Research Article

Minimizing Energy Cost in Electric Arc Furnace Steel Making by Optimal Control Designs

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Production cost in steel industry is a challenge issue and energy optimization is an important part. This paper proposes an optimal control design aiming at minimizing the production cost of the electric arc furnace steel making. In particular, it is shown that with the structure of an electric arc furnace, the production cost which is a linear programming problem can be solved by the tools of linear quadratic regulation control design that not only provides an optimal solution but also is in a feedback form. Modeling and control designs are validated by the actual production data sets.

1. Introduction

Steel industries recycle scrap steel using the electric arc furnace (EAF) by melting it and changing its chemical composition to produce different product grades. Obviously, steel industry is one of the greatest energy consuming sectors and there is a strong demand to decrease the use of electricity and other forms of energy in the EAF steel making.

In general, the current melting process control is manual and the production cost is not optimal. Steel industries have some nominal automation for EAFs but are mostly operator driven. Although operator intuition is invaluable for such industries, it rests on the recipes that have been effective in the past and becomes difficult to take into account all possible uncertainties unless a mathematical framework is developed for the system and an automation process is in place. It has been shown that a substantial part of the energy consumption is wasted in melting the scrap [1–3]. This illustrates that there is a tremendous opportunity for control to play.

Unlike most existing papers in the literature, this paper concerns the production cost which is related to energy consumption for melting process and the way to achieve the optimality. Therefore, the purpose of the model is different from the large part of the existing literature [1–6]. For example, in [1], an adaptive model predictive control was

proposed to follow the preset trajectories. How to design the preset trajectories was not discussed. In [2], a PID controller was proposed to have the electrodes maintain constant power consumption. In [3], a model was developed but there was no discussion on how to control it. In [6], a model was proposed to achieving the maximum power input to the melting process. In all these works, minimizing control consumption was not a concern. In the work reported here, a mathematical model of EAFs is developed and its unknown parameters are then estimated. The goal of the model is to design a control algorithm that minimizes the energy consumption. The model adopted in this paper is a linear time invariant system which seems to work satisfactorily. More importantly, in the second step, optimal control inputs are calculated aiming at minimizing the production cost. Because the actual production cost is linear in the consumptions of energy and graphite electrode, the optimization is a linear programming problem. A problem of the linear programming is that it provides an open loop input design. Though optimal in the absence of model uncertainty and measurement noise, its performance in reality cannot be guaranteed under inevitable model uncertainty and measurement noise. The contribution of the paper is to show that with the particular structure of the EAF, the linear programming problem can in fact be solved by the well-known linear quadratic regulation (LQR) problem

FIGURE 1: Electric arc furnace.

which is in a feedback form and thus is more robust than an open loop solution.

All the data sets used in this study for modeling and control validation are actual production data collected from the Gerdau Ameristeel mill facility in Wilton Iowa that is composed of a scrap shredding facility, an electric arc furnace (EAF) as shown in Figure 1, several continuous billet casters, and a rolling mill. Because of proprietary properties, no actual data value is revealed in the paper, but the modeling results and final performance improvements are shown.

2. Problem Statement

In this section, we describe the system model and the goal of optimal control. Clearly, EAF models could be different for different purposes. Our goal is to optimize the production cost while the quality and efficiency are maintained. To this end, we focus on 7 key variables that provide a fairly reasonable description of the dynamics of an EAF:

- (i) kilowatt-hour consumption (KWH),
- (ii) electrode consumption (I^2h),
- (iii) percentage of scrap melted (PM),
- (iv) average arc current (I_{avg}),
- (v) oxygen input (O_2),
- (vi) gas input (Gas),
- (vii) carbon input (Car).

All 7 variables are either measurable or computable in real time. Among these variables, the kilowatt-hour consumption (KWH) is the consumption of electrical energy that constitutes a substantial part of the total cost, the electrode consumption (I^2h) is the consumption of the graphite electrode that also contributes to the cost significantly, and the percentage of scrap melted (PM) is the percentage of scrap

melted which is a quality constraint on the melting control. At the end of each melting process, PM has to be 100%. Other 4 variables are electrical energy input I_{avg} and chemical energy inputs O_2 (oxygen), Gas (gas), and Car (carbon), respectively. All 4 energy inputs contribute to the total cost and among them I_{avg} is the most expensive one.

