Hindawi Publishing Corporation
Journal of Electrical and Computer Engineering
Volume 2016, Article ID 2108586, 7 pages
http://dx.doi.org/10.1155/2016/2108586

Research Article

Fast Direct Solution of Electromagnetic Scattering from Left-Handed Materials Coated Target over Wide Angle

Guo-hua Wang[1] and Ying-bao Geng[2]

[1] *New Star Institute of Applied Technology, Hefei 230039, China*
[2] *School of Medical Information Engineering, Anhui University of Chinese Medicine, Hefei 230038, China*

Correspondence should be addressed to Ying-bao Geng; gengyingbao@163.com

Received 5 February 2016; Revised 6 April 2016; Accepted 9 May 2016

Academic Editor: John N. Sahalos

When solving the electromagnetic scattering problems over wide angle, the traditional method of moments (MoM) needs to repeat the solving process of dense systems of linear equations using the iteration method at each incident angle, which proved to be quite inefficient. To circumvent this problem, a fast numerical method based on block LDLT factorization accelerated by adaptive cross approximation (ACA) algorithm is presented to analyze the electromagnetic scattering of left-handed materials (LHM) coated target. The ACA algorithm is applied to impedance matrix filling and all steps of block LDLT factorization process, which can accelerate the computation process and reduce the memory consumption. The numerical results proved that the proposed method is efficient in calculating monostatic RCS of LHM coated target with many required sampling angles. Compared with the traditional MoM, computation time and memory consumption are reduced effectively.

1. Introduction

Left-handed materials (LHM) [1, 2] have electromagnetic properties which cannot be generally found in nature; the electric field direction, magnetic field direction, and the propagation vector conform to the laws of the left hand when electromagnetic wave travels in it. LHM are also called double negative materials (DNG) [3] as the dielectric permittivity and permeability at a certain frequency range can be negative together. The application of LHM has been wide in antenna [4], microwave devices, and other fields [5] due to their unusual physical properties, such as negative refraction, inverse Doppler shift, and reversed Cerenkov radiation. For the purpose of stealth, many conductor target surfaces are also coated by LHM; this necessitates a fast and efficient solution to analyze the electromagnetic scattering characteristics of LHM coated targets [6–9].

Researchers have proposed various methods to analyze the scattering problem of LHM coated targets. In literature [7], the radar cross section of metallic spheres covered by DNG based on Mie series solution is studied, but this method is valid only in certain circumstances when the geometric structure of the target is simple. In literature [8], the physical optics (PO) method in combination with the impedance boundary condition (IBC) is used to analyze the near-field electromagnetic scattering characteristics of LHM coated targets. In literature [9], the wide band scattering of perfectly conducting target coated with DNG is computed using finite difference time domain (FDTD) method. Apart from the above methods, this paper mainly discusses the fast method to compute the RCS of LHM coated target over wide angle based on MoM [10] framework and hybrid PEC-dielectric formulation. However, when solving the electromagnetic scattering problems with multiple incident angles, traditional MoM needs to compute the current coefficients as the change of incident angle. Usually iteration method is used to solve the linear system equations which proved to be a quite effective method for bistatic RCS computing, but, for monostatic RCS with many sampling angles, the iteration operations must be repeated when the incident angle changes each time; this is inefficient when the number of sampling angles is large; also there are often convergence issues when dealing with complex structure targets. To circumvent this problem, a new block ACA-LDLT method is proposed in this paper to speed up the filling process of impedance matrix and solve the linear system equations more effectively, which is suitable for computing monostatic RCS with many required sampling angles of complex structure targets.

2. Hybrid PEC-Dielectric Formulation for LHM Coated Target

Recently, a method [11] based on thin dielectric sheet (TDS) approximation with explicit perfect electric conductor (PEC) boundary conditions at the interfaces of PEC and dielectrics was provided, which can get accurate numerical solution of EM scattering but is only suitable for electrical coating materials. In literature [12], a generalized thin coating equivalent model was proposed to be suitable for both electrical and magnetic coating materials, but there is no further discussion when the coating medium is LHM. In this section, we mainly discuss the building up of the hybrid PEC-dielectric formulation for LHM coated target.

