Hindawi Publishing Corporation
Journal of Function Spaces and Applications
Volume 2013, Article ID 425102, 6 pages
http://dx.doi.org/10.1155/2013/425102

Research Article

On the Stability of Nonautonomous Linear Impulsive Differential Equations

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Received 2 May 2013; Accepted 21 June 2013

Academic Editor: Gestur Ólafsson

We introduce two Ulam's type stability concepts for nonautonomous linear impulsive ordinary differential equations. Ulam-Hyers and Ulam-Hyers-Rassias stability results on compact and unbounded intervals are presented, respectively.

1. Introduction

During the past decades, the impulsive differential equations have attracted many authors since it is better to describe dynamics of populations subject to abrupt changes as well as other phenomena such as harvesting and diseases than the corresponding differential equations without impulses. For the basic theory on the impulsive differential equations and impulsive controls, the reader can refer to the monographs of Baĭnov and Simeonov [1], Lakshmikantham et al. [2], Yang [3], and Benchohra et al. [4] and references therein. In particular, exponential, asymptotical, strong, weak and Lyapunov stability of all kinds of impulsive differential equations has been studied extensively in the previous monographs and references therein.

In addition to the previously mentioned stability theory, Ulam stability of functional equation, which was formulated by Ulam on a talk given to a conference at Wisconsin University in 1940, is one of the central subjects in the mathematical analysis area. Many researchers paid much attention to discuss the stability properties of all kinds of equations. In fact, Ulam's type stability problems have been taken up by a large number of mathematicians, and the study of this area has grown to be one of the most important subjects in the mathematical analysis area. For the advanced contribution on such problems, we refer the reader to András and Kolumbán [5], András and Mészáros [6], Burger et al. [7], Cădariu [8], Castro and Ramos [9], Ciepliński [10], Cimpean and Popa [11], Hyers et al. [12], Hegyi and Jung [13], Jung [14, 15], Lungu and Popa [16], Miura et al. [17, 18], Moslehian and Rassias [19], Rassias [20, 21], Rus [22, 23], Takahasi et al. [24], and Wang et al. [25–27].

As far as we know, there are few results on Ulam's type stability of nonautonomous impulsive differential equations. Motivated by recent works [23, 25, 27], we study Ulam's type stability of nonautonomous linear impulsive differential equations:

$$\begin{gathered} x'(t) = A(t)\,x(t) + f(t), \\ t \in J' := J \setminus \{t_1, \ldots, t_m\}, \\ J := [0, L), \\ 0 < L \leq +\infty, \\ \Delta x(t_k) = B_k x(t_k^-) + b_k, \quad k = 1, 2, \ldots, m, \end{gathered} \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$, $f(t) = (f_1(t), f_2(t), \ldots, f_n(t))^T \in C(J, \mathbb{R}^n)$, $A(t) = \operatorname{diag}(A_1(t), A_2(t), \ldots, A_n(t))$ is n-order real diagonal matrix, and $B_k = \operatorname{diag}(B_1^k, B_2^k, \ldots, B_n^k)$ and $b_k = (b_1^k, b_2^k, \ldots, b_n^k)^T$ are n-order bounded diagonal matrix and n-dimensional bounded vector, respectively. Impulsive sequence t_k satisfy $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = L$, $\Delta x(t_k) := x(t_k^+) - x(t_k^-)$, and $x(t_k^+) = \lim_{\epsilon \to 0^+} x(t_k + \epsilon)$

and $x(t_k^-) = \lim_{\epsilon \to 0^-} x(t_k+\epsilon)$ represent the right and left limits of $x(t)$ at $t = t_k$, $k = 1, 2, \ldots, m$.

Firstly, we will modify the Ulam's type stability concepts in [23] and introduce two Ulam's type stability concepts for (1). Secondly, we pay attention to check the Ulam-Hyers and Ulam-Hyers-Rassias stability results on a compact and unbounded intervals, respectively.

