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Research Article

Efficient Compressed Sensing Reconstruction Algorithm for Nonnegative Vectors in Wireless Data Transmission

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With the rapid development of 5G communication and wireless Internet of Things technology, the application of intelligent wearable devices based on wireless data transmission technology is becoming more and more popular. However, due to the bandwidth of wireless data transmission nodes and the power consumption of the device system, the use time and data storage of wearable devices are severely restricted. The compressed sensing (CS) technology has become an effective method to tackle this problem. CS technology includes data compressive sensing at the transmission end and data reconstruction at the receiving end. In this paper, we consider the reconstruction of nonnegative sparse vectors, an important problem in the area of CS for wireless data transmission. It is known that the interval-passing (IP) algorithm is a low-complexity message-passing type method for the problem. However, the reconstruction performance of the IP algorithm is inferior to that of the other state-of-the-art CS reconstruction algorithms such as the orthogonal matching pursuit (OMP) algorithm. In order to address the problem, we propose a two-stage reconstruction algorithm in this paper. The proposed algorithm applies the IP algorithm for the first stage of reconstruction. If the reconstruction fails, the OMP algorithm is then used on the basis of the results in the first stage. The proposed algorithm is evaluated and compared with other state-of-the-art algorithms by the probability of perfect reconstruction under the given sparsity order value. Simulation results suggest that the proposed two-stage algorithm can greatly improve the reconstruction performance of the IP algorithm and can even outperform the OMP algorithm. In addition, the low-complexity advantages of the IP algorithm are maintained in the proposed algorithm.

1. Introduction

The rapid development of 5G communication and wireless Internet of Things (IoT) technology makes intelligent wearable devices more and more popular [1, 2]. However, due to the applications of multiple-type sensors and the requirements of wireless data transmission of multichannel human body signals, the long-term use of intelligent wearable devices is limited by the bandwidth of data transmission nodes and system power consumption. As an effective solution to reduce the bandwidth and system power consumption of wireless data transmission, the compressed sensing (CS) technology has received a lot of interest in the field of wireless data transmission. In the CS framework, the transmission data are measured compressively at the transmission end before transmission. For many practical situations (e.g., image processing), the data after the measurement can be represented as nonnegative vectors. At the receiving end, the measurement data are reconstructed through certain approaches. Through the application of CS technology, wearable intelligent devices based on 5G and wireless IoT technology can meet real-time data acquisition and transmission of the users under the condition of limited data storage resources and battery capacity.

Concretely speaking, the CS problem [3–9] is to reconstruct a high-dimensional signal vector from a lower-dimension set of linear measurements obtained by multiplying the signal by a matrix called a measurement matrix. If the signal vector is sparse (in certain a basis), the number of samples that need to acquire and store can be reduced dramatically under the CS framework. As a consequence, CS has received a considerable amount of attention in various applications of the signal processing area.

The construction of measurement matrices and the design of reconstruction algorithms are two main concerns in CS. To date, several approaches, both random and deterministic, have been proposed for the construction of measurement matrices [10–12]. The random measurement matrices have theoretical reconstruction guarantees, whereas the deterministic measurement matrices are desirable for practical implementation. For the signal reconstruction, there are essentially two classes of popular methods, i.e., basis pursuits (BP) methods and matching pursuits (MP) methods [13–15]. BP methods reconstruct the signal through the solving of an optimization problem [13]. MP is a greedy iterative method, and at each iteration, the method selects the best vector from its redundant dictionary. Among them, the most commonly used is the orthogonal MP (OMP) algorithm [14], which has a relatively small number of iterations by incorporating a least-square (LS) operation for the signal estimation.

Practically, there are many applications [16–18] where the signal vectors are not only sparse but also nonnegative. Therefore, the reconstruction of nonnegative sparse vectors has become a canonical model in CS [19]. Some improvements [20–22] have been proposed for both BP and MP algorithms by taking advantage of the nonnegative property of the signal vectors. In this paper, we also consider the reconstruction of nonnegative sparse vectors, since the problem is of great importance.

