Hindawi Publishing Corporation
Journal of Function Spaces and Applications
Volume 2012, Article ID 947080, 7 pages
doi:10.1155/2012/947080

Research Article

Biseparating Maps on Fréchet Function Algebras

M. S. Hashemi,[1] T. G. Honary,[1] and M. Najafi Tavani[2]

[1] *Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran 14778-93855, Iran*
[2] *Department of Mathematics, Islamic Azad University, Islamshahr Branch, Tehran 33147-67653, Iran*

Correspondence should be addressed to T. G. Honary, honary@tmu.ac.ir

Received 20 August 2012; Accepted 13 November 2012

Academic Editor: Henryk Hudzik

Let A and B be strongly regular normal Fréchet function algebras on compact Hausdorff spaces X and Y, respectively, such that the evaluation homomorphisms are continuous on A and B. Then, every biseparating map $T : A \to B$ is a weighted composition operator of the form $Tf = h \cdot (f \circ \varphi)$, where φ is a homeomorphism from Y onto X and h is a nonvanishing element of B. In particular, T is automatically continuous.

1. Introduction and Preliminaries

Assume that A and B are spaces of complex functions on topological spaces X and Y, respectively. A linear map $T : A \to B$ is called separating or disjointness preserving whenever $\operatorname{coz}(f) \cap \operatorname{coz}(g) = \emptyset$ implies $\operatorname{coz}(Tf) \cap \operatorname{coz}(Tg) = \emptyset$, for all $f, g \in A$, where the cozero set of an element $f \in A$ is defined by $\operatorname{coz}(f) = \{x \in X : f(x) \neq 0\}$. Equivalently, a linear map $T : A \to B$ is separating if for every $f, g \in A$, the equality $f \cdot g = 0$ implies the equality $Tf \cdot Tg = 0$. Moreover, T is called *biseparating* if it is bijective and both T and T^{-1} are separating.

The concept of disjointness preserving operators was introduced for the first time in 1940s (see [1, 2]). Since then, many authors have extended this concept to various kinds of Banach algebras. For example in [3], Jarosz has studied separating maps between spaces of continuous scalar-valued functions. He showed that if X and Y are compact Hausdorff spaces, $A = C(X)$, the space of all continuous scalar-valued functions on X and $B = C(Y)$, then every bijective separating map $T : A \to B$ is a weighted composition operator of the form $Tf(y) = h(y)f(\varphi(y))$, $y \in Y$ and $f \in A$, where φ is a homeomorphism from Y onto X and h is a nonvanishing continuous complex-valued function on Y. Later, Font extended this result to the case where A and B are regular commutative semisimple Banach algebras satisfying Ditkin's condition [4]. On the other hand, Gau et al. used an algebraic method to study separating maps between spaces of continuous scalar as well as vector-valued functions in [5, 6]. For more information about separating maps one, can refer to [7–15].

In this paper, we generalize the results of Jarosz in [3] to Fréchet function algebras using a similar method as in [5, 6]. Then, we define the concept of a cozero preserving map and show that if A and B are Banach function algebras on compact Hausdorff spaces X and Y, respectively, and $T : A \to B$ is a unital cozero preserving map, then T is automatically continuous. Finally, we will find the relation between cozero preserving, separating and biseparating maps between certain Fréchet function algebras. Recently, Li and Wong have obtained several Banach-Stone type theorems for the vector-valued functions, specially in the case that the bijective linear map $T : C(X, E) \to C(Y, F)$ preserves zero set containments, that is,

$$Z(f) \subseteq Z(g) \Longleftrightarrow Z(T(f)) \subseteq Z(T(g)) \quad (f, g \in C(X, E)), \tag{1.1}$$

where X, Y are realcompact or metric spaces and E, F are locally convex spaces [16]. In fact, T preserves zero set containments if and only if T and T^{-1} are cozero preserving. In Corollary 2.6, we obtain similar results for Ff-algebras.

We now present some definitions and known results which we need in the sequel.

