm 1 gives the new bit-loading algorithm with MMSE detector for LP-OFDM systems.

This proposed Algorithm 1 exploits one optimal condition in ZF detector: the minimization of the dispersion of the bit distribution $\{r_u\}_{u\in[1;U]}$. This condition becomes an heuristic with MMSE detector and the convergence of the algorithm to the optimal solution is not ensure. Nevertheless, the MMSE detector can outperforms the ZF detector. Furthermore, the complexity of the algorithm is reduced using the bit-rate obtained with ZF detector as initialisation process.

for all block $b \in [1; B]$ **do**
  Compute $\{r_u\}_{u\in[1;U]}$ from (11)
  Compute $\{\Psi_u\}_{u\in[1;U]}$ from (15)
  while $\sum_u \Psi_u \leq 1$ **do**
    Add 1 bit to $\{r_u\}_{u\in[1;U]}$, while minimizing the dispersion of values
    Compute $\{\Psi_u\}_{u\in[1;U]}$ from (15)
  end while
end for

Algorithm 1: Bit-loading algorithm with MMSE detector.

3. Resource Allocation in Multicarrier Multicast Systems

3.1. Systems Description. Multicast delivers data to a group of users by a single transmission, which is particularly useful for high-data-rate multimedia services due to its ability to save the network resources [3]. Figure 2 illustrates a simple multicast case where the source (in green) sends multimedia data to three receivers (in red). The source sends a known sequence of data that allows the receivers to perform the initial channel estimation. A feedback path from each receiver to the source reports the channel amplitudes $|h_{k,n}|$ on each subcarrier. We assume that the source knows perfectly the channel amplitude of all users. Based on this channel estimation and the QoS requirements, the source can perform the multicast resource allocation based on OFDM or LP-OFDM modulation technique.

Figure 3 shows the PHY-MAC cross-layer modules in the transmitter for the multicast data transmission. The channel state information of receivers is used by the multicast scheduler, the multicast subgroup management, and the resource controller. In multirate multicasting context, the multicast subgroup manager groups the multicast users into subgroups. Moreover, the multicast subgroup manager offers information such as the bit rate to the multicast scheduler module. Multicast scheduler module determines the quantity of data in every frame. The resource controller assigns time slots and subcarriers and determines the modulation order on each subcarrier. Video source encoder module encodes the streaming video data with the determined data rate and coding rate. A sufficiently large size of buffer to store the real-time data is assumed. In the message frame, the data is made from the combination of encoded video and multicast users management information [20]. Then, after processing in the PHY interface module, the multicast TX bits are transmitted to the multicast users. Notice that the guard interval is assumed to be selected to perfectly absorb the multipaths caused by all the channels. Due to the PSD constraint in PLC systems, all users have the same peak power constraint E_{PSD} on each subcarrier. Hence, there is no power allocation. To simplify calculations, it is assumed that all users utilize the same length L of precoding sequences for all blocks.

3.2. Concept of Equivalent Multicast Channel. In multicast systems, all users basically receive the same resources. The multicast bit rate could be considered as the bit rate computed for one user on an equivalent channel. This equivalent channel is derived from the different channels of users. For multicast OFDM systems, the equivalent amplitude of this channel on each subcarrier is given by the worst user subcarrier amplitude [9]. Here, we extend this concept of equivalent channel to LP-OFDM systems. The general formulation of the equivalent channel is given as

$$h_b^{\text{eq}} = f\left(\{h_{k,i}\}_{i\in\mathcal{S}_b\ k\in[1;K]}\right), \tag{18}$$

where b represents the subcarrier index in OFDM case or the block index in LP-OFMD case, and K is the number of multicast receivers. ith OFDM system, the size of $\mathcal{S}_b$ is one and $\mathcal{S}_b = \{b\}$ with $b \in [1; N]$. Computing this equivalent multicast channel, the multicast resource allocation is simplified to a single link resource allocation. Therefore, the bit-loading algorithms can be applied on this equivalent channel.