Recently, it has been shown that there are close connections between the areas of compressed sensing and channel coding [23, 24]. In particular, the successes of sparse channel codes such as low-density parity-check (LDPC) codes suggest that it is deserved to investigate the use of binary sparse matrices as measurement matrices in CS. It can be shown that these matrices have desirable reconstruction performances under the BP and MP methods [25–28]. Besides, the sparseness of these matrices can reduce the storage requirements significantly, which is amicable from a practical point of view. In particular, for the reconstruction of nonnegative sparse vectors, an iterative message-passing type algorithm, called the interval-passing (IP) algorithm, has been provided [29, 30]. The IP algorithm has a very low per-iteration complexity by taking the advantage of the sparseness of these matrices. It can be shown that the performance of the IP algorithm is desirable for the reconstruction of nonnegative sparse vectors using these LDPC matrices as measurement matrices. Several recent studies [31–33] have considered the analysis and optimization of the IP algorithm. In [34], the authors have considered the initial application of the IP algorithm for chemical mixture estimation.

However, compared with the BP and OMP methods, the reconstruction performance of the IP algorithm for nonnegative sparse vectors is degraded [30]. Since binary LDPC measurement matrices have advantages from the practical point of view, it is deserved to explore how to improve the reconstruction performance of the IP algorithm while maintaining its low-complexity merit, which motivates our work. In this paper, we propose a two-stage reconstruction algorithm based on the IP and OMP algorithms. In the first stage of the proposed algorithm, the IP algorithm is applied for the signal reconstruction. If the reconstruction is successful, the proposed algorithm terminates. If the reconstruction is unsuccessful, we can prove a property to determine some entries of the nonnegative signal vector from the output of the IP algorithm. Then, we use the OMP algorithm by taking these entries into account, which can significantly reduce the size of the involved LS problem compared with directly applying the OMP algorithm. Simulation results indicate that the performance of the proposed two-stage algorithm is better than that of the IP algorithm and the OMP algorithm. Moreover, the low-complexity advantages of the IP algorithm are maintained in the proposed algorithm. All these suggest that the proposed reconstruction algorithm is suitable for practical purposes. Overall, the major contributions of the paper are summarized as follows:

(i) We propose a novel reconstruction algorithm for the construction of nonnegative vectors in CS. The proposed algorithm has a low-complexity merit and is suitable for wireless data transmission.

(ii) The proposed algorithm is a two-stage algorithm. The IP algorithm is applied for the signal reconstruction in the first stage. If the reconstruction is successful, the proposed algorithm terminates. If the reconstruction is unsuccessful, we use the OMP algorithm for the reconstruction in the second stage from the output of the IP algorithm.

(iii) Simulations are performed to verify that the performance of the proposed two-stage algorithm is better than that of the IP algorithm and the OMP algorithm.

The rest of the paper is organized as follows. Section 2 reviews the background knowledge of CS. In Section 3, the proposed two-stage reconstruction algorithm is provided. Section 4 presents the simulation results. Finally, Section 5 concludes the paper.

2. Background Knowledge

In this section, we provide the background knowledge of the paper. We first provide the notation definitions of the paper. Then, we review the basic concepts of CS. Finally, we focus on binary LDPC matrices as measurement matrices and the IP reconstruction algorithm designed for these matrices.

