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Research Article

n-Tuplet Coincidence Point Theorems in Intuitionistic Fuzzy Normed Spaces

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For an arbitrary n positive integer, we investigate the existence of n-tuplet coincidence points in intuitionistic fuzzy normed space. Results of the paper are more general than those of the coupled and the tripled fixed point works in intuitionistic fuzzy normed space.

1. Introduction

One of the most important fields in mathematics is the fixed point theory. This theory is used to solve a variety of problems in many areas such as economics, chemistry, computer science, and engineering as well as many branches of mathematics. One of the main theorems in the fixed point theory is the Banach contraction theorem [1]. This theorem states that contraction map in complete metric space has a unique fixed point. Many authors studied the generalization of the Banach contraction theorem. Generalization on the complete partial ordered metric space was given by Ran and Reurings [2] with a weaker condition. In their theorem, contraction condition is provided only for comparable elements with respect to partial order relation in complete metric space. Some fixed point theorems have been obtained by many authors based on [2]. Bhaskar and Lakshmikantham [3] defined the concept of coupled fixed point and used a theorem associated with it for existence and uniqueness of solution of the periodic boundary value problem. Later on, Lakshmikantham and Ćirić [4] introduced the concept of coincidence point which is a generalization of fixed point. By inspiring these works, coupled fixed point theorems have been studied for different type contraction mappings (see [5–12]). The interest on coupled fixed point theorem has motivated the authors to generalize it as tripled fixed point theorem in [13, 14], afterwards as quadruple fixed point theorem in [15–17], and as n-tuplet fixed point theorem in [18–21].

On the other hand, fuzzy theory was introduced by Zadeh [22] and it was generalized by Atanassov [23] as intuitionistic fuzzy theory. While fuzzy theory assigns degree of membership for each element, intuitionistic fuzzy theory assigns degree of membership and nonmembership for each element. Both of them were applied in many fields of sciences.

Introduction of the intuitionistic fuzzy metric space by Park [24] and of the intuitionistic fuzzy normed space by Saadati and Park [25] has enabled many subjects in functional analysis to be studied in intuitionstic fuzzy normed (metric) spaces. Fixed point theory is one of these subjects. Many fixed point theorems have been studied in intuitionistic fuzzy normed (metric) space. Numerous works have been produced since richness of fixed point theory and intuitionistic fuzzy functional analysis. Some of the articles concerning these fields can be found in the literature (such as [26–33]).

Coupled and tripled fixed point theorems in intuitionistic fuzzy normed space were proved via n-property in [29, 30], respectively. The purpose of our paper is to study

n-tuplet coincidence point theorem without n-property in intuitionistic fuzzy normed space, which is the generalization of coupled fixed point theorem [29] and tripled fixed point theorem [30] in intuitionistic fuzzy normed space.

Let us start by recalling some of the concepts used in this paper.

Definition 1 (see [34]). A binary operation $* : [0,1] \times [0,1]$ is a continuous t-norm if it satisfies the following conditions: (i) $*$ is associative and commutative; (ii) $*$ is continuous; (iii) $a * 1 = a$ for all $a \in [0,1]$; (iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for each $a, b, c, d \in [0,1]$.

Definition 2 (see [34]). A binary operation $\diamond : [0,1] \times [0,1]$ is a continuous t-conorm if it satisfies the following conditions: (i) $\diamond$ is associative and commutative; (ii) $\diamond$ is continuous; (iii) $a \diamond 0 = a$ for all $a \in [0,1]$; (iv) $a \diamond b \le c \diamond d$ whenever $a \le c$ and $b \le d$ for each $a, b, c, d \in [0,1]$.

Definition 3 (see [25]). Let $*$ be a continuous t-norm, let $\diamond$ be a continuous t-conorm, and let X be a linear space over the field $IF(\mathbb{R}$ or $\mathbb{C})$. If μ and ν are fuzzy sets on $X \times (0,\infty)$ satisfying the following conditions, the five-tuple $(X, \mu, \nu, *, \diamond)$ is said to be an intuitionistic fuzzy normed space and (μ, ν) is called an intuitionistic fuzzy norm. For every $x, y \in X$ and $s, t > 0$, one has the following:

(i) $\mu(x,t) + \nu(x,t) \le 1$,

(ii) $\mu(x,t) > 0$,

(iii) $\mu(x,t) = 1 \Leftrightarrow x = 0$,

(iv) $\mu(ax,t) = \mu(x, t/|a|)$ for each $a \ne 0$,

(v) $\mu(x,t) * \mu(y,s) \le \mu(x+y, t+s)$,

(vi) $\mu(x,\cdot) : (0,\infty) \to [0,1]$ is continuous,

(vii) $\lim_{t\to\infty}\mu(x,t) = 1$ and $\lim_{t\to 0}\mu(x,t) = 0$,

(viii) $\nu(x,t) < 1$,

(ix) $\nu(x,t) = 0 \Leftrightarrow x = 0$,

(x) $\nu(ax,t) = \nu(x, t/|a|)$ for each $a \ne 0$,

(xi) $\nu(x,t) \diamond \nu(y,s) \ge \nu(x+y, t+s)$,

(xii) $\nu(x,\cdot) : (0,\infty) \to [0,1]$ is continuous,

(xiii) $\lim_{t\to\infty}\nu(x,t) = 0$ and $\lim_{t\to 0}\nu(x,t) = 1$;

we further assume that $(X, \mu, \nu, *, \diamond)$ satisfies the following axiom:

(xiv) $a \diamond a = a$ and $a * a = a$ for all $a \in [0,1]$.

Throughout this paper, expression "intuitionistic fuzzy normed space" will be denoted by IFNS for short.

Similar to Definition 4.12 in [25], Definition 4 can be given as the following:

Definition 4. Let $(X, \mu, \nu, *, \diamond)$ be an IFNS. $(X^n, \Phi, \Psi, *, \diamond)$ is called a cartesian product of intuitionistic fuzzy normed spaces if (Φ, Ψ) is a cartesian product of intuitionistic fuzzy norms defined

$$\Phi(x,t) = \prod_{i=1}^{n} \mu(x_i, t), \qquad \Psi(x,t) = \coprod_{i=1}^{n} \nu(x_i, t), \tag{1}$$

where $x = (x_1, x_2, \ldots, x_n)$, $\prod_{i=1}^{n} a_i = a_1 * a_2 * \cdots * a_n$, $\coprod_{i=1}^{n} a_i = a_1 \diamond a_2 \diamond \cdots \diamond a_n$ and $t > 0$.

Definition 5 (see [25]). Let $(X, \mu, \nu, *, \diamond)$ be an IFNS. Then a sequence (x_k) in X is said to be Cauchy sequence if for each $\epsilon > 0$ and $t > 0$ there exists $k_0 \in \mathbb{N}$ such that

$$\mu(x_k - x_m, t) > 1 - \epsilon, \qquad \nu(x_k - x_m, t) < \epsilon \tag{2}$$

for each $k, m > k_0$.

Definition 6 (see [25]). Let $(X, \mu, \nu, *, \diamond)$ be an IFNS. $(X, \mu, \nu, *, \diamond)$ is said to be complete if every Cauchy sequence in $(X, \mu, \nu, *, \diamond)$ is convergent.

Definition 7 (see [35]). Let X and Y be two IFNSs. $f : X \to Y$ is continuous at $x_0 \in X$ if $(f(x_k))$ in Y convergences to $f(x_0)$ for any (x_k) in X converging to x_0. If $f : X \to Y$ is continuous at each element of X, then $f : X \to Y$ is said to be continuous on X.

Definition 8 (see [18]). Let $(X, \preceq)$ be partially ordered set and $F : X^n \to X$ and $g : X \to X$. It is said that F has the mixed g-monotone property if $F(x_1, x_2, x_3, \ldots, x_n)$ is monotone g-nondecreasing in its odd argument and it is monotone g-nonincreasing in its even argument. That is, for any $x_1, x_2, x_3, \ldots, x_n \in X$,

$$\begin{gathered} y_1, z_1 \in X, \quad g(y_1) \preceq g(z_1) \\ \Longrightarrow F(y_1, x_2, x_3, \ldots, x_n) \preceq F(z_1, x_2, x_3, \ldots, x_n), \\ y_2, z_2 \in X, \quad g(y_2) \preceq g(z_2) \\ \Longrightarrow F(x_1, y_2, x_3, \ldots, x_n) \succeq F(x_1, z_2, x_3, \ldots, x_n), \\ \vdots \\ y_n, z_n \in X, \quad g(y_n) \preceq g(z_n) \\ \Longrightarrow F(x_1, x_2, x_3, \ldots, y_n) \preceq F(x_1, x_2, x_3, \ldots, z_n) \\ (\text{if } n \text{ is odd}), \\ y_n, z_n \in X, \quad g(y_n) \preceq g(z_n) \\ \Longrightarrow F(x_1, x_2, x_3, \ldots, y_n) \succeq F(x_1, x_2, x_3, \ldots, z_n) \\ (\text{if } n \text{ is even}). \end{gathered} \tag{3}$$

Note that if g is the identity mapping, this definition is reduced to Definition 1 in [18].

Definition 9 (see [18]). Let X be a nonempty set and let $F : X^n \to X$ be a given mapping. An element $(x_1, x_2, x_3, \ldots, x_n) \in X^n$ is called an n-tuplet coincidence point of $F : X^n \to X$ and $g : X \to X$ if

$$\begin{gathered} F(x_1, x_2, x_3, \ldots, x_n) = g(x_1), \\ F(x_2, x_3, \ldots, x_n, x_1) = g(x_2), \\ \vdots \\ F(x_n, x_1, x_2, \ldots, x_{n-1}) = g(x_n). \end{gathered} \tag{4}$$

Note that if g is the identity mapping, this definition is reduced to Definition 2 in [18].

Definition 10 (see [18]). Let $F : X^n \to X$ and $g : X \to X$ be two mappings. F and g are called commutative if

$$\begin{aligned} &g(F(x_1, x_2, x_3, \ldots, x_n)) \\ &\quad = F(g(x_1), g(x_2), g(x_3), \ldots, g(x_n)) \end{aligned} \tag{5}$$

for all $x_1, x_2, x_3, \ldots, x_n \in X$.

