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Research Article

Trust-Region Method for Load Flow Solution of Three-Phase Unbalanced Electric Power Distribution System

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At present, the electric power system is getting bigger and more complex, and its loading is also increasing. As a consequence, planning, operation, and control of the power system also become more complicated. It is known that system planning and operation are mostly based on the steady-state condition of the power system, and the system steady-state condition can only be determined from the load flow study. Thus, the development of a reliable and efficient method to solve the load flow problem is necessary so that the system steady-state condition can properly be evaluated. Since the characteristics of the electric distribution system are different from those of the transmission system, special treatments are usually required in the distribution system load flow (DSLF) analysis. In this context, several interesting techniques have been proposed in the analysis. In this paper, the application and extension of the trust-region method to solve the three-phase DSLF problem are proposed and investigated. Case studies using 19-node, 25-node, and 123-node distribution systems are also given in this paper. Results of the studies show that the output values obtained by the proposed method are in excellent agreement with those obtained by previously published methods. These results confirm the validity of the proposed method. Case study results also indicate that the proposed method has better computational performances than the forward/backward sweeping (FBS) method.

1. Introduction

Load flow (or power flow) analysis is basically a solution for the normal operating conditions of a power system. The results of load flow analysis are normally used for system planning, basis data in the operational stage, and electric power system operation and control. Results obtained from load flow studies are also used for system steady-state studies, optimum scheduling of power generation, and system dynamic or stability studies. The significance of load flow analysis has attracted the attention of many engineers for several decades. Many researchers have spent much of their professional careers looking for solutions to load flow problems. A number of efforts that have been made to solve the load flow problem have produced a number of methods reported in many technical publications [1].

The Gauss-Seidel method was the first method that solves the load flow problem digitally. The method was invented by Ward and Hale in 1956. However, the method needs a significant amount of computer memory and iteration number as the power system size increases. The disadvantage of the Gauss-Seidel method triggers the development of the Newton–Raphson method, which was first developed by Van Ness and Griffin [1]. In contrast to the Gauss-Seidel algorithm, the Newton–Raphson-based methods require only a small number of iterations to obtain the solution. Moreover, the methods do not depend on the size of the system network. This advantage makes the method applicable to large power systems and has widely been used to solve the load flow problem of electric power transmission systems [2–15].

However, the application of the Newton–Raphson-based methods to electric power distribution systems can cause some convergence problems. These convergence problems arise because the characteristics of the distribution system are different from those of the electric power transmission system. Electric power distribution systems are usually characterized by (i) radial or weakly meshed network structures; (ii) high ratio of line resistance/reactance (R/X); (iii) very large number of branches and nodes; and (iv) unbalanced network and load. The characteristics (i)–(iii) can cause singularity of the Jacobian matrix in the Newton–Raphson iteration schemes, and the solution will be difficult or impossible to obtain. The characteristic (iv), that is, the system unbalance, requires that the system must be modeled in three phases, and consequently, the three-phase load flow study must be used instead of the single-phase load flow study. This will lead to additional computational effort. This computational effort will further increase if the load flow calculation is carried out repeatedly, for example, the case of service restoration, feeder reconfiguration, and optimal placement of capacitor. Thus, in addition to being reliable, the load flow solution method of an unbalanced electric power distribution system also needs to be efficient.

Other researchers have also conducted investigations into the load flow problem solution methods [16–25]. These researchers use a method or technique that utilizes the radial structure of the electric power distribution system. This technique is also known as the FBS technique. In finding a solution to the load flow problem, the forward/backward sweeping technique does not use the Newton–Raphson iterative scheme. Therefore, it does not have the convergence issue as in the Newton–Raphson method. The drawback of this method is that it requires some complicated branch numbering and bus ordering techniques. In the method investigated and proposed in this paper, the load flow problem is solved using the trust-region method. The trust-region method is commonly used to solve optimization problems. However, several researchers have carried out studies and applied the method to find a solution to a set of nonlinear equations [26–29]. The results of the studies show that this method is very potential to be used as a technique to solve a set of nonlinear equations. Results of the researchers' studies also show that the trust-region method can overcome the case of a singular Jacobian matrix since the optimization method is used in the solution updating process.

In [30], the trust-region method has successfully been applied to find a solution to the single-phase DSLF problem. However, in [30], the distribution system has been assumed to be balanced, and the single-phase load flow method has been used in the analysis. Since distribution systems are inherently unbalanced and to obtain accurate results, the system unbalance needs to be considered and taken into account in the analysis. As a consequence, the distribution system must be modeled in three phases, and the three-phase load flow method must be used in the analysis. Therefore, in this paper, the application and extension of the trust-region method proposed in [30] to three-phase load flow analysis are investigated. The contributions of the present paper can be outlined as follows:

(i) The developed algorithm has a better convergence characteristic, and only a small number of iterations are required in the calculation.

(ii) The proposed method is more efficient; that is, the load flow solution can be obtained with a minimum computation time.

To be more systematic, this paper is organized as follows: Section 2 discusses the formulation of the three-phase DSLF problem. Section 3 continues with an explanation of the proposed method for solving the load flow problem. Case study is presented in Section 4, where validation of the proposed method is also given. Finally, Section 5 points out some important conclusions of the paper.

2. Formulation of Three-Phase DSLF Problem

Load flow problem is usually solved using node analysis where the admittance matrix is frequently used in the analysis. In terms of node quantities, the behavior of a three-phase electric power distribution system can be explained using the following relationship:

$$\mathbf{I}^{\mathrm{abc}} - \mathbf{Y}^{\mathrm{abc}}\mathbf{V}^{\mathrm{abc}} = 0, \tag{1}$$

where $\mathbf{I}^{\mathrm{abc}}$ is the vector of nodal currents, $\mathbf{V}^{\mathrm{abc}}$ is the vector of nodal voltages, and $\mathbf{Y}^{\mathrm{abc}}$ is the system admittance matrix.

For distribution system with n nodes, $\mathbf{I}^{\mathrm{abc}}$, $\mathbf{V}^{\mathrm{abc}}$, and $\mathbf{Y}^{\mathrm{abc}}$ will have the following forms:

$$\mathbf{I}^{\mathrm{abc}} = \begin{bmatrix} \mathbf{I}_1^{\mathrm{abc}} \\ \mathbf{I}_2^{\mathrm{abc}} \\ \vdots \\ \mathbf{I}_n^{abc} \end{bmatrix}; \mathbf{V}^{\mathrm{abc}} = \begin{bmatrix} \mathbf{V}_1^{\mathrm{abc}} \\ \mathbf{V}_2^{\mathrm{abc}} \\ \vdots \\ \mathbf{V}_n^{\mathrm{abc}} \end{bmatrix}, \tag{2}$$

where

$$\mathbf{I}_i^{\mathrm{abc}} = \begin{bmatrix} I_i^a \\ I_i^b \\ I_i^c \end{bmatrix}; \mathbf{V}_i^{\mathrm{abc}} = \begin{bmatrix} V_i^a \\ V_i^b \\ V_i^c \end{bmatrix}, \tag{3a}$$

$$\mathbf{Y}_{ii}^{\mathrm{abc}} = \begin{bmatrix} Y_{ii}^{aa} & Y_{ii}^{ab} & Y_{ii}^{ac} \\ Y_{ii}^{ba} & Y_{ii}^{bb} & Y_{ii}^{bc} \\ Y_{ii}^{ca} & Y_{ii}^{cb} & Y_{ii}^{cc} \end{bmatrix}, \tag{3b}$$

$$\mathbf{Y}_{ij}^{\mathrm{abc}} = \begin{bmatrix} Y_{ij}^{aa} & Y_{ij}^{ab} & Y_{ij}^{ac} \\ Y_{ij}^{ba} & Y_{ij}^{bb} & Y_{ij}^{bc} \\ Y_{ij}^{ca} & Y_{ij}^{cb} & Y_{ij}^{cc} \end{bmatrix}. \tag{3c}$$