2.1. Notations. In the rest of the paper, we denote scalars by normal face letters (e.g., x or X), vectors by lowercase boldface letters (e.g., $\mathbf{x}$), and matrices by uppercase boldface letters (e.g., $\mathbf{A}$). The transpose of matrix $\mathbf{A}$ is $\mathbf{A}^{\mathbf{T}}$. The i-th entry of a vector $\mathbf{x}$ is denoted by x_i and the entry in the j-th row and the i-th column of a matrix $\mathbf{A}$ is denoted by a_{ji}. For a vector $\mathbf{x}$, we define supp$(\mathbf{x})$ =. The l_0-norm, the l_1-norm, and the l_2-norm of $\mathbf{x}$ are given by $\|\mathbf{x}\|_0 = \#\text{supp}(\mathbf{x})$, $\|\mathbf{x}\|_1 = \sum_i |x_i|$, and $\|\mathbf{x}\|_1 = \sum_i \sqrt{|x_i|^2}$, respectively, where # is the cardinality of a set.

2.2. Basic Concepts of CS. We assume that $\mathbf{x}$ is a k-sparse signal of length n, i.e., $\mathbf{x} \in \mathbb{R}^n$ and $\|\mathbf{x}\|_0 \leq k$, where $\mathbb{R}$ is the set of real numbers. The problem of CS is to reconstruct an unknown $\mathbf{x}$ from a linear measurement

$$\mathbf{y} = \mathbf{A}\mathbf{x}, \tag{1}$$

where $\mathbf{A}$ is a matrix over $\mathbb{R}$ with size $m \times n$ $(m \ll n)$, called the measurement matrix. In general, $\mathbf{A}$ can be constructed using either random methods or deterministic methods. It can be shown that a random measurement matrix satisfies the restricted isometry property with a high probability, which indicates that the matrix has a theoretical reconstruction performance guarantee. In practice, however, there are no explicit steps to construct such a matrix. Recently, it has been demonstrated that deterministic measurement matrices also exhibit comparable performances with random matrices. Among them, a special class of binary deterministic measurement matrices, the binary structured LDPC matrices, has received particular attention. The main reason is the belief that an LDPC matrix with a good error correction performance is a potentially good measurement matrix (see [26] for details). In this work, we focus on this class of deterministic measurement matrices, since these matrices have desirable performances as well as practical advantages.

For the signal reconstruction, it is straightforward to estimate x from y using the following minimization problem:

$$\begin{aligned} &\min \|\mathbf{x}\|_0, \\ &\text{s.t.}\mathbf{y} = \mathbf{A}\mathbf{x}. \end{aligned} \tag{2}$$

Practically, it is intractable to solve the abovementioned problem in general due to the nonconvex features of the l_0-norm. The convention is to replace the l_0-norm by the l_1-norm and formulate the relaxed minimization problem

$$\begin{aligned} &\min \|\mathbf{x}\|_1, \\ &\text{s.t.}\mathbf{y} = \mathbf{A}\mathbf{x}. \end{aligned} \tag{3}$$

The problem (3) can be solved by using linear optimization methods, and the obtained method is usually known as BP reconstruction.

Another class of popular reconstruction methods is MP methods, which greedily find the estimation of the original signal through an iterative process. The basic steps of the OMP algorithm are summarized as follows (see Algorithm 1).

In practice, the update $\widehat{\mathbf{x}}$ in Algorithm 1 can be implemented in an LS manner, i.e.,

$$S = \text{supp}(\widehat{\mathbf{x}}), \tag{4}$$

$$\mathbf{x}_S = \mathbf{A}_S\left(\mathbf{A}_S\mathbf{A}_S^T\right)^{-1}\mathbf{y}, \tag{5}$$

where $\mathbf{x}_S$ (respectively, $\mathbf{A}_S$) represents the projection of $\mathbf{x}$ (respectively, $\mathbf{A}$) onto the set S. It has been demonstrated that the OMP method is effective for many classes of measurement matrices (see e.g., [15]). If $\mathbf{x}$ is nonnegative, then the OMP method can be efficiently implemented with the help of matrix factorizations [22].

2.3. LDPC Matrices and the IP Algorithm. Now, we consider a special class of sparse measurement matrices, i.e., binary LDPC matrices. A binary LDPC matrix $\mathbf{A}$ with size $m \times n$ is a sparse matrix defined over the binary field (0 and 1), whose null space specifies a binary linear code of length n.