2. Main Results

Theorem 11. *Let $F : X^n \to X$ be continuous map having mixed g-monotone property on the complete $(X, \mu, \nu, *, \diamond)$ having partial order relation denoted by $\preceq$. Also $F(X^n) \subset g(X)$; g is continuous and commutes with F. Suppose that $F : X^n \to X$ and $g : X \to X$ hold the following conditions, for all $x_1, x_2, x_3, \ldots, x_n, y_1, y_2, y_3, \ldots, y_n \in X$ and $\alpha \in (0, 1)$:*

$$\begin{aligned} &\mu(F(x_1, x_2, x_3, \ldots, x_n) - F(y_1, y_2, y_3, \ldots, y_n), \alpha t) \\ &\quad \geq \mu(g(x_1) - g(y_1), t) \\ &\qquad * \mu(g(x_2) - g(y_2), t) * \cdots * \mu(g(x_n) - g(y_n), t), \end{aligned} \tag{6}$$

$$\begin{aligned} &\nu(F(x_1, x_2, x_3, \ldots, x_n) - F(y_1, y_2, y_3, \ldots, y_n), \alpha t) \\ &\quad \leq \nu(g(x_1) - g(y_1), t) \\ &\qquad \diamond \nu(g(x_2) - g(y_2), t) \diamond \cdots \diamond \nu(g(x_n) - g(y_n), t), \end{aligned} \tag{7}$$

where $g(x_{2i-1}) \preceq g(y_{2i-1})$, $i \in \{1, 2, \ldots, (n+1)/2\}$, and $g(x_{2i}) \succeq g(y_{2i})$, $i \in \{1, 2, \ldots, n/2\}$. If there exist $x_0^1, x_0^2, x_0^3, \ldots, x_0^n \in X$ such that

$$\begin{aligned} &g(x_0^1) \preceq F(x_0^1, x_0^2, x_0^3, \ldots, x_0^n), \\ &g(x_0^2) \succeq F(x_0^2, x_0^3, \ldots, x_0^n, x_0^1), \\ &\quad \vdots \\ &g(x_0^n) \preceq F(x_0^n, x_0^1, x_0^2, \ldots, x_0^{n-1}) \quad (\textit{if } n \textit{ is odd}), \\ &g(x_0^n) \succeq F(x_0^n, x_0^1, x_0^2, \ldots, x_0^{n-1}) \quad (\textit{if } n \textit{ is even}), \end{aligned} \tag{8}$$

then there exist $x_1, x_2, x_3, \ldots, x_n \in X$ such that

$$\begin{gathered} F(x_1, x_2, x_3, \ldots, x_n) = g(x_1), \\ F(x_2, x_3, \ldots, x_n, x_1) = g(x_2), \\ \vdots \\ F(x_n, x_1, x_2, \ldots, x_{n-1}) = g(x_n); \end{gathered} \tag{9}$$

that is, F and g have an n-tuplet coincidence point.

Proof. Proof of this theorem consists of four steps.

Step 1. In first step, let us define $(x_k^1), (x_k^2), \ldots, (x_k^n)$. Let $x_0^1, x_0^2, x_0^3, \ldots, x_0^n \in X$ be as in (8). Since $F(X^n) \subset g(X)$, we construct the sequence $(x_k^1), (x_k^2), \ldots, (x_k^n)$ as in [18]:

$$\begin{aligned} &g(x_k^1) = F(x_{k-1}^1, x_{k-1}^2, \ldots, x_{k-1}^n), \\ &g(x_k^2) = F(x_{k-1}^2, \ldots, x_{k-1}^n, x_{k-1}^1), \\ &\quad \vdots \\ &g(x_k^n) = F(x_{k-1}^n, x_{k-1}^1, \ldots, x_{k-1}^{n-1}) \end{aligned} \tag{10}$$

for $k = 1, 2, 3, \ldots$.

Step 2. We prove that the following inequalities hold:

$$\begin{aligned} &g(x_{k-1}^1) \preceq g(x_k^1), \\ &g(x_{k-1}^2) \succeq g(x_k^2), \\ &\quad \vdots \\ &g(x_{k-1}^n) \preceq g(x_k^n) \quad (\text{if } n \text{ is odd}), \\ &g(x_{k-1}^n) \succeq g(x_k^n) \quad (\text{if } n \text{ is even}) \end{aligned} \tag{11}$$

for all $k \geq 1$. This step is similar to a part of proof of Theorem 1 in [18]. So, we omit it. However, since we use it in the proof, we express it again for the integrity of our proof. We can write (12) from (11) as follows:

$$\begin{aligned} &\cdots \succeq g(x_k^1) \succeq g(x_{k-1}^1) \succeq \cdots \succeq g(x_1^1) \succeq g(x_0^1), \\ &\cdots \preceq g(x_k^2) \preceq g(x_{k-1}^2) \preceq \cdots \preceq g(x_1^2) \preceq g(x_0^2), \\ &\quad \vdots \\ &\cdots \succeq g(x_k^n) \succeq g(x_{k-1}^n) \succeq \cdots \succeq g(x_1^n) \succeq g(x_0^n) \\ &\qquad\qquad (\text{if } n \text{ is odd}), \\ &\cdots \preceq g(x_k^n) \preceq g(x_{k-1}^n) \preceq \cdots \preceq g(x_1^n) \preceq g(x_0^n) \\ &\qquad\qquad (\text{if } n \text{ is even}). \end{aligned} \tag{12}$$

Step 3. In this step, we show that $g(x_k^1), g(x_k^2), \ldots, g(x_k^n)$ are Cauchy sequences in $(X, \mu, \nu, *, \diamond)$.

To do this, we define

$$\begin{aligned}\delta_k^1(t) &= \mu\left(g\left(x_k^1\right)-g\left(x_{k+1}^1\right),t\right)\\&\quad*\mu\left(g\left(x_k^2\right)-g\left(x_{k+1}^2\right),t\right)\\&\quad*\cdots*\mu\left(g\left(x_k^n\right)-g\left(x_{k+1}^n\right),t\right),\\\delta_k^2(t) &= \nu\left(g\left(x_k^1\right)-g\left(x_{k+1}^1\right),t\right)\\&\quad\diamond\nu\left(g\left(x_k^2\right)-g\left(x_{k+1}^2\right),t\right)\\&\quad\diamond\cdots\diamond\nu\left(g\left(x_k^n\right)-g\left(x_{k+1}^n\right),t\right).\end{aligned}\tag{13}$$

By considering (6) and (12), we have in the following inequalities:

$$\begin{aligned}&\mu\left(g\left(x_k^1\right)-g\left(x_{k+1}^1\right),t\right)\\&\quad=\mu\left(F\left(x_{k-1}^1,x_{k-1}^2,\ldots,x_{k-1}^n\right)-F\left(x_k^1,x_k^2,\ldots,x_k^n\right),\alpha\frac{t}{a}\right)\\&\quad\geq\mu\left(g\left(x_{k-1}^1\right)-g\left(x_k^1\right),\frac{t}{\alpha}\right)\\&\qquad*\mu\left(g\left(x_{k-1}^2\right)-g\left(x_k^2\right),\frac{t}{\alpha}\right)\\&\qquad*\cdots*\mu\left(g\left(x_{k-1}^n\right)-g\left(x_k^n\right),\frac{t}{\alpha}\right)\\&\quad=\delta_{k-1}\left(\frac{t}{\alpha}\right),\\&\mu\left(g\left(x_k^2\right)-g\left(x_{k+1}^2\right),t\right)\\&\quad=\mu\left(F\left(x_{k-1}^2,\ldots,x_{k-1}^n,x_{k-1}^1\right)-F\left(x_k^2,\ldots,x_k^n,x_k^1\right),\alpha\frac{t}{\alpha}\right)\\&\quad\geq\mu\left(g\left(x_{k-1}^2\right)-g\left(x_k^2\right),\frac{t}{\alpha}\right)\\&\qquad*\cdots*\mu\left(g\left(x_{k-1}^n\right)-g\left(x_k^n\right),\frac{t}{\alpha}\right)\\&\qquad*\mu\left(g\left(x_{k-1}^1\right)-g\left(x_k^1\right),\frac{t}{\alpha}\right)\\&\quad=\delta_{k-1}^1\left(\frac{t}{\alpha}\right)\\&\quad\vdots\\&\mu\left(g\left(x_k^n\right)-g\left(x_{k+1}^n\right),t\right)\\&\quad=\mu\left(F\left(x_{k-1}^n,x_{k-1}^1,\ldots,x_{k-1}^{n-1}\right)-F\left(x_k^n,x_k^1,\ldots,x_k^{n-1}\right),\alpha\frac{t}{\alpha}\right)\\&\quad\geq\mu\left(g\left(x_{k-1}^n\right)-g\left(x_k^n\right),\frac{t}{\alpha}\right)\\&\qquad*\mu\left(g\left(x_{k-1}^1\right)-g\left(x_k^1\right),\frac{t}{\alpha}\right)\\&\qquad*\cdots*\mu\left(g\left(x_{k-1}^{n-1}\right)-g\left(x_k^{n-1}\right),\frac{t}{\alpha}\right)\\&\quad=\delta_{k-1}^1\left(\frac{t}{\alpha}\right).\end{aligned}\tag{14}$$

Using the property (iv) of t-norm and property (xiv) in Definition 3 together with (14), we get

$$\delta_k^1(t)\geq\delta_{k-1}^1\left(\frac{t}{\alpha}\right)*\delta_{k-1}^1\left(\frac{t}{\alpha}\right)*\cdots*\delta_{k-1}^1\left(\frac{t}{\alpha}\right)=\delta_{k-1}^1\left(\frac{t}{\alpha}\right).\tag{15}$$

Again, by (6) and (12),

$$\begin{aligned}&\mu\left(g\left(x_{k-1}^1\right)-g\left(x_k^1\right),\frac{t}{\alpha}\right)\\&\quad=\mu\left(F\left(x_{k-2}^1,x_{k-2}^2,\ldots,x_{k-2}^n\right)\right.\\&\qquad\left.-F\left(x_{k-1}^1,x_{k-1}^2,\ldots,x_{k-1}^n\right),\alpha\frac{t}{\alpha^2}\right)\\&\quad\geq\mu\left(g\left(x_{k-2}^1\right)-g\left(x_{k-1}^1\right),\frac{t}{\alpha^2}\right)\\&\qquad*\mu\left(g\left(x_{k-2}^2\right)-g\left(x_{k-1}^2\right),\frac{t}{\alpha^2}\right)\\&\qquad*\cdots*\mu\left(g\left(x_{k-2}^n\right)-g\left(x_{k-1}^n\right),\frac{t}{\alpha^2}\right)\\&\quad=\delta_{k-2}^1\left(\frac{t}{\alpha^2}\right),\\&\mu\left(g\left(x_{k-1}^2\right)-g\left(x_k^2\right),\frac{t}{\alpha}\right)\\&\quad=\mu\left(F\left(x_{k-2}^2,\ldots,x_{k-2}^n,x_{k-2}^1\right)\right.\\&\qquad\left.-F\left(x_{k-1}^2,\ldots,x_{k-1}^n,x_{k-1}^1\right),\alpha\frac{t}{\alpha^2}\right)\\&\quad\geq\mu\left(g\left(x_{k-2}^2\right)-g\left(x_{k-1}^2\right),\frac{t}{\alpha^2}\right)\\&\qquad*\cdots*\mu\left(g\left(x_{k-2}^n\right)-g\left(x_{k-1}^n\right),\frac{t}{\alpha^2}\right)\\&\qquad*\mu\left(g\left(x_{k-2}^1\right)-g\left(x_{k-1}^1\right),\frac{t}{\alpha^2}\right)\\&\quad=\delta_{k-2}^1\left(\frac{t}{\alpha^2}\right)\\&\quad\vdots\end{aligned}$$

$$\begin{aligned}
&\mu\left(g\left(x_{k-1}^{n}\right)-g\left(x_{k}^{n}\right),\frac{t}{\alpha}\right)\\
&\quad=\mu\left(F\left(x_{k-2}^{n},x_{k-2}^{1},\dots,x_{k-2}^{n-1}\right)\right.\\
&\qquad\left.-F\left(x_{k-1}^{n},x_{k-1}^{1},\dots,x_{k-1}^{n-1}\right),\alpha\frac{t}{\alpha^{2}}\right)\\
&\quad\geq\mu\left(g\left(x_{k-2}^{n}\right)-g\left(x_{k-1}^{n}\right),\frac{t}{\alpha^{2}}\right)\\
&\qquad*\mu\left(g\left(x_{k-2}^{1}\right)-g\left(x_{k-1}^{1}\right),\frac{t}{\alpha^{2}}\right)\\
&\qquad*\cdots*\mu\left(g\left(x_{k-2}^{n-1}\right)-g\left(x_{k-1}^{n-1}\right),\frac{t}{\alpha^{2}}\right)\\
&\quad=\delta_{k-2}^{1}\left(\frac{t}{\alpha^{2}}\right).
\end{aligned}\tag{16}$$