Nodal current in (1) can be expressed in terms of nodal voltage and nodal power as follows:

$$\mathbf{I}^{\mathrm{abc}} = \left\{ \left[\mathrm{diag}\left(\mathbf{V}^{\mathrm{abc}}\right) \right]^{-1} \left(\mathbf{S}_G^{\mathrm{abc}} - \mathbf{S}_L^{\mathrm{abc}} \right) \right\}^*, \tag{4}$$

where $\mathbf{S}_G^{\mathrm{abc}}$ is the vector of powers entering the node (generation powers) and $\mathbf{S}_G^{\mathrm{abc}}$ is the vector of powers leaving the node (load powers).

For distribution system with n nodes, formulations for $\mathbf{S_G}^{abc}$ and $\mathbf{S_L}^{abc}$ are given by

$$\mathbf{S}_G^{abc} = \begin{bmatrix} \mathbf{S}_{G1}^{abc} \\ \mathbf{S}_{G2}^{abc} \\ \vdots \\ \mathbf{S}_{Gn}^{abc} \end{bmatrix}; \mathbf{S}_L^{abc} = \begin{bmatrix} \mathbf{S}_{L1}^{abc} \\ \mathbf{S}_{L2}^{abc} \\ \vdots \\ \mathbf{S}_{Ln}^{abc} \end{bmatrix}, \quad (5)$$

where

$$\mathbf{S}_{Gi}^{abc} = \begin{bmatrix} S_{Gi}^{a} \\ S_{Gi}^{b} \\ S_{Gi}^{c} \end{bmatrix}; \mathbf{S}_{Li}^{abc} = \begin{bmatrix} S_{Li}^{a} \\ S_{Li}^{b} \\ S_{Li}^{c} \end{bmatrix}. \quad (6)$$

Substituting (4) into (1) results in

$$\left\{ \left[\mathrm{diag}\left(\mathbf{V}^{abc} \right) \right]^{-1} \left(\mathbf{S}_G^{abc} - \mathbf{S}_L^{abc} \right) \right\}^{*} - \mathbf{Y}^{abc}\mathbf{V}^{abc} = 0. \quad (7)$$

Equation (7) is the formulation of the three-phase DSLF problem. All of the variables (known and unknown) in the formulation are shown in Table 1. It is to be noted that distribution systems are normally fed at one node (substation node). Therefore, in DSLF analysis, the substation node is usually considered as a reference node, and the voltage magnitude of this node ($|V_{SS}|$) is specified at a certain value (e.g., 1.0 pu). Moreover, as the system is only fed at the substation node, power generations at the remaining nodes (load nodes) will be zero. It can be seen that (7) is a set of nonlinear equations which has to be solved in load flow analysis to evaluate the steady-state condition of the distribution system. The method of solution to these equations is explained in the next section.

3. Solution Technique

3.1. Trust-Region Method. Similar to the Newton–Raphson method, the iterative technique is also employed in the trust-region method to find a solution to a set of nonlinear equations [30]. Consider a general set of nonlinear equations in terms of vector function $\mathbf{F(x)}$ as follows:

$$\mathbf{F}(\mathbf{x}) = \begin{bmatrix} f_1(x_1, x_2, \ldots, x_n) \\ f_2(x_1, x_2, \ldots, x_n) \\ \vdots \\ f_n(x_1, x_2, \ldots, x_n) \end{bmatrix} = 0, \quad (8)$$

where n is the number of equations, $f_i(\mathbf{x})$ is the i^{th} nonlinear equation, and $\mathbf{x} = [\,x_1 \;\; x_2 \;\; \ldots \;\; x_n\,]^T$ is the vector of unknown variables (to be calculated).

In the iterative method, (8) is solved using

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \mathbf{d}^{(k)}, \quad (9)$$

where k is the iteration count and $\mathbf{d} = [\,d_1 \;\; d_2 \;\; \ldots \;\; d_n\,]^T$ is the vector of correction factors.

In the Newton–Raphson method, the vector of correction factors is calculated directly. However, as the Jacobian matrix for the distribution system is sometimes singular, the calculation does not always succeed, and the solution cannot be obtained. In the trust-region method, a different technique is used to calculate the vector of correction factors. It is determined through the optimization process, where inverting the Jacobian matrix is not required in the process. In this way, the trust-region method can always produce a valid solution. In the trust-region method, the vector of correction factors is determined using

Table 1: Known and unknown variables in DSLF formulation.

Node	Known variable	Unknown variable
Substation	$\mathbf{V}^{abc} = \begin{bmatrix} \lvert V_{SS}\rvert\angle 0^o \\ \lvert V_{SS}\rvert\angle -120^o \\ \lvert V_{SS}\rvert\angle 120^o \end{bmatrix}$	$\mathbf{S}_G^{abc}$
Load	$\mathbf{S}_G^{abc} = [\,0 \;\; 0 \;\; 0\,]^T$	$\mathbf{V}^{abc}$

$$\min_{\mathbf{d}^{(k)}} \left\| q\left(\mathbf{d}^{(k)} \right) \right\|^2, \quad \text{with constraint}: \left\| \mathbf{d}^{(k)} \right\| \le \Delta^{(k)}, \quad (10)$$

where $\Delta^{(k)} > 0$ is the radius of the trust region. Details on how to choose the radius value in every iteration step can be found in [26–30]. Also, in (10), quantity $q(\mathbf{d}^{(k)})$ is determined using

$$q\left(\mathbf{d}^{(k)}\right) = \frac{1}{2} h\left(\mathbf{x}^{(k)}\right) + \mathbf{d}^{(k)T}\mathbf{g}\left(\mathbf{x}^{(k)}\right) + \frac{1}{2}\mathbf{d}^{(k)T}\mathbf{H}\left(\mathbf{x}^{(k)}\right)\mathbf{d}^{(k)}, \quad (11)$$

where

$$h\left(\mathbf{x}^{(k)}\right) = F\left(\mathbf{x}^{(k)}\right)^T F\left(\mathbf{x}^{(k)}\right), \quad (12)$$

$$\mathbf{g}\left(\mathbf{x}^{(k)}\right) = \mathbf{J}\left(\mathbf{x}^{(k)}\right)^T F\left(\mathbf{x}^{(k)}\right), \quad (13)$$

$$\mathbf{H}\left(\mathbf{x}^{(k)}\right) = \mathbf{J}\left(\mathbf{x}^{(k)}\right)^T \mathbf{J}\left(\mathbf{x}^{(k)}\right), \quad (14)$$

and the Jacobian matrix $J(x)$ in (13) and (14) has the following form:

$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}. \quad (15)$$

Elements of $\mathbf{J(x)}$ can be calculated analytically or numerically. However, calculation using the numerical method is more advantageous because the analytic formulations of the partial derivative are sometimes difficult to obtain. An explanation of the numerical method for the determination of the Jacobian matrix is given in Appendix A.