Based on the physics, the increments of the kilowatt-hour consumption (KWH) and the electrode consumption (I^2h) from time $k\delta t$ to $(k+1)\delta t$ are directly related to the current I_{avg} in the interval $[k\delta t, (k+1)\delta t)$. On the other hand, the percentage of scrap melted (PM) is related to all energy inputs during the same period including the electrical energy as well as chemical energy. The EAF assumes the following structure:

$$\begin{pmatrix} \text{KWH} [k+1] \\ I^2h [k+1] \\ \text{PM} [k+1] \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_A \begin{pmatrix} \text{KWH} [k] \\ I^2h [k] \\ \text{PM} [k] \end{pmatrix} + \underbrace{\begin{pmatrix} \varphi_1 & 0 & 0 & 0 \\ \phi_1 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & a_4 \end{pmatrix}}_B \begin{pmatrix} I_{avg} [k] \\ O_2 [k] \\ \text{Gas} [k] \\ \text{Car} [k] \end{pmatrix},$$

$$k = 1, \dots, N, \quad (1)$$

where $\begin{pmatrix} \text{KWH}[0] \\ I^2h[0] \\ \text{PM}[0] \end{pmatrix} = 0$, $k = k\delta t$ with $\delta t = 5$ (sec) being the sampling interval and $(N+1)\delta t$ is the total time by which 100% of scrap should be melted. This model is not surprising and has been used in the literature. To maintain the same productivity and efficiency, $(N+1)\delta$ is the total length of the melting process time and is predetermined by the current practice without optimal control. B is a matrix whose entries depend on a particular EAF and the scrap loads. Note that $I_{avg}[k]$, $O_2[k]$, $\text{Gas}[k]$, $\text{Car}[k]$ are the respective total energy delivered to the EAF during the period $[k\delta t, (k+1)\delta t)$ and $\text{KWH}[k]$ and $I^2h[k]$ are the total electric energy and electrode consumption, respectively, at the end of the k th sampling interval. The production cost is defined as

$$\begin{aligned} J = & q_1 \text{KWH} [N+1] + q_2 I^2h [N+1] + q_3 \sum_{k=1}^N O_2 [k] \\ & + q_4 \sum_{k=1}^N \text{Car} [k] + q_5 \sum_{k=1}^N \text{Gas} [k], \end{aligned} \quad (2)$$

where q_1 , q_2 , q_3 , q_4 , and q_5 are the per unit cost of electric energy, electrode consumption, and the chemical energy of oxygen, gas, and carbon, respectively. The unit cost q_i 's fluctuate according to the market value and are adjusted when the market price changes. The goal of control is to design an input sequence $I_{avg}[k]$, $O_2[k]$, $\text{Gas}[k]$, $\text{Car}[k]$, $k = 1, 2, \dots, N$ so that the above cost function (2) is minimized under the constraint

$$\text{PM} (N+1) = 100. \quad (3)$$

In addition, because of hardware constraints, the input energy must satisfy for each $k = 1, 2, \dots, N$

$$\begin{aligned} 0 < I_l &\leq I_{\text{avg}}[k] \leq I_u, \\ 0 &\leq O_2[k] \leq O_u, \\ 0 &\leq \text{Gas}[k] \leq G_u, \\ 0 &\leq \text{Car}[k] \leq C_u, \end{aligned} \quad (4)$$

where I_l, I_u, O_u, G_u , and C_u denote the physical bounds on the amount of electrical and chemical energy, respectively, that can be delivered to the EAF during each sampling interval δt .

3. Optimal Control Calculation

To simplify notation, in the rest of the paper, we let

$$\begin{aligned} x[k] &= \begin{pmatrix} x_1[k] \\ x_2[k] \\ x_3[k] \end{pmatrix} = \begin{pmatrix} \text{KWH}[k] \\ I^2h[k] \\ \text{PM}[k] \end{pmatrix}, \\ u[k] &= \begin{pmatrix} u_1[k] \\ u_2[k] \\ u_3[k] \\ u_4[k] \end{pmatrix} = \begin{pmatrix} I_{\text{ave}}[k] \\ O_2[k] \\ \text{Gas}[k] \\ \text{Car}[k] \end{pmatrix}. \end{aligned} \quad (5)$$

Further, by some simple calculations, we notice that

$$\begin{aligned} \text{KWH}[N+1] &= \varphi_1 \sum_{k=1}^N I_{\text{avg}}[k], \\ I^2h[N+1] &= \phi_1 \sum_{k=1}^N I_{\text{avg}}[k], \\ \text{PM}[N+1] &= a_1 \sum_{k=1}^N I_{\text{avg}}[k] + a_2 \sum_{k=1}^N O_2[k] \\ &\quad + a_3 \sum_{k=1}^N \text{Gas}[k] + a_4 \sum_{k=1}^N \text{Car}[k]. \end{aligned} \quad (6)$$