By the property (iv) of t-norm together with (16), we get

$$\delta_{k-1}^{1}\left(\frac{t}{\alpha}\right)\geq\delta_{k-2}^{1}\left(\frac{t}{\alpha^{2}}\right).\tag{17}$$

Thus, if we continue this process in this way, we have

$$\delta_{k}^{1}\left(t\right)\geq\delta_{k-1}^{1}\left(\frac{t}{\alpha}\right)\geq\delta_{k-2}^{1}\left(\frac{t}{\alpha^{2}}\right)\geq\cdots\geq\delta_{0}^{1}\left(\frac{t}{\alpha^{k}}\right).\tag{18}$$

Now we will do the same calculations for $\delta_k^2(t)$. By using (7) and (12),

$$\begin{aligned}
&\nu\left(g\left(x_{k}^{1}\right)-g\left(x_{k+1}^{1}\right),t\right)\\
&\quad=\nu\left(F\left(x_{k-1}^{1},x_{k-1}^{2},\dots,x_{k-1}^{n}\right)-F\left(x_{k}^{1},x_{k}^{2},\dots,x_{k}^{n}\right),\alpha\frac{t}{\alpha}\right)\\
&\quad\leq\nu\left(g\left(x_{k-1}^{1}\right)-g\left(x_{k}^{1}\right),\frac{t}{\alpha}\right)\diamond\nu\left(g\left(x_{k-1}^{2}\right)-g\left(x_{k}^{2}\right),\frac{t}{\alpha}\right)\\
&\qquad\diamond\cdots\diamond\nu\left(g\left(x_{k-1}^{n}\right)-g\left(x_{k}^{n}\right),\frac{t}{\alpha}\right)\\
&\quad=\delta_{k-1}^{2}\left(\frac{t}{\alpha}\right),\\
&\nu\left(g\left(x_{k}^{2}\right)-g\left(x_{k+1}^{2}\right),t\right)\\
&\quad=\nu\left(F\left(x_{k-1}^{2},\dots,x_{k-1}^{n},x_{k-1}^{1}\right)-F\left(x_{k}^{2},\dots,x_{k}^{n},x_{k}^{1}\right),\alpha\frac{t}{\alpha}\right)\\
&\quad\leq\nu\left(g\left(x_{k-1}^{2}\right)-g\left(x_{k}^{2}\right),\frac{t}{\alpha}\right)\\
&\qquad\diamond\cdots\diamond\nu\left(g\left(x_{k-1}^{n}\right)-g\left(x_{k}^{n}\right),\frac{t}{\alpha}\right)\\
&\qquad\diamond\nu\left(g\left(x_{k-1}^{1}\right)-g\left(x_{k}^{1}\right),\frac{t}{\alpha}\right)\\
&\quad=\delta_{k-1}^{2}\left(\frac{t}{\alpha}\right)\\
&\quad\vdots\\
&\nu\left(g\left(x_{k}^{n}\right)-g\left(x_{k+1}^{n}\right),t\right)\\
&\quad=\nu\left(F\left(x_{k-1}^{n},x_{k-1}^{1},\dots,x_{k-1}^{n-1}\right)-F\left(x_{k}^{n},x_{k}^{1},\dots,x_{k}^{n-1}\right),\alpha\frac{t}{\alpha}\right)\\
&\quad\leq\nu\left(g\left(x_{k-1}^{n}\right)-g\left(x_{k}^{n}\right),\frac{t}{\alpha}\right)\diamond\nu\left(g\left(x_{k-1}^{1}\right)-g\left(x_{k}^{1}\right),\frac{t}{\alpha}\right)\\
&\qquad\diamond\cdots\diamond\nu\left(g\left(x_{k-1}^{n-1}\right)-g\left(x_{k}^{n-1}\right),\frac{t}{\alpha}\right)\\
&\quad=\delta_{k-1}^{2}\left(\frac{t}{\alpha}\right).
\end{aligned}\tag{19}$$

Using the property (iv) of t-norm and property (xiv) in Definition 3 together with (19), we get

$$\begin{aligned}
\delta_{k}^{2}\left(t\right)&\leq\delta_{k-1}^{2}\left(\frac{t}{\alpha}\right)\diamond\delta_{k-1}^{2}\left(\frac{t}{\alpha}\right)\diamond\cdots\diamond\delta_{k-1}^{2}\left(\frac{t}{\alpha}\right)\\
&=\delta_{k-1}^{2}\left(\frac{t}{\alpha}\right).
\end{aligned}\tag{20}$$

Again, by (7) and (12),

$$\begin{aligned}
&\nu\left(g\left(x_{k-1}^{1}\right)-g\left(x_{k}^{1}\right),\frac{t}{\alpha}\right)\\
&\quad=\nu\left(F\left(x_{k-2}^{1},x_{k-2}^{2},\dots,x_{k-2}^{n}\right)\right.\\
&\qquad\left.-F\left(x_{k-1}^{1},x_{k-1}^{2},\dots,x_{k-1}^{n}\right),\alpha\frac{t}{\alpha^{2}}\right)\\
&\quad\leq\nu\left(g\left(x_{k-2}^{1}\right)-g\left(x_{k-1}^{1}\right),\frac{t}{\alpha^{2}}\right)\\
&\qquad\diamond\nu\left(g\left(x_{k-2}^{2}\right)-g\left(x_{k-1}^{2}\right),\frac{t}{\alpha^{2}}\right)\\
&\qquad\diamond\cdots\diamond\nu\left(g\left(x_{k-2}^{n}\right)-g\left(x_{k-1}^{n}\right),\frac{t}{\alpha^{2}}\right)\\
&\quad=\delta_{k-2}^{2}\left(\frac{t}{\alpha^{2}}\right),\\
&\nu\left(g\left(x_{k-1}^{2}\right)-g\left(x_{k}^{2}\right),\frac{t}{\alpha}\right)\\
&\quad=\nu\left(F\left(x_{k-2}^{2},\dots,x_{k-2}^{n},x_{k-2}^{1}\right)\right.\\
&\qquad\left.-F\left(x_{k-1}^{2},\dots,x_{k-1}^{n},x_{k-1}^{1}\right),\alpha\frac{t}{\alpha^{2}}\right)\\
&\quad\leq\nu\left(g\left(x_{k-2}^{2}\right)-g\left(x_{k-1}^{2}\right),\frac{t}{\alpha^{2}}\right)\\
&\qquad\diamond\cdots\diamond\nu\left(g\left(x_{k-2}^{n}\right)-g\left(x_{k-1}^{n}\right),\frac{t}{\alpha^{2}}\right)\\
&\qquad\diamond\nu\left(g\left(x_{k-2}^{1}\right)-g\left(x_{k-1}^{1}\right),\frac{t}{\alpha^{2}}\right)\\
&\quad=\delta_{k-2}^{2}\left(\frac{t}{\alpha^{2}}\right)\\
&\quad\vdots
\end{aligned}$$

$$
\begin{aligned}
&\nu\left(g\left(x_{k-1}^{n}\right)-g\left(x_{k}^{n}\right),\frac{t}{\alpha}\right)\\
&\quad=\nu\left(F\left(x_{k-2}^{n},x_{k-2}^{1},\ldots,x_{k-2}^{n-1}\right)\right.\\
&\qquad\left.-F\left(x_{k-1}^{n},x_{k-1}^{1},\ldots,x_{k-1}^{n-1}\right),\alpha\frac{t}{\alpha^{2}}\right)\\
&\quad\leq\nu\left(g\left(x_{k-2}^{n}\right)-g\left(x_{k-1}^{n}\right),\frac{t}{\alpha^{2}}\right)\\
&\qquad\diamond\,\nu\left(g\left(x_{k-2}^{1}\right)-g\left(x_{k-1}^{1}\right),\frac{t}{\alpha^{2}}\right)\\
&\qquad\diamond\cdots\diamond\,\nu\left(g\left(x_{k-2}^{n-1}\right)-g\left(x_{k-1}^{n-1}\right),\frac{t}{\alpha^{2}}\right)\\
&\quad=\delta_{k-2}^{2}\left(\frac{t}{\alpha^{2}}\right).
\end{aligned}
\tag{21}
$$

Hence, from (21),

$$
\delta_{k-1}^{2}\left(\frac{t}{\alpha}\right)\leq\delta_{k-2}^{2}\left(\frac{t}{\alpha^{2}}\right).
\tag{22}
$$

Thus, if we continue this process in this way, we have

$$
\delta_{k}^{2}\left(t\right)\leq\delta_{k-1}^{2}\left(\frac{t}{\alpha}\right)\leq\delta_{k-2}^{2}\left(\frac{t}{\alpha^{2}}\right)\leq\cdots\leq\delta_{0}^{2}\left(\frac{t}{\alpha^{k}}\right).
\tag{23}
$$