For LHM coated PEC target under the irradiation of ideal plane wave shown in Figure 1, S stands for the surface of PEC, V is the volume of coating LHM, and τ is the coating thickness. The total tangential component of electric field is zero following the PEC boundary condition of the electric field; we can obtain

$$\left[\mathbf{E}^{\text{inc}}(\mathbf{r}) + \mathbf{E}^{\text{sca}}(\mathbf{r})\right]\Big|_{\tan} = 0 \quad \mathbf{r} \in S; \tag{1}$$

here, the subscript "tan" stands for the tangential component, $\mathbf{r}$ stands for the field point, $\mathbf{E}^{\text{inc}}$ stands for the incident field, and $\mathbf{E}^{\text{sca}}$ is the scattering field which could be described as

$$\mathbf{E}^{\text{sca}}(\mathbf{r}) = \mathbf{E}^{\text{sca}}_{\text{pec}}(\mathbf{r}) + \mathbf{E}^{\text{sca}}_{\text{die}}(\mathbf{r}). \tag{2}$$

The scattering field is produced by conductors and LHM coating medium together, and the scattering field produced by the electric current on S could be described as

$$\mathbf{E}^{\text{sca}}_{\text{pec}}(\mathbf{r}) = j\omega\mu_0 \int_S \left(\mathbf{J}_S(\mathbf{r}') + \frac{1}{k_0^2}\nabla\nabla'\cdot\mathbf{J}_S(\mathbf{r}')\right) g(\mathbf{r},\mathbf{r}')\, dS'; \tag{3}$$

here, $g(\mathbf{r},\mathbf{r}') = e^{-jk|\mathbf{r}-\mathbf{r}'|}/4\pi|\mathbf{r}-\mathbf{r}'|$ stands for scalar Green's function in free space, $\mathbf{J}_S$ is surface current density on S, and μ_0 and k_0 stand for the permeability and wave number in free space, respectively. The scattering field produced by LHM coating medium in V could be described as

$$\begin{aligned}\mathbf{E}^{\text{sca}}_{\text{die}}(\mathbf{r}) = j\omega\mu_0 \Bigg[&\int_V \chi(\mathbf{r}')\, g(\mathbf{r},\mathbf{r}')\, j\omega\mathbf{D}(\mathbf{r}')\, dV' \\ &- \frac{1}{k_0^2}\int_{S_n^+} \chi\nabla g(\mathbf{r},\mathbf{r}')\, j\omega\hat{n}'\cdot\mathbf{D}(\mathbf{r}')\, dS' \\ &+ \frac{1}{k_0^2}\int_{S_n^-} \chi\nabla g(\mathbf{r},\mathbf{r}')\, j\omega\hat{n}'\cdot\mathbf{D}(\mathbf{r}')\, dS' \Bigg] \\ &- \int_V \nabla g(\mathbf{r},\mathbf{r}') \times j\omega\xi(\mathbf{r}')\,\mathbf{B}(\mathbf{r}')\, dV',\end{aligned} \tag{4}$$

where $\chi(\mathbf{r}') = 1/\varepsilon_r(\mathbf{r}') - 1$ and $\xi(\mathbf{r}') = 1/\mu_r(\mathbf{r}') - 1$ are the dielectric contrast ratios; $\hat{n}'$ is the unit vector normal to the upper surface. When the coating thickness τ is very small compared to the wavelength, we can use an approximate method named as TDS approximation:

$$\begin{aligned}\mathbf{D}(\mathbf{r}') &\approx \hat{n}'\nabla'\cdot\frac{\mathbf{J}_S(\mathbf{r}')}{j\omega} \\ \mathbf{B}(\mathbf{r}') &\approx -\mu\hat{n}'\times\mathbf{J}_S(\mathbf{r}') \\ &\qquad \mathbf{r}' \in \mathbf{V}.\end{aligned} \tag{5}$$

Then, the volume integral can be further approximated to surface integral through conversion $dV \approx \tau dS$. Therefore, the scattering field produced by dielectric can be described as

$$\begin{aligned}\mathbf{E}^{\text{sca}}_{\text{die}}(\mathbf{r}) = j\omega\mu_0 \Bigg[&\frac{1-\mu_r}{2}\mathbf{J}_S\tau + \int_S \hat{n}'\tau\chi g\nabla'\cdot\mathbf{J}_S dS' \\ &+ \frac{\nabla}{k_0^2}\int_S \chi(g - g_\tau)\nabla'\cdot\mathbf{J}_S dS' \\ &+ (1-\mu_r)\int_{S-S_0} \tau\nabla g\times\hat{n}'\times\mathbf{J}_S dS' \Bigg].\end{aligned} \tag{6}$$