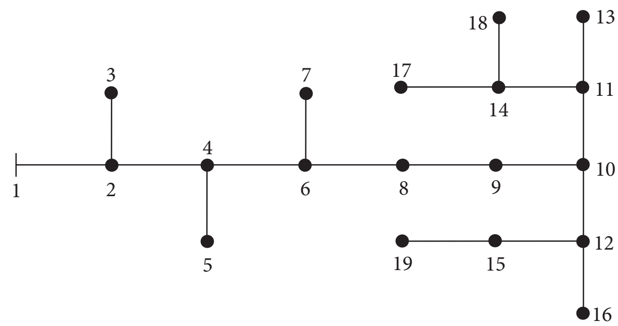

Figure 1: 19-bus network.

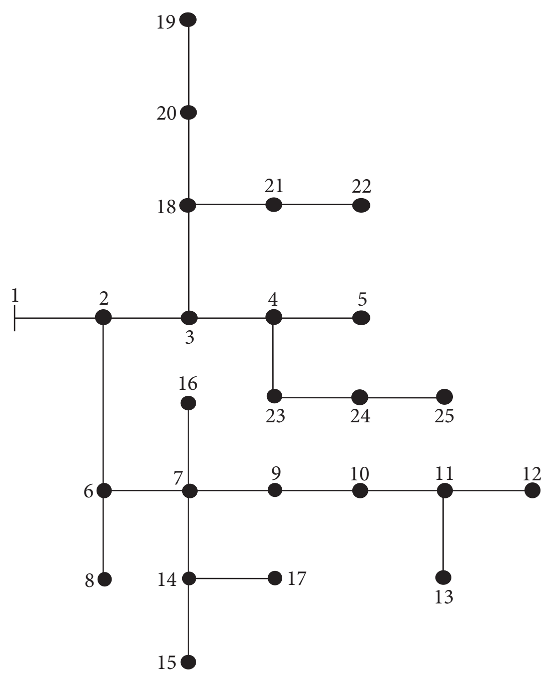

Figure 2: 25-bus network.

3.2. Starting Values for Iteration. To initialize the iteration process in the trust-region method, the following starting values for the unknown variables can be used:

Substation power:

$$\mathbf{S}_G^{\mathrm{abc}\,(0)} = \sum_{Li}^{\mathrm{abc}} (\text{total load in the system}). \qquad (16)$$

Load node voltages:

$$\mathbf{V}_i^{\mathrm{abc}\,(0)} = \begin{bmatrix} 1\angle 0^o \\ 1\angle -120^o \\ 1\angle 120^o \end{bmatrix}. \qquad (17)$$

4. Case Study

4.1. Test Systems and Software Conditions. To validate the method proposed in Section 3, the following three unbalanced distribution networks areused:

(i) 19-bus network [31, 32]

Single line diagram of this 11 kV distribution network is shown in Figure 1. The system (line and load) data are given in (Tables 2 and 3).

(ii) 25-bus network [31]

Single line diagram of this 4.16 kV distribution network is shown in Figure 2. The system (line and load) data are given in (Tables 4–6).

(iii) 123-bus network adopted from [33]

Single line diagram of this 4.16 kV distribution network is shown in Figure 3. The system (line and load) data are given in (Tables 7–9).

It is to be noted that MATLAB® software has been used for all of the computations in the present work (the proposed method algorithm has been implemented as MATLAB® codes or m-files).

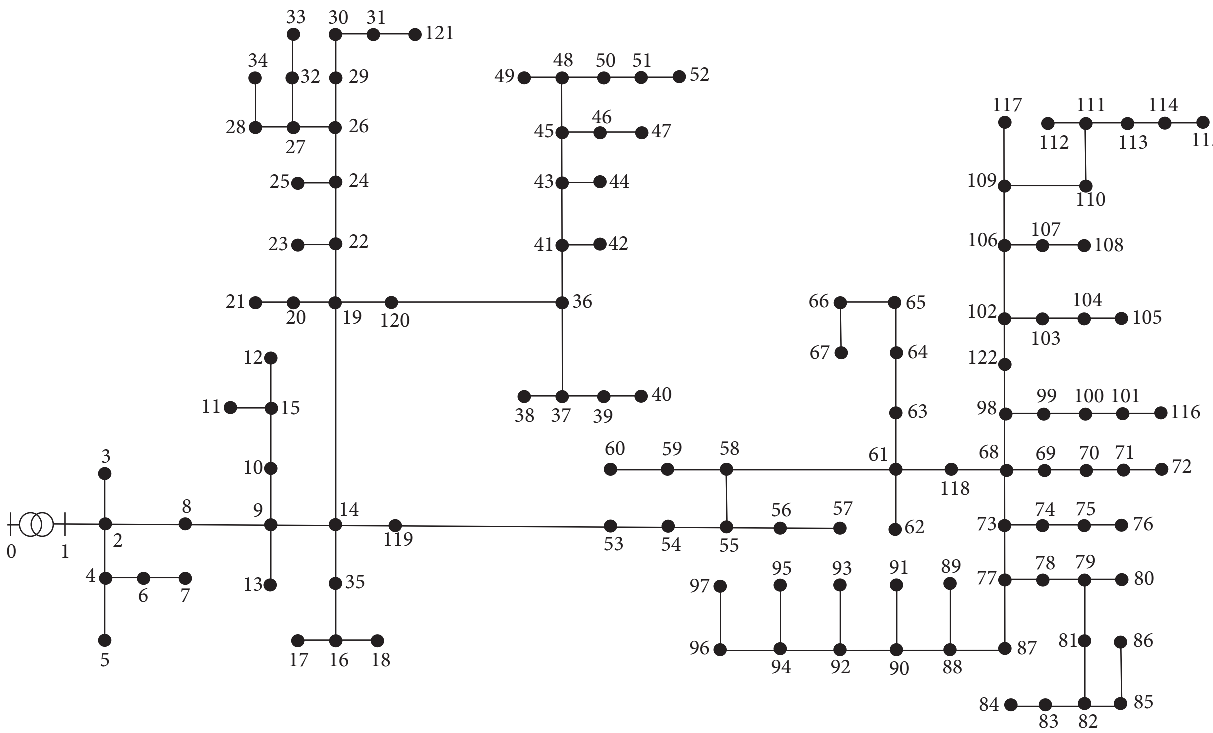

Figure 3: 123-bus network.

Table 2: Line impedance of 19-bus network.