Now, we can show $g(x_k^1), g(x_k^2), \ldots, g(x_k^n)$ are Cauchy sequences in $(X, \mu, \nu, *, \diamond)$ by means of (18) and (23). For each $t > 0$ and $p > 0$,

$$
\begin{aligned}
&\mu\left(g\left(x_{k+p}^{1}\right)-g\left(x_{k}^{1}\right),t\right)*\mu\left(g\left(x_{k+p}^{2}\right)-g\left(x_{k}^{2}\right),t\right)\\
&\quad*\cdots*\mu\left(g\left(x_{k+p}^{n}\right)-g\left(x_{k}^{n}\right),t\right)\\
&\quad=\mu\left(g\left(x_{k+p}^{1}\right)-g\left(x_{k+p-1}^{1}\right)+g\left(x_{k+p-1}^{1}\right)-g\left(x_{k+p-2}^{1}\right)\right.\\
&\qquad+g\left(x_{k+p-2}^{1}\right)-\cdots-g\left(x_{k-1}^{1}\right)+g\left(x_{k-1}^{1}\right)\\
&\qquad\left.-g\left(x_{k}^{1}\right),\frac{t}{p}+\frac{t}{p}+\cdots+\frac{t}{p}\right)\\
&\quad*\mu\left(g\left(x_{k+p}^{2}\right)-g\left(x_{k+p-1}^{2}\right)+g\left(x_{k+p-1}^{2}\right)\right.\\
&\qquad-g\left(x_{k+p-2}^{2}\right)+g\left(x_{k+p-2}^{2}\right)-\cdots-g\left(x_{k-1}^{2}\right)\\
&\qquad\left.+g\left(x_{k-1}^{2}\right)-g\left(x_{k}^{2}\right),\frac{t}{p}+\frac{t}{p}+\cdots+\frac{t}{p}\right)\\
&\quad*\cdots*\mu\left(g\left(x_{k+p}^{n}\right)-g\left(x_{k+p-1}^{n}\right)+g\left(x_{k+p-1}^{n}\right)\right.\\
&\qquad-g\left(x_{k+p-2}^{n}\right)+g\left(x_{k+p-2}^{n}\right)-\cdots-g\left(x_{k-1}^{n}\right)\\
&\qquad\left.+g\left(x_{k-1}^{n}\right)-g\left(x_{k}^{n}\right),\frac{t}{p}+\frac{t}{p}+\cdots+\frac{t}{p}\right)\\
&\quad\geq\mu\left(g\left(x_{k+p}^{1}\right)-g\left(x_{k+p-1}^{1}\right),\frac{t}{p}\right)\\
&\qquad*\mu\left(g\left(x_{k+p-1}^{1}\right)-g\left(x_{k+p-2}^{1}\right),\frac{t}{p}\right)\\
&\qquad*\cdots*\mu\left(g\left(x_{k-1}^{1}\right)-g\left(x_{k}^{1}\right),\frac{t}{p}\right)\\
&\qquad*\cdots*\mu\left(g\left(x_{k+p}^{n}\right)-g\left(x_{k+p-1}^{n}\right),\frac{t}{p}\right)\\
&\qquad*\mu\left(g\left(x_{k+p-1}^{n}\right)-g\left(x_{k+p-2}^{n}\right),\frac{t}{p}\right)\\
&\qquad*\cdots*\mu\left(g\left(x_{k-1}^{n}\right)-g\left(x_{k}^{n}\right),\frac{t}{p}\right)\\
&\quad=\mu\left(g\left(x_{k+p}^{1}\right)-g\left(x_{k+p-1}^{1}\right),\frac{t}{p}\right)\\
&\qquad*\mu\left(g\left(x_{k+p}^{2}\right)-g\left(x_{k+p-1}^{2}\right),\frac{t}{p}\right)\\
&\qquad*\cdots*\mu\left(g\left(x_{k+p}^{n}\right)-g\left(x_{k+p-1}^{n}\right),\frac{t}{p}\right)\\
&\qquad*\mu\left(g\left(x_{k+p-1}^{1}\right)-g\left(x_{k+p-2}^{1}\right),\frac{t}{p}\right)\\
&\qquad*\mu\left(g\left(x_{k+p-1}^{2}\right)-g\left(x_{k+p-2}^{2}\right),\frac{t}{p}\right)\\
&\qquad*\cdots*\mu\left(g\left(x_{k+p-1}^{n}\right)-g\left(x_{k+p-2}^{n}\right),\frac{t}{p}\right)\\
&\qquad*\cdots*\mu\left(g\left(x_{k-1}^{1}\right)-g\left(x_{k}^{1}\right),\frac{t}{p}\right)\\
&\qquad*\mu\left(g\left(x_{k-1}^{2}\right)-g\left(x_{k}^{2}\right),\frac{t}{p}\right)\\
&\qquad*\cdots*\mu\left(g\left(x_{k-1}^{n}\right)-g\left(x_{k}^{n}\right),\frac{t}{p}\right)\\
&\quad\geq\mu\left(g\left(x_{0}^{1}\right)-g\left(x_{1}^{1}\right),\frac{t}{p\alpha^{k+p-1}}\right)\\
&\qquad*\mu\left(g\left(x_{0}^{2}\right)-g\left(x_{1}^{2}\right),\frac{t}{p\alpha^{k+p-1}}\right)\\
&\qquad*\cdots*\mu\left(g\left(x_{0}^{n}\right)-g\left(x_{1}^{n}\right),\frac{t}{p\alpha^{k+p-1}}\right)\\
&\qquad*\mu\left(g\left(x_{0}^{1}\right)-g\left(x_{1}^{1}\right),\frac{t}{p\alpha^{k+p-2}}\right)\\
&\qquad*\mu\left(g\left(x_{0}^{2}\right)-g\left(x_{1}^{2}\right),\frac{t}{p\alpha^{k+p-2}}\right)\\
&\qquad*\cdots*\mu\left(g\left(x_{0}^{n}\right)-g\left(x_{1}^{n}\right),\frac{t}{p\alpha^{k+p-2}}\right)\\
&\qquad*\cdots*\mu\left(g\left(x_{0}^{1}\right)-g\left(x_{1}^{1}\right),\frac{t}{p\alpha^{k}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&* \mu\left(g\left(x_0^2\right)-g\left(x_1^2\right), \frac{t}{p\alpha^k}\right) \\
&* \cdots * \mu\left(g\left(x_0^n\right)-g\left(x_1^n\right), \frac{t}{p\alpha^k}\right), \\
&\nu\left(g\left(x_{k+p}^1\right)-g\left(x_k^1\right), t\right) \diamond \nu\left(g\left(x_{k+p}^2\right)-g\left(x_k^2\right), t\right) \\
&\diamond \cdots \diamond \nu\left(g\left(x_{k+p}^n\right)-g\left(x_k^n\right), t\right) \\
&= \nu\Big(g\left(x_{k+p}^1\right)-g\left(x_{k+p-1}^1\right)+g\left(x_{k+p-1}^1\right) \\
&\quad -g\left(x_{k+p-2}^1\right)+g\left(x_{k+p-2}^1\right)-\cdots-g\left(x_{k-1}^1\right) \\
&\quad +g\left(x_{k-1}^1\right)-g\left(x_k^1\right), \frac{t}{p}+\frac{t}{p}+\cdots+\frac{t}{p}\Big) \\
&\diamond \nu\Big(g\left(x_{k+p}^2\right)-g\left(x_{k+p-1}^2\right)+g\left(x_{k+p-1}^2\right) \\
&\quad -g\left(x_{k+p-2}^2\right)+g\left(x_{k+p-2}^2\right)-\cdots-g\left(x_{k-1}^2\right) \\
&\quad +g\left(x_{k-1}^2\right)-g\left(x_k^2\right), \frac{t}{p}+\frac{t}{p}+\cdots+\frac{t}{p}\Big) \\
&\diamond \cdots \diamond \nu\Big(g\left(x_{k+p}^n\right)-g\left(x_{k+p-1}^n\right)+g\left(x_{k+p-1}^n\right) \\
&\quad -g\left(x_{k+p-2}^n\right)+g\left(x_{k+p-2}^n\right)-\cdots-g\left(x_{k-1}^n\right) \\
&\quad +g\left(x_{k-1}^n\right)-g\left(x_k^n\right), \frac{t}{p}+\frac{t}{p}+\cdots+\frac{t}{p}\Big) \\
&\leq \nu\left(g\left(x_{k+p}^1\right)-g\left(x_{k+p-1}^1\right), \frac{t}{p}\right) \\
&\diamond \nu\left(g\left(x_{k+p-1}^1\right)-g\left(x_{k+p-2}^1\right), \frac{t}{p}\right) \\
&\diamond \cdots \diamond \nu\left(g\left(x_{k-1}^1\right)-g\left(x_k^1\right), \frac{t}{p}\right) \\
&\diamond \cdots \diamond \nu\left(g\left(x_{k+p}^n\right)-g\left(x_{k+p-1}^n\right), \frac{t}{p}\right) \\
&\diamond \nu\left(g\left(x_{k+p-1}^n\right)-g\left(x_{k+p-2}^n\right), \frac{t}{p}\right) \\
&\diamond \cdots \diamond \nu\left(g\left(x_{k-1}^n\right)-g\left(x_k^n\right), \frac{t}{p}\right) \\
&= \nu\left(g\left(x_{k+p}^1\right)-g\left(x_{k+p-1}^1\right), \frac{t}{p}\right) \\
&\diamond \nu\left(g\left(x_{k+p}^2\right)-g\left(x_{k+p-1}^2\right), \frac{t}{p}\right) \\
&\diamond \cdots \diamond \nu\left(g\left(x_{k+p}^n\right)-g\left(x_{k+p-1}^n\right), \frac{t}{p}\right) \\
&\diamond \nu\left(g\left(x_{k+p-1}^1\right)-g\left(x_{k+p-2}^1\right), \frac{t}{p}\right) \\
&\diamond \nu\left(g\left(x_{k+p-1}^2\right)-g\left(x_{k+p-2}^2\right), \frac{t}{p}\right) \\
&\diamond \cdots \diamond \nu\left(g\left(x_{k+p-1}^n\right)-g\left(x_{k+p-2}^n\right), \frac{t}{p}\right) \\
&\diamond \cdots \diamond \nu\left(g\left(x_{k-1}^1\right)-g\left(x_k^1\right), \frac{t}{p}\right) \\
&\diamond \nu\left(g\left(x_{k-1}^2\right)-g\left(x_k^2\right), \frac{t}{p}\right) \\
&\diamond \cdots \diamond \nu\left(g\left(x_{k-1}^n\right)-g\left(x_k^n\right), \frac{t}{p}\right) \\
&\leq \nu\left(g\left(x_0^1\right)-g\left(x_1^1\right), \frac{t}{p\alpha^{k+p-1}}\right) \\
&\diamond \nu\left(g\left(x_0^2\right)-g\left(x_1^2\right), \frac{t}{p\alpha^{k+p-1}}\right) \\
&\diamond \cdots \diamond \nu\left(g\left(x_0^n\right)-g\left(x_1^n\right), \frac{t}{p\alpha^{k+p-1}}\right) \\
&\diamond \nu\left(g\left(x_0^1\right)-g\left(x_1^1\right), \frac{t}{p\alpha^{k+p-2}}\right) \\
&\diamond \nu\left(g\left(x_0^2\right)-g\left(x_1^2\right), \frac{t}{p\alpha^{k+p-2}}\right) \\
&\diamond \cdots \diamond \nu\left(g\left(x_0^n\right)-g\left(x_1^n\right), \frac{t}{p\alpha^{k+p-2}}\right) \\
&\diamond \cdots \diamond \nu\left(g\left(x_0^1\right)-g\left(x_1^1\right), \frac{t}{p\alpha^k}\right) \\
&\diamond \nu\left(g\left(x_0^2\right)-g\left(x_1^2\right), \frac{t}{p\alpha^k}\right) \\
&\diamond \cdots \diamond \nu\left(g\left(x_0^n\right)-g\left(x_1^n\right), \frac{t}{p\alpha^k}\right),
\end{aligned} \tag{24}
$$