Here, the last term in (6) is the principal value integral. Finally, we substitute (3) and (6) in (2); in combination with (1), we can obtain the hybrid PEC-dielectric formulation which can be expressed by the only unknown $\mathbf{J}_S$:

$$\begin{aligned}\mathbf{E}^{\text{inc}}(\mathbf{r}) = -j\omega\mu_0 \Bigg[&\frac{1-\mu_r}{2}\mathbf{J}_S\tau \\ &+ \int_S g\left(\mathbf{J}_S + \hat{n}'\tau\chi\nabla'\cdot\mathbf{J}_S\right) dS' \\ &+ \frac{\nabla}{k_0^2}\int_S (g + \chi g - \chi g_\tau)\nabla'\cdot\mathbf{J}_S dS' \\ &+ (1-\mu_r)\int_S \tau\nabla g\times\hat{n}'\times\mathbf{J}_S dS' \Bigg] \quad \mathbf{r}\in S.\end{aligned} \tag{7}$$

Here, ε_r and μ_r are permittivity and permeability of coating LHM given by Drude model [13, 14]; g_τ is scalar Green's function defined by

$$g_\tau = \frac{e^{-jk|\mathbf{r}-(\mathbf{r}'+\tau\hat{n}')|}}{4\pi\left|\mathbf{r}-(\mathbf{r}'+\tau\hat{n}')\right|}. \tag{8}$$

3. Solutions

We will refer to (8) as a hybrid PEC-dielectric formulation, which can be solved using the MoM or MLFMA with iteration method. But, for monostatic RCS with many required sampling angles, iteration method must be repeated with many right-hand sides (RHS), so this part becomes expensive. The block LDLT factorization method in combination with adaptive cross approximation (ACA) is introduced to this paper which can reduce time and memory storage and is proven to be more efficient in computing the monostatic RCS than the method in [12].

3.1. Block LDLT Factorization Method. By discretizing the surface currents using RWG vector basis functions [15, 16], the integral equation (8) can be transformed into a dense, complex linear equation:

$$\mathbf{Z} \cdot \mathbf{J} = \mathbf{V}, \tag{9}$$

where $\mathbf{Z}$ is the impedance matrix of dimension $N \times N$, $\mathbf{J}$ is induced current density, and $\mathbf{V}$ is excitation voltage matrix; N is the unknowns number of closed scattering mesh. When using MoM to solve the equation, the storage complexity and matrix filling time is $O(N^2)$ and the direct LU factorization complexity is $O(N^3)$; both grow rapidly for electrically large target and bring a heavy burden to computer. To overcome the problem, the unknowns in this work have been grouped into multiple local regions, and the rank of the submatrices decreases with the increase of distance of two blocks; then the compressed algorithm can be used to reduce the storage and operations count. When unknowns are grouped into local spatial regions, (9) can be converted into the following form:

$$\begin{pmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \cdots & \mathbf{Z}_{1M} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \cdots & \mathbf{Z}_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z}_{M1} & \mathbf{Z}_{M2} & \cdots & \mathbf{Z}_{MM} \end{pmatrix} \begin{pmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \\ \vdots \\ \mathbf{J}_M \end{pmatrix} = \begin{pmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \vdots \\ \mathbf{V}_M \end{pmatrix}, \tag{10}$$

where $\mathbf{Z}_{ij}$ $(i, j = 1, 2, 3, \ldots, M)$ represent the submatrix of impedance matrix. By using RWG vector basis functions as the test functions, the impedance matrix in (10) is a complex coefficient symmetric matrix, which can be expressed as block LDLT form:

$$\begin{pmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \cdots & \mathbf{Z}_{1M} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \cdots & \mathbf{Z}_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z}_{M1} & \mathbf{Z}_{M2} & \cdots & \mathbf{Z}_{MM} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ \mathbf{L}_{21} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{L}_{M1} & \mathbf{L}_{M2} & \cdots & 1 \end{pmatrix} \begin{pmatrix} \mathbf{D}_{11} & 0 & \cdots & 0 \\ 0 & \mathbf{D}_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{D}_{MM} \end{pmatrix} \begin{pmatrix} 1 & \mathbf{L}_{21}^T & \cdots & \mathbf{L}_{M1}^T \\ 0 & 1 & \cdots & \mathbf{L}_{M2}^T \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}, \tag{11}$$

where $\mathbf{L}_{ij}$ are the lower triangular entities and $\mathbf{D}_{ii}$ are the diagonal entities; then making the substitutions $\mathbf{U}_{ii} = \mathbf{D}_{ii}$ and $\mathbf{U}_{ij} = \mathbf{D}_{ii}\mathbf{L}_{ji}^T$, we can obtain the standard block LU form:

$$\begin{aligned} &\begin{pmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \cdots & \mathbf{Z}_{1M} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \cdots & \mathbf{Z}_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z}_{M1} & \mathbf{Z}_{M2} & \cdots & \mathbf{Z}_{MM} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & \cdots & 0 \\ \mathbf{L}_{21} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{L}_{M1} & \mathbf{L}_{M2} & \cdots & 1 \end{pmatrix} \begin{pmatrix} \mathbf{U}_{11} & \mathbf{U}_{12} & \cdots & \mathbf{U}_{1M} \\ 0 & \mathbf{U}_{22} & \cdots & \mathbf{U}_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{U}_{MM} \end{pmatrix}. \end{aligned} \tag{12}$$