No.	Line	Self-impedance (ohm)	Mutual impedance (ohm)
1	1–2	$3.0 \times (1.56090 + j0.67155)$	$3.0 \times (0.52030 + j0.22385)$
2	2–3	$5.0 \times (1.56090 + j0.67155)$	$5.0 \times (0.52030 + j0.22385)$
3	2–4	$1.5 \times (1.56090 + j0.67155)$	$1.5 \times (0.52030 + j0.22385)$
4	4–5	$1.5 \times (1.56090 + j0.67155)$	$1.5 \times (0.52030 + j0.22385)$
5	4–6	$1.0 \times (1.56090 + j0.67155)$	$1.0 \times (0.52030 + j0.22385)$
6	6–7	$2.0 \times (1.56090 + j0.67155)$	$2.0 \times (0.52030 + j0.22385)$
7	6–8	$2.5 \times (1.56090 + j0.67155)$	$2.5 \times (0.52030 + j0.22385)$
8	8–9	$3.0 \times (1.56090 + j0.67155)$	$3.0 \times (0.52030 + j0.22385)$
9	9–10	$5.0 \times (1.56090 + j0.67155)$	$5.0 \times (0.52030 + j0.22385)$
10	10–11	$1.5 \times (1.56090 + j0.67155)$	$1.5 \times (0.52030 + j0.22385)$
11	10–12	$1.5 \times (1.56090 + j0.67155)$	$1.5 \times (0.52030 + j0.22385)$
12	11–13	$5.0 \times (1.56090 + j0.67155)$	$5.0 \times (0.52030 + j0.22385)$
13	11–14	$1.0 \times (1.56090 + j0.67155)$	$1.0 \times (0.52030 + j0.22385)$
14	12–15	$5.0 \times (1.56090 + j0.67155)$	$5.0 \times (0.52030 + j0.22385)$
15	12–16	$6.0 \times (1.56090 + j0.67155)$	$6.0 \times (0.52030 + j0.22385)$
16	14–17	$3.5 \times (1.56090 + j0.67155)$	$3.5 \times (0.52030 + j0.22385)$
17	14–18	$4.0 \times (1.56090 + j0.67155)$	$4.0 \times (0.52030 + j0.22385)$
18	15–19	$4.0 \times (1.56090 + j0.67155)$	$4.0 \times (0.52030 + j0.22385)$

4.2. Results and Discussion. Output of the load flow analysis in terms of system voltage magnitudes are shown in Tables 10–12. For comparison purposes, output from other methods (i.e., FBS method [16–25, 31, 32]) and results from the OpenDSS tool [34] are also shown in the tables. It can be seen that the output of the proposed method is in excellent agreement with those from other methods. These results confirm the validity of the proposed method for solving the three-phase DSLF problem. In addition, Table 13 gives computational performances of the proposed method and the FBS method. The computations were run on a PC with Intel Core 2 2.4 GHz processor. It can be seen from Table 13 that the proposed method requires fewer iterations, which indicates that it has a better convergence characteristic. Moreover, the proposed trust-region method is more efficient than the BFS method since the BFS method consumes more computation time than the proposed method.

Table 3: Loads of 19-bus network.

Node	Phase a		Phase b		Phase c	
	P (kW)	Q (kVAR)	P (kW)	Q (kVAR)	P (kW)	Q (kVAR)
1	0	0	0	0	0	0
2	10.38	5.01	5.19	2.52	10.38	5.01
3	11.01	5.34	5.19	2.52	9.72	4.71
4	4.05	1.95	5.67	2.76	6.48	3.15
5	6.48	3.15	5.19	2.52	4.53	2.19
6	4.20	2.04	3.09	1.50	2.91	1.41
7	9.72	4.71	8.10	3.93	8.10	3.93
8	7.44	3.60	5.34	2.58	3.39	1.65
9	12.30	5.97	14.91	7.23	13.29	6.42
10	3.39	1.65	4.20	2.04	2.58	1.26
11	7.44	3.60	7.44	3.60	11.01	5.34
12	9.72	4.71	8.10	3.93	8.10	3.93
13	4.38	2.13	5.34	2.58	6.48	3.15
14	3.09	1.50	3.09	1.50	4.05	1.95
15	4.38	2.13	4.86	2.34	6.96	3.36
16	7.77	3.78	10.38	5.01	7.77	3.78
17	6.48	3.15	4.86	2.34	4.86	2.34
18	5.34	2.58	5.34	2.58	5.52	2.67
19	8.76	4.23	10.05	4.86	7.14	3.45

Table 4: Line impedance of 25-bus network.

Conductor type		Impedance (ohm/mile)		
		a	b	c
1	a	$0.3686 + j0.6852$	$0.0169 + j0.1515$	$0.0155 + j0.1098$
	b	$0.0169 + j0.1515$	$0.3757 + j0.6715$	$0.0188 + j0.2072$
	c	$0.0155 + j0.1098$	$0.0188 + j0.2072$	$0.3723 + j0.6783$
2	a	$0.9775 + j0.8717$	$0.0167 + j0.1697$	$0.0152 + j0.1264$
	b	$0.0167 + j0.1697$	$0.9844 + j0.8654$	$0.0186 + j0.2275$
	c	$0.0152 + j0.1264$	$0.0186 + j0.2275$	$0.9810 + j0.8648$
3	a	$1.9280 + j1.4194$	$0.0161 + j0.1183$	$0.0161 + j0.1183$
	b	$0.0161 + j0.1183$	$1.9308 + j1.4215$	$0.0161 + j0.1183$
	c	$0.0161 + j0.1183$	$0.0161 + j0.1183$	$1.9337 + j1.4236$

Table 5: Type and length of conductor lines of 25-bus network.

No.	Line	Conductor type	Length (ft)
1	1–2	1	1000
2	2–3	1	500
3	2–6	2	500
4	3–4	1	500
5	3–18	2	500
6	4–5	2	500
7	4–23	2	400
8	6–7	2	500
9	6–8	2	1000
10	7–9	2	500
11	7–14	2	500
12	7–16	2	500
13	9–10	2	500
14	10–11	2	300
15	11–12	3	200
16	11–13	3	200
17	14–15	2	300
18	14–17	3	300
19	18–20	2	500
20	18–21	3	400
21	20–19	3	400
22	21–22	3	400
23	23–24	2	400
24	24–25	3	400

Table 6: Loads of 25-bus network.

Bus	Phase a		Phase b		Phase c	
	P (kW)	Q (kVAR)	P (kW)	Q (kVAR)	P (kW)	Q (kVAR)
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	35	25	40	30	45	32
4	50	40	60	45	50	35
5	40	30	40	30	40	30
6	40	30	45	32	35	25
7	0	0	0	0	0	0
8	40	30	40	30	40	30
9	60	45	50	40	50	35
10	35	25	40	30	45	32
11	45	32	35	25	40	30
12	50	35	60	45	50	40
13	35	25	45	32	40	30
14	50	35	50	40	60	45
15	133	100	133	100	133	100
16	40	30	40	30	40	30
17	40	30	35	25	45	32
18	40	30	40	30	40	30
19	60	45	50	35	50	40
20	35	25	40	30	45	32
21	40	30	35	25	45	32
22	50	35	60	45	50	40
23	60	45	50	40	50	35
24	35	25	45	32	40	30
25	60	45	50	30	50	35

Table 7: Line impedance of 123-bus network.