A Fréchet algebra (F-algebra) is a locally multiplicatively convex algebra (LMC-algebra) A which is also a complete metrizable space. The topology of a Fréchet algebra can be defined by an increasing sequence (p_n) of submultiplicative seminorms and without loss of generality; we may assume that $p_n(1) = 1$, for all $n \in \mathbb{N}$, if A has unit. An F-algebra A with a defining sequence of seminorms (p_n) is denoted by $(A, (p_n))$. The set of all characters (nonzero complex homomorphisms) of an F-algebra $(A, (p_n))$ is denoted by S_A, and the continuous character space, or the spectrum of $(A, (p_n))$, denoted by M_A, is the set of all continuous characters on A. We always endow S_A and M_A with the Gelfand topology, and $\widehat{A}$ is the set of all Gelfand transforms $\widehat{f}$ of elements f in A. The algebra A is called functionally continuous whenever $S_A = M_A$.

Note that a sequence $(f_k)_k$ in an F-algebra $(A, (p_n))$ converges to an element $f \in A$ if and only if for each $n \in \mathbb{N}$, $p_n(f_k - f) \to 0$ as $k \to \infty$.

Definition 1.1. Let X be a nonempty topological space. A subalgebra A of $C(X)$ is a function algebra on X if A contains the constants and separates the points of X. The algebra A is a Fréchet function algebra (Ff-algebra) or a Banach function algebra (Bf-algebra) on X if A is a function algebra which is also an F-algebra or a Banach algebra, respectively, with respect to some topology.

Clearly every Bf-algebra is a Ff-algebra. Let A be an Ff-algebra (Bf-algebra) on X such that the evaluation homomorphisms $\delta_x : A \to \mathbb{C}$ are all continuous, where $\delta_x(f) = f(x)$ for $f \in A$ and $x \in X$. It is clear that the map $J : X \to M_A$, $x \mapsto \delta_x$ is continuous and injective. If this map is also surjective and its inverse is continuous, then it is a homeomorphism, and in this case, we say that A is a natural Ff-algebra (Bf-algebra) on X, and we identify X with M_A, through this map.

Note that the evaluation homomorphisms are always continuous in Bf-algebras, but they may not be continuous in Ff-algebras. By [17, Lemma 3.2.5], the class of natural Ff-algebras and the class of unital commutative semisimple Fréchet algebras are the same. Moreover, all Ff-algebras as well as Bf-algebras are semisimple.

Example 1.2. Let (X,d) be a compact metric space and $\alpha > 0$. The algebra of all complex-valued functions f on X for which

$$p_\alpha(f) = \sup\left\{\frac{|f(x) - f(y)|}{d^\alpha(x,y)} : x, y \in X,\ x \neq y\right\} < \infty \tag{1.2}$$

is denoted by $\mathrm{Lip}(X,\alpha)$, and its subalgebra of those functions with the property $\lim_{d(x,y)\to 0}(|f(x)-f(y)|/d^\alpha(x,y)) = 0$ is denoted by $\mathrm{lip}(X,\alpha)$. It is known that $\mathrm{Lip}(X,\alpha)$ for $0 < \alpha \leq 1$ and $\mathrm{lip}(X,\alpha)$ for $0 < \alpha < 1$ are Bf-algebras on X under the norm $\|f\|_\alpha = \|f\|_X + p_\alpha(f)$.

In the case that X is a perfect compact plane set which is a finite union of regular sets (see [18] for the definition), the algebra of all functions f with derivatives of all orders (resp., $f^{(k)} \in \mathrm{Lip}(X,\alpha) f^{(k)} \in \mathrm{lip}(X,\alpha)$ for all $k \in \mathbb{N}$) is denoted by $D^\infty(X)$ (resp., $\mathrm{Lip}^\infty(X,\alpha)$ $\mathrm{lip}^\infty(X,\alpha)$) (see, e.g., [19]). It is interesting to note that $D^\infty(X)$, $\mathrm{Lip}^\infty(X,\alpha)$ and $\mathrm{lip}^\infty(X,\alpha)$ are natural Ff-algebras on X which are not Bf-algebras.

