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Research Article

Approximation Theorems for Functions of Two Variables via σ-Convergence

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Çakan et al. (2006) introduced the concept of σ-convergence for double sequences. In this work, we use this notion to prove the Korovkin-type approximation theorem for functions of two variables by using the test functions 1, x, y, and $x^2 + y^2$ and construct an example by considering the Bernstein polynomials of two variables in support of our main result.

1. Introduction and Preliminaries

In [1], Pringsheim introduced the following concept of convergence for double sequences. A double sequence $x = (x_{jk})$ is said to be *convergent* to the number L in *Pringsheim's sense* (shortly, *p-convergent* to L) if for every $\varepsilon > 0$ there exists an integer N such that $|x_{jk} - L| < \varepsilon$ whenever $j, k > N$. In this case L is called the *p-limit* of x.

A double sequence $x = (x_{jk})$ of real or complex numbers is said to be *bounded* if $\|x\|_\infty = \sup_{j,k}|x_{jk}| < \infty$. We denote the space of all bounded double sequences by $\mathscr{M}_u$.

If $x \in \mathscr{M}_u$ and is p-convergent to L, then x is said to be *boundedly p-convergent* to L (shortly, *bp-convergent* to L). In this case L is called the *bp-limit* of the double sequences (x_{jk}). The assumption of being bp-convergent was made because a double sequence which is p-convergent is not necessarily bounded.

Assume that σ is a one-to-one mapping from the set $\mathbb{N}$ (the set of natural numbers) into itself. A continuous linear functional φ on the space ℓ_∞ of bounded single sequences is said to be an *invariant mean* or a *σ-mean* if and only if (i) $\varphi(x) \geq 0$ when the sequence $x = (x_k)$ has $x_k \geq 0$ for all k, (ii) $\varphi(e) = 1$, where $e = (1, 1, 1, \ldots)$, and (iii) $\varphi(x) = \varphi(x_{\sigma(k)})$ for all $x \in \ell_\infty$.

Throughout this paper we consider the mapping σ which has no finite orbits; that is, $\sigma^p(k) \neq k$ for all integer $k \geq 0$ and $p \geq 1$, where $\sigma^p(k)$ denotes the pth iterate of σ at k. Note that a σ-mean extends the limit functional on the space c of convergent single sequences in the sense that $\varphi(x) = \lim x$ for all $x \in c$ (see [2]). Consequently, $c \subset V_\sigma$ the set of bounded sequences all of whose σ-means are equal. We say that a sequence $x = (x_k)$ is *σ-convergent* if and only if $x \in V_\sigma$. Schaefer [3] defined and characterized the σ-conservative, σ-regular, and σ-coercive matrices for single sequences by using the notion of σ-convergence. If $\sigma(n) = n + 1$, then the set V_σ is reduced to the set f of almost convergent sequences due to Lorentz [4].

In 2006, Çakan et al. [5] presented the following definition of σ-convergence for double sequences and established core theorem for σ-convergence and later on this notion was studied by Mursaleen and Mohiuddine [6–8]. A double sequence $x = (x_{jk})$ of real numbers is said to be *σ-convergent* to a number L if and only if $x \in \mathscr{V}_\sigma$, where

$$\mathscr{V}_\sigma = \Big\{x \in \mathscr{M}_u : \lim_{p,q\to\infty} \zeta_{pqst}(x) = L \text{ uniformly in } s, t; L = \sigma\text{-}\lim x\Big\}, \quad (1)$$

$$\zeta_{pqst}(x) = \frac{1}{(p+1)(q+1)} \sum_{j=0}^{p} \sum_{k=0}^{q} x_{\sigma^j(s),\sigma^k(t)},$$

while here the limit means bp-limit. Let us denote by $\mathscr{V}_\sigma$ the space of σ-convergent double sequences $x = (x_{jk})$. If

σ is translation mapping, then the set $\mathscr{V}_\sigma$ is reduced to the set $\mathscr{F}$ of almost convergent double sequences [9]. Note that $\mathscr{C}_{bp} \subset \mathscr{V}_\sigma \subset \mathscr{M}_u$.