Therefore, the cost function of (2) can be rewritten as

$$\begin{aligned} J &= p_1 \sum_{k=1}^N I_{\text{avg}}[k] + p_2 \sum_{k=1}^N O_2[k] \\ &\quad + p_3 \sum_{k=1}^N \text{Car}[k] + p_4 \sum_{k=1}^N \text{Gas}[k], \end{aligned} \quad (7)$$

where $p_1 = q_1\varphi_1 + q_2\phi_1$, $p_2 = q_3$, $p_3 = q_4$, and $p_4 = q_5$. To recap, the goal is to design an input sequence $u_i[k]$, $i = 1, 2, 3, 4$, $k = 1, 2, \dots, N$ for a given N so that the cost function

$$\begin{aligned} J &= p_1 \sum_{k=1}^N u_1[k] + p_2 \sum_{k=1}^N u_2[k] + p_3 \sum_{k=1}^N u_3[k] \\ &\quad + p_4 \sum_{k=1}^N u_4[k] \end{aligned} \quad (8)$$

is minimized under the equality and inequality constraints,

$$\begin{aligned} x[k+1] &= \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_A x[k] \\ &\quad + \underbrace{\begin{pmatrix} \varphi_1 & 0 & 0 & 0 \\ \phi_1 & 0 & 0 & 0 \\ \theta_1 & \theta_2 & \theta_3 & \theta_4 \end{pmatrix}}_B u[k], \\ x[0] &= 0, \quad k = 1, 2, \dots, N, \\ 0 &< I_l \leq u_1[k] \leq I_u, \\ 0 &\leq u_2[k] \leq O_u, \\ 0 &\leq u_3[k] \leq G_u, \\ 0 &\leq u_4[k] \leq C_u, \end{aligned} \quad (9)$$

$$\begin{aligned} 100 &= a_1 \sum_{k=1}^N u_1[k] + a_2 \sum_{k=1}^N u_2[k] \\ &\quad + a_3 \sum_{k=1}^N u_3[k] + a_4 \sum_{k=1}^N u_4[k]. \end{aligned}$$

Here, KWH, I^2h , and PM describe the 3-dimensional state space and I_{avg} , O_2 , Car, and Gas describe the 4-dimensional input space. Matrices A and B are given as before.

3.1. Solution Using Linear Programming. The optimal control design problem described above is exactly a linear programming problem with a linear objective function and linear equality and inequality constraints. This can be solved by commercially available software packages. For real time applications, one issue is its computational complexity. In reality, N is approximately 1200 which implies that there are $3 \cdot 1200 = 3600$ state variables, $4 \cdot 1200 = 4800$ input variables, $2 \cdot 4 \cdot 1200 = 9600$ inequality constraints, and $3 \cdot 1200 = 3600$ equality constraints. Simply put, it is a linear programming problem with 8400 variables and 14400 constraints. The linear programming has to be recalculated every time when the model experiences some changes. The computational power required for a real time application is not favorable. The second problem is that the linear programming provides an open loop control. Yes, the solution $u^*[k]$, $k = 1, 2, \dots, N$ of the linear programming is optimal if everything is ideal. In reality, model uncertainty and measurement noise are always present and the linear programming solution is open loop. It is impossible to guarantee a good performance of an open loop control system in the presence of uncertainty and noise. What we prefer is a solution that minimizes the cost function (8) under the constraints (9) but in a closed loop form. To this end, note that the linear quadratic regulation (LQR) control design has been extensively studied in the literature. The solution of LQR is in a feedback form which is more robust than the open loop solution provided by the linear programming. Moreover, the LQR control has many desirable

properties. We will show in the next section that by proper choices of the weights in the LQR calculation, the solution of the LQR also solves the linear programming problem or equivalently the cost function (8) under constraints (9) but in a closed loop form.

Though the linear programming solution may not be practically useful because of its open loop nature, it does reveal some important insights. Let

$$U_i = \sum_{k=1}^N u_i[k], \quad i = 1, 2, 3, 4. \quad (10)$$

Then, the cost function and the terminal constraint can be rewritten, respectively, as

$$(J =) J_{LP} = p_1 U_1 + p_2 U_2 + p_3 U_3 + p_4 U_4, \quad (11)$$

$$100 = a_1 U_1 + a_2 U_2 + a_3 U_3 + a_4 U_4.$$

Also from the state space equation, we have $x_1[N+1] = \phi_1 U_1$ and $x_2[N+1] = \phi U_1$. Now suppose $u_i^*[k]$'s, $k = 1, 2, \dots, N$, $i = 1, 2, 3, 4$ are the solution of the linear programming. Define

$$u_i[k] = \frac{1}{N} \sum_{m=1}^N u_i^*[m] = \frac{1}{N} U_i, \quad (12)$$

$$i = 1, 2, 3, 4, \quad k = 1, 2, \dots, N.$$

Clearly, $\sum_{k=1}^N u_i[k] = \sum_{k=1}^N u_i^*[k] = U_i$ and $\min_m u_i^*[m] \leq u_i[k] \leq \max_m u_i^*[m]$ for each $i = 1, 2, 3, 4$ and every $k = 1, 2, \dots, N$. Therefore, the solutions $u_i[k]$'s of (12) are constants that minimize the cost function (8) and satisfy all the equality and inequality constraints (9). In short, the solution of the linear programming may not be unique, but there always exists a constant solution. We summarize this observation in a theorem form that will be used later.