For a function algebra A on a nonempty topological space X and for each nonempty closed subset S of X, we consider the following subsets of A:

$$\begin{aligned} A_{00} &= \{f \in A : \operatorname{supp} f \text{ is compact in } X\}, \\ I(S) &= \{f \in A_{00} : (\operatorname{supp} f) \cap S = \emptyset\}, \\ M(S) &= \{f \in A : f(S) = 0\}. \end{aligned} \tag{1.3}$$

For $x \in X$, we usually write I_x for $I(\{x\})$ and M_x for $M(\{x\})$. Note that A_{00} is an ideal in A and $A = A_{00}$ whenever X is compact.

Definition 1.3. Let A be an Ff-algebra on a topological space X.

(i) A is said to be regular on X if for any nonempty closed subset S of X and each $x \in X \setminus S$, there exists $f \in A$ such that $f(x) = 1$ and $f(S) = \{0\}$, and it is normal if for each nonempty closed subset E and nonempty compact subset F of X with $E \cap F = \emptyset$, there exists $f \in A$ such that $f(E) = \{1\}$ and $f(F) = \{0\}$.

A commutative F-algebra is regular (normal) if $S_A \neq \emptyset$ and $\widehat{A}$ is a regular (normal) Ff-algebra on S_A.

(ii) A is said to be a strongly regular algebra if for every $f \in A$ and $x \in X$ with $f(x) = 0$, there exists a sequence $\{f_n\}$ in A_{00} and open neighborhoods V_n of x such that $f_n|_{V_n} = 0$ for all $n \in \mathbb{N}$, and $f_n \to f$ as $n \to \infty$, or equivalently, $M_x = \overline{I_x}$ for each $x \in X$.

(iii) An Ff-algebra A on X is said to satisfy Ditkin's condition if for every $f \in A$ and $x \in X$ with $f(x) = 0$, there exists a sequence $\{f_n\}$ in A_{00} and open neighborhoods V_n of x such that $f_n|_{V_n} = 0$ for all $n \in \mathbb{N}$, and $f_n f \to f$ as $n \to \infty$, or equivalently, $f \in \overline{f I_x}$ for all $x \in X$ and $f \in M_x$.

It is clear that every strongly regular algebra is regular. Moreover, if an Ff-algebra satisfies Ditkin's condition, then A is strongly regular. In general, the converse is not true as the following example shows.

Recall that a Banach sequence algebra on a nonempty set S is a Banach algebra A such that $c_{00}(S) \subset A \subset \mathbb{C}^S$, where $c_{00}(S)$ is the linear span of the set $\{\chi_s : s \in S\}$ consisting of all characteristic functions χ_s of the singleton subsets $\{s\}$ of S.

Example 1.4 (see [20, Example 4.5.33]). Consider the Banach space $(L^2(\mathbb{T}), \|\cdot\|_2)$ where $\mathbb{T} = [-\pi, \pi]$. Let $S = [-\pi/2, \pi/2]$ and set

$$W = \left\{ f \in L^2(\mathbb{T}) : f|_S \in C(S) \right\}. \tag{1.4}$$

For each $f, g \in W$ and $\theta \in [-\pi, \pi]$, set

$$\begin{gathered} (f \star g)(\theta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\theta - \phi) g(\phi) d\phi, \\ \|f\| = \|f\|_2 + \|f\|_S = \frac{1}{\sqrt{2\pi}} \left(\int_{-\pi}^{\pi} |f(\theta)|^2 d\theta \right)^{1/2} + \|f\|_S. \end{gathered} \tag{1.5}$$

Then, $(W, \star, \|\cdot\|)$ is a commutative Banach algebra (for an equivalent norm).