The iterative formula of $\mathbf{U}_{ij}$ is

$$\mathbf{U}_{ij} = \mathbf{Z}_{ij} - \sum_{p=1}^{i-1} \mathbf{U}_{pi}^T \mathbf{D}_{pp}^{-1} \mathbf{U}_{pj}. \tag{13}$$

The surface currents can be obtained through the forward iterative step

$$\mathbf{X}_i = \mathbf{V}_i - \sum_{p=1}^{i-1} \mathbf{U}_{pi}^T \mathbf{D}_{pp}^{-1} \mathbf{X}_p \tag{14}$$

and back iterative step

$$\mathbf{J}_i = \mathbf{D}_{ii}^{-1} \left(\mathbf{X}_i - \sum_{p=i+1}^{n} \mathbf{U}_{ip} \mathbf{J}_p \right). \tag{15}$$

3.2. ACA Acceleration. The impedance matrix in (10) is composed of multiple submatrices, and the off-diagonal blocks are of low rank and can be compressed. In literature [17, 18], Shaeffer points out that the upper triangular matrix $\mathbf{U}$ and multiple plane wave excitation RHS voltage matrix $\mathbf{V}$ also have low rank characteristics and can be compressed too, so the compression operation can be used for all steps of the solutions including impedance filling, LU factorization, and LU solving. In numerous matrix compression methods, the ACA algorithm in literature [19] is widely used [20–22]. The ACA algorithm operates by scanning a row followed by a column of the matrix at each iteration and progressively builds up a low rank estimate of the matrix based on the rows and columns that have been scanned. The algorithm operates as follows:

(1) Initialize the approximate matrix as $\widetilde{\mathbf{A}}^{m\times n} \approx 0^{m\times n}$ and the iteration count as $j = 1$; arbitrarily choose a row of the matrix.

(2) Scan the jth row of the matrix.

(3) Find the error of the previous field approximation at the jth row $\mathbf{R} = \mathbf{A}^{m\times n}(j\text{th row}) - \widetilde{\mathbf{A}}^{m\times n}(j\text{th row})$ and choose the jth column to be the one containing the maximum element of $|\mathbf{R}|$.

(4) Assign $v_j^{1\times n} = \mathbf{R}/\mathbf{R}$ (jth column).

(5) Scan the jth column and find the error of the previous estimate at this column: $\mathbf{C} = \mathbf{A}^{m\times n}$($j$th column) $-$ $\widetilde{\mathbf{A}}^{m\times n}$($j$th column). Choose the $(j+1)$th row to be the one containing the maximum element of $|\mathbf{C}|$ (ensuring that the jth row is not chosen again).

(6) Assign $u_j^{m\times 1} = \mathbf{C}$.

(7) Update the field approximation $\widetilde{\mathbf{A}}^{m\times n} = \widetilde{\mathbf{A}}^{m\times n} + u_j^{m\times 1} v_j^{1\times n}$.

(8) If $\|u_j^{m\times 1}\|\|v_j^{1\times n}\| < \varepsilon\|\widetilde{\mathbf{A}}^{m\times n}\|$, stop scanning; else increment j and repeat steps (2) to (8).

A low rank matrix can be well approximated by the product of two full rank matrices using ACA method:

$$\mathbf{A}^{m\times n} \approx \widetilde{\mathbf{A}}^{m\times n} = \mathbf{A}_U^{m\times r}\mathbf{A}_V^{r\times n}, \tag{16}$$

where r is the effective rank of matrix and the memory requirement decreased from $m\times n$ entries to $r(m+n)$ entries. We can get the block decomposition expression by using the ACA to the iterative process of $\mathbf{U}_{ij}$:

$$\begin{aligned}\mathbf{U}_{ij} &= [\mathbf{U}_U\mathbf{U}_V]_{i,j} \\ &= [\mathbf{Z}_U\mathbf{Z}_V]_{ij} - \sum_{p=1}^{i-1}[\mathbf{U}_V^T\mathbf{U}_U^T]_{pi}\mathbf{D}_{pp}^{-1}[\mathbf{U}_U\mathbf{U}_V]_{pj}.\end{aligned} \tag{17}$$