Example 1. Let $w = (w_{mn})$ be a double sequence such that

$$w_{mn} = \begin{cases} 1 & \text{if } n \text{ is odd,} \\ -1 & \text{if } n \text{ is even,} \end{cases} \quad (2)$$

for all m. Then w is σ-convergent to zero (for $\sigma(n) = n + 1$) but not convergent.

Suppose that $C[a, b]$ is the space of all functions f continuous on $[a, b]$. It is well known that $C[a, b]$ is a Banach space with the norm

$$\|f\|_\infty := \sup_{x\in[a,b]} |f(x)|, \quad f \in C[a, b]. \quad (3)$$

The classical Korovkin approximation theorem is given as follows (see [10, 11]).

Theorem 2. *Let (T_n) be a sequence of positive linear operators from $C[a, b]$ into $C[a, b]$ and $\lim_n \|T_n(f_i, x) - f_i(x)\|_\infty = 0$, for $i = 0, 1, 2$, where $f_0(x) = 1$, $f_1(x) = x$, and $f_2(x) = x^2$. Then $\lim_n \|T_n(f, x) - f(x)\|_\infty = 0$, for all $f \in C[a, b]$.*

In [12], Mohiuddine obtained the Korovkin-type approximation theorem through the notion of almost convergence for single sequences and proved some interesting results. Such type of approximation theorems for the function of two variables is proved in [13, 14] through the concept of almost convergence and statistical convergence of double sequences, respectively. Recently, Mohiuddine et al. [15] determined the Korovkin-type approximation theorem by using the test functions 1, e^{-x}, and e^{-2x} through the notion of statistical summability $(C, 1)$. Quite recently, by using the concept of (λ, μ)-statistical convergence, Mohiuddine and Alotaibi [16] proved the Korovkin-type approximation theorem for functions of two variables. For more details and some recent work on this topic, we refer to [17–21] and references therein. In this work, we apply the notion of σ-convergence to prove the Korovkin-type approximation theorem by using the test functions $1, x, y$, and $x^2 + y^2$. We apply the classical Bernstein polynomials of two variables to construct an example in support of our result.

2. Main Result

Now, we prove the classical Korovkin-type approximation theorem for the functions of two variables through σ-convergence.

By $C(I \times I)$, we denote the set of all two dimensional continuous functions on $I^2 = I \times I$, where $I = [a, b]$. Let T be a linear operator from $C(I^2)$ into $C(I^2)$. Then, a linear operator T is said to be positive provided that $f(x, y) \geq 0$ implies $T(f; x, y) \geq 0$.

Theorem 3. *Suppose that $(T_{j,k})$ is a double sequence of positive linear operators from $C(I^2)$ into $C(I^2)$ and $D_{m,n,p,q}(f; x, y) = (1/pq) \sum_{j=0}^{p-1} \sum_{k=0}^{q-1} T_{\sigma^j(m),\sigma^k(n)}(f; x, y)$ satisfying the following conditions:*

$$\begin{gathered} \lim_{p,q\to\infty} \left\|D_{m,n,p,q}(1; x, y) - 1\right\|_\infty = 0, \\ \lim_{p,q\to\infty} \left\|D_{m,n,p,q}(s; x, y) - x\right\|_\infty = 0, \\ \lim_{p,q\to\infty} \left\|D_{m,n,p,q}(t; x, y) - y\right\|_\infty = 0, \\ \lim_{p,q\to\infty} \left\|D_{m,n,p,q}(s^2 + t^2; x, y) - (x^2 + y^2)\right\|_\infty = 0, \end{gathered} \quad (4)$$

which hold uniformly in m, n. Then for any function $f \in C(I^2)$ bounded on the whole plane, one has

$$\begin{gathered} \sigma\text{-}\lim_{j,k\to\infty} \left\|T_{j,k}(f; x, y) - f(x, y)\right\|_\infty = 0. \quad \textit{That is,} \\ \lim_{p,q\to\infty} \left\|D_{m,n,p,q}(f; x, y) - f(x, y)\right\|_\infty = 0, \end{gathered} \quad (5)$$

uniformly in m, n.