It is convenient to associate $\mathbf{A}$ with a bipartite graph (called the Tanner graph [35]) that has m check nodes $\{c_1, \cdots, c_m\}$ and n variable nodes $\{v_1, \cdots, v_n\}$ corresponding to the m rows and n columns of $\mathbf{A}$, respectively. A variable node v_i is connected to a check node c_j if and only if the entry of a_{ji} is 1. We denote the set of variable nodes connected to the j-th check node as $N(j) = \{i\colon a_{ji} = 1\}$ and the set of check nodes connected to the i-th variable node as $M(i) = \{j\colon a_{ji} = 1\}$ (more detailed discussions on the related concepts can be found in [35]).

For nonnegative sparse signals, the IP algorithm iteratively computes the upper and lower bounds for each entry in the signal vector. It has been shown that either the lower or the upper bound can converge to the same value for each variable node under certain conditions, which indicates that the algorithm can successfully estimate the original signal vector. With the help of a Tanner graph, the IP algorithm can be described in a message-passing form that consists of two alternative update rules for messages along the edges of the Tanner graph, one for check nodes and the other for variable nodes. We suppose that $L_{j\longrightarrow i}$ (respectively, $U_{j\longrightarrow i}$) is the lower (respectively, upper) bound on the message from the j-th check node to the i-th variable node and $L_{i\longrightarrow j}$ (respectively, $U_{i\longrightarrow j}$) is the lower (respectively, upper) bound on the message from the i-th variable node to the j-th check node. The basic steps of the IP algorithm are described as follows (see Algorithm 2).

We can see that the complicated matrix multiplications and matrix inversions of the OMP algorithm are avoided in the IP algorithm. Besides, the IP algorithm has a nice property through which a nonnegative vector $\mathbf{x}$ can be reconstructed if and only if the binary vector $\mathbf{x}_b$ with supp$(\mathbf{x})$ = supp$(\mathbf{x}_b)$ can be reconstructed by the algorithm [30, 33]. This property can facilitate the analysis of the algorithm.

3. Proposed Reconstruction Algorithm

Although the IP algorithm has a lower complexity than that of the OMP algorithm, the reconstruction performance is degraded in general, especially in the case where the sparsity of the original vector $\mathbf{x}$ is relatively large. This is undesirable in situations where high reconstruction performance is required. In this section, we apply the idea of multistage reconstruction to improve the performance.

Input: $\mathbf{A}$, $\mathbf{y}$, and the error threshold ε. Initialize: $\mathbf{x} = \mathbf{0}$, $\mathbf{r} = \mathbf{y}$, and $S = \varnothing$.
Repeat:
(1) Find the column index i_0 by
$i_0 = \operatorname{argmax}_i \langle \mathbf{r}, \mathbf{a}_i \rangle$
where $\mathbf{a}_i$ is the i-th column of $\mathbf{A}$ and $\langle , \rangle$ is the inner product of the two vectors
(2) Set $S \leftarrow S \cup \{i_0\}$
(3) Update $\hat{\mathbf{x}}$ by
$\hat{\mathbf{x}} = \operatorname{argmin}_{S=\operatorname{supp}(\mathbf{x})} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2$
(4) Update $\mathbf{r}$ by
$\mathbf{r} = \mathbf{y} - \mathbf{A}\hat{\mathbf{x}}$
until $\|\mathbf{r}\|_2 < \varepsilon$
Output $\hat{\mathbf{x}}$

Algorithm 1: OMP algorithm.