2. Preliminaries

Let $C(J, \mathbb{R}^n)$ be the Banach space of all continuous functions from J into $\mathbb{R}^n$ with the norm $\|x\| := \max\{\|x_1\|_C, \|x_2\|_C, \ldots, \|x_n\|_C\}$ for $x \in C(J, \mathbb{R}^n)$, where $\|x_k\|_C := \sup_{t\in J}|x_k(t)|$. Also, we use the Banach space $PC(J, \mathbb{R}^n) := \{x : J \to \mathbb{R}^n : x \in C((t_k, t_{k+1}), \mathbb{R}^n), k = 0, 1, \ldots, m$, and there exist $x(t_k^-)$ and $x(t_k^+)$, $k = 1, \ldots, m$, with $x(t_k^-) = x(t_k)\}$ with the norm $\|x\|_{PC} := \max\{\|x_1\|_{PC}, \|x_2\|_{PC}, \ldots, \|x_n\|_{PC}\}$. Denote $PC^1(J, \mathbb{R}^n) := \{x \in PC(J, \mathbb{R}^n) : x' \in PC(J, \mathbb{R}^n)\}$. Set $\|x\|_{PC^1} := \max\{\|x\|_{PC}, \|x'\|_{PC}\}$. It can be seen that endowed with the norm $\|\cdot\|_{PC^1}$, $PC^1(J, \mathbb{R}^n)$ is also a Banach space.

If $x, y \in \mathbb{R}^n$, $x = (x_1, x_2, \ldots x_n)$, $y = (y_1, y_2, \ldots y_n)$, by $x \le y$, we mean that $x_i \le y_i$ for $i = 1, 2, \ldots, n$.

It follows [3], we introduce the concept of piecewise continuous solutions.

Definition 1. By a PC^1, from solution of the following impulsive Cauchy problem

$$\begin{gathered} x'(t) = A(t)x(t) + f(t), \quad t \in J' := J \setminus \{t_k\}, \\ \Delta x(t_k) = B_k x(t_k^-) + b_k, \quad k = 1, 2, \ldots, m, \\ x(0) = x_0 \in \mathbb{R}^n, \end{gathered} \tag{2}$$

we mean that the function $x \in PC^1(J, \mathbb{R}^n)$ which satisfies

$$\begin{aligned} x(t) = \Psi(t, 0)x_0 + \int_0^t \Psi(t, s) f(s)\, ds \\ + \sum_{0<t_k<t} \Psi(t, t_k^+) b_k, \quad t \in J, \end{aligned} \tag{3}$$

where Ψ is called impulsive evolution matrix which is given by

$$\Psi(t,\theta) = \begin{cases} \Phi(t,\theta), & t_{k-1} \le \theta \le t \le t_k, \\ \Phi(t,t_k^+)(I + B_k)\Phi(t_k,\theta), & \\ & t_{k-1} \le \theta < t_k < t \le t_{k+1}, \\ \Phi(t,t_k^+)\left[\prod_{\theta<t_j<t}(I + B_j)\Phi(t_j,t_{j-1}^+)\right] & \\ \times(I + B_i)\Phi(t_i,\theta), & \\ & t_{i-1} \le \theta < t_i \le \cdots < t_k < t \le t_{k+1}, \\ & k = 1, 2, \ldots, m, \end{cases} \tag{4}$$

Φ is the evolution matrix for the system $x' = A(t)x$ and I denotes the identity matrix.

If there exists $M > 0$, such that $\|A(t)\| = \max_{t\in J}\{|A_i(t)|, i = 1, 2, \ldots, n\} \le M$ for any $t \in J$, then Φ satisfy

$$\|\Phi(t, s)\| \le e^{M(t-s)}, \quad \forall s, t \in J,\ s \le t. \tag{5}$$

By proceeding with the same elementary computation in Lemma 2.5(5) of [28], we have

$$\|\Psi(t, s)\| \le e^{M(t-s)} \prod_{i=1}^{m} (1 + \|B_i\|) \quad \forall s, t \in J,\ s < t. \tag{6}$$

Next, we introduce two Ulam's type stability definitions for (1) which can be regarded as the extension of the Ulam's type stability concepts for ordinary differential equations in [23].

Let $\epsilon_i > 0$, $\psi_i \ge 0$, and $\varphi_i \in PC(J, \mathbb{R}_+)$ be nondecreasing functions where $i = 1, 2, \ldots, n$. For $t \in J$, denote

$$\begin{gathered} \zeta(t) = \begin{cases} 1, & \text{if } L < +\infty, \\ e^{-Mt}, & \text{if } L = +\infty, \end{cases} \\ \xi(t) = \begin{cases} 1, & \text{if } L < +\infty, \\ e^{-M(t-t_j)}, & \text{if } L = +\infty, \\ & t_j \in \{t_1, t_2, \ldots, t_m\}. \end{cases} \end{gathered} \tag{7}$$