By using the ACA to the LU solving solution, the induced current density $\mathbf{J}_i$ on S can be obtained by the forward iterative steps

$$\mathbf{X}_i = \mathbf{V}_i - \sum_{p=1}^{i-1}[\mathbf{U}_V^T\mathbf{U}_U^T]_{pi}\mathbf{D}_{pp}^{-1}[\mathbf{X}_U\mathbf{X}_V]_p \tag{18}$$

and back iterative steps

$$\mathbf{J}_i = \mathbf{D}_{ii}^{-1}\left(\mathbf{X}_i - \sum_{p=i+1}^{n}[\mathbf{U}_U\mathbf{U}_V]_{ip}[\mathbf{J}_U\mathbf{J}_V]_p\right). \tag{19}$$

4. Numerical Results

In this section, we simulated three numerical examples to demonstrate the accuracy and validity of the proposed method. All the results are computed on an Intel® Core™2 Duo PC with 3.40 GHz processor and 16 GB RAM; only one core is used; the ACA iteration error threshold is $1e-3$. The percentage error of advised algorithm is defined as

$$\sqrt{\frac{\|\mathbf{V}-\mathbf{Z}\cdot\mathbf{J}\|}{\|\mathbf{V}\|}} \times 100\%. \tag{20}$$

(A) A Dielectric Coated PEC Sphere. We start by considering the scattering problem from a dielectric coated PEC sphere with radius 1λ. Incident frequency is 300 MHz from $\theta = 0°$ ($\varphi = 0°$). The thickness of the coating material is $\tau = 0.01\lambda$. The relative permittivity of LHM is $\varepsilon_r = 9.0$ and relative permeability is $\mu_r = 1.77 - j4.06$. The geometry is divided into 2220 triangular patches with an average length of $\lambda/10$, and this gives rise to 3330 unknowns. In Figure 2, the bistatic radar cross section (RCS) is shown from $\theta = 0°$ to $\theta = 180°$. The dashed lines by our proposed TDS-LDLT-ACA method agree well with the solid lines by the commercial EM software FEKO.

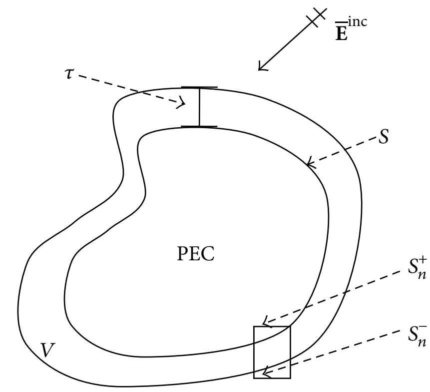


Figure 1: A PEC target coated by LHM.

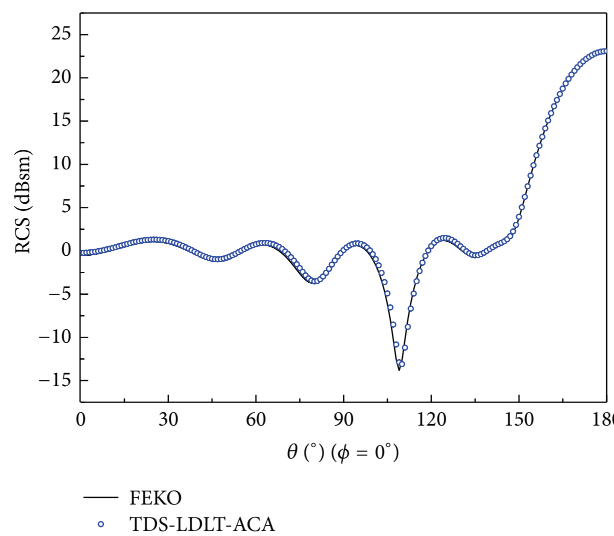


Figure 2: LHM coated PEC sphere.