Conductor type		Impedance (ohm/mile)		
		a	b	c
1	a	$0.4576 + j1.0780$	$0.1560 + j0.5017$	$0.1535 + j0.3849$
	b	$0.1560 + j0.5017$	$0.4666 + j1.0482$	$0.1580 + j0.4236$
	c	$0.1560 + j0.5017$	$0.1580 + j0.4236$	$0.4615 + j1.0651$
2	a	$0.4666 + j1.0482$	$0.1580 + j0.4236$	$0.1560 + j0.5017$
	b	$0.1580 + j0.4236$	$0.4615 + j1.0651$	$0.1535 + j0.3849$
	c	$0.1560 + j0.5017$	$0.1535 + j0.3849$	$0.4576 + j1.0780$
3	a	$0.4615 + j1.0651$	$0.1535 + j0.3849$	$0.1580 + j0.4236$
	b	$0.1535 + j0.3849$	$0.4576 + j1.0780$	$0.1560 + j0.5017$
	c	$0.1580 + j0.4236$	$0.1560 + j0.5017$	$0.4666 + j1.0482$
4	a	$0.4615 + j1.0651$	$0.1580 + j0.4236$	$0.1535 + j0.3849$
	b	$0.1580 + j0.4236$	$0.4666 + j1.0482$	$0.1560 + j0.5017$
	c	$0.1535 + j0.3849$	$0.1560 + j0.5017$	$0.4576 + j1.0780$
5	a	$0.4666 + j1.0482$	$0.1560 + j0.5017$	$0.1580 + j0.4236$
	b	$0.1560 + j0.5017$	$0.4576 + j1.0780$	$0.1535 + j0.3849$
	c	$0.1580 + j0.4236$	$0.1535 + j0.3849$	$0.4615 + j1.0651$
6	a	$0.4576 + j1.0780$	$0.1535 + j0.3849$	$0.1560 + j0.5017$
	b	$0.1535 + j0.3849$	$0.4615 + j1.0651$	$0.1580 + j0.4236$
	c	$0.1560 + j0.5017$	$0.1580 + j0.4236$	$0.4666 + j1.0482$
7	a	$1.5209 + j0.7521$	$0.5198 + j0.2775$	$0.4924 + j0.2157$
	b	$0.5198 + j0.2775$	$1.5329 + j0.7162$	$0.5198 + j0.2775$
	c	$0.4924 + j0.2157$	$0.5198 + j0.2775$	$1.5209 + j0.7521$

Table 8: Type and length of conductor lines of 123-bus network.

No.	Line	Conductor type	Length (ft)
1	0–1	*	*
2	1–2	1	400
3	2–3	5	175
4	2–4	6	250
5	2–8	1	300
6	4–5	6	200
7	4–6	6	325
8	6–7	6	250
9	8–9	1	200
10	9–13	5	225
11	9–10	4	225
12	9–14	1	300
13	10–15	4	425
14	14–35	6	150
15	14–19	1	825
16	12–15	4	250
17	11–15	4	250
18	16–17	6	375
19	16–18	6	350
20	19–20	4	250
21	19–22	1	300
22	20–21	4	325
23	22–23	5	525
24	22–24	1	250
25	24–25	6	550
26	24–26	1	275
27	26–27	2	350
28	26–29	1	200
29	27–28	2	275
30	27–32	6	225
31	28–34	4	500
32	29–30	1	300
33	30–31	1	350
34	31–121	1	200
35	32–33	6	300
36	35–16	6	100
37	36–37	3	650
38	36–41	1	250
39	37–38	4	300
40	37–39	5	250
41	39–40	5	325
42	41–42	6	325
43	41–43	1	250
44	43–44	5	500
45	43–45	1	200
46	45–46	4	200
47	45–48	1	250
48	46–47	4	300
49	48–49	1	150
50	48–50	1	250
51	50–51	1	250
52	51–52	1	250
53	53–54	1	200
54	54–55	1	125
55	55–56	1	275
56	55–58	1	350
57	56–57	1	275
58	58–59	5	250
59	58–61	1	750
60	59–60	5	250
61	61–62	1	550
62	61–63	7	250

Table 8: Continued.

No.	Line	Conductor type	Length (ft)
63	63–64	7	175
64	64–65	7	350
65	65–66	7	425
66	66–67	7	325
67	68–69	4	200
68	68–73	1	275
69	68–98	1	250
70	69–70	4	275
71	70–71	4	325
72	71–72	4	275
73	73–74	6	275
74	73–77	1	200
75	74–75	6	350
76	75–76	6	400
77	77–78	1	400
78	77–87	1	700
79	78–79	1	100
80	79–80	1	225
81	79–81	1	475
82	81–82	1	475
83	82–83	1	250
84	82–85	6	675
85	83–84	1	250
86	85–86	6	475
87	87–88	1	450
88	88–89	4	175
89	88–90	1	275
90	90–91	5	225
91	90–92	1	225
92	92–93	6	300
93	92–94	1	225
94	94–95	4	275
95	94–96	1	300
96	96–97	5	200
97	98–99	1	275
98	99–100	1	550
99	100–101	1	300
100	101–116	1	800
101	102–103	6	225
102	102–106	1	275
103	103–104	6	325
104	104–105	6	700
105	106–107	5	225
106	106–109	1	325
107	107–108	5	575
108	109–110	4	450
109	109–117	1	1000
110	110–111	4	300
111	111–112	4	575
112	111–113	4	125
113	113–114	4	525
114	114–115	4	325
115	36–120	1	375
116	53–119	1	400
117	68–118	1	350
118	102–122	1	250
119	61–118	1	250
120	19–120	1	250
121	14–119	1	250
122	98–122	1	250

*Transformer with impedance: $Z_{aa} = Z_{bb} = Z_{cc} = 0.017306 + j0.138444$ ohms and $Z_{ab} = Z_{ac} = Z_{bc} = 0$ ohms.

Table 9: Loads of 123-bus network.

Bus	Phase a		Phase b		Phase c	
	P (kW)	Q (kVAR)	P (kW)	Q (kVAR)	P (kW)	Q (kVAR)
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	40	20	0	0	0	0
3	0	0	20	10	0	0
4	0	0	0	0	0	0
5	0	0	0	0	40	20
6	0	0	0	0	20	10
7	0	0	0	0	40	20
8	20	10	0	0	0	0
9	0	0	0	0	0	0
10	40	20	0	0	0	0
11	20	10	0	0	0	0
12	40	20	0	0	0	0
13	0	0	20	10	0	0
14	0	0	0	0	0	0
15	0	0	0	0	0	0
16	0	0	0	0	0	0
17	0	0	0	0	40	20
18	0	0	0	0	20	10
19	0	0	0	0	0	0
20	40	20	0	0	0	0
21	40	20	0	0	0	0
22	0	0	0	0	0	0
23	0	0	40	20	0	0
24	0	0	0	0	0	0
25	0	0	0	0	40	20
26	0	0	0	0	0	0
27	0	0	0	0	0	0
28	0	0	0	0	0	0
29	40	20	0	0	0	0
30	40	20	0	0	0	0
31	0	0	0	0	40	20
32	0	0	0	0	20	10
33	0	0	0	0	20	10
34	40	20	0	0	0	0
35	0	0	0	0	40	20
36	40	20	0	0	0	0
37	0	0	0	0	0	0
38	40	20	0	0	0	0
39	0	0	20	10	0	0
40	0	0	20	10	0	0
41	0	0	0	0	0	0
42	0	0	0	0	20	10
43	20	10	0	0	0	0
44	0	0	40	20	0	0
45	0	0	0	0	0	0
46	20	10	0	0	0	0
47	20	10	0	0	0	0
48	35	25	35	25	35	25
49	70	50	70	50	70	50
50	35	25	70	50	35	20
51	0	0	0	0	40	20
52	20	10	0	0	0	0
53	40	20	0	0	0	0
54	40	20	0	0	0	0
55	0	0	0	0	0	0
56	20	10	0	0	0	0
57	0	0	20	10	0	0
58	0	0	0	0	0	0
59	0	0	20	10	0	0
60	0	0	20	10	0	0
61	20	10	0	0	0	0

Table 9: Continued.