Identifying W with its algebra of Fourier transforms on $\mathbb{Z}$, W is a strongly regular Banach sequence algebra on $\mathbb{Z}$. Moreover, $W^\#$, the unitization of W, is a strongly regular Bf-algebra on $\mathbb{Z}_\infty$, the one point compactification of $\mathbb{Z}$. Now, we show that $W^\#$ does not satisfy Ditkin's condition. In the following, we write $r\mathbb{Z} + s$ for the subset $\{rn + s : n \in \mathbb{Z}\}$ of $\mathbb{Z}$. Set $F_1 = 4\mathbb{Z} \cup \{\infty\}$, $F_2 = (4\mathbb{Z} + 2) \cup \{\infty\}$, and $F = F_1 \cup F_2 = 2\mathbb{Z} \cup \{\infty\}$. Define g_0 on $\mathbb{T}$ by

$$g_0(\theta) = 1 \quad \left(|\theta| \le \frac{\pi}{2} \right), \qquad g_0(\theta) = -1 \quad \left(\frac{\pi}{2} < |\theta| \le \pi \right). \tag{1.6}$$

Then, $g_0 \in W$ and

$$\widehat{g_0}(k) = \frac{1}{\pi} \int_0^{\pi} g_0(\theta) \cos k\theta d\theta = \frac{2}{k\pi} \sin\left(\frac{k\pi}{2} \right) \quad (k \in \mathbb{Z},\ k \neq 0), \tag{1.7}$$

with $\widehat{g_0}(0) = 0$, and so $\widehat{g_0} \in M(F)$. By [20, Example 4.5.33(v)], $\widehat{g_0} \in M(F) \setminus \overline{I(F)}$. Since $\widehat{g_0} \in M(F)$, necessarily $\overline{\widehat{g_0} I_\infty} \subset \overline{I(F)}$, and so $\widehat{g_0} \notin \overline{\widehat{g_0} I_\infty}$, where I_∞ is the set of all functions in A_{00} which are zero on a neighborhood of ∞.

2. Main Results

We first state the following useful result, which is, in fact, the generalization of [6, Lemma 2.1 and Theorem 2.2].

Lemma 2.1. *Let X and Y be compact Hausdorff spaces, A and B normal Ff-algebras on X and Y, respectively, and $T : A \to B$ a biseparating map. Then, for each $x \in X$, there exists a unique $y \in Y$ such that $TI_x = I_y$. If we define $\varphi : Y \to X$ by $\varphi(y) = x$, then φ is a homeomorphism.*

Proof. We omit the proof, since it is similar to the proofs of [6, Lemma 2.1 and Theorem 2.2]. □

We now bring the following theorem, which is an extension of the results of Jarosz and Font.

Theorem 2.2. *Let $(A,(p_n))$ and $(B,(q_n))$ be strongly regular normal Ff-algebras on compact Hausdorff spaces X and Y, respectively, such that the evaluation homomorphisms on A and B are continuous. Then, every biseparating map $T : A \to B$ is a weighted composition operator of the form*

$$Tf = h \cdot (f \circ \varphi), \quad (f \in A), \tag{2.1}$$

where φ is a homeomorphism from Y onto X and h is a nonvanishing element of B. In particular, T is automatically continuous.

Proof. By Lemma 2.1, there exists a homeomorphism φ from Y onto X defined by $\varphi(y) = x$, where $TI_x = I_y$. We first show that $TM_x \subseteq M_y$. Suppose on the contrary that there exists $f \in M_x$ such that $Tf(y) \neq 0$. If x belongs to the interior of $f^{-1}(0)$, then $f \in I_x$, and thus $Tf(y) = 0$, since $TI_x = I_y$. Therefore, we may assume that there exists a net $\{x_\lambda\}_\lambda$ of distinct elements of X converging to x such that $f(x_\lambda)$ is never zero. Consider the net $\{y_\lambda\}_\lambda$ in Y such that $\varphi(y_\lambda) = x_\lambda$. Clearly, y_λ converges to y, and by passing through a subnet if necessary, we may assume that there exists a constant ε such that