(B) LHM Coated PEC Cylinder. The second example is the problem of scattering from LHM coated PEC cylinder with radius of 0.5λ and height of 2λ as shown in Figure 3, the plane wave is incident from $\theta = 0°$ ($\varphi = 0°$) to $\theta = 180°$ ($\varphi = 0°$), the relative permittivity of LHM is $\varepsilon_r = -9.0$, and relative permeability is $\mu_r = -1.77 - j4.06$. As a comparison, we have examined the RHM coated case with the relative permittivity $\varepsilon_r = 9.0$ and relative permeability $\mu_r = 1.77 - j4.06$. The geometry is divided into 1456 triangular patches with an average length of $\lambda/10$, and this gives rise to 2184 unknowns. In ACA operation, the geometry is subdivided into 8 spatial regions as shown in Figure 3. The monostatic

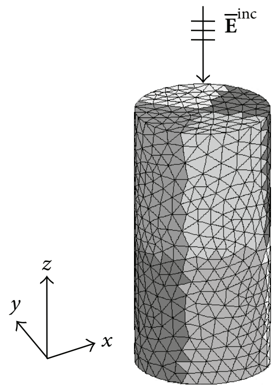
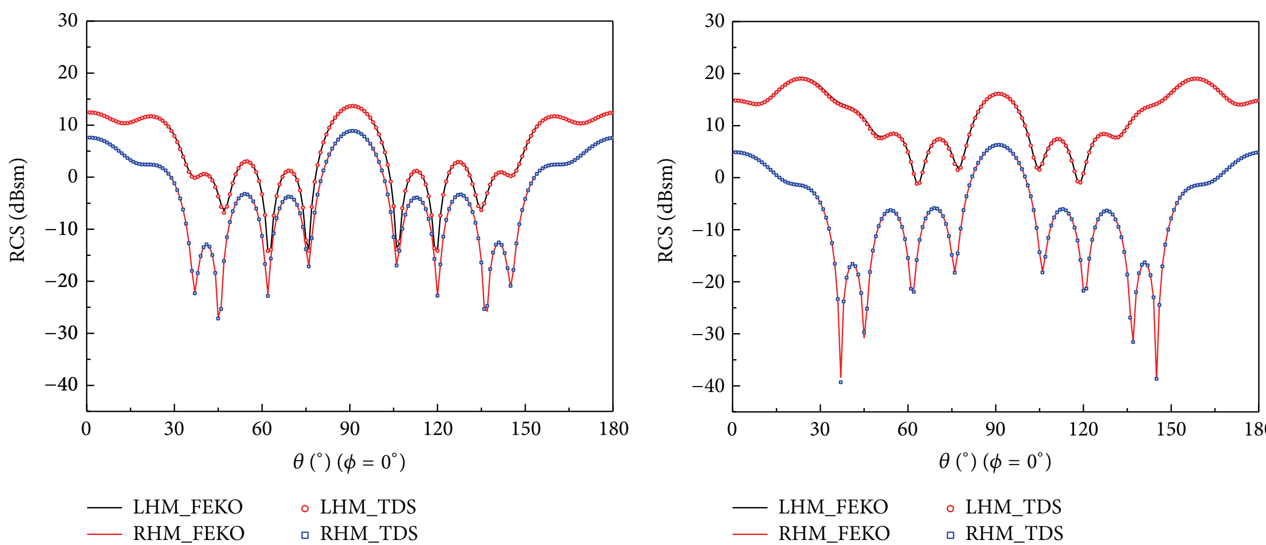


FIGURE 3: LHM coated PEC cylinder.



(a) Coating thickness: $0.005\,\lambda$

(b) Coating thickness: $0.01\,\lambda$

FIGURE 4: VV polarization monostatic RCS of coated cylinder.

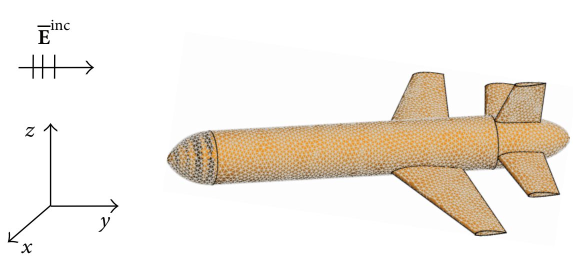


FIGURE 5: LHM coated PEC missile model.

RCS calculated for VV polarization is shown in Figure 3 along with the reference solution provided by the FEKO.

In Figure 4(a), the coating thickness is $0.01\,\lambda$; a quite good agreement between the reference FEKO solution and the results of advised method can be observed. Also, we can see that the monostatic RCS value of coated cylinder with RHM is less than that of LHM coated circumstance in most of angle range.

The monostatic RCS is shown in Figure 4(b) when coating thickness τ reduced to $0.005\,\lambda$. We can find that the RCS change rule with incident plane wave angle is similar to Figure 4(a). When coating thickness reduced, the RCS value of RHM drops about 3 dB in most of incident angle area, while the RCS values of LHM increase about 2 dB in most of incident angle area. The phenomenon is also consistent with the conclusion in literature [23] that the reflectivity and RCS will vary with the coating thickness.