Theorem 1. Consider the optimal control problem of (8) under the constraints (9). Then, there always exist constant sequences $u_i[k] \equiv u_i[m]$'s, $i = 1, 2, 3, 4$, and $1 \leq k, m \leq N$ that solve the problem.

3.2. LQR Design. Linear quadratic regulator problem is extensively studied in the literature and its solution can be readily found in a number of textbooks [7, 8]. In addition, its properties in terms of stability, robustness, and so forth, are well documented and in fact are favorable [7, 8]. The difference is however that LQR minimizes a quadratic performance function

$$J_{LQR} = x[N+1]^T S x[N+1] + \sum_{k=1}^N (x[k]^T Q x[k] + u[k]^T R u[k]), \quad (13)$$

where $S = S^T \geq 0$, $Q = Q^T \geq 0$, and $R = R^T > 0$ are the terminal state, state and input cost matrices, respectively. Our goal in this section is to show that by some properly chosen

matrixes S , Q , and R , the LQR solution also solves the linear programming problem which is the goal of the control design. It is emphasized that the LQR is not only more robust but also numerically efficient. Note that the LQR is recursive and at each time kt , only a fixed dimension Riccati equation (in our case, 4 dimensional) needs to be calculated [7, 8].

To this end, we set

$$S = 0, \quad Q = 0, \quad R = \begin{pmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 \\ 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & r_4 \end{pmatrix} \quad (14)$$

for some $r_i > 0$ or equivalently

$$J_{LQR}(u(r_1, r_2, r_3, r_4)) = \sum_{k=1}^N u[k]^T R u[k]$$

$$= r_1 \sum_{k=1}^N u_1^2[k] + r_2 \sum_{k=1}^N u_2^2[k] \quad (15)$$

$$+ r_3 \sum_{k=1}^N u_3^2[k] + r_4 \sum_{k=1}^N u_4^2[k],$$

where $J_{LQR}(u(r_1, r_2, r_3, r_4))$ indicates that the cost depends on the optimal input sequence $u[k]$ which in turn depends on the weights r_i 's. Together with the terminal constraint in (9), the LQR solution assumes the form of [7, 8]

$$u[k] = K[k] x[k] + H[k] x_3[N+1], \quad (16)$$

where $K[\cdot]$ and $H[\cdot]$ are functions of r_1, r_2, r_3, r_4 . Note that the control sequence is in a feedback form. Interested readers may find more details in many textbooks, for example, [7, 8]. It is easy to verify the following theorem.

Theorem 2. Consider the above LQR problem of (15) under the constraints (9). Then, the input sequences $u_i^*[k]$'s, $i = 1, 2, 3, 4$, $k = 1, 2, \dots, N$ that solve the LQR problem must be constant sequences $u_i^*[k] \equiv u_i^*[m]$ for all $i = 1, 2, 3, 4$ and $1 \leq k, m \leq N$.

Proof. The proof is by contradiction. Suppose not, there exists at least one k so that $u_i^*[k] \neq u_i^*[k+1]$ for some i . Let $u_i[m]$'s, $i = 1, 2, 3, 4$, and $m = 1, 2, \dots, N$ be the exact sequences of $u_i^*[m]$ except at k and $k+1$,

$$u_i[k] = u_i[k+1] = \frac{1}{2} (u_i^*[k] + u_i^*[k+1]). \quad (17)$$

Then,

$$\begin{aligned}
 & r_i \left(u_i^2[k] + u_i^2[k+1] \right) \\
 &= r_i \left(\left(\frac{(u_i^*[k] + u_i^*[k+1])}{2} \right)^2 \right. \\
 &\quad \left. + \left(\frac{(u_i^*[k] - u_i^*[k+1])}{2} \right)^2 \right) \quad (18) \\
 &= r_i \frac{(u_i^*[k] + u_i^*[k+1])^2}{2} \\
 &< r_i (u_i^{*2}[k] + u_i^{*2}[k+1]).
 \end{aligned}$$

In other words, all the constraints of (9) are satisfied by the new sequences $u_i[k]$, but the cost function generated by $u_i[k]$ is strictly smaller than the cost function generated by $u_i^*[k]$. Therefore, the optimal sequences have to be constant sequences. This finishes the proof. $\square$

Now let the solution sequence of the LQR problem (15) under the constraints (9) be denoted by

$$\begin{aligned}
 & u_1^{\text{LQR}}(r_i), u_2^{\text{LQR}}(r_i), u_3^{\text{LQR}}(r_i), u_4^{\text{LQR}}(r_i), \\
 & i = 1, 2, 3, 4, \quad k = 1, 2, \dots, N. \quad (19)
 \end{aligned}$$

Further, let

$$(r_1^*, r_2^*, r_3^*, r_4^*) = \arg \min_{r_i} J_{\text{LQR}}(u(r_1, r_2, r_3, r_4)). \quad (20)$$

Now we claim the following.