4. Unirate Multicast Systems

The unirate multicast systems refer to the multicast systems where all users receive exactly the same bit rate. To ensure such a common bit rate to all users, the resource allocations methods must adjust the PHY layer parameters to the worst user link conditions. The modulation orders for each dimension are then computed using the worst user channel gains on this dimension. Notice that the dimension corresponds to the subcarrier in OFDM case or the block of subcarriers in LP-OFDM case.

4.1. Conventional Multicast Resource Allocation. The conventional method in multicast OFDM, LCG (low channel gain [7]), consists in allocating resources while satisfying requirements of all users. This method sets the number of loaded bits per subcarrier with the lowest number of loaded bits over this subcarrier, considering all the channels of users. The equivalent channel, considered as the equivalent OFDM channel, then writes [11]

$$\left|h_n^{\text{eq}}\right|^2 = \min_u \left|h_{u,n}\right|^2. \tag{19}$$

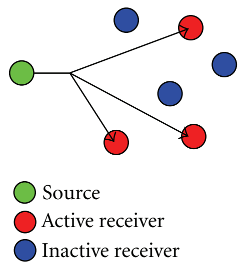

FIGURE 2: Multicast communication scenario.

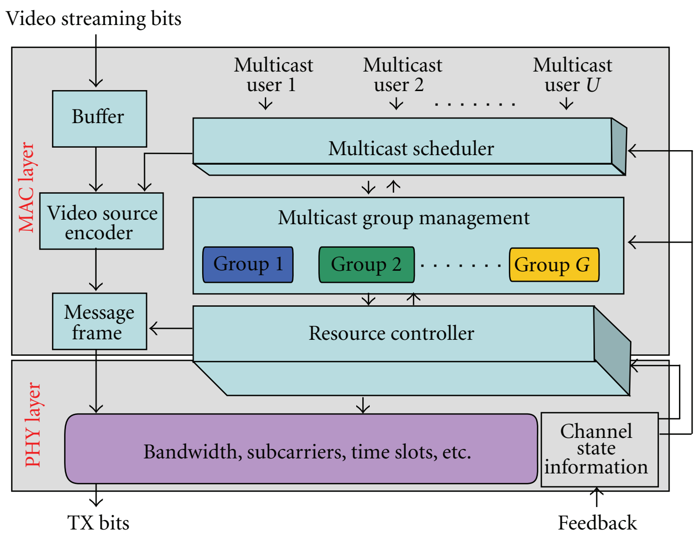

FIGURE 3: PHY-MAC cross-layer modules in the transmitter side [10].

Based on this channel, the multicast bit rate achieved by the LCG method, on subcarrier n, writes under PSD constraint and unconstrained modulation

$$\mathcal{R}_n^{\text{LCG}} = \log_2\left(1 + \frac{E}{\Gamma N_0}\left|h_n^{\text{eq}}\right|^2\right). \tag{20}$$

It has been shown that the conventional multicast bit rate is saturated as the number of users increases due to the different channel selectivity [7, 10].

4.2. Resource Allocation for Unirate Multicast LP-OFDM Systems. To increase the bit rate offered by the LCG method, we proposed new resource allocation algorithms for unirate multicast OFDM systems in indoor powerline communication context. These algorithms jointly use the LP-OFDM modulation technique with zero forcing detector and an adaptation of the LCG approach to exploit the transmission link diversities of users [9, 10]. In this part, we propose a new resource allocation scheme with MMSE detection technique for unirate multicast LP-OFDM systems. In multicast systems, when considering the LP-OFDM modulation technique, the loaded bits over the block S_b of subcarriers will be the lowest bit rate of users over this block.