$$\left|Tf(y_\lambda)\right| \geq \varepsilon > 0, \tag{2.2}$$

for all λ. Since $f(x_\lambda) \to 0$, we can find a subsequence $\{f(x_n)\}$ such that $f(x_n) \to 0$. Since $x_\lambda \to x$, it follows that $x_n \to x$. By the normality of X, there exists a neighborhood W_n of x_n such that $W_n \cap W_m = \emptyset$ if $n \neq m$. Let V_n be a neighborhood of x_n such that $V_n \subseteq \overline{V}_n \subseteq W_n$. Consider the sequence $\{y_n\}$ in Y such that $\varphi(y_n) = x_n$, for each $n \in \mathbb{N}$. By (2.2), without loss of generality, we may assume that $|T(f - f(x_n)1)(y_n)| \geq \delta$ for some positive δ and for all $n \in \mathbb{N}$. Since A is normal, for each $n \in \mathbb{N}$, there exists $s_n \in A$ such that $s_n = 1$ on V_n and $s_n = 0$ on $X \setminus W_n$. If we take $f_n = n(f - f(x_n)1)$, then $f_n(x_n) = 0$, and since A is strongly regular, we can find h_n in A and a neighborhood U_n of x_n in X such that $U_n \subseteq V_n$, $h_n = 0$ on U_n and $p_n(f_n - h_n) \leq 1/n^2(p_n(s_n) + 1)$. If we set $\psi_n = (f_n - h_n)s_n$, then $p_n(\psi_n) \leq 1/n^2$, and hence for each $k \in \mathbb{N}$, $k \geq 2$, we have

$$\sum_{n=1}^{\infty} p_k(\psi_n) \leq \sum_{n=1}^{k-1} p_k(\psi_n) + \sum_{n=k}^{\infty} p_n(\psi_n) \leq \sum_{n=1}^{k-1} p_k(\psi_n) + \sum_{n=k}^{\infty} \frac{1}{n^2} < \infty. \tag{2.3}$$

Therefore, $\sum_{n=1}^{\infty} \psi_n$ converges to an element $\psi \in A$. On the other hand, for each $n \in \mathbb{N}$, $\psi_n = f_n$ on U_n, which implies that $\psi_n - f_n \in I_{x_n}$ and $T(\psi_n - f_n) \in I_{y_n}$. Consequently, $T\psi_n(y_n) = Tf_n(y_n)$. Since the evaluation homomorphisms are continuous on A, the series $\sum_{n=1}^{\infty} \psi_n(x)$ converges to $\psi(x)$ for each $x \in X$. Hence, $\psi = \psi_n$ on W_n, since the elements of the sequence $\{W_n\}$ are pairwise disjoint and $\mathrm{coz}\psi_n \subset W_n$. Therefore,

$$\left|T\psi(y_n)\right| = \left|T\psi_n(y_n)\right| = \left|Tf_n(y_n)\right| = n\left|T(f - f(x_n)1)(y_n)\right| \geq n\delta, \tag{2.4}$$

for all $n \in \mathbb{N}$, which is a contradiction, since $y_n = \varphi^{-1}(x_n) \to \varphi^{-1}(x)$ and $T\psi(y_n) \to T\psi(\varphi^{-1}(x))$. Therefore, $TM_x \subseteq M_y$.

By a similar argument, we can show that $T^{-1}M_y \subseteq M_x$ and hence $TM_x = M_y$. Thus $\ker \delta_x = \ker(\delta_y \circ T)$, and so there exists a scalar $h(y)$ such that $\delta_y \circ T = h(y)\delta_x$. Equivalently, $Tf(y) = h(y)f(\varphi(y))$ for all f in A and y in Y. In particular, when $f = 1$, we have $h = T1$, which is a nonvanishing element of B, since T is surjective. □

Definition 2.3. Let A and B be Ff-algebras on compact Hausdorff spaces X and Y respectively. A linear map $T : A \to B$ is called cozero preserving, whenever $\mathrm{coz}(f) \subseteq \mathrm{coz}(g)$ implies $\mathrm{coz}(Tf) \subseteq \mathrm{coz}(Tg)$.