We consider the following inequalities:

$$\begin{gathered} \begin{pmatrix} \left|y_1'(t) - A_1(t)y_1(t) - f_1(t)\right| \\ \left|y_2'(t) - A_2(t)y_2(t) - f_2(t)\right| \\ \vdots \\ \left|y_n'(t) - A_n(t)y_n(t) - f_n(t)\right| \end{pmatrix} \\ \le \zeta(t)(\epsilon_1, \epsilon_2, \ldots, \epsilon_n)^T, \quad t \in J', \\ \begin{pmatrix} \left|\Delta y_1(t_k) - B_1^k y_1(t_k^-) - b_1^k\right| \\ \left|\Delta y_2(t_k) - B_2^k y_2(t_k^-) - b_2^k\right| \\ \vdots \\ \left|\Delta y_n(t_k) - B_n^k y_n(t_k^-) - b_n^k\right| \end{pmatrix} \\ \le \xi(t)(\epsilon_1, \epsilon_2, \ldots, \epsilon_n)^T, \quad k = 1, 2, \ldots, m, \end{gathered} \tag{8}$$

$$\begin{pmatrix} \left|y_1'(t) - A_1(t)\,y_1(t) - f_1(t)\right| \\ \left|y_2'(t) - A_2(t)\,y_2(t) - f_2(t)\right| \\ \vdots \\ \left|y_n'(t) - A_n(t)\,y_n(t) - f_n(t)\right| \end{pmatrix} \le \zeta(t)\left(\varphi_1(t)\,\epsilon_1, \varphi_2(t)\,\epsilon_2, \ldots, \varphi_n(t)\,\epsilon_n\right)^T, \quad t \in J',$$

$$\begin{pmatrix} \left|\Delta y_1(t_k) - B_1^k y_1(t_k^-) - b_1^k\right| \\ \left|\Delta y_2(t_k) - B_2^k y_2(t_k^-) - b_2^k\right| \\ \vdots \\ \left|\Delta y_n(t_k) - B_n^k y_n(t_k^-) - b_n^k\right| \end{pmatrix} \le \xi(t)\left(\psi_1\epsilon_1, \psi_2\epsilon_2, \ldots, \psi_n\epsilon_n\right)^T, \quad k = 1, 2, \ldots, m. \tag{9}$$

Definition 2. Equation (1) is Ulam-Hyers stable, if there exist constants $C_{f,m}^i > 0$, $i = 1, 2, \ldots, n$, such that for each $\epsilon_i > 0$ and for each solution $y \in PC^1(J, \mathbb{R}^n)$ of inequality (8) there exists a solution $x \in PC^1(J, \mathbb{R}^n)$ of (1) with

$$\begin{pmatrix} \left|y_1(t) - x_1(t)\right| \\ \left|y_2(t) - x_2(t)\right| \\ \vdots \\ \left|y_n(t) - x_n(t)\right| \end{pmatrix} \le \left(C_{f,m}^1\epsilon_1, C_{f,m}^2\epsilon_2, \ldots, C_{f,m}^n\epsilon_n\right)^T, \quad t \in J. \tag{10}$$

Definition 3. Equation (1) is Ulam-Hyers-Rassias stable with respect to (φ_i, ψ_i) if there exist $C_{f,m,\varphi_i} > 0$, $i = 1, 2, \ldots, n$ such that for each $\epsilon_i > 0$ and for each solution $y \in PC^1(J, \mathbb{R}^n)$ of inequality (9) there exists a solution $x \in PC^1(J, \mathbb{R}^n)$ of (1) with

$$\begin{pmatrix} \left|y_1(t) - x_1(t)\right| \\ \left|y_2(t) - x_2(t)\right| \\ \vdots \\ \left|y_n(t) - x_n(t)\right| \end{pmatrix} \le \begin{pmatrix} C_{f,m,\varphi_1}\left(\varphi_1(t) + \psi_1\right)\epsilon_1 \\ C_{f,m,\varphi_2}\left(\varphi_2(t) + \psi_2\right)\epsilon_2 \\ \vdots \\ C_{f,m,\varphi_n}\left(\varphi_n(t) + \psi_n\right)\epsilon_n \end{pmatrix}, \quad t \in J. \tag{11}$$

3. Stability Results in the Case $L<+\infty$

We introduce the following assumptions.

(H_1) $f \in C(J, \mathbb{R}^n)$.

(H_2) $A_i \in C(J, \mathbb{R})$, $i = 1, 2, \ldots, n$.