To define the equivalent LP-OFDM channel, we need first to determine the distribution of subcarriers in each block. Let $\mathbf{D} = (d_{b,n})$ be a decision matrix of size $B \times N$. $\mathbf{D}$ determines the distribution of the N subcarriers into the B blocks and satisfies the following constraints:

$$d_{b,n} = \begin{cases} 1 & \text{if } n \in S_b \\ 0 & \text{else} \end{cases} \quad \forall n \in [1; N] \ \sum_{b=1}^{B} d_{b,n} = 1. \tag{21}$$

The optimal decision matrix $\mathbf{D}$ is such that the distortion is low for each block and is determined by solving an NP-hard combinatorial problem [10]. The proposed suboptimal definition of $\mathbf{D}$ consists in two steps. First, the subcarriers of the equivalent OFDM channel are sorted in descending order. Then, the adjacent subcarrier indices after the sorting operation are used to define the blocks. Let $\mathbf{O}$ be the vector of sorted indices of $|h_n^{\text{eq}}|^2$ in descending order. The decision matrix is then

$$d_{b,n} = \begin{cases} 1 & \text{if } n \in \left\{O_j \ \middle|\ (b-1)L + 1 \le j \le bL\right\}, \\ 0 & \text{else.} \end{cases} \tag{22}$$

Here is an example with $N = 8$, $L = 4$, $B = 2$, and $\mathbf{O} = [2\ 4\ 6\ 7\ 1\ 3\ 5\ 8]$. Hence, it follows that

$$\mathbf{D} = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}. \tag{23}$$

For a given decision matrix $\mathbf{D}$, the equivalent LP-OFDM channel for the block b writes

$$\left| h_b^{\text{eq}}(\mathbf{D}) \right|^2 = \min_k \frac{L}{\sum_{n=1}^{N} \left(d_{b,n} / \left| h_{k,n} \right|^2 \right)}. \tag{24}$$

4.3. Application of LP-OFDM Bit-Loading Algorithms. Based on the equivalent LP-OFDM channel, the multicast bit rate offered by the low block channel gain (LBCG) method proposed in [9] for ZF detector can then be rewritten as

$$\mathcal{R}_b^{\text{LP}}(\mathbf{D}) = L \log_2 \left(1 + \frac{E}{\Gamma N_0} \left| h_b^{\text{eq}}(\mathbf{D}) \right|^2 \right), \tag{25}$$

where $\mathbf{D}$ is defined in (22). The bit distributions $r_{u,b}$ for the different precoding sequences of the block are computed using (13). This method is called LBCG-ZF in reference to the detection technique used.

To determine the bit distribution with the MMSE detector for multicast LP-OFDM systems, Algorithm 1 is applied on the equivalent LP-OFDM channel (24). The newest method, which is based on MMSE detector, is called LBCG-MMSE.

5. Multirate Multicast Systems

Assuming that the multicast data are encoded into layers and any combination of the layers can be decoded at the receiver, the multicast bit rate can be increased by separating users according to their channel conditions. Under this assumption, the sender provides data in several layers organized in a hierarchy. Receivers subscribe to the layers cumulatively to provide progressive refinements [6–8]. If only the first layer is received by the user with the lowest data rate, the decoder produces the worst quality version. As more layers are received by more capable users, the decoder combines the layers to produce improved quality. To increase the total multicast bit rate, a first approach is based on the separation of users in frequency domain [3, 7]. Actually, each subcarrier is assigned to a subgroup of users which receive the same data symbols on this subcarrier. And then, the number of loaded bits on each subcarrier is determined considering the lowest one among the channel amplitudes of all the users allocated to this subcarrier. Notice that the subgroups of users are not the same for each subcarrier. This method, called frequency domain multirate multicast (FDMM) method, significantly increases the total multicast bit rate compared to the conventional LCG method in wireless communications [7].

It has been shown that this FDMM approach yields a good average bit rate, but the QoS requirements of users are not ensured and the fairness among users is degraded [9]. To enforce the fairness performance while minimizing throughput degradation, it has been proposed resource allocation algorithm with proportional fairness (PF) for multirate multicast OFDM systems [7]. In this part, we propose to jointly use the linear precoding technique with this PF-based approach to increase the multirate multicast bit rate. The proposed resource allocation algorithms take into account both ZF and MMSE detectors for multirate multicast LP-OFDM systems.