Proof. Since $f \in C(I^2)$ and f is bounded on the whole plane, we have

$$|f(x, y)| \leq M, \quad -\infty < x, y < \infty. \quad (6)$$

Therefore,

$$|f(s, t) - f(x, y)| \leq 2M, \quad -\infty < s, t, x, y < \infty. \quad (7)$$

Also we have that f is continuous on $I \times I$; that is,

$$|f(s, t) - f(x, y)| < \epsilon, \quad \forall |s - x| < \delta, \quad |t - y| < \delta. \quad (8)$$

From (7) and (8), putting $\psi_1 = \psi_1(s, x) = (s - x)^2$ and $\psi_2 = \psi_2(t, y) = (t - y)^2$, we obtain

$$\begin{gathered} |f(s, t) - f(x, y)| < \epsilon + \frac{2M}{\delta^2}(\psi_1 + \psi_2), \\ \forall |s - x| < \delta, \quad |t - y| < \delta, \end{gathered} \quad (9)$$

or

$$\begin{aligned} -\epsilon - \frac{2M}{\delta^2}(\psi_1 + \psi_2) &< f(s, t) - f(x, y) \\ &< \epsilon + \frac{2M}{\delta^2}(\psi_1 + \psi_2). \end{aligned} \quad (10)$$

Now, we operate $T_{\sigma^j(m),\sigma^k(n)}(1; x, y)$ on the above inequality since $T_{\sigma^j(m),\sigma^k(n)}(f; x, y)$ is monotone and linear. We obtain

$$\begin{aligned} &T_{\sigma^j(m),\sigma^k(n)}(1; x, y)\left(-\epsilon - \frac{2M}{\delta^2}(\psi_1 + \psi_2)\right) \\ &\quad < T_{\sigma^j(m),\sigma^k(n)}(1; x, y)(f(s, t) - f(x, y)) \\ &\quad < T_{\sigma^j(m),\sigma^k(n)}(1; x, y)\left(\epsilon + \frac{2M}{\delta^2}(\psi_1 + \psi_2)\right). \end{aligned} \quad (11)$$

Therefore

$$-\epsilon T_{\sigma^j(m),\sigma^k(n)}(1;x,y)-\frac{2M}{\delta^2}T_{\sigma^j(m),\sigma^k(n)}(\psi_1+\psi_2;x,y)$$
$$<T_{\sigma^j(m),\sigma^k(n)}(f;x,y)-f(x,y)T_{j,k}(1;x,y)$$
$$<\epsilon T_{\sigma^j(m),\sigma^k(n)}(1;x,y)+\frac{2M}{\delta^2}T_{\sigma^j(m),\sigma^k(n)}(\psi_1+\psi_2;x,y). \tag{12}$$

But

$$\begin{aligned}T_{\sigma^j(m),\sigma^k(n)}&(f;x,y)-f(x,y)\\&=T_{\sigma^j(m),\sigma^k(n)}(f;x,y)-f(x,y)T_{\sigma^j(m),\sigma^k(n)}(1;x,y)\\&\quad+f(x,y)T_{\sigma^j(m),\sigma^k(n)}(1;x,y)-f(x,y)\\&=\left[T_{\sigma^j(m),\sigma^k(n)}(f;x,y)-f(x,y)T_{\sigma^j(m),\sigma^k(n)}(1;x,y)\right]\\&\quad+f(x,y)\left[T_{\sigma^j(m),\sigma^k(n)}(1;x,y)-1\right].\end{aligned}\tag{13}$$