Bus	Phase a		Phase b		Phase c	
	P (kW)	Q (kVAR)	P (kW)	Q (kVAR)	P (kW)	Q (kVAR)
62	0	0	0	0	0	0
63	0	0	0	0	40	20
64	40	20	0	0	0	0
65	0	0	75	35	0	0
66	0	0	0	0	0	0
67	35	25	35	25	70	50
68	0	0	0	0	0	0
69	20	10	0	0	0	0
70	40	20	0	0	0	0
71	20	10	0	0	0	0
72	40	20	0	0	0	0
73	0	0	0	0	0	0
74	0	0	0	0	40	20
75	0	0	0	0	40	20
76	0	0	0	0	40	20
77	105	80	70	50	70	50
78	0	0	40	20	0	0
79	0	0	0	0	0	0
80	40	20	0	0	0	0
81	0	0	40	20	0	0
82	0	0	0	0	0	0
83	40	20	0	0	0	0
84	0	0	0	0	20	10
85	0	0	0	0	20	10
86	0	0	0	0	40	20
87	0	0	20	10	0	0
88	0	0	40	20	0	0
89	40	20	0	0	0	0
90	0	0	0	0	0	0
91	0	0	40	20	0	0
92	0	0	0	0	0	0
93	0	0	0	0	40	20
94	0	0	0	0	0	0
95	40	20	0	0	0	0
96	0	0	20	10	0	0
97	0	0	20	10	0	0
98	0	0	0	0	0	0
99	40	20	0	0	0	0
100	0	0	40	20	0	0
101	0	0	0	0	40	20
102	0	0	0	0	0	0
103	0	0	0	0	20	10
104	0	0	0	0	40	20
105	0	0	0	0	40	20
106	0	0	0	0	0	0
107	0	0	40	20	0	0
108	0	0	40	20	0	0
109	0	0	0	0	0	0
110	40	20	0	0	0	0
111	0	0	0	0	0	0
112	20	10	0	0	0	0
113	20	10	0	0	0	0
114	40	20	0	0	0	0
115	20	10	0	0	0	0
116	0	0	0	0	0	0
117	0	0	0	0	0	0
118	0	0	0	0	0	0
119	0	0	0	0	0	0
120	0	0	0	0	0	0
121	0	0	0	0	0	0
122	0	0	0	0	0	0

Table 10: Voltage profile of 19-bus network.

Node	Proposed method			FBS		
	$\|V_a\|$	$\|V_b\|$	$\|V_c\|$	$\|V_a\|$	$\|V_b\|$	$\|V_c\|$
1	1.00000	1.00000	1.00000	1.0000	1.0000	1.0000
2	0.98746	0.98910	0.98798	0.9875	0.9891	0.9880
3	0.98542	0.98869	0.98633	0.9854	0.9887	0.9863
4	0.98235	0.98390	0.98301	0.9824	0.9839	0.9830
5	0.98201	0.98366	0.98283	0.9820	0.9837	0.9828
6	0.97928	0.98078	0.98005	0.9793	0.9808	0.9801
7	0.97861	0.98029	0.97956	0.9786	0.9803	0.9796
8	0.97281	0.97381	0.97347	0.9728	0.9738	0.9735
9	0.96592	0.96598	0.96575	0.9659	0.9660	0.9657
10	0.95625	0.95549	0.95500	0.9563	0.9555	0.9550
11	0.95499	0.95429	0.95330	0.9550	0.9543	0.9533
12	0.95478	0.95377	0.95358	0.9548	0.9538	0.9536
13	0.95440	0.95344	0.95210	0.9544	0.9534	0.9521
14	0.95449	0.95388	0.95282	0.9545	0.9539	0.9528
15	0.95274	0.95122	0.95126	0.9527	0.9512	0.9513
16	0.95339	0.95147	0.95217	0.9534	0.9515	0.9522
17	0.95365	0.95377	0.95232	0.9537	0.9534	0.9523
18	0.95380	0.95319	0.95209	0.9538	0.9532	0.9521
19	0.95159	0.94976	0.95047	0.9516	0.9498	0.9505

Table 11: Voltage profile of 25-bus network.

Node	Proposed method			FBS		
	$\|V_a\|$	$\|V_b\|$	$\|V_c\|$	$\|V_a\|$	$\|V_b\|$	$\|V_c\|$
1	1.00000	1.00000	1.00000	1.0000	1.0000	1.0000
2	0.97020	0.97110	0.97545	0.9702	0.9711	0.9755
3	0.96323	0.96444	0.96984	0.9632	0.9644	0.9698
4	0.95978	0.96219	0.96739	0.9598	0.9613	0.9674
5	0.95872	0.96025	0.96644	0.9587	0.9603	0.9664
6	0.95948	0.95587	0.96148	0.9550	0.9559	0.9615
7	0.94191	0.94283	0.94923	0.9419	0.9428	0.9492
8	0.95286	0.95378	0.95957	0.9529	0.9538	0.9596
9	0.93588	0.93668	0.94379	0.9359	0.9367	0.9438
10	0.93149	0.93186	0.93953	0.9315	0.9319	0.9395
11	0.92941	0.92963	0.93763	0.9294	0.9296	0.9376
12	0.92841	0.92839	0.93659	0.9284	0.9284	0.9366
13	0.92871	0.92872	0.93682	0.9287	0.9287	0.9368
14	0.93594	0.93699	0.94338	0.9359	0.9370	0.9434
15	0.93377	0.93487	0.94144	0.9338	0.9349	0.9414
16	0.94083	0.94177	0.94826	0.9408	0.9418	0.9483
17	0.93473	0.93595	0.94203	0.9347	0.9360	0.9420
18	0.95732	0.95864	0.96432	0.9573	0.9586	0.9643
19	0.95241	0.95443	0.95998	0.9524	0.9544	0.9600
20	0.95482	0.95634	0.96201	0.9548	0.9563	0.9620
21	0.95379	0.95487	0.96053	0.9537	0.9549	0.9605
22	0.95184	0.95246	0.95852	0.9518	0.9525	0.9585
23	0.95646	0.95838	0.96479	0.9565	0.9584	0.9648
24	0.95443	0.95651	0.96311	0.9544	0.9565	0.9631
25	0.95202	0.95469	0.96117	0.9520	0.9547	0.9612

Table 12: Voltage profile of 123-bus network.

