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Research Article

Long Time Decay Rate to a Bipolar Quantum Drift-Diffusion Model

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This paper is devoted to studying the long time behavior of solutions to a bipolar quantum hydrodynamic in one-dimensional space for general pressure functions. The model is usually applied to simulate some quantum effects in semiconductor devices. The decay rate for time variable is obtained by the entropy functional method and semidiscrete technique.

1. Introduction

By performing relaxation time limit in the quantum hydrodynamic equation, the semiconductor quantum drift-diffusion model can be obtained. Usually, it is applied to simulate the quantum effects, for example, resonant tunneling in semiconductor devices. Formally, the model also belongs to the field of the fourth-order parabolic equations (see [1]) including the thin film equation (see [2–4]) and the Cahn-Hilliard equation. In the paper, we are mainly concerned with the following bipolar quantum drift-diffusion model in one-dimensional space:

$$n_t + \frac{\varepsilon^2}{2}\left(n\left(\log n\right)_{xx}\right)_{xx} - \left(P_n\left(n\right)\right)_{xx} + \left(nV_x\right)_x = 0, \quad (1)$$

$$p_t + \frac{\varepsilon^2}{2}\left(p\left(\log p\right)_{xx}\right)_{xx} - \left(P_p\left(p\right)\right)_{xx} - \left(pV_x\right)_x = 0, \quad (2)$$

$$\lambda^2 V_{xx} = n - p - C\left(x\right), \quad (x,t) \in Q_T = \Omega \times (0,T) \quad (3)$$

with the initial-boundary conditions as follows:

$$\begin{aligned} &n = p = 1, \\ &n_x = p_x = 0, \\ &V = V_D, \qquad \text{on } \partial\Omega = \{0,1\}, \\ &n(\cdot,0) = n_0, \\ &p(\cdot,0) = p_0, \qquad \text{in } \Omega, \end{aligned} \quad (4)$$

where $\Omega = (0,1)$, V_D is a constant, n is the electron density, p is the positively charged ion (or hole) density, and V is the electron static potential. P_n and P_p are the pressure functions and the function $C(x)$ is the doping profile. The parameter ε is the scaled Plank constant and $\lambda > 0$ is the Debye length.

Dolbeault et al. [1] studied the existence and uniqueness of the fourth-order parabolic equation $n_t + (n(\log n)_{xx})_{xx} = 0$ with periodic boundary conditions. For the same equation, Jüngel and Toscani [5] used the entropy functional method and the semidiscrete technique to construct an iteration and obtained the exponential decay results. By employing the semidiscrete method, Jüngel and Violet obtained the existence of weak solution and gave the quasineutral limit in [6] to the bipolar quantum drift-diffusion model.

Generally, the bipolar model is more meaningful in physics and we will treat the case with a general pressure function. By applying the entropy method (see [7]) and iteration procedure which have already been used successfully in [5], we will get the long time exponential decay rate to the quantum drift-diffusion model (1)–(4). It is a key to deal with the coupling relationship in the Poisson equation (3). Moreover, since the maximum principle does not hold again for the high order partial differential equations, we need to overcome this difficulty for the purpose of getting uniform energy estimates.

As [6] has shown, by letting $\tau = T/n$, $n_k = n(x, kh)$, $p_k = p(x, kh)$, and $k = 1, 2, \dots, n$, we still borrow the semidiscrete system. Consider

$$\frac{1}{\tau}(n_k - n_{k-1}) + \frac{\varepsilon^2}{2}(n_k(\log n_k)_{xx})_{xx} - (P_n(n_k))_{xx} + (n_k V_{kx})_x = 0, \tag{5}$$

$$\frac{1}{\tau}(p_k - p_{k-1}) + \frac{\varepsilon^2}{2}(p_k(\log p_k)_{xx})_{xx} - (P_p(p_k))_{xx} - (p_k V_{kx})_x = 0, \tag{6}$$

$$\lambda^2 V_{kxx} = n_k - p_k - C(x), \quad \text{in } \Omega, \tag{7}$$

$$n_k = p_k = 1, \quad n_{kx} = p_{kx} = 0, \quad V_k = V_D, \quad \text{on } \partial\Omega. \tag{8}$$

For the problem (1)–(4) and the semidiscrete system (5)–(8), we list some results (Theorems 1–3) which had been proved in [6].