From (12) and (13), we get

$$\begin{aligned}T_{\sigma^j(m),\sigma^k(n)}&(f;x,y)-f(x,y)\\&<\epsilon T_{\sigma^j(m),\sigma^k(n)}(1;x,y)\\&\quad+\frac{2M}{\delta^2}T_{\sigma^j(m),\sigma^k(n)}(\psi_1+\psi_2;x,y)\\&\quad+f(x,y)\left(T_{\sigma^j(m),\sigma^k(n)}(1;x,y)-1\right).\end{aligned}\tag{14}$$

Now

$$\begin{aligned}T_{\sigma^j(m),\sigma^k(n)}&(\psi_1+\psi_2;x,y)\\&=T_{\sigma^j(m),\sigma^k(n)}\left((s-x)^2+(t-y)^2;x,y\right)\\&=T_{\sigma^j(m),\sigma^k(n)}\left(s^2-2sx+x^2+t^2-2ty+y^2;x,y\right)\\&=T_{\sigma^j(m),\sigma^k(n)}\left(s^2+t^2;x,y\right)-2xT_{\sigma^j(m),\sigma^k(n)}(s;x,y)\\&\quad-2yT_{\sigma^j(m),\sigma^k(n)}(t;x,y)+\left(x^2+y^2\right)T_{\sigma^j(m),\sigma^k(n)}(1;x,y)\\&=\left[T_{\sigma^j(m),\sigma^k(n)}\left(s^2+t^2;x,y\right)-\left(x^2+y^2\right)\right]\\&\quad-2x\left[T_{\sigma^j(m),\sigma^k(n)}(s;x,y)-x\right]\\&\quad-2y\left[T_{\sigma^j(m),\sigma^k(n)}(t;x,y)-y\right]\\&\quad+\left(x^2+y^2\right)\left[T_{\sigma^j(m),\sigma^k(n)}(1;x,y)-1\right].\end{aligned}\tag{15}$$

Using (14), we obtain

$$\begin{aligned}T_{\sigma^j(m),\sigma^k(n)}&(f;x,y)-f(x,y)\\&<\epsilon T_{\sigma^j(m),\sigma^k(n)}(1;x,y)\\&\quad+\frac{2M}{\delta^2}\left\{\left[T_{\sigma^j(m),\sigma^k(n)}\left(s^2+t^2;x,y\right)-\left(x^2+y^2\right)\right]\right.\\&\qquad-2x\left[T_{\sigma^j(m),\sigma^k(n)}(s;x,y)-x\right]\\&\qquad-2y\left[T_{\sigma^j(m),\sigma^k(n)}(t;x,y)-y\right]\\&\qquad\left.+\left(x^2+y^2\right)\left[T_{\sigma^j(m),\sigma^k(n)}(1;x,y)-1\right]\right\}\\&\quad+f(x,y)\left(T_{\sigma^j(m),\sigma^k(n)}(1;x,y)-1\right)\\&=\epsilon\left[T_{\sigma^j(m),\sigma^k(n)}(1;x,y)-1\right]+\epsilon\\&\quad+\frac{2M}{\delta^2}\left\{\left[T_{\sigma^j(m),\sigma^k(n)}\left(s^2+t^2;x,y\right)-\left(x^2+y^2\right)\right]\right.\\&\qquad-2x\left[T_{\sigma^j(m),\sigma^k(n)}(s;x,y)-x\right]\\&\qquad-2y\left[T_{\sigma^j(m),\sigma^k(n)}(t;x,y)-y\right]\\&\qquad\left.+\left(x^2+y^2\right)\left[T_{\sigma^j(m),\sigma^k(n)}(1;x,y)-1\right]\right\}\\&\quad+f(x,y)\left(T_{\sigma^j(m),\sigma^k(n)}(1;x,y)-1\right).\end{aligned}\tag{16}$$