$t/p\alpha^{k+p-1}, t/p\alpha^{k+p-2}, \ldots, t/p\alpha^k$ tend to infinity when $k \to \infty$ in (24). So, we get (25) with properties (vii) and (xiii) of intuitionistic fuzzy norm:

$$
\begin{aligned}
&\lim_{k\to\infty} \mu\left(g\left(x_{k+p}^1\right)-g\left(x_k^1\right), t\right) * \mu\left(g\left(x_{k+p}^2\right)-g\left(x_k^2\right), t\right) \\
&\quad * \cdots * \mu\left(g\left(x_{k+p}^n\right)-g\left(x_k^n\right), t\right) \geq 1 * 1 * \cdots * 1 = 1, \\
&\lim_{k\to\infty} \nu\left(g\left(x_{k+p}^1\right)-g\left(x_k^1\right), t\right) \diamond \nu\left(g\left(x_{k+p}^2\right)-g\left(x_k^2\right), t\right) \\
&\quad \diamond \cdots \diamond \nu\left(g\left(x_{k+p}^n\right)-g\left(x_k^n\right), t\right) \leq 0 \diamond 0 \diamond \cdots \diamond 0 = 0.
\end{aligned} \tag{25}
$$

Hence, $g(x_k^1), g(x_k^2), \ldots, g(x_k^n)$ are Cauchy sequences in $(X, \mu, \nu, *, \diamond)$.

Step 4. In final step, we prove that g and F have an n-tuplet coincidence point. Since X is complete, then there exist $x^1, x^2, \ldots, x^n$ such that

$$\begin{aligned}
&\lim_{k\to\infty}\mu\left(g\left(x_k^1\right)-x^1,t\right)=1, \qquad \lim_{k\to\infty}\nu\left(g\left(x_k^1\right)-x^1,t\right)=0,\\
&\lim_{k\to\infty}\mu\left(g\left(x_k^2\right)-x^2,t\right)=1, \qquad \lim_{k\to\infty}\nu\left(g\left(x_k^2\right)-x^2,t\right)=0,\\
&\qquad\vdots\\
&\lim_{k\to\infty}\mu\left(g\left(x_k^n\right)-x^n,t\right)=1, \qquad \lim_{k\to\infty}\nu\left(g\left(x_k^n\right)-x^n,t\right)=0.
\end{aligned} \tag{26}$$

By using intuitionistic fuzzy continuity of g, we write

$$\begin{aligned}
&\lim_{k\to\infty}\mu\left(g\left(g\left(x_k^1\right)\right)-g\left(x^1\right),t\right)=1,\\
&\lim_{k\to\infty}\nu\left(g\left(g\left(x_k^1\right)\right)-g\left(x^1\right),t\right)=0,\\
&\lim_{k\to\infty}\mu\left(g\left(g\left(x_k^2\right)\right)-g\left(x^2\right),t\right)=1,\\
&\lim_{k\to\infty}\nu\left(g\left(g\left(x_k^2\right)\right)-g\left(x^2\right),t\right)=0,\\
&\qquad\vdots\\
&\lim_{k\to\infty}\mu\left(g\left(g\left(x_k^n\right)\right)-g\left(x^n\right),t\right)=1,\\
&\lim_{k\to\infty}\nu\left(g\left(g\left(x_k^n\right)\right)-g\left(x^n\right),t\right)=0.
\end{aligned} \tag{27}$$

Since commutativity of F and g, it follows that

$$\begin{aligned}
g\left(g\left(x_{k+1}^1\right)\right)&=g\left(F\left(x_k^1,x_k^2,\ldots,x_k^n\right)\right)\\
&=F\left(g\left(x_k^1\right),g\left(x_k^2\right),\ldots,g\left(x_k^n\right)\right),\\
g\left(g\left(x_{k+1}^2\right)\right)&=g\left(F\left(x_k^2,\ldots,x_k^n,x_k^1\right)\right)\\
&=F\left(g\left(x_k^2\right),\ldots,g\left(x_k^n\right),g\left(x_k^1\right)\right)\\
&\ \,\vdots\\
g\left(g\left(x_{k+1}^n\right)\right)&=g\left(F\left(x_k^n,x_k^1,\ldots,x_k^{n-1}\right)\right)\\
&=F\left(g\left(x_k^n\right),g\left(x_k^1\right),\ldots,g\left(x_k^{n-1}\right)\right).
\end{aligned} \tag{28}$$

Using continuous of t-norm and t-conorm, we get

$$\begin{aligned}
&\lim_{k\to\infty}\Phi\left(\left(g\left(x_k^1\right),g\left(x_k^2\right),\ldots,g\left(x_k^n\right)\right)-\left(x^1,x^2,\ldots,x^n\right),t\right)\\
&\quad=\lim_{k\to\infty}\Phi\left(\left(g\left(x_k^1\right)-x^1,g\left(x_k^2\right)-x^2,\ldots,g\left(x_k^n\right)-x^n\right),t\right)\\
&\quad=\lim_{k\to\infty}\left[\mu\left(g\left(x_k^1\right)-x^1,t\right)*\mu\left(g\left(x_k^2\right)-x^2,t\right)\right.\\
&\qquad\qquad\left.*\cdots*\mu\left(g\left(x_k^n\right)-x^n,t\right)\right]\\
&\quad=\lim_{k\to\infty}\mu\left(g\left(x_k^1\right)-x^1,t\right)*\lim_{k\to\infty}\mu\left(g\left(x_k^2\right)-x^2,t\right)\\
&\qquad\qquad*\cdots*\lim_{k\to\infty}\mu\left(g\left(x_k^n\right)-x^n,t\right)\\
&\quad=1*1*\cdots*1=1,\\
&\lim_{k\to\infty}\Psi\left(\left(g\left(x_k^1\right),g\left(x_k^2\right),\ldots,g\left(x_k^n\right)\right)-\left(x^1,x^2,\ldots,x^n\right),t\right)\\
&\quad=\lim_{k\to\infty}\Psi\left(\left(g\left(x_k^1\right)-x^1,g\left(x_k^2\right)-x^2,\ldots,g\left(x_k^n\right)-x^n\right),t\right)\\
&\quad=\lim_{k\to\infty}\left[\nu\left(g\left(x_k^1\right)-x^1,t\right)\diamond\nu\left(g\left(x_k^2\right)-x^2,t\right)\right.\\
&\qquad\qquad\left.\diamond\cdots\diamond\nu\left(g\left(x_k^n\right)-x^n,t\right)\right]\\
&\quad=\lim_{k\to\infty}\nu\left(g\left(x_k^1\right)-x^1,t\right)\diamond\lim_{k\to\infty}\nu\left(g\left(x_k^2\right)-x^2,t\right)\\
&\qquad\qquad\diamond\cdots\diamond\lim_{k\to\infty}\nu\left(g\left(x_k^n\right)-x^n,t\right)\\
&\quad=0\diamond 0\diamond\cdots\diamond 0=0.
\end{aligned} \tag{29}$$

That is $(g(x_k^1), g(x_k^2), \ldots, g(x_k^n)) \xrightarrow{(\Phi,\Psi)} (x^1, x^2, \ldots, x^n)$. Similarly $(g(x_k^2), \ldots, g(x_k^n), g(x_k^1)) \xrightarrow{(\Phi,\Psi)} (x^2, \ldots, x^n, x^1), \ldots, (g(x_k^n), g(x_k^1), \ldots, g(x_k^{n-1})) \xrightarrow{(\Phi,\Psi)} (x^n, x^1, \ldots, x^{n-1})$. From the intuitionistic fuzzy continuous assumption of F, we write in the followings

$$\begin{aligned}
&F\left(g\left(x_k^1\right),g\left(x_k^2\right),\ldots,g\left(x_k^n\right)\right)\xrightarrow{(\mu,\nu)}F\left(x^1,x^2,\ldots,x^n\right)\\
&F\left(g\left(x_k^2\right),\ldots,g\left(x_k^n\right),g\left(x_k^1\right)\right)\xrightarrow{(\mu,\nu)}F\left(x^2,\ldots,x^n,x^1\right)\\
&\qquad\vdots\\
&F\left(g\left(x_k^n\right),g\left(x_k^1\right),\ldots,g\left(x_k^{n-1}\right)\right)\xrightarrow{(\mu,\nu)}F\left(x^n,x^1,\ldots,x^{n-1}\right).
\end{aligned} \tag{30}$$

Considering (28) and (29), we have

$$
\begin{aligned}
&\mu\left(g\left(x^1\right)-F\left(x^1,x^2,\ldots,x^n\right),t\right)\\
&=\mu\left(g\left(x^1\right)-g\left(g\left(x^1_{k+1}\right)\right)\right.\\
&\qquad\left.+g\left(g\left(x^1_{k+1}\right)\right)-F\left(x^1,x^2,\ldots,x^n\right),t\right)\\
&\geq\mu\left(g\left(x^1\right)-g\left(g\left(x^1_{k+1}\right)\right),\frac{t}{2}\right)\\
&\quad*\mu\left(g\left(g\left(x^1_{k+1}\right)\right)-F\left(x^1,x^2,\ldots,x^n\right),\frac{t}{2}\right)\\
&=\mu\left(g\left(x^1\right)-g\left(g\left(x^1_{k+1}\right)\right),\frac{t}{2}\right)\\
&\quad*\mu\left(g\left(F\left(x^1_k,x^2_k,\ldots,x^n_k\right)\right)\right.\\
&\qquad\left.-F\left(x^1,x^2,\ldots,x^n\right),\frac{t}{2}\right)\\
&=\mu\left(g\left(x^1\right)-g\left(g\left(x^1_{k+1}\right)\right),\frac{t}{2}\right)\\
&\quad*\mu\left(F\left(g\left(x^1_k\right),g\left(x^2_k\right),\ldots,g\left(x^n_k\right)\right)\right.\\
&\qquad\left.-F\left(x^1,x^2,\ldots,x^n\right),\frac{t}{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
&\nu\left(g\left(x^1\right)-F\left(x^1,x^2,\ldots,x^n\right),t\right)\\
&=\nu\left(g\left(x^1\right)-g\left(g\left(x^1_{k+1}\right)\right)\right.\\
&\qquad\left.+g\left(g\left(x^1_{k+1}\right)\right)-F\left(x^1,x^2,\ldots,x^n\right),t\right)\\
&\leq\nu\left(g\left(x^1\right)-g\left(g\left(x^1_{k+1}\right)\right),\frac{t}{2}\right)\\
&\diamond\,\nu\left(g\left(g\left(x^1_{k+1}\right)\right)-F\left(x^1,x^2,\ldots,x^n\right),\frac{t}{2}\right)\\
&=\nu\left(g\left(x^1\right)-g\left(g\left(x^1_{k+1}\right)\right),\frac{t}{2}\right)\\
&\diamond\,\nu\left(g\left(F\left(x^1_k,x^2_k,\ldots,x^n_k\right)\right)-F\left(x^1,x^2,\ldots,x^n\right),\frac{t}{2}\right)\\
&=\nu\left(g\left(x^1\right)-g\left(g\left(x^1_{k+1}\right)\right),\frac{t}{2}\right)\\
&\quad\diamond\,\nu\left(F\left(g\left(x^1_k\right),g\left(x^2_k\right),\ldots,g\left(x^n_k\right)\right)\right.\\
&\qquad\left.-F\left(x^1,x^2,\ldots,x^n\right),\frac{t}{2}\right),
\end{aligned}
$$