Theorem 1. *Assume $C(x) \in L^\infty(\Omega)$, $0 \le n_0, p_0 \in L^1(\Omega)$, and*

$$\int_\Omega (n_0 - \log n_0)\,dx + \int_\Omega (p_0 - \log p_0)\,dx + \int_\Omega (n_0(\log n_0 - 1) + 1)\,dx + \int_\Omega (p_0(\log p_0 - 1) + 1)\,dx < \infty. \tag{9}$$

Let $P_i \in C^1[0,\infty)$ $(i = n, p)$ be nondecreasing and assume that there exist two constants $0 < q < 7/2$ and $C_P > 0$ such that $P_i(x) \le C_P(1 + |x|^q)$ for $x \ge 0$, $i = n, p$. Then there exists a weak solution (n, p, V) to (1)–(4) such that

$$\begin{gathered} n, p \ge 0, \quad \text{in } Q_T, \\ n, p \in L^{7/2}(Q_T), \\ \log n, \log p \in L^2(0, T; H_0^2(\Omega)), \\ n_t, p_t \in L^1(0, T; H^{-3}(\Omega)), \\ V \in L^\infty(0, T; H^2(\Omega)). \end{gathered} \tag{10}$$

Theorem 2. *Under the assumptions of Theorem 1, there exists a weak solution $(n_k, p_k, V_k) \in H^2(\Omega) \times H^2(\Omega) \times H^2(\Omega)$ of (5)–(8) satisfying $(\log n_k, \log p_k, V_k - V_D) \in H_0^2(\Omega) \times H_0^2(\Omega) \times H_0^1(\Omega)$ and $n_k, p_k > 0$ in Ω.*

By defining the approximate solutions

$$\begin{aligned} n^{(\tau)}(x,t) &= n_k(x), \\ \widetilde{n^{(\tau)}}(x,t) &= n_{k-1}(x), \\ p^{(\tau)}(x,t) &= p_k(x), \\ \widetilde{p^{(\tau)}}(x,t) &= p_{k-1}(x), \\ V^{(\tau)}(x,t) &= V_k(x) \end{aligned} \tag{11}$$

for $x \in \Omega$, $t \in ((k-1)\tau, k\tau]$, [6] gave the following convergence results.

Theorem 3. *Under the assumptions of Theorem 1, there exists a subsequence of $(n^{(\tau)}, p^{(\tau)}, V^{(\tau)})$ (not relabeled) such that*

$$\begin{aligned} &\log n^{(\tau)} \rightharpoonup \log n, \quad \log p^{(\tau)} \rightharpoonup \log p, \quad \text{weakly in } L^2(0, T; H^2(\Omega)), \\ &n^{(\tau)} \longrightarrow n, \quad p^{(\tau)} \longrightarrow p, \quad \text{strongly in } L^2(Q_T), \\ &P_n(n^{(\tau)}) \longrightarrow P_n(n), \quad P_p(p^{(\tau)}) \longrightarrow P_p(p), \quad \text{strongly in } L^1(Q_T), \\ &\frac{1}{\tau}\left(n^{(\tau)} - \widetilde{n^{(\tau)}}\right) \rightharpoonup n_t, \quad \text{weakly in } L^s(0, T; H^{-3}(\Omega)), \\ &\frac{1}{\tau}\left(p^{(\tau)} - \widetilde{p^{(\tau)}}\right) \rightharpoonup p_t, \quad \text{weakly in } L^s(0, T; H^{-3}(\Omega)), \\ &V^{(\tau)} \rightharpoonup V, \quad \text{weakly in } L^2(0, T; H^1(\Omega)) \end{aligned} \tag{12}$$

and it holds

$$\int_0^T \langle \partial_t n, \phi \rangle\,dt = -\iint_{Q_T} n(\log n)_{xx}\phi_{xx}\,dx\,dt + \iint_{Q_T} P_n(n)\phi_{xx}\,dx\,dt + \iint_{Q_T} n V_x \phi_x\,dx\,dt,$$

$$\int_0^T \langle \partial_t p, \phi \rangle \, dt = -\iint_{Q_T} p (\log p)_{xx} \phi_{xx} dx\, dt + \iint_{Q_T} P_p(p) \phi_{xx} dx\, dt - \iint_{Q_T} p V_x \phi_x dx\, dt,$$
$$\lambda^2 \iint_{Q_T} V_x \phi_x dx\, dt = -\iint_{Q_T} (n-p) \phi \, dx\, dt \tag{13}$$

for all $\phi \in C_0^\infty(Q_T)$, where $s = \min\{7/2q, 14/11\} > 1$ and $\langle \cdot, \cdot \rangle$ is the duality product between $H^{-3}(\Omega)$ and $H_0^3(\Omega)$.