Since ϵ is arbitrary, we can write

$$\begin{aligned}T_{\sigma^j(m),\sigma^k(n)}&(f;x,y)-f(x,y)\\&\le\epsilon\left[T_{\sigma^j(m),\sigma^k(n)}(1;x,y)-1\right]\\&\quad+\frac{2M}{\delta^2}\left\{\left[T_{\sigma^j(m),\sigma^k(n)}\left(s^2+t^2;x,y\right)-\left(x^2+y^2\right)\right]\right.\\&\qquad-2x\left[T_{\sigma^j(m),\sigma^k(n)}(s;x,y)-x\right]\\&\qquad-2y\left[T_{\sigma^j(m),\sigma^k(n)}(t;x,y)-y\right]\\&\qquad\left.+\left(x^2+y^2\right)\left[T_{\sigma^j(m),\sigma^k(n)}(1;x,y)-1\right]\right\}\\&\quad+f(x,y)\left(T_{\sigma^j(m),\sigma^k(n)}(1;x,y)-1\right).\end{aligned}\tag{17}$$

Similarly,

$$\begin{aligned}D_{m,n,p,q}&(f;x,y)-f(x,y)\\&\le\epsilon\left[D_{m,n,p,q}(1;x,y)-1\right]\\&\quad+\frac{2M}{\delta^2}\left\{\left[D_{m,n,p,q}\left(s^2+t^2;x,y\right)-\left(x^2+y^2\right)\right]\right.\\&\qquad-2x\left[D_{m,n,p,q}(s;x,y)-x\right]\\&\qquad-2y\left[D_{m,n,p,q}(t;x,y)-y\right]\\&\qquad\left.+\left(x^2+y^2\right)\left[D_{m,n,p,q}(1;x,y)-1\right]\right\}\\&\quad+f(x,y)\left(D_{m,n,p,q}(1;x,y)-1\right).\end{aligned}\tag{18}$$

Thus, we have

$$\begin{aligned}&\left\|D_{m,n,p,q}(f;x,y)-f(x,y)\right\|_{\infty}\\&\le\left(\epsilon+\frac{2M\left(a^2+b^2\right)}{\delta^2}+M\right)\left\|D_{m,n,p,q}(1;x,y)-1\right\|_{\infty}\\&-\frac{4Ma}{\delta^2}\left\|D_{m,n,p,q}(s;x,t)-x\right\|_{\infty}\\&-\frac{4Mb}{\delta^2}\left\|D_{m,n,p,q}(t;x,y)-y\right\|_{\infty}\\&+\frac{2M}{\delta^2}\left\|D_{m,n,p,q}\left(s^2+t^2;x,y\right)-\left(x^2+y^2\right)\right\|_{\infty}.\end{aligned} \tag{19}$$

Taking the limit $p,q\to\infty$ and from (4), we obtain

$$\lim_{p,q\to\infty}\left\|D_{m,n,p,q}(f;x,y)-f(x,y)\right\|_{\infty}=0,\quad\text{uniformly in } m,n. \tag{20}$$

□

Theorem 4. *Suppose a double sequence $(T_{m,n})$ of positive linear operators on $C(I^2)$ such that*

$$\lim_{m,n}\sup_{s,t}\frac{1}{mn}\sum_{j=0}^{m-1}\sum_{k=0}^{n-1}\left\|T_{m,n}-T_{\sigma^j(s),\sigma^k(t)}\right\|=0. \tag{21}$$

If

$$\sigma\text{-}\lim_{m,n}\left\|T_{m,n}(t_\nu,x)-t_\nu\right\|_{\infty}=0\quad(\nu=0,1,2,3), \tag{22}$$

where $t_0(x,y)=1$, $t_1(x,y)=x$, $t_2(x,y)=y$, and $t_3(x,y)=x^2+y^2$, then

$$\lim_{m,n}\left\|T_{m,n}(f;x,y)-f(x,y)\right\|_{\infty}=0, \tag{23}$$

for any function $f\in C(I^2)$ bounded on the whole plane.