Theorem 3. *The solution of the LQR problem $u_i^{\text{LQR}}(r_1^*, r_2^*, r_3^*, r_4^*)$, $i = 1, 2, 3, 4$, defined above also solves the linear programming problem of (8) and (9).*

Proof. Since both the linear programming problem and the LQR problem assume constant solutions, the goal is to find constant u_i 's for all k that solve, respectively, the linear programming problem and the LQR problem as shown in Table 1.

For the linear programming problem, at least one of the optimal solution lies at one of the corners formed by the intersections of the hyperplanes of the cost function, the terminal constraint, and the inequality constraints as shown in Figure 2 for a two-dimension case. On the other hand, the solution of the LQR problem is at the intersection of the same hyperplanes formed by the terminal and inequality constraints and an ellipsoid

$$\begin{aligned}
 1 &= \frac{(u_1^{\text{LQR}}(r_i))^2}{(\sqrt{J_{\text{LQR}}/r_1 N})^2} + \frac{(u_2^{\text{LQR}}(r_i))^2}{(\sqrt{J_{\text{LQR}}/r_2 N})^2} \\
 &\quad + \frac{(u_3^{\text{LQR}}(r_i))^2}{(\sqrt{J_{\text{LQR}}/r_3 N})^2} + \frac{(u_4^{\text{LQR}}(r_i))^2}{(\sqrt{J_{\text{LQR}}/r_4 N})^2} \quad (21)
 \end{aligned}$$

TABLE 1

LP	LQR
$\min_{u_i} J_{\text{LP}}(u_i) = N \sum_{i=1}^4 p_i u_i$	$\min_{u_i} J_{\text{LQR}}(u_i(r_1, r_2, r_3, r_4)) = N \sum_{i=1}^4 r_i u_i^2(r_1, r_2, r_3, r_4)$
$x_1[N+1] = N\phi_1 u_1$	$x_1[N+1] = N\phi_1 u_1(r_i)$
s.t. $x_2[N+1] = N\phi_1 u_1$	$x_2[N+1] = N\phi_1 u_1(r_i)$
$x_3[N+1] = N \sum_{i=1}^4 a_i u_i$	$x_3[N+1] = N \sum_{i=1}^4 a_i u_i(r_1, r_2, r_3, r_4)$
$0 < I_l \leq u_1 \leq I_u$	$0 < I_l \leq u_1(r_i) \leq I_u$
$0 \leq u_2 \leq O_u$	$0 \leq u_2(r_i) \leq O_u$
$0 \leq u_3 \leq G_u$	$0 \leq u_3(r_i) \leq G_u$
$0 \leq u_4 \leq C_u$	$0 \leq u_4(r_i) \leq C_u$

FIGURE 2: X = solution of linear programming.

for a given set of r_i . By varying r_i , the axes of the ellipsoid can be adjusted arbitrarily and there always exist some r_i^* 's so that the LQR solution is at one of the corners that is also a solution of the linear programming problem as illustrated in Figure 3 for a two-dimensional case. Denote such solutions by $u_1^{\text{LQR}}(r_i^*), u_2^{\text{LQR}}(r_i^*), u_3^{\text{LQR}}(r_i^*), u_4^{\text{LQR}}(r_i^*)$. This implies

$$\begin{aligned}
 & J_{\text{LP}}(u_1^{\text{LQR}}(r_i^*), u_2^{\text{LQR}}(r_i^*), u_3^{\text{LQR}}(r_i^*), u_4^{\text{LQR}}(r_i^*)) \\
 & \leq J_{\text{LP}}(u_1, u_2, u_3, u_4) \quad (22)
 \end{aligned}$$

for any u_i or the linear programming cost function J_{LP} evaluated at the LQR solution $u_1^{\text{LQR}}(r_i^*), u_2^{\text{LQR}}(r_i^*), u_3^{\text{LQR}}(r_i^*), u_4^{\text{LQR}}(r_i^*)$ is no larger than $J_{\text{LP}}(u_1, u_2, u_3, u_4)$ for any u_i . On the other hand,

$$\begin{aligned}
 & \min_{u_i} J_{\text{LP}}(u_i) \\
 & \leq J_{\text{LP}}(u_1^{\text{LQR}}(r_i^*), u_2^{\text{LQR}}(r_i^*), u_3^{\text{LQR}}(r_i^*), u_4^{\text{LQR}}(r_i^*)). \quad (23)
 \end{aligned}$$

Thus, the solution $u_1^{\text{LQR}}(r_i^*), u_2^{\text{LQR}}(r_i^*), u_3^{\text{LQR}}(r_i^*), u_4^{\text{LQR}}(r_i^*)$ of the LQR also solves the LP problem. This completes the proof.