The main result of the paper is as follows.

Theorem 4. *Under the assumptions of Theorem 1, let $C(x) \equiv 0$ and let (n, p, V) be the weak solution to (1)–(4). Then*

$$\|n(t) - 1\|_{L^1(\Omega)} + \|p(t) - 1\|_{L^1(\Omega)} + \|V(t) - V_D\|_{H^1(\Omega)} \le C_1 (\eta_0 + \xi_0) e^{-C_2 t} \tag{14}$$

for $t \ge 0$, where the constants C_1 and $C_2 > 0$ only depend on Ω, n_0, and p_0.

Here, we need the condition $C(x) \equiv 0$ for the purpose of integration by parts and nonpositivity for some terms.

The paper is arranged as follows. We will prove some auxiliary lemmas at first in Section 2. The exponential decay rate will be established in Section 3.

2. Semidiscrete Solutions

Introduce some discrete entropies

$$\eta_k = \int_\Omega (n_k (\log n_k - 1) + 1) \, dx,$$
$$\xi_k = \int_\Omega (p_k (\log p_k - 1) + 1) \, dx,$$
$$q_k = \int_\Omega (n_k - \log n_k) \, dx,$$
$$\zeta_k = \int_\Omega (p_k - \log p_k) \, dx, \tag{15}$$

for $k \in N$. For the positive entropies η_k and ξ_k, we have the following iteration estimate.

Lemma 5. *Assume $\eta_{k-1}, \xi_{k-1} < \infty$, $k \ge 1$. Then*

$$\frac{1}{\tau}(\eta_k - \eta_{k-1}) + \frac{1}{\tau}(\xi_k - \xi_{k-1}) + \frac{\varepsilon^2}{2} \int_\Omega n_k (\log n_k)_{xx}^2 dx + \frac{\varepsilon^2}{2} \int_\Omega p_k (\log p_k)_{xx}^2 dx \le 0. \tag{16}$$

Proof. Multiplying (5) and (6) by $\log n_k$ and $\log p_k$, respectively, we have

$$\frac{1}{\tau} \int_\Omega (n_k - n_{k-1}) \log n_k dx + \frac{\varepsilon^2}{2} \int_\Omega n_k (\log n_k)_{xx}^2 dx = -\int_\Omega ((P_n(n_k))_x - n_k V_{kx}) (\log n_k)_x \, dx,$$
$$\frac{1}{\tau} \int_\Omega (p_k - p_{k-1}) \log p_k dx + \frac{\varepsilon^2}{2} \int_\Omega p_k (\log p_k)_{xx}^2 dx = -\int_\Omega \left((P_p(p_k))_x + p_k V_{kx}\right) (\log p_k)_x \, dx. \tag{17}$$

A direct calculation yields

$$-\int_\Omega (P_n(n_k))_x (\log n_k)_x \, dx = -\int_\Omega P_n'(n_k) \frac{n_{kx}^2}{n_k} dx \le 0 \tag{18}$$

and similarly $-\int_\Omega (P_p(p_k))_x (\log p_k)_x dx \le 0$. On the other hand, a simple calculation gives

$$\int_\Omega n_k V_{kx} (\log n_k)_x \, dx = \int_\Omega V_{kx} n_{kx} dx, \tag{19}$$

and similarly $-\int_\Omega p_k V_{kx} (\log p_k)_x dx = -\int_\Omega V_{kx} p_{kx} dx$. Letting $n_k - p_k$ be a test function in the Poisson equation (7), we get

$$\lambda^2 \int_\Omega V_{kx} (n_k - p_k)_x \, dx = -\int_\Omega (n_k - p_k)^2 \, dx \le 0. \tag{20}$$

Combining (18)–(20) with (17), we have

$$\frac{1}{\tau} \int_\Omega (n_k - n_{k-1}) \log n_k dx + \frac{\varepsilon^2}{2} \int_\Omega n_k (\log n_k)_{xx}^2 dx + \frac{1}{\tau} \int_\Omega (p_k - p_{k-1}) \log p_k dx + \frac{\varepsilon^2}{2} \int_\Omega p_k (\log p_k)_{xx}^2 dx \le 0. \tag{21}$$