5.1. Bit Rate Optimization. The PF-based method adapts the bit rates of multicast users at each time slot according to previous allocated bit rates. Let $R_k(iT)$ be the bit rate of the kth user and r_n be the number of bits that are assigned to the nth subcarrier or the nth precoding sequence during the ith time slot of duration T. Let $\{r_{k,n}(iT)\}_{n\in[1;N]}$ be the bit distribution, resulting from bit-loading algorithms in OFDM case or LP-OFDM case, for the kth user in time slot i of duration T. For a low computational complexity, a simplified PF algorithm is developed by employing the average data rate, which is given by [21]

$$\begin{aligned} R_k(iT) = {} & \left(1 - \frac{1}{T_W}\right) R_k((i-1)T) \\ & + \frac{1}{T_W} \sum_{n=1}^{N} r_n(iT)\, \mathcal{U}\left(r_{k,n}(iT) - r_n(iT)\right), \end{aligned} \tag{26}$$

where T_W indicates the average window size and $\mathcal{U}$ the Heaviside step function, defined by

$$\mathcal{U}(x) = \begin{cases} 0 & \text{if } x < 0, \\ 1 & \text{if } x \geq 0. \end{cases} \tag{27}$$

The optimization problem, derived from [7], is written as

$$\max_{r_n} \sum_{k=1}^{K} R_k(iT) = \max_{r_n} \prod_{k=1}^{K} R_k(iT), \tag{28}$$

subject to given $R_k((i-1)T)$. As in [7], we show that the optimization problem (28) is asymptotically equivalent to the following one

$$\max_{r_n} \sum_{k=1}^{K} \sum_{n=1}^{N} \frac{r_n(iT)\, \mathcal{U}\left(r_{k,n}(iT) - r_n(iT)\right)}{R_k((i-1)T)}, \tag{29}$$

```
T_W; ∀k, R_k(0) = 1
for all user k ∈ [1; K] do
    Compute {r_{k,n}}_{n∈[1,N]} using ZF or MMSE detection bit-loading
end for
for all time slot index i ∈ [1; T_W] do
    Select the user index k* as defined in (30)
    Compute {r_n(iT)}_{n∈[1,N]} using (29)
    Compute R_k(iT) using (24)
end for
```

Algorithm 2: Bit-loading algorithm for multirate multicast systems.

subject to given $R_k((i-1)T)$. In fact, with $\mathcal{U}_{k,n} = \mathcal{U}(r_{k,n}(iT) - r_n(iT))$, the product in (28) is given by (30)

$$\begin{aligned}\prod_{k=1}^{K} R_k(iT) &= \underbrace{\left[\left(1-\frac{1}{T_W}\right)^K \prod_{k=1}^{K} R_k((i-1)T)\right]}_{=c_K \text{ (constant)}} \\ &\quad \times \prod_{k=1}^{K}\left(1+\frac{\sum_n r_n(iT)\mathcal{U}_{k,n}}{(T_W-1)R_k((i-1)T)}\right) \\ &= c_K\left[1+\frac{1}{T_W-1}\sum_{k=1}^{K}\sum_{n=1}^{N}\frac{r_n(iT)\mathcal{U}_{k,n}}{R_k((i-1)T)}\right. \\ &\quad +\left(\frac{1}{T_W-1}\right)^2 \\ &\quad \times \sum_{l\neq j}\frac{\sum_n r_n(iT)\mathcal{U}_{l,n}\sum_m r_m(iT)\mathcal{U}_{j,m}}{R_l((i-1)T)R_j((i-1)T)} \\ &\quad \left. + \cdots \right],\end{aligned} \tag{30}$$

and, for large T_W, the terms of orders greater than two can be neglected.

Since the considered Heaviside step function $\mathcal{U}$ in (27) is not continuous at zero, a local maximum cannot be found by using the first derivative test or second derivative test. A solution for the problem (29) is

$$r_n(iT) = r_{k^*,n}(iT), \tag{31}$$

where

$$k^* = \arg\max_{k} r_{k,n}(iT)\sum_{j=1}^{K}\frac{\mathcal{U}\left(r_{j,n}(iT) - r_{k,n}(iT)\right)}{R_j((i-1)T)}. \tag{32}$$

Algorithm 2 gives the corresponding bit-loading algorithm to provide the solution (31).