Input: $\mathbf{A}$ and $\mathbf{y}$. Initialize: $L_{i\longrightarrow j} = 0$ and $U_{i\longrightarrow j} = y_j$.
Repeat:
(1) Check node processing:
$L_{j\longrightarrow i} = \max\left\{0, y_j - \sum_{i' \in N(j)\setminus i} U_{i' \longrightarrow j}\right\}$
$U_{j\longrightarrow i} = y_j - \sum_{i' \in N(j)\setminus i} L_{i' \longrightarrow j}$
(2) Variable node processing:
$L_{i\longrightarrow j} = \max_{j' \in M(i)\setminus j} L_{j' \longrightarrow i} := L_i$
$U_{i\longrightarrow j} = \max_{j' \in M(i)\setminus j} U_{j' \longrightarrow i} := U_i$
until the algorithm converges or the maximum iteration number reaches
Output $\hat{\mathbf{x}} = [\hat{x}_i]$, where $\hat{x}_i = L_i$

Algorithm 2: IP Algorithm.

3.1. Algorithm Design. Our reconstruction algorithm consists of two stages. First, we use the IP algorithm to reconstruct the signal. If the algorithm converges, the whole reconstruction process halts and outputs the estimated vector. Otherwise, the reconstruction switches to the second stage by using the OMP algorithm. It is worth mentioning that although the IP algorithm fails, we can derive a property to determine some entries of the signal vector from its results.

Theorem 1. *Suppose L_i and U_i are the estimation results of the IP algorithm when the maximum iteration number is reached. If $L_i = U_i$, then $x_i = L_i = U_i$.*

Proof of Theorem 1. Since L_i and U_i are the lower and upper bounds of x_i, respectively, it holds $L_i \le x_i \le U_i$. Hence, we have $x_i = L_i = U_i$ if $L_i = U_i$. Thus, the theorem is proved.

In the following, the entry x_i is said to be determined by the IP algorithm if $L_i = U_i$, where L_i and U_i are the final results of the IP algorithm. Through computer simulations, we can find the following interesting phenomenon: even if the IP algorithm fails to reconstruct the original signal, a relatively large proportion of the entries of the original vector can be determined. In order to illustrate this fact, we consider the array-based quasi-cyclic (QC) LDPC matrix with size 68×289, which is a 4×17 array of cyclic matrices of size 17×17 [36]. Table 1 shows the average proportion of the determined entries under various values of sparsity order k.

It is known from Table 1 that the average proportion of the determined entries is larger than one-half when the sparsity order k is less than 0.1 n. As a consequence, we can design a reduced-complexity OMP algorithm for the second-stage reconstruction using these determined entries.

We assume D (respectively, $\overline{D}$) is the set of column indices whose corresponding entries are determined (respectively, not determined). We suppose that $\hat{\mathbf{x}}_D$ is the projection of $\hat{\mathbf{x}}$ onto the set D, where $\hat{\mathbf{x}}$ is the output of the IP algorithm in the first stage and $\mathbf{A}_D$ (respectively, $\mathbf{A}_{\overline{D}}$) is the projection of $\mathbf{A}$ onto the set D (respectively, $\overline{D}$).

$$\overline{\mathbf{y}} = \mathbf{y} - \mathbf{A}_D \hat{\mathbf{x}}_D, \tag{6}$$

In the second-stage reconstruction, we use $\overline{\mathbf{y}}$ and $\mathbf{A}_{\overline{D}}$ as the inputs of the OMP algorithm and reconstruct a vector of length $n - \#D$. We denote the output by $\hat{\mathbf{x}}_{\overline{D}}$. From the abovementioned analysis, we know that the length of $\overline{\mathbf{y}}$ is less than that of $\mathbf{y}$. Hence, the reconstruction complexity is reduced when compared with the use of $\mathbf{y}$ and $\mathbf{A}$ as the inputs of the OMP algorithm. Finally, we construct a vector $\tilde{\mathbf{x}}$ whose projections onto the set D and $\overline{D}$ are $\hat{\mathbf{x}}_D$ and $\hat{\mathbf{x}}_{\overline{D}}$, respectively. The details of the proposed two-stage reconstruction algorithm are described as follows (see Algorithm 3). □

Table 1: The average proportion of the determined entries for a quasi-cyclic LDPC matrix with size 68×289.