(C) LHM Coated PEC Missile Model. The third example is LHM coated PEC missile model shown in Figure 5, which has a length of 9.91 m, a wingspan of 6.28 m, and a height of 2.19 m. The geometry is divided into 11300 triangular patches with an average length of $\lambda/10$, and this gives rise to 16950 unknowns.

The thickness of the material is $\tau = 0.01\,\lambda$. The relative permittivity of LHM is $\varepsilon_r = -1.5 - j0.15$ and relative permeability is $\mu_r = -1.0 - j0.1$. The missile model is illuminated by an incident plane wave with the incident direction of

Table 1: Time and RAM requirements.

The targets and the unknowns	Number of incident plane waves	Method	RAM requirements/MB	Time/s
Coated sphere, 3330	1	FEKO	146	56
		TDS-LDLT-ACA	132	46
LHM coated cylinder, 2184	180	FEKO	118	948
		TDS-LDLT-ACA	28	385
LHM coated missile model, 16950	1	FEKO	1347	924
		TDS-LDLT-ACA	1355	816

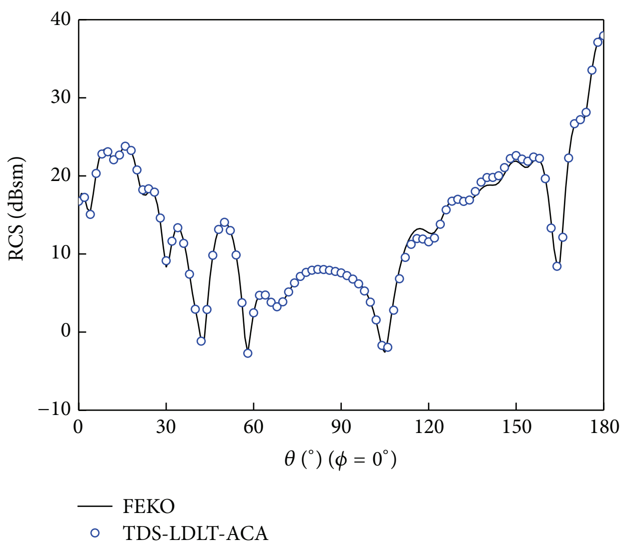


Figure 6: Bistatic RCS of LHM coated PEC missile model.

$\theta = 90^\circ$ and $\varphi = 270^\circ$. Bistatic RCS is shown in Figure 6 along with the reference solution provided by FEKO. We observe that the agreement between the FEKO results and those obtained with the TDS-LDLT-ACA method is excellent. There exists difference between the incident angles $110^\circ < \theta < 160^\circ$; however, the difference of scattering results is relatively smaller.

The total CPU time and RAM requirements of above three examples are shown in Table 1. It can be seen that the proposed method has compressed much more CPU time and random memory demand than conventional MoM.

5. Conclusion

A TDS approach together with explicit PEC boundary conditions has been proposed to handle the EM scattering problems of thin LHM coating target efficiently. The modeling process has been greatly simplified compared with the surface or volume integral equation approach. The ACA method is incorporated into block LDLT factorization algorithm to reduce the filling time of impedance matrix and speed up the factorization and solving processes. The numerical results demonstrate that the backscattering monostatic RCS values in specific angle area can be reduced by selecting the appropriate LHM coating thickness. The method can be used for electromagnetic scattering problems of complex structure LHM coated PEC target and has an important reference value in practical engineering applications of arbitrary shapes targets.

Competing Interests

The authors declare that they have no competing interests.

References

[1] H. Chen, B.-I. Wu, and J. A. Kong, "Review of electromagnetic theory in left-handed materials," *Journal of Electromagnetic Waves and Applications*, vol. 20, no. 15, pp. 2137–2151, 2006.

[2] V. G. Veselago, "The electrodynamics of substances with simultaneously negative valueS OF ϵ and μ," *Soviet Physics Uspekhi*, vol. 10, no. 4, pp. 509–514, 1968.

[3] N. Engheta and R. W. Ziolkowski, "A positive future for double-negative metamaterials," *IEEE Transactions on Microwave Theory and Techniques*, vol. 53, no. 4, pp. 1535–1556, 2005.

[4] K. Z. Rajab, R. Mittra, and M. T. Lanagan, "Size reduction of microstrip antennas using metamaterials," in *Proceedings of the IEEE Antennas and Propagation Society International Symposium*, pp. 296–299, Washington, DC, USA, July 2005.