Denote $M_i = \max_{t\in J}\{|A_i(t)|\}$, and denote $M = \max\{M_1, M_2, \ldots, M_n\}$.

Now, we are ready to state our first Ulam-Hyers stable result on a compact interval.

Theorem 4. *Assume that* (H_1)*-*(H_2) *are satisfied. Then* (1) *is Ulam-Hyers stable.*

Proof. Let $y \in PC^1(J, \mathbb{R}^n)$ be a solution of inequality (8). Define

$$\begin{aligned} g(t) &= \left(g_1(t), g_2(t), \ldots, g_n(t)\right)^T \\ &:= y'(t) - A(t)\,y(t) - f(t), \\ &\qquad t \in J', \\ g_k &= \left(g_1^k, g_2^k, \ldots, g_n^k\right)^T \\ &:= \Delta y(t_k) - B_k y(t_k^-) - b_k, \\ &\qquad k = 1, 2, \ldots, m. \end{aligned} \tag{12}$$

Then we have

$$\begin{aligned} \|g(t)\| &\le \epsilon_{\max} := \max\{\epsilon_1, \epsilon_2, \ldots, \epsilon_n\}, \quad t \in J, \\ \|g_k\| &\le \epsilon_{\max}, \quad k = 1, 2, \ldots, m. \end{aligned} \tag{13}$$

According to Definition 1, for each $t \in (t_k, t_{k+1}]$, we have

$$\begin{aligned} y(t) = {} & \Psi(t, 0)\,y(0) \\ & + \int_0^t \Psi(t, s)\,f(s)\,ds \\ & + \int_0^t \Psi(t, s)\,g(s)\,ds \\ & + \sum_{i=1}^k \Psi(t, t_i^+)\left(b_k + g_k\right). \end{aligned} \tag{14}$$

Suppose that x be the unique solution of the impulsive Cauchy problem:

$$\begin{aligned} x'(t) &= A(t)\,x(t) + f(t), \quad t \in J', \\ \Delta x(t_k) &= B_k x(t_k^-) + b_k, \quad k = 1, 2, \ldots, m, \\ x(0) &= y(0), \quad y(0) \in \mathbb{R}^n. \end{aligned} \tag{15}$$

Then for each $t \in (t_k, t_{k+1}]$, we have

$$\begin{aligned} x(t) = {} & \Psi(t, 0)\,y(0) \\ & + \int_0^t \Psi(t, s)\,f(s)\,ds \\ & + \sum_{i=1}^k \Psi(t, t_i^+)\,b_k. \end{aligned} \tag{16}$$

It follows that, for each $t \in (t_k, t_{k+1}]$, we can derive

$$\begin{aligned}\|y(t) - x(t)\| &= \Big\| y(t) - \Psi(t,0)\, y(0) \\ &\quad - \sum_{i=1}^{k} \Psi(t, t_i^+)\, b_k \\ &\quad - \int_0^t \Psi(t,s)\, f(s)\, ds \Big\| \\ &\le \sum_{i=1}^{m} \|\Psi(t,t_i^+)\| \, \|g_k\| \\ &\quad + \int_0^t \|\Psi(t,s)\| \, \|g(s)\|\, ds \\ &\le m e^{ML} \prod_{k=1}^{m} (1 + \|B_k\|)\, \epsilon_{\max} \\ &\quad + L e^{ML} \prod_{k=1}^{m} (1 + \|B_k\|)\, \epsilon_{\max} \\ &= e^{ML} \prod_{k=1}^{m} (1 + \|B_k\|)\,(m+L)\, \epsilon_{\max}.\end{aligned} \tag{17}$$

Thus,

$$\begin{pmatrix} |y_1(t) - x_1(t)| \\ |y_2(t) - x_2(t)| \\ \vdots \\ |y_n(t) - x_n(t)| \end{pmatrix} \le \left(C_{f,m}\epsilon_{\max}, C_{f,m}\epsilon_{\max}, \dots, C_{f,m}\epsilon_{\max}\right)^T, \quad t \in J, \tag{18}$$

where

$$C_{f,m} = e^{ML}(m+L) \prod_{k=1}^{m} (1 + \|B_k\|). \tag{19}$$

So (1) is Ulam-Hyers stable. The proof is completed. □

In order to discuss Ulam-Hyers-Rassias stability, we need the following condition.