In [21], Font has studied the automatic continuity of cozero preserving maps between Fourier algebras. In the following theorems, we generalize the results of Font to Bf-algebras as well as Ff-algebras.

Theorem 2.4. *Let A and B be Bf-algebras on compact Hausdorff spaces X and Y, respectively, such that B is inverse closed. If $T : A \to B$ is a unital cozero preserving surjective map, then T is automatically continuous.*

Proof. Let $\lambda \notin sp_A(f)$. Then, $(\lambda 1 - f) \in \mathrm{Inv}(A)$, and so $\mathrm{coz}(\lambda 1 - f) = X$. Therefore, we have $X = \mathrm{coz}(1) \subseteq \mathrm{coz}(\lambda 1 - f)$. Since $T1 = 1$ and T is cozero preserving, we conclude that $Y = \mathrm{coz}(1) \subseteq \mathrm{coz}(\lambda 1 - Tf)$. Since B is inverse closed and $(\lambda 1 - Tf)(y) \neq 0$ for all $y \in Y$, it follows that $\lambda \notin sp_B(Tf)$, which implies that $sp_B(Tf) \subseteq sp_A(f)$, for every $f \in A$. Thus by [20, Theorem 5.1.9(iii)], $\mathfrak{S}(T) \subseteq \mathrm{rad}(B) = \{0\}$, and hence T is automatically continuous. □

We now adopt a similar method as in the proof of [6, Lemma 3.3] to obtain the following results.

Theorem 2.5. *Let A and B be function algebras on compact Hausdorff spaces X and Y, respectively, and $T : A \to B$ a cozero preserving injection. If A is regular, then T^{-1} is separating.*

Proof. Suppose on the contrary that there exist f and g in A such that $TfTg = 0$ but $f(x)g(x) \neq 0$ for some $x \in X$. So, we can find an open neighborhood V of x such that $V \subseteq \mathrm{coz}(f) \cap \mathrm{coz}(g)$. Since A is regular, there exists $h \in A$ such that $h(x) = 1$ and $h|_{X\setminus V} = 0$. It is clear that $\mathrm{coz}(h) \subseteq \mathrm{coz}(f) \cap \mathrm{coz}(g)$. So by hypothesis, $\mathrm{coz}(Th) \subseteq \mathrm{coz}(Tf) \cap \mathrm{coz}(Tg)$. On the other hand, $TfTg = 0$ implies that $\mathrm{coz}(Tf) \cap \mathrm{coz}(Tg) = \emptyset$. It follows that $\mathrm{coz}(Th) = \emptyset$, that is, $Th = 0$. Now injectivity of T shows that $h = 0$, which is a contradiction. Therefore, T^{-1} is separating. □

Corollary 2.6. *Let A and B be strongly regular normal Ff-algebras on compact Hausdorff spaces X and Y, respectively, such that evaluation homomorphisms are continuous on A and B. If $T : A \to B$ is a linear bijection, then the following statements are equivalent:*

(i) *T is separating and cozero preserving;*

(ii) *T is biseparating;*

(iii) *T and T^{-1} are both cozero preserving;*

(iv) *T and T^{-1} are weighted composition operators.*

Proof. It suffices to prove (ii) $\Rightarrow$ (i). The other implications are direct consequences of Theorems 2.2 and 2.5. If (ii) is satisfied, then all hypotheses of Theorem 2.2 are satisfied.

Let for $f, g \in A$ and $y \in Y$, we have $\mathrm{coz}(f) \subseteq \mathrm{coz}(g)$ and $y \notin \mathrm{coz}(Tg)$. By Theorem 2.2, $TM_x = M_y$. Since $Tg(y) = 0$, it follows that $g(x) = 0$, and hence $f(x) = 0$, that is, $f \in M_x$. Thus, $Tf(y) = 0$ and consequently $y \notin \mathrm{coz}(Tf)$. □

References

[1] B. Z. Vulkh, "On linear multiplicative operations," *Doklady Akademii Nauk USSR*, vol. 41, pp. 148–151, 1943.