Furthermore, the inequality $-\log x + x - 1 \ge 0$ for all $x > 0$ implies

$$\frac{1}{\tau} \int_\Omega (n_k - n_{k-1}) \log n_k dx = \frac{1}{\tau}(\eta_k - \eta_{k-1}) + \frac{1}{\tau} \int_\Omega n_{k-1} \left(-\log \frac{n_k}{n_{k-1}} + \frac{n_k}{n_{k-1}} - 1 \right) dx \ge \frac{1}{\tau}(\eta_k - \eta_{k-1}) \tag{22}$$

and similarly $(1/\tau) \int_\Omega (p_k - p_{k-1}) \log p_k dx \ge (1/\tau)(\xi_k - \xi_{k-1})$. Now we have completed the proof. □

Lemma 6. *Assume $q_{k-1}, \zeta_{k-1} < \infty$, $k \geq 1$. Then*

$$\frac{1}{\tau}(q_k - q_{k-1}) + \frac{\varepsilon^2}{2}\int_\Omega (\log n_k)_{xx}^2\,dx + \frac{1}{\tau}(\zeta_k - \zeta_{k-1}) + \frac{\varepsilon^2}{2}\int_\Omega (\log p_k)_{xx}^2\,dx \leq 0. \tag{23}$$

Proof. Multiply (5) and (6) by $1 - 1/n_k$ and $1 - 1/p_k$, respectively, to get

$$\begin{aligned}\frac{1}{\tau}\int_\Omega \left(n_k - n_{k-1} - 1 + \frac{n_{k-1}}{n_k}\right)dx + \frac{\varepsilon^2}{2}\int_\Omega (\log n_k)_{xx}^2\,dx &= \frac{\varepsilon^2}{2}\int_\Omega (\log n_k)_{xx}(\log n_k)_x^2\,dx - \int_\Omega P_n'(n_k)\frac{n_{kx}^2}{n_k^2}dx + \int_\Omega V_{kx}(\log n_k)_x\,dx \\ &= \frac{\varepsilon^2}{2}\int_\Omega \left((\log n_k)_x^3\right)_x^2 dx - \int_\Omega P_n'(n_k)\frac{n_{kx}^2}{n_k^2}dx + \int_\Omega V_{kx}(\log n_k)_x\,dx \leq \int_\Omega V_{kx}(\log n_k)_x\,dx,\end{aligned} \tag{24}$$

$$\frac{1}{\tau}\int_\Omega \left(p_k - p_{k-1} - 1 + \frac{p_{k-1}}{p_k}\right)dx + \frac{\varepsilon^2}{2}\int_\Omega (\log p_k)_{xx}^2\,dx \leq -\int_\Omega V_{kx}(\log p_k)_x\,dx. \tag{25}$$

Using $\log n_k - \log p_k$ as a test function in (7) and applying the inequality $(x_1 - x_2)(\log x_1 - \log x_2) \geq 0$ for all $x_1, x_2 > 0$, we have

$$\lambda^2\int_\Omega V_{kx}(\log n_k - \log p_k)_x\,dx = -\int_\Omega (n_k - p_k)(\log n_k - \log p_k)\,dx \leq 0. \tag{26}$$

The inequality $x - 1 \geq \log x$ for $x > 0$ yields

$$\begin{aligned}\int_\Omega \left(n_k - n_{k-1} - 1 + \frac{n_{k-1}}{n_k}\right)dx &= q_k - q_{k-1} + \int_\Omega \left(-\log\frac{n_{k-1}}{n_k} + \frac{n_{k-1}}{n_k} - 1\right)dx \\ &\geq q_k - q_{k-1}\end{aligned} \tag{27}$$

and similarly $\int_\Omega (p_k - p_{k-1} - 1 + p_{k-1}/p_k)dx \geq \zeta_k - \zeta_{k-1}$. By (24)–(27), we obtain

$$\begin{aligned}&\frac{1}{\tau}(q_k - q_{k-1}) + \frac{\varepsilon^2}{2}\int_\Omega (\log n_k)_{xx}^2\,dx + \frac{1}{\tau}(\zeta_k - \zeta_{k-1}) + \frac{\varepsilon^2}{2}\int_\Omega (\log p_k)_{xx}^2\,dx \\ &\quad\leq \int_\Omega V_{kx}(\log n_k - \log p_k)_x\,dx \leq 0.\end{aligned} \tag{28}$$