$$
\begin{aligned}
&\mu\left(g\left(x^2\right)-F\left(x^2,\ldots,x^n,x^1\right),t\right)\\
&=\mu\left(g\left(x^2\right)-g\left(g\left(x^2_{k+1}\right)\right)\right.\\
&\qquad\left.+g\left(g\left(x^2_{k+1}\right)\right)-F\left(x^2,\ldots,x^n,x^1\right),t\right)\\
&\geq\mu\left(g\left(x^2\right)-g\left(g\left(x^2_{k+1}\right)\right),\frac{t}{2}\right)\\
&\quad*\mu\left(g\left(g\left(x^2_{k+1}\right)\right)-F\left(x^2,\ldots,x^n,x^1\right),\frac{t}{2}\right)\\
&=\mu\left(g\left(x^2\right)-g\left(g\left(x^2_{k+1}\right)\right),\frac{t}{2}\right)\\
&\quad*\mu\left(g\left(F\left(x^2_k,\ldots,x^n_k,x^1_k\right)\right)-F\left(x^2,\ldots,x^n,x^1\right),\frac{t}{2}\right)\\
&=\mu\left(g\left(x^2\right)-g\left(g\left(x^2_{k+1}\right)\right),\frac{t}{2}\right)\\
&\quad*\mu\left(F\left(g\left(x^2_k\right),\ldots,g\left(x^n_k\right),g\left(x^1_k\right)\right)\right.\\
&\qquad\left.-F\left(x^2,\ldots,x^n,x^1\right),\frac{t}{2}\right),
\end{aligned}
$$

$$
\begin{aligned}
&\nu\left(g\left(x^2\right)-F\left(x^2,\ldots,x^n,x^1\right),t\right)\\
&=\nu\left(g\left(x^2\right)-g\left(g\left(x^2_{k+1}\right)\right)\right.\\
&\qquad\left.+g\left(g\left(x^2_{k+1}\right)\right)-F\left(x^2,\ldots,x^n,x^1\right),t\right)\\
&\leq\nu\left(g\left(x^2\right)-g\left(g\left(x^2_{k+1}\right)\right),\frac{t}{2}\right)\\
&\quad\diamond\,\nu\left(g\left(g\left(x^2_{k+1}\right)\right)-F\left(x^2,\ldots,x^n,x^1\right),\frac{t}{2}\right)\\
&=\nu\left(g\left(x^2\right)-g\left(g\left(x^2_{k+1}\right)\right),\frac{t}{2}\right)\\
&\diamond\,\nu\left(g\left(F\left(x^2_k,\ldots,x^n_k,x^1_k\right)\right)-F\left(x^2,\ldots,x^n,x^1\right),\frac{t}{2}\right)\\
&=\nu\left(g\left(x^2\right)-g\left(g\left(x^2_{k+1}\right)\right),\frac{t}{2}\right)\\
&\quad\diamond\,\nu\left(F\left(g\left(x^2_k\right),\ldots,g\left(x^n_k\right),g\left(x^1_k\right)\right)\right.\\
&\qquad\left.-F\left(x^2,\ldots,x^n,x^1\right),\frac{t}{2}\right),
\end{aligned}
$$

$$\vdots$$

$$
\begin{aligned}
&\mu\left(g\left(x^n\right)-F\left(x^n,x^1,\ldots,x^{n-1}\right),t\right)\\
&=\mu\left(g\left(x^n\right)-g\left(g\left(x^n_{k+1}\right)\right)\right.\\
&\qquad\left.+g\left(g\left(x^n_{k+1}\right)\right)-F\left(x^n,x^1,\ldots,x^{n-1}\right),t\right)\\
&\geq\mu\left(g\left(x^n\right)-g\left(g\left(x^n_{k+1}\right)\right),\frac{t}{2}\right)\\
&\quad*\mu\left(g\left(g\left(x^n_{k+1}\right)\right)-F\left(x^n,x^1,\ldots,x^{n-1}\right),\frac{t}{2}\right)\\
&=\mu\left(g\left(x^n\right)-g\left(g\left(x^n_{k+1}\right)\right),\frac{t}{2}\right)\\
&\quad*\mu\left(g\left(F\left(x^n_k,x^1_k,\ldots,x^{n-1}_k\right)\right)\right.\\
&\qquad\left.-F\left(x^n,x^1,\ldots,x^{n-1}\right),\frac{t}{2}\right)
\end{aligned}
$$

$$= \mu\left(g\left(x^{n}\right) - g\left(g\left(x_{k+1}^{n}\right)\right), \frac{t}{2}\right)$$
$$* \mu\left(F\left(g\left(x_{k}^{n}\right), g\left(x_{k}^{1}\right), \ldots, g\left(x_{k}^{n-1}\right)\right)\right.$$
$$\left. -F\left(x^{n}, x^{1}, \ldots, x^{n-1}\right), \frac{t}{2}\right),$$

$$\begin{aligned}
\nu\left(g\left(x^{n}\right) - F\left(x^{n}, x^{1}, \ldots, x^{n-1}\right), t\right)
&= \nu\left(g\left(x^{n}\right) - g\left(g\left(x_{k+1}^{n}\right)\right)\right. \\
&\quad \left. + g\left(g\left(x_{k+1}^{n}\right)\right) - F\left(x^{n}, x^{1}, \ldots, x^{n-1}\right), t\right) \\
&\le \nu\left(g\left(x^{n}\right) - g\left(g\left(x_{k+1}^{n}\right)\right), \frac{t}{2}\right) \\
&\quad \diamond \nu\left(g\left(g\left(x_{k+1}^{n}\right)\right) - F\left(x^{n}, x^{1}, \ldots, x^{n-1}\right), \frac{t}{2}\right) \\
&= \nu\left(g\left(x^{n}\right) - g\left(g\left(x_{k+1}^{n}\right)\right), \frac{t}{2}\right) \\
&\quad \diamond \nu\left(g\left(F\left(x_{k}^{n}, x_{k}^{1}, \ldots, x_{k}^{n-1}\right)\right) - F\left(x^{n}, x^{1}, \ldots, x^{n-1}\right), \frac{t}{2}\right) \\
&= \nu\left(g\left(x^{n}\right) - g\left(g\left(x_{k+1}^{n}\right)\right), \frac{t}{2}\right) \\
&\quad \diamond \nu\left(F\left(g\left(x_{k}^{n}\right), g\left(x_{k}^{1}\right), \ldots, g\left(x_{k}^{n-1}\right)\right) - F\left(x^{n}, x^{1}, \ldots, x^{n-1}\right), \frac{t}{2}\right).
\end{aligned} \tag{31}$$

□

By taking the limit as $k \to \infty$ in (31), we get

$$\begin{aligned}
F\left(x^{1}, x^{2}, x^{3}, \ldots, x^{n}\right) &= g\left(x^{1}\right), \\
F\left(x^{2}, x^{3}, \ldots, x^{n}, x^{1}\right) &= g\left(x^{2}\right), \\
&\vdots \\
F\left(x^{n}, x^{1}, x^{2}, \ldots, x^{n-1}\right) &= g\left(x^{n}\right).
\end{aligned} \tag{32}$$

Theorem 12. *Let $F : X^n \to X$ be map having mixed g-monotone property on the complete $(X, \mu, \nu, *, \diamond)$ having partial order relation denoted by $\preceq$. Also $F(X^n) \subset g(X)$; g is continuous and commutes with F. Suppose that X has the following property:*

(a) *if non decreasing sequence $x_k \xrightarrow{(\mu,\nu)} x$, then $x_k \preceq x$ for all k,*

(b) *if non increasing sequence $y_k \xrightarrow{(\mu,\nu)} y$, then $y_k \succeq y$ for all k.*

Also, suppose that $F : X^n \to X$ and $g : X \to X$ hold the following conditions, for all $x_1, x_2, x_3, \ldots, x_n, y_1, y_2, y_3, \ldots, y_n \in X$ and $\alpha \in (0,1)$:

$$\begin{aligned}
&\mu\left(F\left(x_1, x_2, x_3, \ldots, x_n\right) - F\left(y_1, y_2, y_3, \ldots, y_n\right), \alpha t\right) \\
&\quad \ge \mu\left(g\left(x_1\right) - g\left(y_1\right), t\right) * \mu\left(g\left(x_2\right) - g\left(y_2\right), t\right) \\
&\qquad * \cdots * \mu\left(g\left(x_n\right) - g\left(y_n\right), t\right), \\
&\nu\left(F\left(x_1, x_2, x_3, \ldots, x_n\right) - F\left(y_1, y_2, y_3, \ldots, y_n\right), \alpha t\right) \\
&\quad \le \nu\left(g\left(x_1\right) - g\left(y_1\right), t\right) \diamond \nu\left(g\left(x_2\right) - g\left(y_2\right), t\right) \\
&\qquad \diamond \cdots \diamond \nu\left(g\left(x_n\right) - g\left(y_n\right), t\right),
\end{aligned} \tag{33}$$

where $g(x_{2i-1}) \preceq g(y_{2i-1})$, $i \in \{1, 2, \ldots, (n+1)/2\}$, and $g(x_{2i}) \succeq g(y_{2i})$, $i \in \{1, 2, \ldots, n/2\}$. If there exist $x_0^1, x_0^2, x_0^3, \ldots, x_0^n \in X$ such that

$$\begin{aligned}
g\left(x_0^1\right) &\preceq F\left(x_0^1, x_0^2, x_0^3, \ldots, x_0^n\right), \\
g\left(x_0^2\right) &\succeq F\left(x_0^2, x_0^3, \ldots, x_0^n, x_0^1\right), \\
&\vdots \\
g\left(x_0^n\right) &\preceq F\left(x_0^n, x_0^1, x_0^2, \ldots, x_0^{n-1}\right) \quad (\text{if } n \text{ is odd}), \\
g\left(x_0^n\right) &\succeq F\left(x_0^n, x_0^1, x_0^2, \ldots, x_0^{n-1}\right) \quad (\text{if } n \text{ is even}),
\end{aligned} \tag{34}$$

then there exist $x_1, x_2, x_3, \ldots, x_n \in X$ such that

$$\begin{aligned}
F\left(x_1, x_2, x_3, \ldots, x_n\right) &= g\left(x_1\right), \\
F\left(x_2, x_3, \ldots x_n, x_1\right) &= g\left(x_2\right), \\
&\vdots \\
F\left(x_n, x_1, x_2, \ldots, x_{n-1}\right) &= g\left(x_n\right);
\end{aligned} \tag{35}$$

that is, F and g have an n-tuplet coincidence point.