k	12	16	20	24	28
Proportion	0.8381	0.7830	0.7513	0.6560	0.5624

Input: $\mathbf{A}$, $\mathbf{y}$, and the error threshold ε
(1) Perform the IP algorithm using $\mathbf{y}$ and $\mathbf{A}$
(2) **If** the IP algorithm converges then
(3) Obtain the estimated signal vector $\hat{\mathbf{x}}$
(4) **else**
(5) Construct D, $\overline{D}$, $\mathbf{A}_D$, $\mathbf{A}_{\overline{D}}$, and $\hat{\mathbf{x}}_D$ based on the outputs of the IP algorithm
(6) Compute $\overline{\mathbf{y}}$
(7) Perform the OMP algorithm using $\overline{\mathbf{y}}$ and $\mathbf{A}_{\overline{D}}$
(8) Obtain the estimated signal vector $\hat{\mathbf{x}} = \tilde{\mathbf{x}}$ from $\hat{\mathbf{x}}_D$ and $\hat{\mathbf{x}}_{\overline{D}}$
(9) **end**
(10) Output $\hat{\mathbf{x}}$

Algorithm 3: Two-stage reconstruction algorithm.

3.2. Complexity Assessment. We know from the process of the proposed reconstruction algorithm that its complexity depends on the complexity of the IP and OMP algorithms as well as the rate at which the OMP algorithm is invoked. It is known from the description provided in the last section that the IP algorithm is an iterative process. Hence, its complexity can be estimated as

$$C_{\mathrm{IP}} = I_{\max} \cdot C_{\mathrm{av}}, \tag{7}$$

where $I_{\max}$ is the maximum iteration number and C_{av} is the per-iteration complexity of the algorithm. For an LDPC matrix $\mathbf{A}$ of size $m \times n$ with an average column weight of γ and an average row weight of ρ, we need $2m\rho(\rho-1)$ additions (subtractions) as well as $2m\rho(\rho-1)$ comparisons in each iteration [37].

For the OMP algorithm, the complexity can be written as the sum of the two terms [38], i.e.,

$$C_{\mathrm{OMP}} = C_1 + C_2, \tag{8}$$

where the first term C_1 is dependent on the size of $\mathbf{A}$ and the second term C_2 is independent on the size of $\mathbf{A}$. The term C_1 involves about $n(m-1)k + (2m-1)(k+1)k/2$ additions (subtractions), $(n-1)k$ comparisons, and $nmk + m(k+1)k$ multiplications. The complexity C_2 can be estimated as $\Theta(k^3)$, which is mainly caused by the calculations in the LS step (8). It should be noted that although we have $k \ll n$, the operations involved in this term contain divisions, which are undesirable from the implementation point of view [38].

Finally, the complexity of the proposed algorithm is given by

$$C_{\mathrm{TS}} = C_{\mathrm{OMP}} + \alpha C_{\mathrm{OMP}}, \tag{9}$$

where the parameter α is the rate at which the OMP algorithm is invoked when the IP algorithm fails. Since the OMP algorithm is invoked in the proposed algorithm when the IP algorithm fails, the rate α is equal to $1-p$, where p is the perfect reconstruction probability of the IP algorithm. It can be shown from the simulation results in the subsequent section that the reconstruction probability p is close to 1 when the sparsity order is small, which indicates that the rate α is close to zero. In addition, the vector length in our second-stage OMP algorithm is reduced compared with the direct application of the OMP algorithm for signal recovery. All these suggest that the proposed two-stage algorithm has low-complexity properties.