Hence, (23) has been proved. □

Lemma 7. *Assume $q_0 + \zeta_0 < \infty$. Then*

$$0 < e^{-(q_0+\zeta_0)} \leq \int_\Omega (n_k + p_k)\,dx \leq 2(q_0 + \zeta_0) < \infty. \tag{29}$$

Proof. The inequality $x - \log x \geq x/2$ for $x > 0$ gives

$$\int_\Omega (n_k + p_k)\,dx \leq 2(q_k + \zeta_k) \leq 2(q_0 + \zeta_0) \tag{30}$$

and Jensen's inequality yields

$$\begin{aligned}-\log\int_\Omega (n_k + p_k)\,dx &\leq -\int_\Omega \log(n_k + p_k)\,dx + \int_\Omega (n_k + p_k)\,dx \\ &\leq \frac{1}{2}\int_\Omega (n_k + p_k)\,dx \leq q_0 + \zeta_0 \\ &< \infty.\end{aligned} \tag{31}$$

The assertion finishes the proof of the lemma. □

3. Exponential Decay

In order to prove Theorem 4, we list some known results (see [5]) at first.

Lemma 8. *Assume the function $u \in H^2(\Omega)$, $u > 0$, in Ω and $u = 1$, $u_x = 0$, on $\partial\Omega$. Then*

$$\int_\Omega u(\log u)_{xx}^2\,dx \geq 8\left\|\sqrt{u} - 1\right\|_{L^\infty(\Omega)}^2. \tag{32}$$

Lemma 9. *Assume the function $u \in L^\infty(\Omega)$, $u > 0$, in Ω. Then*

$$\int_\Omega (u(\log u - 1) + 1)\,dx \leq \left(\int_\Omega u\,dx + 2\right)\left\|\sqrt{u} - 1\right\|_{L^\infty(\Omega)}^2. \tag{33}$$

Lemma 10 (Criszar-Kullback-type inequality). *Assume the function $0 < u \in L^1(\Omega)$ and $q = \int_\Omega (u - \log u)dx < \infty$. Then*

$$\int_\Omega (u(\log u - 1) + 1)\,dx \geq \frac{1}{(1 + \sqrt{2q})^2}\left(\int_\Omega |u - 1|\,dx\right)^2. \tag{34}$$

Proof of Theorem 4. According to Lemmas 5, 8, 9, and 7, we get

$$\begin{aligned}(\eta_k-\eta_{k-1})+(\xi_k-\xi_{k-1}) &\le -\frac{\tau}{2}\varepsilon^2\left(\int_\Omega n_k(\log n_k)_{xx}^2\,dx + \frac{\varepsilon^2}{2}\int_\Omega p_k(\log p_k)_{xx}^2\,dx\right)\\ &\le -4\tau\varepsilon^2\left(\|\sqrt{n_k}-1\|^2_{L^\infty(\Omega)}+\|\sqrt{p_k}-1\|^2_{L^\infty(\Omega)}\right)\\ &\le -4\tau\varepsilon^2\left(\frac{\eta_k}{\int_\Omega n_k dx+2}+\frac{\xi_k}{\int_\Omega p_k dx+2}\right)\\ &\le \frac{-2\tau\varepsilon^2}{q_0+\zeta_0+1}(\eta_k+\xi_k)\end{aligned} \tag{35}$$

and then, by iterating the above inequality, we deduce that

$$\begin{aligned}\eta_k+\xi_k &\le \left(1+\frac{2\tau\varepsilon^2}{q_0+\zeta_0+1}\right)^{-1}(\eta_{k-1}+\xi_{k-1})\\ &\le \left(1+\frac{2\tau\varepsilon^2}{q_0+\zeta_0+1}\right)^{-k}(\eta_0+\xi_0).\end{aligned} \tag{36}$$

Moreover, for $(k-1)\tau < t \le k\tau$, we have

$$\eta_k+\xi_k \le \left(1+\frac{2\tau\varepsilon^2}{q_0+\zeta_0+1}\right)^{-t/\tau}(\eta_0+\xi_0). \tag{37}$$