6. Results and Discussions

In this section, the performances of the resource allocation algorithms with the MMSE detector in multicast systems are presented. First, a performance comparison in terms of achieved total bit rate is realized for both unirate and multirate multicast systems. The different systems are summarized in Table 1. In unirate case, the LP-OFDM systems with both ZF and MMSE detection technique (LBCG-ZF and LBCG-MMSE) are compared with the conventional OFDM approach (LCG). In multirate case, the performance of the LP-OFDM systems (FDMM-LP-ZF and FDMM-LP-MMSE) are compared to the frequency domain multirate multicast approach for OFDM systems (FDMM-OFDM). Then, the fairness and complexities issues are presented.

Table 1: System characteristics and acronyms.

		Unirate	Multirate
OFDM-based		LCG	FDMM-OFDM
LP-based	ZF	LBCG-ZF	FDMM-LP-ZF
	MMSE	LBCG-MMSE	FDMM-LP-MMSE

The generated signal is composed of $N = 1024$ subcarriers transmitted in the (2; 27) MHz band. Perfect synchronization and channel estimation are assumed. A high background noise level of -110 dBm/Hz is considered, and the signal is transmitted with respect to a flat PSD of -50 dBm/Hz for all users. Results are given for a fixed target SER of 10^{-3} without channel coding. To determine the performances of the different algorithms, measured transfer functions of PLC channels are used. Here, the proposed multipath channel models for PLC in [22] are considered, where a classification of PLC channels is realized. PLC channels for indoor networks are classified into 9 classes per ascending order of their capacities, that is, the higher the channel class number, the better the channel amplitudes. A model of transfer function is associated to each class. Figure 4 shows three examples of PLC transfer function models. One channel of the category "good" (i.e., class 9 channel), one channel of the category "average" (i.e., class 5 channel), and one channel of the category "bad" (i.e., class 2 channel) are represented in the (2; 27) MHz band. In the following, a multicast system with a maximum of 9 users is considered, and each user experiences one different class of channel within the 9 classes.

6.1. Precoding Sequence Length Influence. We begin the simulation by highlighting the bit rate improvement provided

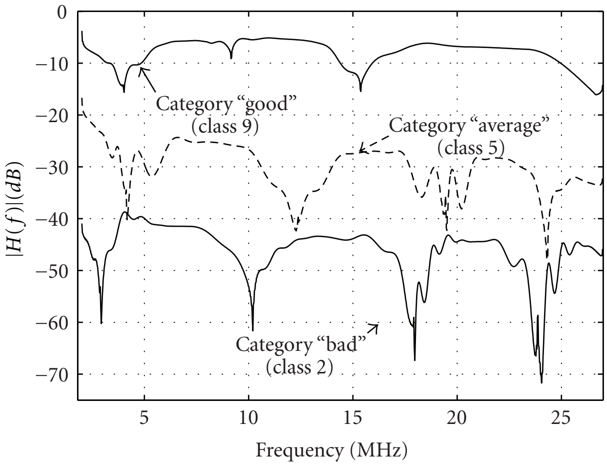

Figure 4: Transfer functions of three channel classes (class 2, class 5, and class 9 channels).