Proof. From Theorem 3, we have that if (22) holds then

$$\sigma\text{-}\lim_{m,n}\left\|T_{m,n}(f;x,y)-f(x,y)\right\|_{\infty}=0, \tag{24}$$

which is equivalent to

$$\lim_{m,n}\left\|\sup_{s,t}D_{s,t,m,n}(f;x,y)-f(x,y)\right\|_{\infty}=0. \tag{25}$$

Now

$$\begin{aligned}T_{m,n}-D_{s,t,m,n}&=T_{m,n}-\frac{1}{mn}\sum_{j=0}^{m-1}\sum_{k=0}^{n-1}T_{\sigma^j(s),\sigma^k(t)}\\&=\frac{1}{mn}\sum_{j=0}^{m-1}\sum_{k=0}^{n-1}\left(T_{m,n}-T_{\sigma^j(s),\sigma^k(t)}\right).\end{aligned} \tag{26}$$

Therefore

$$T_{m,n}-\sup_{s,t}D_{s,t,m,n}=\sup_{s,t}\frac{1}{mn}\sum_{j=0}^{m-1}\sum_{k=0}^{n-1}\left(T_{m,n}-T_{\sigma^j(s),\sigma^k(t)}\right). \tag{27}$$

Hence, using the hypothesis, we get

$$\begin{aligned}&\lim_{m,n}\left\|T_{m,n}(f;x,y)-f(x,y)\right\|_{\infty}\\&=\lim_{m,n}\left\|\sup_{s,t}D_{s,t,m,n}(f;x,y)-f(x,y)\right\|_{\infty}=0.\end{aligned} \tag{28}$$

That is, (23) holds. □

3. Example and the Concluding Remark

In this section, we prove that our theorem is stronger than Theorem 2. Let us consider the following Bernstein polynomials (see [22]) of two variables:

$$\begin{aligned}&B_{m,n}(f;x,y)\\&:=\sum_{j=0}^{m}\sum_{k=0}^{n}f\left(\frac{j}{m},\frac{k}{n}\right)\binom{m}{j}\binom{n}{k}x^j(1-x)^{m-j}y^k(1-y)^{n-k},\\&\qquad 0\le x,y\le 1.\end{aligned} \tag{29}$$

Let $\Lambda_{m,n}:C(I^2)\to C(I^2)$ be defined by

$$\Lambda_{m,n}(f;x,y)=(1+w_{mn})B_{m,n}(f;x,y), \tag{30}$$

where the double sequence (w_{mn}) is defined by (2) in Section 1. Then

$$\begin{aligned}B_{m,n}(1;x,y)&=1,\\B_{m,n}(s;x,y)&=x,\\B_{m,n}(t;x,y)&=y,\\B_{m,n}\left(s^2+t^2;x,y\right)&=x^2+y^2+\frac{x-x^2}{m}+\frac{y-y^2}{n}.\end{aligned} \tag{31}$$

Also, $(\Lambda_{m,n})$ satisfies (4). Hence, we have

$$\sigma\text{-}\lim_{m,n\to\infty}\left\|\Lambda_{m,n}(f;x,y)-f(x,y)\right\|_{\infty}=0. \tag{32}$$

Since $B_{m,n}(f;0,0)=f(0,0)$, we have $\Lambda_{m,n}(f;0,0)=(1+w_{mn})f(0,0)$. Thus

$$\begin{aligned}\left\|\Lambda_{m,n}(f;x,y)-f(x,y)\right\|_{\infty}&\ge\left|\Lambda_{m,n}(f;0,0)-f(0,0)\right|\\&=w_{mn}\left|f(0,0)\right|.\end{aligned} \tag{33}$$

But Theorem 3 does not hold, since the limit w_{mn} does not exist as $m,n\to\infty$. Therefore we conclude that our Theorem 3 is stronger than the classical Korovkin theorem for functions of two variables due to Volkov [23].

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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