Proof. Proof of the present theorem is also in four steps. However, three steps of poof are similar to Theorem 11. We now prove the last step. Considering the hypotheses (a)-(b) given in the theorem and $g(x_k^1) \xrightarrow{(\mu,\nu)} x^1, g(x_k^2) \xrightarrow{(\mu,\nu)} x^2, \ldots, g(x_k^n) \xrightarrow{(\mu,\nu)} x^n$, we have

$$\begin{aligned}
g\left(x_k^1\right) &\preceq x^1, \\
g\left(x_k^2\right) &\succeq x^2, \\
&\vdots \\
g\left(x_k^n\right) &\preceq x^n \quad (\text{if } n \text{ is odd}), \\
g\left(x_k^n\right) &\succeq x^n \quad (\text{if } n \text{ is even})
\end{aligned} \tag{36}$$

with (12) for all k. Due to intuitionistic fuzzy continuity of g, we write

$$\begin{aligned} g\left(g\left(x_k^1\right)\right) &\xrightarrow{(\mu,\nu)} g\left(x^1\right), \\ g\left(g\left(x_k^2\right)\right) &\xrightarrow{(\mu,\nu)} g\left(x^2\right), \\ &\vdots \\ g\left(g\left(x_k^n\right)\right) &\xrightarrow{(\mu,\nu)} g\left(x^n\right). \end{aligned} \tag{37}$$

Then, by (6) and (7), we have

$$\begin{aligned} &\mu\left(g\left(x^1\right) - F\left(x^1, x^2, \ldots, x^n\right), \alpha t\right) \\ &\quad = \mu\left(g\left(x^1\right) - g\left(g\left(x_{k+1}^1\right)\right) + g\left(g\left(x_{k+1}^1\right)\right) - F\left(x^1, x^2, \ldots, x^n\right), \alpha t\right) \\ &\quad \geq \mu\left(g\left(x^1\right) - g\left(g\left(x_{k+1}^1\right)\right), \frac{\alpha t}{2}\right) * \mu\left(g\left(g\left(x_{k+1}^1\right)\right) - F\left(x^1, x^2, \ldots, x^n\right), \frac{\alpha t}{2}\right) \\ &\quad = \mu\left(g\left(x^1\right) - g\left(g\left(x_{k+1}^1\right)\right), \frac{\alpha t}{2}\right) * \mu\left(g\left(F\left(x_k^1, x_k^2, \ldots, x_k^n\right)\right) - F\left(x^1, x^2, \ldots, x^n\right), \frac{\alpha t}{2}\right) \\ &\quad = \mu\left(g\left(x^1\right) - g\left(g\left(x_{k+1}^1\right)\right), \frac{\alpha t}{2}\right) * \mu\left(F\left(g\left(x_k^1\right), g\left(x_k^2\right), \ldots, g\left(x_k^n\right)\right) - F\left(x^1, x^2, \ldots, x^n\right), \frac{\alpha t}{2}\right) \\ &\quad \geq \mu\left(g\left(x^1\right) - g\left(g\left(x_{k+1}^1\right)\right), \frac{\alpha t}{2}\right) * \mu\left(g\left(g\left(x_k^1\right)\right) - g\left(x^1\right), \frac{t}{2}\right) \\ &\qquad * \mu\left(g\left(g\left(x_k^2\right)\right) - g\left(x^2\right), \frac{t}{2}\right) * \cdots * \mu\left(g\left(g\left(x_k^n\right)\right) - g\left(x^n\right), \frac{t}{2}\right), \\ &\nu\left(g\left(x^1\right) - F\left(x^1, x^2, \ldots, x^n\right), \alpha t\right) \\ &\quad = \nu\left(g\left(x^1\right) - g\left(g\left(x_{k+1}^1\right)\right) + g\left(g\left(x_{k+1}^1\right)\right) - F\left(x^1, x^2, \ldots, x^n\right), \alpha t\right) \\ &\quad \leq \nu\left(g\left(x^1\right) - g\left(g\left(x_{k+1}^1\right)\right), \frac{\alpha t}{2}\right) \diamond \nu\left(g\left(g\left(x_{k+1}^1\right)\right) - F\left(x^1, x^2, \ldots, x^n\right), \frac{\alpha t}{2}\right) \\ &\quad = \nu\left(g\left(x^1\right) - g\left(g\left(x_{k+1}^1\right)\right), \frac{\alpha t}{2}\right) \diamond \nu\left(g\left(F\left(x_k^1, x_k^2, \ldots, x_k^n\right)\right) - F\left(x^1, x^2, \ldots, x^n\right), \frac{\alpha t}{2}\right) \\ &\quad = \nu\left(g\left(x^1\right) - g\left(g\left(x_{k+1}^1\right)\right), \frac{\alpha t}{2}\right) \diamond \nu\left(F\left(g\left(x_k^1\right), g\left(x_k^2\right), \ldots, g\left(x_k^n\right)\right) - F\left(x^1, x^2, \ldots, x^n\right), \frac{\alpha t}{2}\right) \\ &\quad \leq \nu\left(g\left(x^1\right) - g\left(g\left(x_{k+1}^1\right)\right), \frac{\alpha t}{2}\right) \diamond \nu\left(g\left(g\left(x_k^1\right)\right) - g\left(x^1\right), \frac{t}{2}\right) \\ &\qquad \diamond \nu\left(g\left(g\left(x_k^2\right)\right) - g\left(x^2\right), \frac{t}{2}\right) \diamond \cdots \diamond \nu\left(g\left(g\left(x_k^n\right)\right) - g\left(x^n\right), \frac{t}{2}\right) \end{aligned} \tag{38}$$

by taking limit as $k \to \infty$ and using (37), we obtain the following results from calculation mentioned above:

$$\begin{aligned} \mu\left(g\left(x^1\right) - F\left(x^1, x^2, \ldots, x^n\right), \alpha t\right) &= 1, \\ \nu\left(g\left(x^1\right) - F\left(x^1, x^2, \ldots, x^n\right), \alpha t\right) &= 0. \end{aligned} \tag{39}$$

Hence, $g(x^1) = F(x^1, x^2, \ldots, x^n)$.

In a similar way to the previous calculations,

$$\begin{aligned} &\mu\left(g\left(x^2\right) - F\left(x^2, \ldots, x^n, x^1\right), \alpha t\right) \\ &\quad = \mu\left(g\left(x^2\right) - g\left(g\left(x_{k+1}^2\right)\right) + g\left(g\left(x_{k+1}^2\right)\right) - F\left(x^2, \ldots, x^n, x^1\right), \alpha t\right) \\ &\quad \geq \mu\left(g\left(x^2\right) - g\left(g\left(x_{k+1}^2\right)\right), \frac{\alpha t}{2}\right) * \mu\left(g\left(g\left(x_{k+1}^2\right)\right) - F\left(x^2, \ldots, x^n, x^1\right), \frac{\alpha t}{2}\right) \\ &\quad = \mu\left(g\left(x^2\right) - g\left(g\left(x_{k+1}^2\right)\right), \frac{\alpha t}{2}\right) * \mu\left(g\left(F\left(x_k^2, \ldots, x_k^n, x_k^1\right)\right) - F\left(x^2, \ldots, x^n, x^1\right), \frac{\alpha t}{2}\right) \\ &\quad = \mu\left(g\left(x^2\right) - g\left(g\left(x_{k+1}^2\right)\right), \frac{\alpha t}{2}\right) \end{aligned}$$

$$
\begin{aligned}
&* \mu\Big(F\left(g\left(x_k^2\right),\dots,g\left(x_k^n\right),g\left(x_k^1\right)\right) \\
&\qquad -F\left(x^2,\dots,x^n,x^1\right),\frac{\alpha t}{2}\Big) \\
&\geq \mu\Big(g\left(x^2\right)-g\left(g\left(x_{k+1}^2\right)\right),\frac{\alpha t}{2}\Big) \\
&\quad * \mu\Big(g\left(g\left(x_k^2\right)\right)-g\left(x^2\right),\frac{t}{2}\Big) \\
&\quad * \cdots * \mu\Big(g\left(g\left(x_k^n\right)\right)-g\left(x^n\right),\frac{t}{2}\Big) \\
&\quad * \mu\Big(g\left(g\left(x_k^1\right)\right)-g\left(x^1\right),\frac{t}{2}\Big), \\
&\nu\left(g\left(x^2\right)-F\left(x^2,\dots,x^n,x^1\right),\alpha t\right) \\
&= \nu\Big(g\left(x^2\right)-g\left(g\left(x_{k+1}^2\right)\right) \\
&\qquad +g\left(g\left(x_{k+1}^2\right)\right)-F\left(x^2,\dots,x^n,x^1\right),\alpha t\Big) \\
&\leq \nu\Big(g\left(x^2\right)-g\left(g\left(x_{k+1}^2\right)\right),\frac{\alpha t}{2}\Big) \\
&\quad \diamond \nu\Big(g\left(x_{k+1}^2\right)-F\left(x^2,\dots,x^n,x^1\right),\frac{\alpha t}{2}\Big) \\
&= \nu\Big(g\left(x^2\right)-g\left(g\left(x_{k+1}^2\right)\right),\frac{\alpha t}{2}\Big) \\
&\quad \diamond \nu\Big(g\left(F\left(x_k^2,\dots,x_k^n,x_k^1\right)\right) \\
&\qquad -F\left(x^2,\dots,x^n,x^1\right),\frac{\alpha t}{2}\Big) \\
&= \nu\Big(g\left(x^2\right)-g\left(g\left(x_{k+1}^2\right)\right),\frac{\alpha t}{2}\Big) \\
&\quad \diamond \nu\Big(F\left(g\left(x_k^2\right),\dots,g\left(x_k^n\right),g\left(x_k^1\right)\right) \\
&\qquad -F\left(x^2,\dots,x^n,x^1\right),\frac{\alpha t}{2}\Big) \\
&\leq \nu\Big(g\left(x^2\right)-g\left(g\left(x_{k+1}^2\right)\right),\frac{\alpha t}{2}\Big) \\
&\quad \diamond \nu\Big(g\left(g\left(x_k^2\right)\right)-g\left(x^2\right),\frac{t}{2}\Big) \\
&\quad \diamond \cdots \diamond \nu\Big(g\left(g\left(x_k^n\right)\right)-g\left(x^n\right),\frac{t}{2}\Big) \\
&\quad \diamond \nu\Big(g\left(g\left(x_k^1\right)\right)-g\left(x^1\right),\frac{t}{2}\Big)
\end{aligned} \tag{40}
$$

by taking limit as $k \to \infty$ and using (37), we get the following equalities:

$$
\begin{aligned}
&\mu\left(g\left(x^2\right)-F\left(x^2,\dots,x^n,x^1\right),\alpha t\right)=1, \\
&\nu\left(g\left(x^2\right)-F\left(x^2,\dots,x^n,x^1\right),\alpha t\right)=0.
\end{aligned} \tag{41}
$$

Hence, $F(x^2,x^3,\dots,x^n,x^1)=g(x^2),\dots$.