by the linear precoding component. For this purpose, we focus on results obtained when L varies. Notice that, finding the optimal precoding sequence lengths amounts to a complex combinatorial optimization problem that cannot be reduced to an equivalent convex problem. Thus, no analytical solution exists and optimal solution can only be obtained following exhaustive search [18]. Figure 5 shows the evolution of multicast LP-OFDM bit rate as a function of the length of the precoding sequences in (a) unirate case and (b) in multirate case. The parameter T_W is fixed to 10 to reduce the simulation time. It has been shown that the performance difference for small and large values of T_W is negligible [7]. The bit rate offered by the OFDM-based methods is given for $L = 1$. It is clear that the achievable bit rate with the LP-OFDM methods, whatever the equalization criteria, is improved when the length of the precoding sequences is greater than 1. This bit rate reaches a maximum value for $L = 64$ and $L = 32$, respectively in unirate and multirate multicast cases. In unirate case, the achieved multicast bit rate increases from approximately 5.42 kbit/OFDM symbol for LCG method with OFDM system to at least 9.35 kbit/OFDM symbol for LBCG methods with LP-OFDM system and ZF detector. In multirate case, this achieved bit rate increases from 33.35 kbit/OFDM symbol for FDMM-OFDM method to 35.5 kbit/OFDM symbol for FDMM-LP method with ZF and MMSE detectors. These improvements correspond to the bit rate gains of 42% and 6.4%, respectively for unirate and multirate multicast cases. Based on these results, we can state that the utilization of the linear precoding technique increases the bit rate of the multicast systems. The reason for the better performance of the LP-OFDM systems compared to OFDM systems is the efficient utilization of the PSD limit. The precoding component accumulates the residual energies of a given block of subcarriers to transmit additional bits.

In addition, the MMSE detection technique offers better performance than the ZF detection technique. The bit rate improvement reaches up to 8% in unirate multicast case, while the performance difference between ZF and MMSE

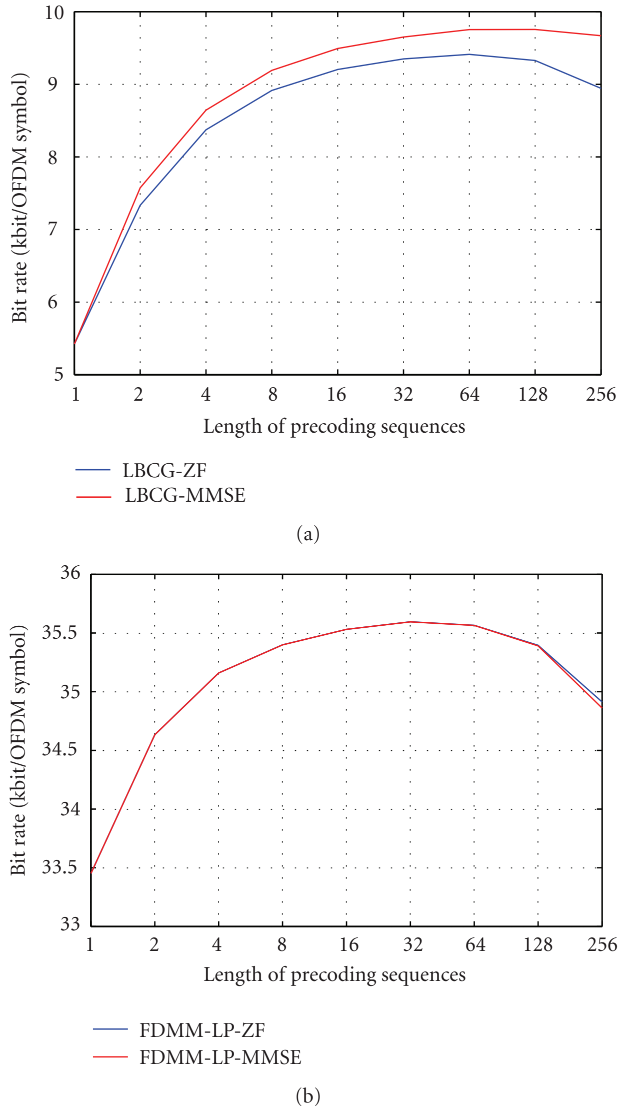

Figure 5: Total bit rate in bit per OFDM symbol versus the length of the precoding sequences for 9 users: (a) in unirate case and (b) in multirate case.

detectors is minor in multirate multicast case. The low MMSE gain compared to the performance of ZF detector is due to the decision matrix **D** designed to reduce the distortion within each block. With low level of distortion, the powerfulness of the MMSE detector cannot be highlighted. Furthermore, the bit-loading algorithm for multirate multicast systems also reduces the distortion within each block to increase the multicast bit rate. The MMSE detector cannot improve with high gain the performance of the ZF detector.