4. Simulation Results and Discussions

4.1. Simulation Results. In this section, we provide the simulation results for the proposed two-stage reconstruction algorithm. Three binary-structured LDPC matrices are adopted as measurement matrices. It should be noted that structured LDPC matrices have advantages from the practical point of view. The first matrix $\mathbf{A}_1$ is the 68×289array-based QC LDPC matrix provided in the previous section. The second matrix $\mathbf{A}_2$ is a 64×256 LDPC matrix constructed from the incidence structure of the points and lines of the two-dimensional Euclidean geometry over the field of GF (16), which can be written as a 4×17 array of permutation matrices of size 17×17 [35]. The third matrix $\mathbf{A}_3$ is a 252×504 QC LDPC matrix constructed from the additive group of finite fields using the method mentioned in [39]. It should be noted that the Tanner graphs of all these three LDPC matrices are free of cycles of length 4.

The maximum iteration number of the IP algorithm is set as 100. The sparse signals in our simulation are generated as follows: first, the support of cardinality k is randomly generated. Then, each of the k nonzero elements is drawn according to a normal distribution. For comparison purposes, the reconstruction performances of the standard BP algorithm for all three measurement matrices are also evaluated using the MATLAB-embedded function linprog. All the algorithms are evaluated by the probability of perfect reconstruction under the given sparsity order value [30].

Figure 1 shows the performances of various reconstruction algorithms for the matrix $\mathbf{A}_1$. It is known from the figure that the proposed two-stage reconstruction

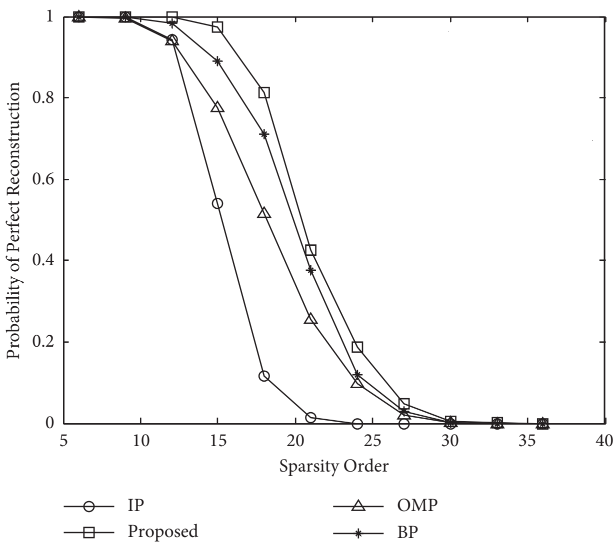

Figure 1: Reconstruction performances of the measurement matrix $\mathbf{A}_1$.

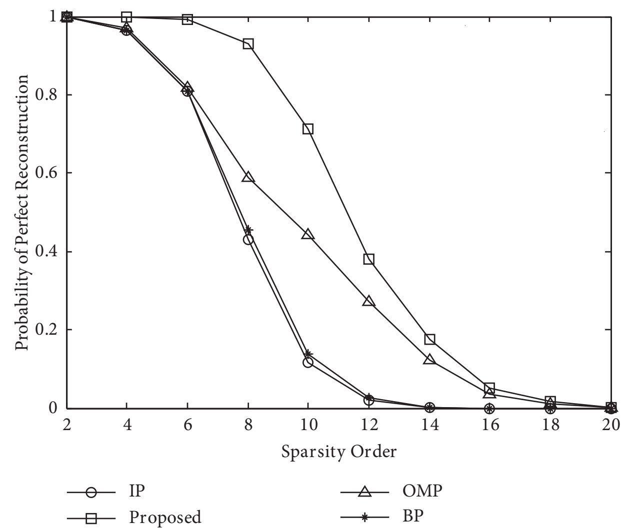

Figure 2: Reconstruction performances of the measurement matrix $\mathbf{A}_2$.