(H_3) There exist $\lambda_{\varphi_i} > 0$, $i = 1, 2, \dots, n$, such that

$$\int_0^t e^{M(t-s)} \varphi_i(s)\, ds \le \lambda_{\varphi_i} \varphi_i(t) \quad \text{for each } t \in J, \tag{20}$$

where $\varphi_i \in PC(J, \mathbb{R}_+)$ is nondecreasing.

Theorem 5. *Assume that (H_1)–(H_3) are satisfied. Then (1) is Ulam-Hyers-Rassias stable.*

Proof. Let $y \in PC^1(J, \mathbb{R}^n)$ be a solution of inequality (9). For g and g_k defined in (12), we have

$$\begin{aligned}\|g(t)\| &\le \varphi_i(t)\, \epsilon_{\max}, \quad t \in J, \\ \|g_k\| &\le \psi_i \epsilon_{\max}, \quad k = 1, 2, \dots, m.\end{aligned} \tag{21}$$

Let x be the unique solution of (15). Hence, for each $t \in (t_k, t_{k+1}]$, it follows from (H_3) that

$$\begin{aligned}&\|y(t) - x(t)\| \\ &\quad \le m e^{ML} \prod_{k=1}^{m} (1 + \|B_k\|)\, \psi_i \epsilon_{\max} \\ &\qquad + \epsilon_{\max} \prod_{k=1}^{m} (1 + \|B_k\|) \int_0^t e^{M(t-s)} \varphi_i(t)\, ds \\ &\quad \le m e^{ML} \prod_{k=1}^{m} (1 + \|B_k\|)\, \psi_i \epsilon_{\max} \\ &\qquad + \epsilon_{\max} \prod_{k=1}^{m} (1 + \|B_k\|)\, \lambda_{\varphi_i} \varphi_i(t) \\ &\quad = \prod_{k=1}^{m} (1 + \|B_k\|) \left(m e^{ML} + \lambda_{\varphi_i}\right) \\ &\qquad \times (\varphi_i(t) + \psi_i)\, \epsilon_{\max}, \quad i = 1, 2, \dots, n.\end{aligned} \tag{22}$$

Thus, we obtain

$$\begin{pmatrix} |y_1(t) - x_1(t)| \\ |y_2(t) - x_2(t)| \\ \vdots \\ |y_n(t) - x_n(t)| \end{pmatrix} \le \begin{pmatrix} C_{f,m,\varphi_1} (\varphi_1(t) + \psi_1)\, \epsilon_{\max} \\ C_{f,m,\varphi_2} (\varphi_2(t) + \psi_2)\, \epsilon_{\max} \\ \vdots \\ C_{f,m,\varphi_n} (\varphi_n(t) + \psi_n)\, \epsilon_{\max} \end{pmatrix}, \quad t \in J, \tag{23}$$

where

$$C_{f,m,\varphi_i} = \left(m e^{ML} + \lambda_{\varphi_i}\right) \prod_{k=1}^{m} \left(1 + \|B^k\|\right), \quad i = 1, 2, \dots, n. \tag{24}$$

Thus, (1) is Ulam-Hyers-Rassias stable. The proof is completed. □

4. Stability Results in the Case $L = +\infty$

In this section, we will present stability results on an unbounded interval.

We change (H_2) to the following strong condition.

$(H_2{}')$ A_i is continuous and uniformly bounded function on J, $i = 1, 2, \dots, n$. Thus, there exists $M > 0$ such that $\|A(t)\| \le M$ for any $t \in J$.

Theorem 6. *Assume that* (H_1)–$(H_2{}')$ *are satisfied. Then* (1) *is Ulam-Hyers stable.*

Proof. Let $y \in PC^1(J, \mathbb{R}^n)$ be a solution of inequality (8). For g and g_k, defined in (12), we have

$$\|g(t)\| \le e^{-Mt}\epsilon_{\max}, \quad t \in J,$$
$$\|g_k\| \le e^{-M(t-t_i)}\epsilon_{\max}, \quad k = 1, 2, \ldots, m. \tag{25}$$

Let x be the unique solution of (15). Hence, for each $t \in (t_k, t_{k+1}]$, we have

$$\begin{aligned}\|y(t) - x(t)\| &\le \sum_{i=1}^{m} \|\Psi(t, t_i^+)\| \|g^k\| + \int_0^t \|\Psi(t,s)\| \|g(s)\| ds \\ &\le \prod_{k=1}^{m} \left(1 + \|B^k\|\right) \epsilon_{\max} + \epsilon_{\max} \prod_{k=1}^{m} \left(1 + \|B^k\|\right) \int_0^t e^{-Ms} ds \\ &= \prod_{k=1}^{m} \left(1 + \|B^k\|\right) \left(1 + \frac{1}{M}\right) \epsilon_{\max}.\end{aligned} \tag{26}$$