Introduce the following functions:

$$\begin{aligned}\eta^{(\tau)}(t) &= \int_\Omega\left(n^{(\tau)}(t)\left(\log n^{(\tau)}(t)-1\right)+1\right)dx,\\ \xi^{(\tau)}(t) &= \int_\Omega\left(p^{(\tau)}(t)\left(\log p^{(\tau)}(t)-1\right)+1\right)dx.\end{aligned} \tag{38}$$

Equation (37) implies

$$\eta^{(\tau)}(t)+\xi^{(\tau)}(t) \le (\eta_0+\xi_0)\left(1+\frac{2\tau\varepsilon^2}{q_0+\zeta_0+1}\right)^{-t/\tau}. \tag{39}$$

By applying Theorem 3, we conclude that there exists a subsequence of $(n^{(\tau)}, p^{(\tau)})$ such that $n^{(\tau)}(t) \to n$ and $p^{(\tau)}(t) \to p$ a.e. in Ω. Furthermore, we have $n^{(\tau)}\log n^{(\tau)} \to n\log n$ and $p^{(\tau)}\log p^{(\tau)} \to p\log p$ a.e. in Ω. On the other hand, $\eta^{(\tau)} + \xi^{(\tau)}$ is bounded uniformly in τ from (39) and then Lebesgue's convergence theorem yields $\eta^{(\tau)} + \xi^{(\tau)} \to \eta + \xi$ for $t \in (0, T)$. Therefore, we have

$$\begin{aligned}\eta(t)+\xi(t) &\le (\eta_0+\xi_0)\lim_{\tau\to 0}\left(1+\frac{2\tau\varepsilon^2}{q_0+\zeta_0+1}\right)^{-t/\tau}\\ &= (\eta_0+\xi_0)e^{-(2/(q_0+\zeta_0+1))t}.\end{aligned} \tag{40}$$

Applying Lemma 10, we have

$$\|n(t)-1\|_{L^1(\Omega)}+\|p(t)-1\|_{L^1(\Omega)} \le C_1(\eta_0+\xi_0)e^{-C_2 t}, \tag{41}$$

where C_1 and C_2 are both positive constants. Finally, multiply (7) by $V_k - V_D$ to get

$$\begin{aligned}\lambda^2\int_\Omega\left|(V_k-V_D)_x\right|^2dx &= -\int_\Omega(n_k-p_k)(V_k-V_D)\,dx\\ &\le \int_\Omega|n_k-p_k|\,dx\,\|V_k-V_D\|_{L^\infty(\Omega)}\\ &\le C\|V_k-V_D\|_{H^1(\Omega)}\int_\Omega|n_k-p_k|\,dx\end{aligned} \tag{42}$$

and then

$$\lambda^2\left\|V^{(\tau)}-V_D\right\|_{H^1(\Omega)} \le C\int_\Omega\left|n^{(\tau)}-p^{(\tau)}\right|dx. \tag{43}$$

By letting $\tau \to 0$, we can get the result of the theorem. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] J. Dolbeault, I. Gentil, and A. Jüngel, "A logarithmic fourth-order parabolic equation and related logarithmic Sobolev inequalities," *Communications in Mathematical Sciences*, vol. 4, no. 2, pp. 275–290, 2006.

[2] F. Bernis and A. Friedman, "Higher order nonlinear degenerate parabolic equations," *Journal of Differential Equations*, vol. 83, no. 1, pp. 179–206, 1990.

[3] L. Giacomelli, "A fourth-order degenerate parabolic equation describing thin viscous flows over an inclined plane," *Applied Mathematics Letters*, vol. 12, no. 8, pp. 107–111, 1999.

[4] T. G. Myers, "Thin films with high surface tension," *SIAM Review*, vol. 40, no. 3, pp. 441–462, 1998.

[5] A. Jüngel and G. Toscani, "Exponential time decay of solutions to a nonlinear fourth-order parabolic equation," *Zeitschrift für angewandte Mathematik und Physik ZAMP*, vol. 54, no. 3, pp. 377–386, 2003.

[6] A. Jüngel and I. Violet, "The quasineutral limit in the quantum drift-diffusion equations," *Asymptotic Analysis*, vol. 53, no. 3, pp. 139–157, 2007.

[7] B. Liang and S. Zheng, "Exponential decay to a quantum hydrodynamic model for semiconductors," *Nonlinear Analysis: Real World Applications*, vol. 9, no. 2, pp. 326–337, 2008.