FIGURE 3: X = solution of LQR.

3.3. Computation and Validation of r_i^* 's. What the results of Theorem 3 say is that there exist constants u_i 's that solve both the LQR and LP problems. Furthermore, let $u_1(r_i)$, $u_2(r_i)$, $u_3(r_i)$, $u_4(r_i)$ be the solution of LQR for given r_1 , r_2 , r_3 , and r_4 . Then,

$$(r_1^*, r_2^*, r_3^*, r_4^*) = \arg \min_{r_i} J_{LP}(u_1(r_i), u_2(r_i), u_3(r_i), u_4(r_i)) \quad (24)$$

and this provides a way to compute r_i^* .

For simplicity, let us leave out all the inequality constraint in (9) first. Consider the terminal constraint and the cost function,

$$\begin{aligned} J_{LQR} &= N(r_1 u_1^2 + r_2 u_2^2 + r_3 u_3^2 + r_4 u_4^2), \\ 100 &= N(a_1 u_1 + a_2 u_2 + a_3 u_3 + a_4 u_4). \end{aligned} \quad (25)$$

To find the LQR solution, we introduce the Lagrange multiplier

$$\begin{aligned} F &= N(r_1 u_1^2 + r_2 u_2^2 + r_3 u_3^2 + r_4 u_4^2) \\ &+ \lambda(100 - N(a_1 u_1 + a_2 u_2 + a_3 u_3 + a_4 u_4)). \end{aligned} \quad (26)$$

Setting the derivatives of F with respect to u_1 , u_2 , u_3 , u_4 and λ to zero implies that

$$\begin{aligned} u_1 &= \frac{\lambda a_1}{2r_1}, & u_2 &= \frac{\lambda a_2}{2r_2}, \\ u_3 &= \frac{\lambda a_3}{2r_3}, & u_4 &= \frac{\lambda a_4}{2r_4}, \\ \lambda &= \frac{200}{N} \left[\frac{a_1^2}{r_1} + \frac{a_2^2}{r_2} + \frac{a_3^2}{r_3} + \frac{a_4^2}{r_4} \right]^{-1}. \end{aligned} \quad (27)$$

By solving these 5 equations, we have

$$\begin{aligned} u_1 &= \frac{100a_1}{r_1 N} \left[\frac{a_1^2}{r_1} + \frac{a_2^2}{r_2} + \frac{a_3^2}{r_3} + \frac{a_4^2}{r_4} \right]^{-1}, \\ u_2 &= \frac{100a_2}{r_2 N} \left[\frac{a_1^2}{r_1} + \frac{a_2^2}{r_2} + \frac{a_3^2}{r_3} + \frac{a_4^2}{r_4} \right]^{-1}, \\ u_3 &= \frac{100a_3}{r_3 N} \left[\frac{a_1^2}{r_1} + \frac{a_2^2}{r_2} + \frac{a_3^2}{r_3} + \frac{a_4^2}{r_4} \right]^{-1}, \\ u_4 &= \frac{100a_4}{r_4 N} \left[\frac{a_1^2}{r_1} + \frac{a_2^2}{r_2} + \frac{a_3^2}{r_3} + \frac{a_4^2}{r_4} \right]^{-1}. \end{aligned} \quad (28)$$

Now putting back these values into J_{LP} , we have

$$J_{LP} = 100 \frac{p_1 a_1 r_2 r_3 r_4 + p_2 a_2 r_1 r_3 r_4 + p_3 a_3 r_1 r_2 r_4 + p_4 a_4 r_1 r_2 r_3}{a_1^2 r_2 r_3 r_4 + a_2^2 r_1 r_3 r_4 + a_3^2 r_1 r_2 r_4 + a_4^2 r_1 r_2 r_3}. \quad (29)$$

We make two observations.

- (i) If $r_1, r_2, r_3, r_4 \geq 0$, then the two sets of weights (r_1, r_2, r_3, r_4) and $\alpha(r_1, r_2, r_3, r_4)$ would produce the identical LQR optimal input sequences for any $\alpha > 0$.
- (ii) Practically, the usage of electric energy is always positive which implies $r_1^* > 0$.