[5] C. J. Zapata-Rodríguez, D. Pastor, L. E. Martínez, and J. J. Miret, "Left-handed metamaterial coatings for subwavelength-resolution imaging," *Journal of the Optical Society of America A: Optics and Image Science, and Vision*, vol. 29, no. 9, pp. 1992–1998, 2012.

[6] L. K. Hady and A. A. Kishk, "Electromagnetic scattering from conducting circular cylinder coated by metamaterials and loaded with helical strips under oblique incidence," *Progress In Electromagnetics Research B*, vol. 3, pp. 189–206, 2008.

[7] M.-Y. Wang, J. Xu, J. Wu, and W.-F. Wu, "Electromagnetic scattering of metallic sphere covered by double-negative metamaterials based on Mie series," *Systems Engineering and Electronics*, vol. 30, no. 11, pp. 2082–2086, 2008.

[8] R. Zhao, H.-L. Yang, N. Wu, and D.-G. Xie, "Near-field EM scattering by coated objects with left-handed materials," in *Proceedings of the Cross Strait Quad-Regional Radio Science and Wireless Technology Conference (CSQRWC '11)*, vol. 1, pp. 44–47, Harbin, China, July 2011.

[9] B. Wei, D.-B. Ge, and M.-Y. Wang, "Wide band scattering of perfectly conducting object coated with DNM," *Systems Engineering and Electronics*, vol. 28, no. 6, pp. 840–843, 2006.

[10] R. F. Harrington, *Field Computation by Moment Methods*, MacMillan, New York, NY, USA, 1968.

[11] C. P. Davis and W. C. Chew, "An alternative to impedance boundary conditions for dielectric-coated PEC surfaces," in *Proceedings of the IEEE Antennas and Propagation Society International Symposium*, pp. 2785–2788, IEEE, Honolulu, Hawaii, USA, June 2007.

[12] S. He, *Theoretical Modeling and Efficient Algorithm Research on Electromagnetic Scattering from Inhomogeneous and Complex Structures*, University of Electronic Science and Technology of China (UESTC), Chengdu, China, 2011.

[13] Z. Liu, J. Cao, and Y. Geng, "Simulation study on the EM scattering by left-hand-material," *Chinese Journal of Electron Devices*, vol. 34, no. 2, pp. 154–158, 2011.

[14] M. S. Wheeler, J. S. Aitchison, and M. Mojahedi, "Coated non-magnetic spheres with a negative index of refraction at infrared frequencies," *Physical Review B—Condensed Matter and Materials Physics*, vol. 73, no. 4, Article ID 045105, 2006.

[15] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Transactions on Antennas and Propagation*, vol. 30, no. 3, pp. 409–418, 1982.

[16] A. F. Peterson, S. L. Ray, and R. Mittra, *Computational Methods for Electromagnetics*, IEEE, New York, NY, USA, 1998.

[17] J. Shaeffer, "Direct solve of electrically large integral equations for problem sizes to 1 M unknowns," *IEEE Transactions on Antennas and Propagation*, vol. 56, no. 8, pp. 2306–2313, 2008.

[18] J. Shaeffer and F. Canning, "Adaptive cross approximation for MoM matrix fill for PC problem sizes to 157000 unknowns," in *Proceedings of the IEEE/ACES International Conference on Wireless Communications and Applied Computational Electromagnetics Society (ACES '05)*, pp. 748–753, Honolulu, Hawaii, USA, April 2005.

[19] M. Bebendorf, "Approximation of boundary element matrices," *Numerische Mathematik*, vol. 86, no. 4, pp. 565–589, 2000.

[20] K. Zhao, M. Vouvakis, and J.-F. Lee, "The adaptive cross approximation algorithm for accelerated method of moments computations of EMC problems," *IEEE Transactions on Electromagnetic Compatibility*, vol. 47, no. 4, pp. 763–773, 2005.

[21] R. Maaskant, R. M. Mittra, and A. Tijhuis, "Fast analysis of large antenna arrays using the characteristic basis function method and the adaptive cross approximation algorithm," *IEEE Transactions on Antennas and Propagation*, vol. 56, no. 11, pp. 3440–3451, 2008.

[22] C. Li, S.-Y. He, J. Yang, Z. Zhang, B.-X. Xiao, and G.-Q. Zhu, "Monostatic scattering from two-dimensional two-layer rough surfaces using hybrid 3DMLUV-ACA method," *International Journal of Applied Electromagnetics and Mechanics*, vol. 42, no. 1, pp. 1–11, 2013.

[23] D. Lü and C. Tong, "RCS simulation of target coated with chiral medium," *Fire Control & Command Control*, vol. 36, no. 6, pp. 67–70, 2011.