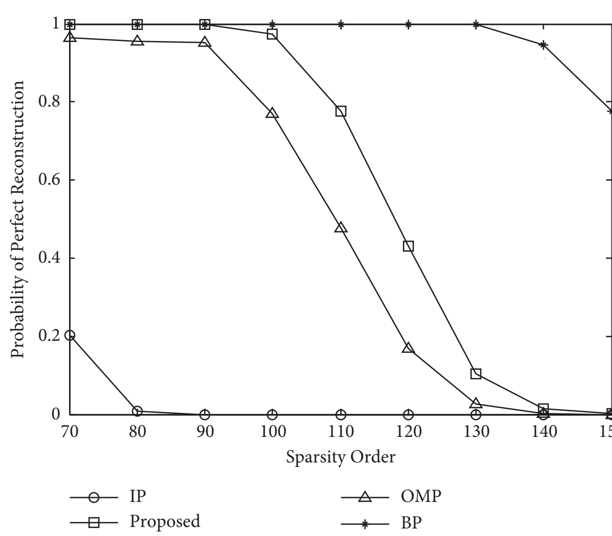

Figure 3: Reconstruction performances of the measurement matrix $\mathbf{A}_3$.

algorithm outperforms both the original IP algorithm and the OMP algorithm. For example, when the sparsity order is 20, the reconstruction probability of the original IP algorithm is less than 0.1, whereas the reconstruction probability of the proposed two-stage reconstruction algorithm is higher than 0.6. Furthermore, the proposed two-stage reconstruction algorithm can also slightly outperform the standard BP algorithm for this matrix.

Figure 2 shows the performances of various reconstruction algorithms for the matrix $\mathbf{A}_2$. We know from the figure that the performance of the IP algorithm degrades severely when the sparse order reaches about 8. We also know from the figure that the performance of the proposed reconstruction algorithm is also better than that of the IP and OMP algorithms.

Figure 3 shows the performances of various reconstruction algorithms for the matrix $\mathbf{A}_3$. It is known from the figure that the IP algorithm has a poor reconstruction performance when the sparsity order reaches 70. Instead, the OMP algorithm has a desirable performance in this range. Compared with $\mathbf{A}_1$ and $\mathbf{A}_2$, the measurement matrix $\mathbf{A}_3$ has a better performance when the sparsity order is large. This is due to the fact that $\mathbf{A}_3$ has a large ratio of m/n.

We can see from the figure that the standard BP algorithm has little performance degradation in the simulated range of sparsity orders. In addition, we know from the figure that the performance of the proposed reconstruction algorithm is greatly improved compared with that of the IP and OMP algorithms.

4.2. Discussions. We can see from the simulation results that our proposed reconstruction algorithm can outperform both the IP algorithm and the OMP algorithm for all three measurement matrices. We think this is reasonable, since there exist random sparse vectors that cannot be reconstructed by both the IP algorithm and the OMP algorithm but can be reconstructed by our proposed algorithm (note that our proposed algorithm applies the OMP algorithm on the basis of the determined entries of the IP algorithm result). This, together with the low-complexity property of the proposed two-stage algorithm, suggests that the algorithm is suitable for practical purposes.

It is also deserved to mention that we can further improve the performance of the proposed algorithm if the quality of the wireless channel for data transmission can be detected in practice. We can increase the measurement number in low channel quality situations and decrease the measurement number in high channel quality situations. This is an interesting future research direction for our work.

5. Conclusion

In this paper, we investigated the performance improvement of the IP algorithm for the reconstruction of nonnegative sparse vectors. Based on the IP and OMP algorithms, we propose a two-stage reconstruction algorithm. In the first stage, the IP algorithm is applied to the proposed algorithm. If the algorithm converges, the whole reconstruction process halts. Otherwise, the OMP algorithm is then used on the basis of the results of the IP algorithm. Simulation results on several structured binary LDPC measurement matrices suggest that the proposed two-stage algorithm can greatly improve the reconstruction performance of the IP algorithm and can even outperform the OMP algorithm. Besides, the proposed algorithm has a low-complexity property, which indicates that the algorithm is suitable for implementation and application in intelligent wearable device systems based on wireless transmission technology.

Data Availability

The data used to support the findings of the study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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