Based on these two observations, we may set $r_1 = 1$ and

$$\begin{aligned} r_1 &= 1, & r_2 &= r_2 r_1 = r_2, \\ r_3 &= r_3 r_1 = r_3, & r_4 &= r_4 r_1 = r_4 \end{aligned} \quad (30)$$

which in turn gives rise to

$$J_{LP} = 100 \frac{p_1 a_1 r_2 r_3 r_4 + p_2 a_2 r_3 r_4 + p_3 a_3 r_2 r_4 + p_4 a_4 r_2 r_3}{a_1^2 r_2 r_3 r_4 + a_2^2 r_3 r_4 + a_3^2 r_2 r_4 + a_4^2 r_2 r_3}. \quad (31)$$

Suppose r_i^* 's minimize the above J_{LP} . Then, the optimal u_i 's are those in (28) by replacing r_i 's by r_i^* 's. These are the solutions without the inequality constraints (9). With (9), the best r_i 's are those that minimize J_{LP} under the constraints that the resultant u_i 's are in the box defined by the inequality constraints (9).

To support the above theoretical derivation, a large number of actual production data sets (over 2 thousands) are collected. Based on 30 randomly chosen data sets, we first solve the linear programming problem (2) and (4) directly using the command "linprog" embedded in MATLAB. We then solve the same problem by finding the optimal r_i^* 's and the corresponding optimal input sequences $u_i^*[k]$'s through the LQR optimization as discussed before. In each case, the two optimization results are not identical but very close about 1-2% difference in the cost functions. We believe that this small numerical difference of 1-2% is due to numerical imperfection of MATLAB for a large dimensional linear programming (8400 variables and 14000 constraints).

4. Parameter Estimation

In the previous sections, the entries of the matrix B are assumed to be available. In reality, B depends on the melting process and in particular the load weight of scraps. In general, the entries of B have to be estimated. To put parameter estimation into a perspective, we first describe the melting process. The melting process is a batch process. One batch of steel is called a heat. The time between taps is the amount of time required to produce a batch of liquid steel. Produced tons is measured using charged scrap tons. First, scrap is brought into the melt shop and an overhead crane is used to load the scrap into a charging bucket. When the charge bucket has been loaded, the loading crane picks up the charge bucket and places the scrap into the EAF. The EAF is typically operated utilizing a 3-charge practice and a melting cycle is composed of the following stages.

- (1) Furnace Turn Around—the EAF is inspected and prepared for the next heat.
- (2) Charge—the first scrap is put into the EAF.
- (3) Melting—power is turned on and the scrap is melted until the volume is sufficiently reduced to fit the second charge into the EAF.
- (4) Charge and melting cycle is repeated for the second and third scrap charges.
- (5) Refine—alloys are added to the liquid steel as necessary and steel is 100% melted.
- (6) Tap—the liquid steel is transferred from the EAF to a ladle for casting.

Obviously depending on the weight of scrap in three charges, the entry values of B vary. To this end, we provide an identifier, l = low, m = medium, and h = high to each charge to reflect the amount of scrap loaded in each charge.

Therefore, there are totally 9 different scrap loading scenarios as shown in Table 2. For instance, if three charges for a particular heat are 32 tons (1st charge), 32 tons (2nd charge), and 18 tons (3rd charge), respectively, then it is identified as (1l,2l,3l). If three charges are 32 tons, 34 tons, and 23 tons, it is identified as (1l,2m,3h). For each scenario, the entries of B are estimated separately. For actual estimation, let

$$\Delta x[k] = x[k+1] - x[k]. \quad (32)$$

Then, we have for $k = 1, 2, \dots, N$,

$$\begin{aligned} \Delta x_1[k] &= u_1[k] \phi_i, & \Delta x_2[k] &= u_1[k] \phi_1, \\ \Delta x_3[k] &= (u_1[k], u_2[k], u_3[k], u_4[k]) \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}. \end{aligned} \quad (33)$$

TABLE 2: Algorithm for choosing charge identifiers based on charge weights.

Charge number	CW (charge weight, in tons)		
	l (low)	m (medium)	h (high)
1	CW < 33	$33 \leq \text{CW} \leq 35$	CW > 35
2	CW < 33	$33 \leq \text{CW} \leq 35$	CW > 35
3	CW < 19	$19 \leq \text{CW} \leq 22$	CW > 22

The estimates $\hat{\phi}_1$, $\hat{\phi}_1$, and $\hat{a}_i$, $i = 1, 2, 3, 4$, of ϕ_1 , ϕ_1 , a_i defined as solutions of the least squares fits

$$\begin{aligned} \hat{\phi}_1 &= \arg \min_{\hat{\phi}} \sum_{k=1}^N (\Delta x_1[k] - u_1[k] \hat{\phi})^2, \\ \hat{\phi}_1 &= \arg \min_{\hat{\phi}} \sum_{k=1}^N (\Delta x_2[k] - u_1[k] \hat{\phi})^2, \\ (\hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4) &= \arg \min_{\hat{a}_i} \sum_{k=1}^N \left(\Delta x_3[k] - (u_1[k], u_2[k], u_3[k], u_4[k]) \right. \\ &\quad \left. \times \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \\ \hat{a}_4 \end{pmatrix} \right)^2, \end{aligned} \quad (34)$$