Node	Proposed method			OpenDSS		
	$\|V_a\|$	$\|V_b\|$	$\|V_c\|$	$\|V_a\|$	$\|V_b\|$	$\|V_c\|$
0	1.00000	1.0000	1.00000	0.99994	0.99994	0.99996
1	0.99187	0.99480	0.99351	0.99185	0.99476	0.99345
2	0.98680	0.99368	0.98959	0.98676	0.99366	0.98955
3	0.98680	0.99364	0.98961	0.98670	0.99360	0.98952
4	0.98696	0.99365	0.98932	0.98691	0.99360	0.98923
5	0.98701	0.99364	0.98923	0.98696	0.99363	0.98920
6	0.98709	0.99362	0.98910	0.98706	0.99356	0.98904
7	0.98715	0.99361	0.98899	0.98710	0.99359	0.98893
8	0.98301	0.99287	0.98696	0.98295	0.99283	0.98691
9	0.98053	0.99230	0.98521	0.98046	0.99224	0.98512
10	0.98028	0.99242	0.98519	0.98024	0.99240	0.98516
11	0.97994	0.99259	0.98516	0.97990	0.99256	0.98513
12	0.97988	0.99261	0.98515	0.97978	0.99255	0.98514
13	0.98052	0.99225	0.98523	0.98052	0.99223	0.98514
14	0.97716	0.99133	0.98259	0.97707	0.99125	0.98252
15	0.97999	0.99256	0.98516	0.97990	0.99246	0.98511
16	0.97729	0.99130	0.98236	0.97721	0.99123	0.98229
17	0.97738	0.99128	0.98219	0.97737	0.99125	0.98214
18	0.97733	0.99129	0.98228	0.97731	0.99124	0.98221
19	0.97321	0.99072	0.98041	0.97317	0.99071	0.98035
20	0.97298	0.99083	0.98039	0.97291	0.99074	0.98032
21	0.97284	0.99090	0.98038	0.97282	0.99081	0.98032
22	0.97296	0.99078	0.98004	0.97288	0.99069	0.97994
23	0.97292	0.99054	0.98014	0.97291	0.99052	0.98012
24	0.97277	0.99093	0.97967	0.97270	0.99087	0.97966
25	0.97291	0.99090	0.97943	0.97286	0.99090	0.97942
26	0.97251	0.99111	0.97940	0.97243	0.99107	0.97939
27	0.97244	0.99117	0.97922	0.97237	0.99114	0.97917
28	0.97232	0.99123	0.97920	0.97223	0.99122	0.97915
29	0.97237	0.99121	0.97929	0.97228	0.99119	0.97925
30	0.97229	0.99127	0.97915	0.97226	0.99122	0.97907
31	0.97236	0.99125	0.97899	0.97229	0.99124	0.97893
32	0.9725	0.99116	0.97912	0.97248	0.99110	0.97904
33	0.97253	0.99116	0.97905	0.97253	0.99111	0.97896
34	0.97209	0.99134	0.97918	0.97202	0.99127	0.97908
35	0.97725	0.99131	0.98242	0.97720	0.99124	0.98240
36	0.97130	0.98983	0.97958	0.97125	0.98977	0.97957
37	0.97098	0.98967	0.97972	0.97089	0.98967	0.97965
38	0.97084	0.98974	0.97970	0.97078	0.98973	0.97970
39	0.97096	0.98956	0.97977	0.97090	0.98953	0.97971
40	0.97095	0.98949	0.97980	0.97086	0.98944	0.97974
41	0.97078	0.98946	0.97922	0.97070	0.98939	0.97914
42	0.97082	0.98945	0.97915	0.97076	0.98941	0.97910
43	0.97023	0.98909	0.97891	0.97021	0.98900	0.97888
44	0.97019	0.98887	0.97901	0.97017	0.98879	0.97894
45	0.96986	0.98885	0.97863	0.96977	0.98876	0.97856
46	0.96977	0.98890	0.97862	0.96976	0.98884	0.97857
47	0.96970	0.98893	0.97862	0.96965	0.98890	0.97858
48	0.96950	0.98850	0.97828	0.96948	0.98849	0.97827
49	0.96940	0.98842	0.97820	0.96930	0.98836	0.97814
50	0.96940	0.98833	0.97816	0.96933	0.98825	0.97813
51	0.96939	0.98835	0.97804	0.96934	0.98831	0.97804
52	0.96933	0.98838	0.97804	0.96929	0.98838	0.97796
53	0.97266	0.98979	0.97934	0.97265	0.98976	0.97932
54	0.97136	0.98926	0.97836	0.97129	0.98925	0.97831
55	0.97061	0.98890	0.97774	0.97061	0.98887	0.97767
56	0.97054	0.98888	0.97777	0.97053	0.98883	0.97773
57	0.97053	0.98882	0.97780	0.97048	0.98876	0.97772
58	0.96860	0.98793	0.97600	0.96859	0.98788	0.97596
59	0.96858	0.98782	0.97605	0.96850	0.98773	0.97598
60	0.96857	0.98776	0.97607	0.96849	0.98771	0.97600

Table 12: Continued.

Node	Proposed method			OpenDSS		
	$\|V_a\|$	$\|V_b\|$	$\|V_c\|$	$\|V_a\|$	$\|V_b\|$	$\|V_c\|$
61	0.96435	0.98617	0.97210	0.96428	0.98608	0.97206
62	0.96435	0.98617	0.97210	0.96433	0.98611	0.97210
63	0.96417	0.98583	0.97127	0.96410	0.98574	0.97123
64	0.96402	0.98557	0.97083	0.96396	0.98555	0.97079
65	0.96402	0.98500	0.96991	0.96392	0.98493	0.96988
66	0.96392	0.98496	0.96867	0.96385	0.98493	0.96865
67	0.96400	0.98504	0.96815	0.96392	0.98497	0.96807
68	0.96121	0.98526	0.97001	0.96117	0.98519	0.96996
69	0.96094	0.98540	0.96998	0.96090	0.98539	0.96989
70	0.96063	0.98555	0.96995	0.96055	0.98552	0.96992
71	0.96041	0.98565	0.96993	0.96040	0.98563	0.96986
72	0.96028	0.98571	0.96992	0.96027	0.98565	0.96988
73	0.96058	0.98473	0.96939	0.96057	0.98464	0.96930
74	0.96080	0.98468	0.96902	0.96076	0.98465	0.96894
75	0.96098	0.98465	0.96870	0.96089	0.98457	0.96867
76	0.96108	0.98463	0.96852	0.96100	0.98456	0.96850
77	0.96001	0.98437	0.96920	0.96000	0.98437	0.96912
78	0.95974	0.98418	0.96898	0.95970	0.98412	0.96889
79	0.95968	0.98418	0.96891	0.95963	0.98414	0.96887
80	0.95958	0.98424	0.96890	0.95954	0.98414	0.96883
81	0.95962	0.98404	0.96856	0.95956	0.98404	0.96852
82	0.95960	0.98412	0.96812	0.95953	0.98407	0.96803
83	0.95951	0.98418	0.96805	0.95948	0.98414	0.96797
84	0.95953	0.98417	0.96799	0.95949	0.98413	0.96798
85	0.95986	0.98407	0.96766	0.95986	0.98399	0.96758
86	0.95998	0.98404	0.96745	0.95988	0.98401	0.96735
87	0.95932	0.98361	0.96936	0.95931	0.98353	0.96931
88	0.95890	0.98323	0.96941	0.95889	0.98318	0.96932
89	0.95882	0.98327	0.96940	0.95879	0.98326	0.96935
90	0.95879	0.98304	0.96940	0.95877	0.98302	0.96938
91	0.95877	0.98294	0.96944	0.95872	0.98287	0.96942
92	0.95872	0.98299	0.96933	0.95868	0.98294	0.96929
93	0.95879	0.98297	0.96920	0.95870	0.98296	0.96912
94	0.95860	0.98295	0.96937	0.95850	0.98291	0.96929
95	0.95847	0.98301	0.96936	0.95847	0.98295	0.96928
96	0.95857	0.98282	0.96944	0.95850	0.98280	0.96941
97	0.95857	0.98277	0.96946	0.95854	0.98270	0.96940
98	0.96082	0.98518	0.96973	0.96077	0.98516	0.96972
99	0.96073	0.98512	0.96965	0.96067	0.98503	0.96964
100	0.96079	0.98485	0.96953	0.96070	0.98482	0.96951
101	0.96085	0.98483	0.96939	0.96081	0.98476	0.96932
102	0.96019	0.98513	0.96931	0.96009	0.98511	0.96926
103	0.96034	0.98510	0.96906	0.96031	0.98507	0.96904
104	0.96050	0.98507	0.96877	0.96043	0.98506	0.96872
105	0.96069	0.98503	0.96846	0.96062	0.98498	0.96844
106	0.95971	0.98514	0.96939	0.95966	0.98507	0.96939
107	0.95968	0.98494	0.96948	0.95961	0.98488	0.96939
108	0.95963	0.98468	0.96959	0.95957	0.98464	0.96954
109	0.95919	0.98543	0.96935	0.95917	0.98537	0.96925
110	0.95848	0.98578	0.96928	0.95846	0.98572	0.96921
111	0.95813	0.98595	0.96925	0.95804	0.98588	0.96919
112	0.95800	0.98601	0.96924	0.95799	0.98594	0.96916
113	0.95802	0.98600	0.96924	0.95802	0.98591	0.96915
114	0.95766	0.98617	0.96921	0.95761	0.98615	0.96911
115	0.95759	0.98621	0.96920	0.95750	0.98614	0.96920
116	0.96085	0.98483	0.96939	0.96079	0.98481	0.96930
117	0.95919	0.98543	0.96935	0.95917	0.98542	0.96928
118	0.96304	0.98579	0.97123	0.96300	0.98573	0.97113
119	0.97542	0.99074	0.98134	0.97538	0.99069	0.98129
120	0.97244	0.99036	0.98008	0.97234	0.99032	0.98003
121	0.97236	0.99125	0.97899	0.97234	0.99118	0.97891
122	0.96050	0.98516	0.96952	0.96042	0.98508	0.96950