6.2. Statistical Results of the Total Multicast Bit Rate. This part deals with the statistical results of the total multicast bit rate obtained through thousand simulations over a 9-user multicast system. They concern the cumulative distribution functions (CDF) of the maximal total multicast bit rates for $L = 32$ given in Figure 6. This CDF is also the bit rate

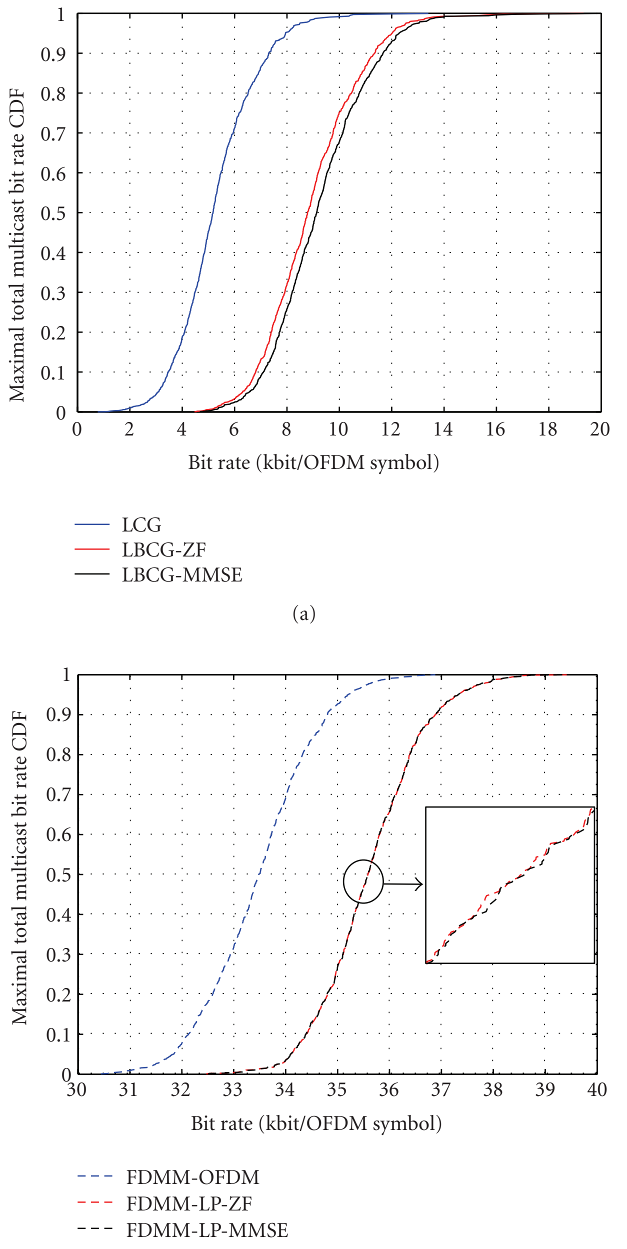

Figure 6: Cumulative distribution function (CDF) of the maximal total multicast bit rates: (a) in unirate case and (b) in multirate case for $L = 32$.

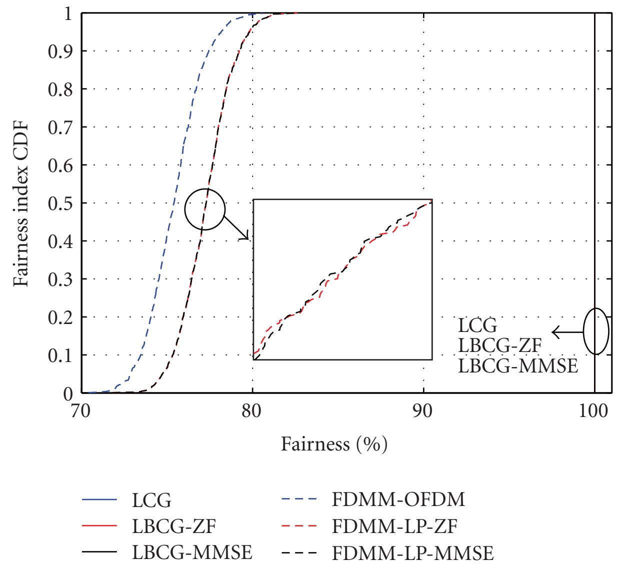

Figure 7: Cumulative distribution function of the fairness index of multicast users for $L = 32$.