can be readily solved as

$$\begin{aligned} \hat{\phi}_1 &= \frac{\sum_{k=1}^N \Delta x_1[k] u_1[k]}{\sum_{k=1}^N u_1[k]^2}, \\ \hat{\phi}_1 &= \frac{\sum_{k=1}^N \Delta x_2[k] u_1[k]}{\sum_{k=1}^N u_1[k]^2}, \end{aligned} \quad (35)$$

$$\begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \\ \hat{a}_4 \end{pmatrix} = \left(\sum_{k=1}^N u[k] u^T[k] \right)^{-1} \sum_{k=1}^N u[k] \Delta x_3[k].$$

To validate the parameter estimation results, we let

$$\begin{aligned} \widehat{\Delta x}_1[k] &= u_1[k] \hat{\phi}_1, & \widehat{\Delta x}_2[k] &= u_1[k] \hat{\phi}_1, \\ \widehat{\Delta x}_3[k] &= u^T[k] \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \\ \hat{a}_4 \end{pmatrix} \end{aligned} \quad (36)$$

and define the Goodness-of Fit as

$$\text{GoF}_i = 1 - \sqrt{\sum_{k=1}^N \frac{(\Delta x_i[k] - \widehat{\Delta x}_i[k])^2}{(\Delta x_i[k] - (1/N) \sum_{m=1}^N \Delta x_i[m])^2}}, \quad (37)$$

$i = 1, 2, 3.$

TABLE 3: GoFs.

	GoF of Δx_1 (KWH)	GoF of Δx_2 (I^2h)	GoF of Δx_3 (PM)
Max GoF	0.9992	0.9995	0.9948
Min GoF	0.9984	0.9992	0.9930
Avg GoF	0.9990	0.9994	0.9941

TABLE 4: Production cost reduction of 15 heat files.

Heat file number	Ratio of the costs by optimal control and manual control	Cost saving
1	0.86	14%
2	0.9	10%
3	0.9	10%
4	0.9	10%
5	0.9	10%
6	0.86	14%
7	0.93	7%
8	0.84	16%
9	0.87	13%
10	0.91	9%
11	0.78	22%
12	0.9	10%
13	0.81	19%
14	0.82	18%
15	0.84	16%

The parameters are derived from one heat file and validated by 30 other fresh heat files, which were not used for estimation but with the same identifier. The validation results are shown in Table 3.

Clearly from Table 3, the results are satisfactory and the values of $\hat{\varphi}_1$, $\hat{\phi}_1$, and $\hat{a}_i$ will be used in the optimal control design to replace the unknown φ_1 , ϕ_1 , and a_i 's.

5. Control Design Validation

As discussed before, the control design involves two steps: (1) estimate the entries of B for different scrap load weights and (2) based on the estimates calculate the optimal r_i for the LQR problem and the corresponding optimal inputs $u_i^*(r_1^*, r_2^*, r_3^*, r_4^*)$. Recall that our goal is to design optimal input sequences so that the production cost (2) is minimized as compared to the current manual operation in which experiences play a key role. In theory, it has been demonstrated that the LQR solution $u_i^*(r_1^*, r_2^*, r_3^*, r_4^*)$, $i = 1, 2, 3, 4$, solves the LP problem and consequently minimizes the production cost (2) under the constraints. Whether it works in reality needs to be validated. To this end, we randomly choose 15 actual heat files with known production costs. For each heat file, a fresh heat file with the same identifier but not in the original 15 is chosen and used to estimate the entries of B to build the EAF model and calculate the optimal LQR solutions

$u_i^*(r_1^*, r_2^*, r_3^*, r_4^*)$, $i = 1, 2, 3, 4$. The optimal inputs are applied back to the original file to compute the production cost by the optimal control design. Based on the original production cost (2), the ratios of the costs by the optimal control designs derived by the LQR in which the weights r_i 's are adjusted to minimize the LP cost and by the current manual practice are shown in Table 4 along with their percentage of cost saving. In all 15 experiments (files), the optimal inputs meet the constraints (9) and reduce the production cost ranging from 7% to 22%. This supports our theoretical derivation and demonstrates that the production cost in which the energy consumption is a main part can be reduced substantially by changing the practice from manual to automatic.

6. Conclusion and Future Work

The method proposed for the optimal control strategy of an arc furnace provides a closed loop robust scheme that optimizes the production cost of the EAF steel making. In this study, we focus on 7 key variables and a linear modeling. In the future, other minor variables and possibly nonlinear effects could and should be considered. It is expected that by combining all the factors, the production cost can be reduced further. Also, the strategy developed in this paper can be extended to other applications in which the cost function is linear and the control design used is quadratic.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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