We continue process

$$
\begin{aligned}
&\mu\left(g\left(x^n\right)-F\left(x^n,x^1,x^2\cdots,x^{n-1}\right),\alpha t\right) \\
&= \mu\Big(g\left(x^n\right)-g\left(g\left(x_{k+1}^n\right)\right)+g\left(g\left(x_{k+1}^n\right)\right) \\
&\qquad -F\left(x^n,x^1,x^2\cdots,x^{n-1}\right),\alpha t\Big) \\
&\geq \mu\Big(g\left(x^n\right)-g\left(g\left(x_{k+1}^n\right)\right),\frac{\alpha t}{2}\Big) \\
&\quad * \mu\Big(g\left(g\left(x_{k+1}^n\right)\right)-F\left(x^n,x^1,x^2\cdots,x^{n-1}\right),\frac{\alpha t}{2}\Big) \\
&= \mu\Big(g\left(x^n\right)-g\left(g\left(x_{k+1}^n\right)\right),\frac{\alpha t}{2}\Big) \\
&\quad * \mu\Big(g\left(F\left(x_k^n,x_k^1,\dots,x_k^{n-1}\right)\right) \\
&\qquad -F\left(x^n,x^1,x^2\cdots,x^{n-1}\right),\frac{\alpha t}{2}\Big) \\
&= \mu\Big(g\left(x^n\right)-g\left(g\left(x_{k+1}^n\right)\right),\frac{\alpha t}{2}\Big) \\
&\quad * \mu\Big(F\left(g\left(x_k^n\right),g\left(x_k^1\right),\dots,g\left(x_k^{n-1}\right)\right) \\
&\qquad -F\left(x^n,x^1,x^2\cdots,x^{n-1}\right),\frac{\alpha t}{2}\Big) \\
&\geq \mu\Big(g\left(x^n\right)-g\left(g\left(x_{k+1}^n\right)\right),\frac{\alpha t}{2}\Big) \\
&\quad * \mu\Big(g\left(g\left(x_k^n\right)\right)-g\left(x^n\right),\frac{t}{2}\Big) \\
&\quad * \mu\Big(g\left(g\left(x_k^1\right)\right)-g\left(x^1\right),\frac{t}{2}\Big) \\
&\quad * \cdots * \mu\Big(g\left(g\left(x_k^{n-1}\right)\right)-g\left(x^{n-1}\right),\frac{t}{2}\Big), \\
&\nu\left(g\left(x^n\right)-F\left(x^n,x^1,x^2\cdots,x^{n-1}\right),\alpha t\right) \\
&= \nu\Big(g\left(x^n\right)-g\left(g\left(x_{k+1}^n\right)\right)+g\left(g\left(x_{k+1}^1\right)\right) \\
&\qquad -F\left(x^n,x^1,x^2\cdots,x^{n-1}\right),\alpha t\Big) \\
&\leq \nu\Big(g\left(x^n\right)-g\left(g\left(x_{k+1}^n\right)\right),\frac{\alpha t}{2}\Big) \\
&\quad \diamond \nu\Big(g\left(g\left(x_{k+1}^n\right)\right)-F\left(x^n,x^1,x^2\cdots,x^{n-1}\right),\frac{\alpha t}{2}\Big) \\
&= \nu\Big(g\left(x^n\right)-g\left(g\left(x_{k+1}^n\right)\right),\frac{\alpha t}{2}\Big) \\
&\quad \diamond \nu\Big(g\left(F\left(x_k^n,x_k^1,\dots,x_k^{n-1}\right)\right) \\
&\qquad -F\left(x^n,x^1,x^2\cdots,x^{n-1}\right),\frac{\alpha t}{2}\Big) \\
&= \nu\Big(g\left(x^n\right)-g\left(g\left(x_{k+1}^n\right)\right),\frac{\alpha t}{2}\Big)
\end{aligned}
$$

$$
\begin{aligned}
&\diamond \nu\Big(F\left(g\left(x_k^n\right), g\left(x_k^1\right), \ldots, g\left(x_k^{n-1}\right)\right) \\
&\qquad -F\left(x^n, x^1, x^2 \cdots, x^{n-1}\right), \frac{\alpha t}{2}\Big) \\
&\leq \nu\left(g\left(x^n\right)-g\left(g\left(x_{k+1}^n\right)\right), \frac{\alpha t}{2}\right) \\
&\quad \diamond \nu\left(g\left(g\left(x_k^n\right)\right)-g\left(x^n\right), \frac{t}{2}\right) \\
&\quad \diamond \nu\left(g\left(g\left(x_k^1\right)\right)-g\left(x^1\right), \frac{t}{2}\right) \\
&\quad \diamond \cdots \diamond \nu\left(g\left(g\left(x_k^{n-1}\right)\right)-g\left(x^{n-1}\right), \frac{t}{2}\right)
\end{aligned} \tag{42}
$$

by taking limit as $k \to \infty$ and using (37), we obtain the last equalities. That is,

$$
\begin{aligned}
&\mu\left(g\left(x^n\right)-F\left(x^n, x^1, x^2, \ldots, x^{n-1}\right), \alpha t\right)=1, \\
&\nu\left(g\left(x^n\right)-F\left(x^n, x^1, x^2, \ldots, x^{n-1}\right), \alpha t\right)=0.
\end{aligned} \tag{43}
$$

Hence, $F(x^n, x^1, x^2, \ldots, x^{n-1}) = g(x^n)$.

Thus, we proved that F and g have an n-tuplet coincidence point. □

Remark 13. Theorems 11 and 12 are restricted to Theorem 2.5 in [29] for $n = 2$; it is restricted to Theorem 3.1 in [36] for $n = 2$ and $g = I$. For $n = 3$, Theorems 11 and 12 are restricted to Theorem 2.1 in [30].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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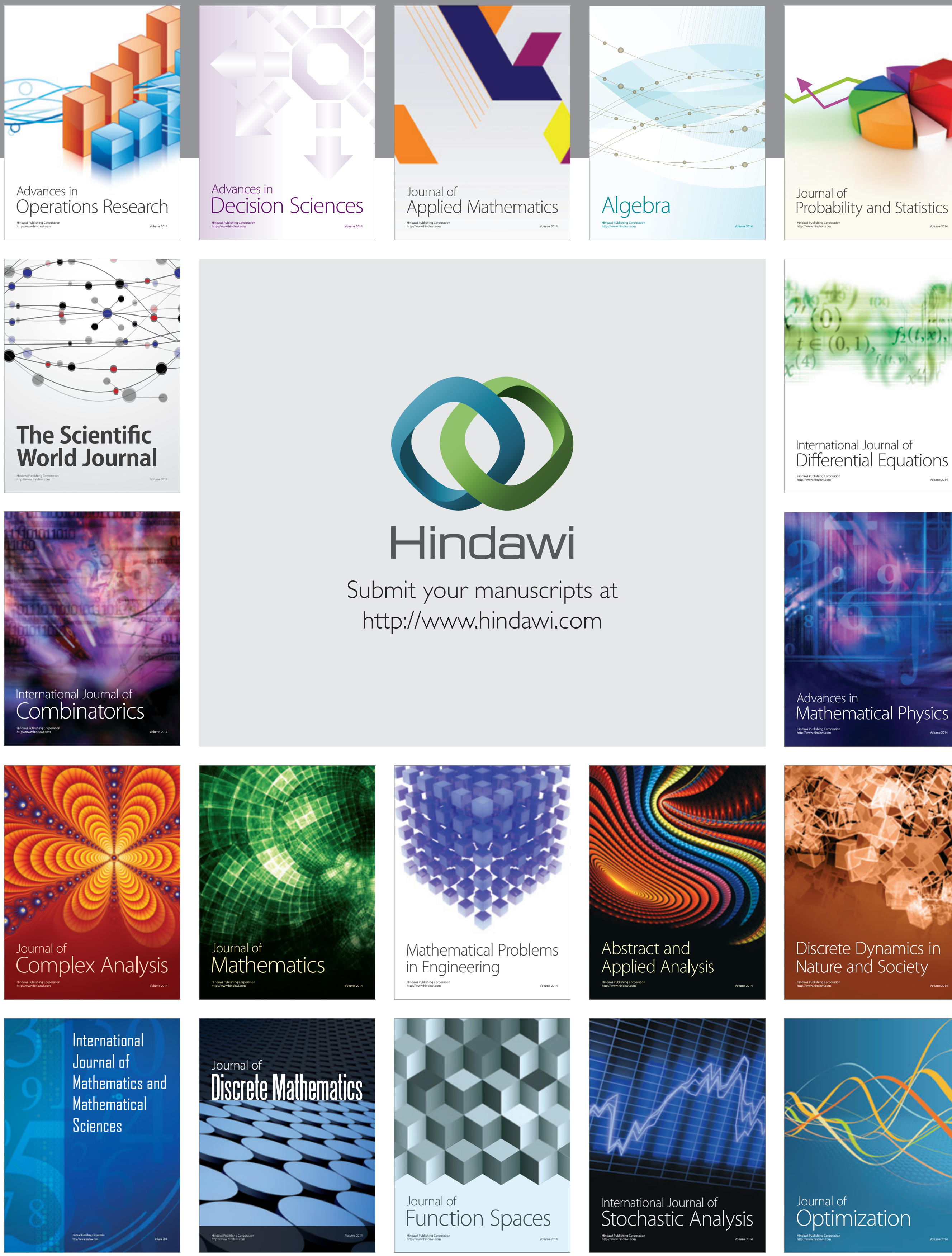
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