[2] B. Z. Vulkh, "Multiplication in linear semi-ordered spaces and its application to the theory of operations," *Matematicheskii Sbornik*, vol. 22, pp. 267–317, 1948.

[3] K. Jarosz, "Automatic continuity of separating linear isomorphisms," *Canadian Mathematical Bulletin*, vol. 33, no. 2, pp. 139–144, 1990.

[4] J. J. Font, "Automatic continuity of certain isomorphisms between regular Banach function algebras," *Glasgow Mathematical Journal*, vol. 39, no. 3, pp. 333–343, 1997.

[5] H.-L. Gau, J.-S. Jeang, and N.-C. Wong, "An algebraic approach to the Banach-Stone theorem for separating linear bijections," *Taiwanese Journal of Mathematics*, vol. 6, no. 3, pp. 399–403, 2002.

[6] H.-L. Gau, J.-S. Jeang, and N.-C. Wong, "Biseparating linear maps between continuous vector-valued function spaces," *Journal of the Australian Mathematical Society*, vol. 74, no. 1, pp. 101–109, 2003.

[7] Y. A. Abramovich and A. K. Kitover, "A solution to a problem on invertible disjointness preserving operators," *Proceedings of the American Mathematical Society*, vol. 126, no. 5, pp. 1501–1505, 1998.

[8] Y. A. Abramovich and A. K. Kitover, "Inverses of disjointness preserving operators," *Memoirs of the American Mathematical Society*, vol. 143, no. 679, 2000.

[9] J. Araujo, "Separating maps and linear isometries between some spaces of continuous functions," *Journal of Mathematical Analysis and Applications*, vol. 226, no. 1, pp. 23–39, 1998.

[10] J. Araujo, "Realcompactness and spaces of vector-valued functions," *Fundamenta Mathematicae*, vol. 172, no. 1, pp. 27–40, 2002.

[11] J. Araujo and L. Dubarbie, "Biseparating maps between Lipschitz function spaces," *Journal of Mathematical Analysis and Applications*, vol. 357, no. 1, pp. 191–200, 2009.

[12] J. Araujo and K. Jarosz, "Automatic continuity of biseparating maps," *Studia Mathematica*, vol. 155, no. 3, pp. 231–239, 2003.

[13] J. J. Font and S. Hernández, "On separating maps between locally compact spaces," *Archiv der Mathematik*, vol. 63, no. 2, pp. 158–165, 1994.

[14] J.-S. Jeang and N.-C. Wong, "Weighted composition operators of $C_0(X)'$s," *Journal of Mathematical Analysis and Applications*, vol. 201, no. 3, pp. 981–993, 1996.

[15] D. H. Leung, "Biseparating maps on generalized Lipschitz spaces," *Studia Mathematica*, vol. 196, no. 1, pp. 23–40, 2010.

[16] L. Li and N.-C. Wong, "Banach-Stone theorems for vector valued functions on completely regular spaces," *Journal of Mathematical Analysis and Applications*, vol. 395, no. 1, pp. 265–274, 2012.

[17] H. Goldmann, *Uniform Fréchet Algebras*, vol. 162 of *North-Holland Mathematics Studies*, North-Holland, Amsterdam, The Netherlands, 1990.

[18] H. G. Dales and A. M. Davie, "Quasianalytic Banach function algebras," *Journal of Functional Analysis*, vol. 13, pp. 28–50, 1973.

[19] T. G. Honary and H. Mahyar, "Approximation in Lipschitz algebras of infinitely differentiable functions," *Bulletin of the Korean Mathematical Society*, vol. 36, no. 4, pp. 629–636, 1999.

[20] H. D. Dales, *Banach Algebras and Automatic Continuity*, LMS Monographs 24, Clarendon Press, Oxford, UK, 2000.

[21] J. J. Font, "Disjointness preserving mappings between Fourier algebras," *Colloquium Mathematicum*, vol. 77, no. 2, pp. 179–187, 1998.