Thus, we obtain

$$\begin{pmatrix} |y_1(t) - x_1(t)| \\ |y_2(t) - x_2(t)| \\ \vdots \\ |y_n(t) - x_n(t)| \end{pmatrix} \le \left(C_{f,m}\epsilon_{\max}, C_{f,m}\epsilon_{\max}, \ldots, C_{f,m}\epsilon_{\max}\right)^T, \quad t \in J, \tag{27}$$

where

$$C_{f,m} = \prod_{k=1}^{m} \left(1 + \|B^k\|\right) \left(1 + \frac{1}{M}\right). \tag{28}$$

Thus, (1) is Ulam-Hyers stable. The proof is completed. □

Next, we suppose the following.

$(H_3{}')$ There exist $\lambda_{\varphi_i} > 0$, $i = 1, 2, \ldots, n$, such that

$$\int_0^t e^{-Ms} \varphi_i(s)\, ds \le \lambda_{\varphi_i} \varphi_i(t) \quad \text{for each } t \in J, \tag{29}$$

where $\varphi_i \in PC(J, \mathbb{R}_+)$ is nondecreasing.

We have

Theorem 7. *Assume that* (H_1)–$(H_2{}')$ *and* $(H_3{}')$ *are satisfied. Then* (1) *is Ulam-Hyers-Rassias stable.*

Proof. Let $y \in PC^1(J, \mathbb{R}^n)$ be a solution of inequality (9). For g and g_k defined in (12), we have

$$\|g(t)\| \le e^{-Mt}\varphi_i(t)\epsilon_{\max}, \quad t \in J,$$
$$\|g_k\| \le e^{-M(t-t_i)}\psi_i\epsilon_{\max}, \quad k = 1, 2, \ldots, m. \tag{30}$$

Let x be the unique solution of (15). Hence for each $t \in (t_k, t_{k+1}]$, it follows from $(H_3{}')$ that

$$\begin{aligned}\|y(t) - x(t)\| &\le \prod_{k=1}^{m} \left(1 + \|B^k\|\right) \psi_i\epsilon_{\max} + \epsilon_{\max} \prod_{k=1}^{m} \left(1 + \|B^k\|\right) \int_0^t e^{-Ms}\varphi_i(t)\, ds \\ &\le \prod_{k=1}^{m} \left(1 + \|B^k\|\right) \psi_i\epsilon_{\max} + \epsilon_{\max} \prod_{k=1}^{m} \left(1 + \|B^k\|\right) \lambda_{\varphi_i}\varphi_i(t) \\ &= \prod_{k=1}^{m} \left(1 + \|B^k\|\right) \left(1 + \lambda_{\varphi_i}\right) \times \left(\varphi_i(t) + \psi_i\right)\epsilon_{\max}, \quad i = 1, 2, \ldots, n.\end{aligned} \tag{31}$$

Thus, we obtain

$$\begin{pmatrix} |y_1(t) - x_1(t)| \\ |y_2(t) - x_2(t)| \\ \vdots \\ |y_n(t) - x_n(t)| \end{pmatrix} \le \begin{pmatrix} C_{f,m,\varphi_1}\left(\varphi_1(t) + \psi_1\right)\epsilon_{\max} \\ C_{f,m,\varphi_2}\left(\varphi_2(t) + \psi_2\right)\epsilon_{\max} \\ \vdots \\ C_{f,m,\varphi_n}\left(\varphi_n(t) + \psi_n\right)\epsilon_{\max} \end{pmatrix}, \quad t \in J, \tag{32}$$

where

$$C_{f,m,\varphi_i} = \left(1 + \lambda_{\varphi_i}\right) \prod_{k=1}^{m} \left(1 + \|B^k\|\right), \quad i = 1, 2, \ldots, n. \tag{33}$$

Thus, (1) is Ulam-Hyers-Rassias stable. The proof is completed. □

Acknowledgments

The authors thank the referee for valuable comments and suggestions which improved their paper. This work is supported by the National Natural Science Foundation of China (11201091), Key Projects of Science and Technology Research in the Chinese Ministry of Education (211169) and Key Support Subject (Applied Mathematics) of Guizhou Normal College.

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