outage probability. Results confirm the better performances of the linear precoding technique in multicast systems. First, in unirate multicast case, the gap at 0.5 of the CDF between the maximal total bit rate of the LBCG and the LCG methods is at least 3.6 kbit/OFDM symbol, corresponding to a bit rate gain of 70%. In addition, the new resource allocation with MMSE detector (LBCG-MMSE) offers more bit rate than the LBCG-ZF method with low gains, around 3%. Second, in multirate multicast case, the gap at 0.5 of the CDF between the maximal total bit rate of the FDMM-LP and the FDMM-OFDM methods is at least 2.1 kbit/OFDM symbol, corresponding to a bit rate gain of 6%. The performance difference between the FDMM-LP-ZF and the FDMM-LP-MMSE methods remain low in this context.

These results suggest that the linear precoding technique with MMSE detector is the best solution for resource allocation in unirate multicast systems. By cons, the linear precoding technique with ZF detector is sufficient in multirate multicast systems. Notice that the utilization of a more powerful equalizer such as minimum mean square error equalizer improves the LP-OFDM systems bit rate. However, the utilization of a ZF detector leads to fairly simple manipulations resolving the bit rate optimization problem in multirate context.

6.3. Fairness and Complexity Considerations. The unirate multicast methods equally distribute the resources, and all users receive the same bit rate. When the multicast users experience very different channel conditions, it is justifiable to give more resources to some users than others [10]. It is this idea that underlies the separation of users based on their channel conditions. The proposed algorithms in such a context have already demonstrated their better performance in terms of total multicast bit rate. It is also necessary to measure their performance in terms of fairness among users. As a performance metric, the fairness index defined in [23] is used as follows:

$$\mathrm{FI} = \frac{1}{U}\frac{\left(\sum_k R_k\right)^2}{\sum_k R_k^2}. \tag{33}$$

This fairness index measures the equality of user allocation and it is continuous so that any change in allocation changes the fairness also, contrary to the max-min fairness. Figure 7 shows the cumulative distribution function of the fairness index of multicast users. As expected, the unirate multicast methods give the best fairness index, FI = 1. The LP-based methods, in addition to increase the total multicast bit rate, improve the fairness among the users. The fairness improvement is almost 2%.

In all cases, the MMSE detector outperforms the ZF detector. However, the ZF detector enhancement is low because the channel distortion has already been reduced by the decision matrix **D** and by the bit-loading algorithm for multirate multicast systems.

Besides the comparison of the performance in terms of bit rate and fairness index, the required downlink signaling overheads are compared. In unirate multicast systems, only the modulation order on each subcarrier or each precoding sequence needs to be signaled to users. In addition to information about the modulation order, the multirate multicast systems need to transmit information about the subgroups of users. Thus, it follows that the downlink signaling overhead of the FDMM based methods is higher than the other methods due to the subcarrier and bit allocation information. Furthermore, under the assumption that any combination of layers consisting of multicast data can be decoded at the receiver, an intelligent mapping algorithm for efficiently recovering the original data from different layers is needed [7]. This may bring additional signaling overhead.

7. Conclusion

In this paper, we have addressed the bit rate optimization problem in multicast linearly precoded OFDM systems in PLC context. A new resource allocation method with MMSE detector for multicast LP-OFDM systems has been proposed. The proposed method jointly uses linear precoded OFDM modulation technique and an adaptation of the OFDM-based multicast approaches to exploit the channel frequency selectivity experienced by each user in multicast OFDM systems. It has been shown through simulations that the proposed LP-based methods outperform the OFDM-based methods for both unirate and multirate multicast systems. Additionally, it is shown that the proposed bit-loading algorithm with MMSE detector offers the best performances in terms of total multicast bit rate and fairness among users.

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