Table 13: Comparison of computational performances (proposed/FBS).

System	Iteration number	Computation time (s)
19-node	2/4	0.344/0.385
25-node	2/4	0.906/1.311
123-node	4/6	2.753/3.920

5. Conclusions

Load flow analysis is basically a solution for the normal operating conditions of an electric power system. In general, the results of load flow calculation are used for power system planning, basis data in the operational stage, and power system operation and control. In the present paper, the trust-region method has been investigated and proposed to solve the load flow problem of the three-phase unbalanced electric power distribution system. The trust-region method is commonly used to solve the optimization problem. However, this method can be used as a technique to solve nonlinear equation systems arising from the load flow problem formulation. SpecialB treatments that are usually required in the distribution system load flow (DSLF) analysis are not needed in the proposed method. Moreover, the method can always obtain a solution even if the system is ill-conditioned. Case studies using 19-node, 25-node, and 123-node distribution systems have also been given in this paper. Results of the studies show that the output values obtained by the proposed method are in excellent agreement with those obtained by the previously published method. These results confirm the validity of the proposed method for solving the three-phase unbalanced DSLF problem. Case study results also indicate that the proposed method has better computational performances than the FBS method. In future work, the extension of the method so that it can be implemented in distribution system with distributed energy resources (DERs) can be investigated. This is probably an interesting topic since the penetration of DERs in the distribution network is presently increasing, which complicates the system load flow analysis.

Appendix

A

The following is the formulation to calculate the Jacobian matrix $\mathbf{J}(\mathbf{x})$ of a vector function $\mathbf{F}(\mathbf{x})$ at the point $\mathbf{x}*$:

$$\mathbf{J}(\mathbf{x}*) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}. \tag{A.1}$$

Elements of $\mathbf{J}(\mathbf{X}*)$ are computed as follows:

First row:

$$\begin{aligned} \frac{\partial}{\partial x_1} f_1(\mathbf{x}*) &\approx \frac{f_1(x_1^* + h, x_2^*, x_3^*, \ldots, x_n^*) - f_1(x_1^* - h, x_2^*, x_3^*, \ldots, x_n^*)}{2h} \\ \frac{\partial}{\partial x_2} f_1(\mathbf{x}*) &\approx \frac{f_1(x_1^*, x_2^* + h, x_3^*, \ldots, x_n^*) - f_1(x_1^*, x_2^* - h, x_3^*, \ldots, x_n^*)}{2h} \\ &\vdots \\ &\vdots \\ \frac{\partial}{\partial x_n} f_1(\mathbf{x}*) &\approx \frac{f_1(x_1^*, x_2^*, x_3^*, \ldots, x_n^* + h) - f_1(x_1^*, x_2^*, x_3^*, \ldots, x_n^* - h)}{2h}. \end{aligned} \tag{A.2}$$

Second row:

$$\begin{aligned}
\frac{\partial}{\partial x_1} f_2(\mathbf{x}*) &\approx \frac{f_2(x_1^* + h, x_2^*, x_3^*, \ldots, x_n^*) - f_2(x_1^* - h, x_2^*, x_3^*, \ldots, x_n^*)}{2h} \\
\frac{\partial}{\partial x_2} f_2(\mathbf{x}*) &\approx \frac{f_2(x_1^*, x_2^* + h, x_3^*, \ldots, x_n^*) - f_2(x_1^*, x_2^* - h, x_3^*, \ldots, x_n^*)}{2h} \\
&\vdots \\
&\vdots \\
\frac{\partial}{\partial x_n} f_2(\mathbf{x}*) &\approx \frac{f_2(x_1^*, x_2^*, x_3^*, \ldots, x_n^* + h) - f_2(x_1^*, x_2^*, x_3^*, \ldots, x_n^* - h)}{2h}.
\end{aligned} \tag{A.3}$$

n th row:

$$\begin{aligned}
\frac{\partial}{\partial x_1} f_n(\mathbf{x}*) &\approx \frac{f_n(x_1^* + h, x_2^*, x_3^*, \ldots, x_n^*) - f_n(x_1^* - h, x_2^*, x_3^*, \ldots, x_n^*)}{2h} \\
\frac{\partial}{\partial x_2} f_n(\mathbf{x}*) &\approx \frac{f_n(x_1^*, x_2^* + h, x_3^*, \ldots, x_n^*) - f_n(x_1^*, x_2^* - h, x_3^*, \ldots, x_n^*)}{2h} \\
&\vdots \\
&\vdots \\
\frac{\partial}{\partial x_n} f_n(\mathbf{x}*) &\approx \frac{f_n(x_1^*, x_2^*, x_3^*, \ldots, x_n^* + h) - f_n(x_1^*, x_2^*, x_3^*, \ldots, x_n^* - h)}{2h}.
\end{aligned} \tag{A.4}$$

In the above formulas, h is a constant and has a small numerical value (e.g., 0.01, 0.001, or 0.0001).

Data Availability

The 19-node, 25-node, and 123-node distribution systems data used in the verification of the proposed method are included in the paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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