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Research Article

Caristi's Fixed Point Theorem and Subrahmanyam's Fixed Point Theorem in ν-Generalized Metric Spaces

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We discuss the completeness of ν-generalized metric spaces in the sense of Branciari. We also prove generalizations of Subrahmanyam's and Caristi's fixed point theorem.

1. Introduction

In 2000, Branciari introduced the following very interesting concept.

Definition 1 (Branciari [1]). Let X be a set, let d be a function from $X \times X$ into $[0, \infty)$, and let $\nu \in \mathbb{N}$. Then, (X, d) is said to be a *ν-generalized metric space* if the following hold.

(N1) $d(x, y) = 0$ iff $x = y$ for any $x, y \in X$.

(N2) $d(x, y) = d(y, x)$ for any $x, y \in X$.

(N3) $d(x, y) \leq d(x, u_1) + d(u_1, u_2) + \cdots + d(u_{\nu-1}, u_\nu) + d(u_\nu, y)$ for any $x, u_1, u_2, \ldots, u_\nu, y \in X$ such that $x, u_1, u_2, \ldots, u_\nu, y$ are all different.

Remark 2. Every metric space (X, d) is a 1-generalized metric space.

Branciari in [1] also proved a generalization of the Banach contraction principle; however, the proof is not correct; see [2–4]. When proving theorems on ν-generalized metric spaces X, we may have to be careful because X does not necessarily have the compatible topology; see [4]. See also [5, 6].

In this paper, we first discuss the completeness of ν-generalized metric spaces. We next prove generalizations of Subrahmanyam's and Caristi's fixed point theorem.

2. Preliminaries

Throughout this paper we denote by $\mathbb{N}$ the set of all positive integers and by $\mathbb{R}$ the set of all real numbers.

In this section, we give some preliminaries.

As we mentioned in Section 1, ν-generalized metric spaces do not necessarily have the compatible topology. So we have to define something connected with convergence.

Definition 3. Let (X, d) be a ν-generalized metric space.

(i) A sequence $\{x_n\}$ in X is said to *converge* to x iff $\lim_n d(x, x_n) = 0$.

(ii) A sequence $\{x_n\}$ in X is said to *converge only* to x iff $\lim_n d(x, x_n) = 0$ holds and $\lim_n d(y, x_n) = 0$ does not hold for $y \in X \setminus \{x\}$.

(iii) A mapping T on X is said to be *sequentially continuous* iff $\{Tx_n\}$ converges to Tx whenever $\{x_n\}$ converges to x.

(iv) A function f from X into $(-\infty, \infty]$ is said to be *sequentially lower semicontinuous* iff $f(x) \leq \liminf_n f(x_n)$ whenever $\{x_n\}$ converges to x.

We introduce new concepts.

Definition 4. Let (X, d) be a ν-generalized metric space and let $\kappa \in \mathbb{N}$.

(i) A sequence $\{x_n\}$ in X is said to be *κ-Cauchy* iff

$$\lim_{n\to\infty} \sup \left\{ d\left(x_n, x_{n+1+j\kappa}\right) : j = 0, 1, 2, \ldots \right\} = 0 \tag{1}$$

holds.

(ii) X is *κ-complete* iff every κ-Cauchy sequence converges.

Remark 5. We sometimes write "Cauchy" instead of "1-Cauchy" and "complete" instead of "1-complete."

The following is obvious.

Proposition 6. *Let (X, d) be a ν-generalized metric space and let $\kappa, \lambda \in \mathbb{N}$ such that λ is divisible by κ. Then, the following hold.*

(i) *Every κ-Cauchy sequence is λ-Cauchy.*

(ii) *If X is λ-complete, then X is κ-complete.*

The following are partially converse to Proposition 6(i).

Proposition 7. *Let (X, d) be a ν-generalized metric space where ν is odd. Let $\{x_n\}$ be a ν-Cauchy sequence such that x_n are all different. Then, $\{x_n\}$ is Cauchy.*

Proof. In the case where $\nu = 1$, the conclusion clearly holds. So we assume $\nu \geq 3$. Fix $\varepsilon > 0$. Then, there exists $\ell \in \mathbb{N}$ such that

$$d\left(x_n, x_{n+1+j\nu}\right) < \varepsilon \tag{2}$$

for any $j \in \mathbb{N} \cup \{0\}$ and $n \in \mathbb{N}$ with $n \geq \ell$. Fix $j \in \mathbb{N} \cup \{0\}$ and $n \in \mathbb{N}$ with $n \geq \ell$. We first show

$$d\left(x_n, x_{n+1+j\nu+2k}\right) < (k\nu + 1)\,\varepsilon \tag{3}$$

for $k = 0, 1, \ldots, (\nu-1)/2$. It is obvious that (3) holds when $k = 0$. We assume that (3) holds for some k with $0 \leq k < (\nu-1)/2$. Then, we have by (N3)

$$\begin{aligned} d\left(x_n, x_{n+1+j\nu+2(k+1)}\right) &\leq d\left(x_n, x_{n+1+j\nu+2k}\right) \\ &\quad + d\left(x_{n+1+j\nu+2k}, x_{n+2+(j+1)\nu+2k}\right) \\ &\quad + \sum_{i=n+1+j\nu+2(k+1)}^{n+1+(j+1)\nu+2k} d\left(x_i, x_{i+1}\right) \\ &\leq (k\nu + 1)\,\varepsilon + \varepsilon + (\nu - 1)\,\varepsilon \\ &= ((k+1)\,\nu + 1)\,\varepsilon. \end{aligned} \tag{4}$$

Hence, (3) holds when $k := k + 1$. Therefore, (3) holds for every k, which implies

$$d\left(x_n, x_{n+1+j\nu+2k}\right) < \left(\frac{\nu^2}{2} - \frac{\nu}{2} + 1\right)\varepsilon \tag{5}$$

for any $j \in \mathbb{N} \cup \{0\}$, $k = 0, 1, \ldots, (\nu - 1)/2$, and $n \in \mathbb{N}$ with $n \geq \ell$. Using this, we have

$$\begin{aligned} d\left(x_n, x_{n+1+j\nu+2k+1}\right) &\leq d\left(x_n, x_{n+1+j\nu+2k}\right) \\ &\quad + d\left(x_{n+1+j\nu+2k}, x_{n+2+j\nu+2k+\nu-1}\right) \\ &\quad + \sum_{i=n+2+j\nu+2k}^{n+(j+1)\nu+2k} d\left(x_i, x_{i+1}\right) \\ &< 2\left(\frac{\nu^2}{2} - \frac{\nu}{2} + 1\right)\varepsilon + (\nu - 1)\,\varepsilon \\ &= \left(\nu^2 + 1\right)\varepsilon, \end{aligned} \tag{6}$$

for any $j \in \mathbb{N} \cup \{0\}$, $k = 0, 1, \ldots, (\nu - 3)/2$, and $n \in \mathbb{N}$ with $n \geq \ell$. So $\{x_n\}$ is Cauchy. □

Proposition 8. *Let (X, d) be a ν-generalized metric space where ν is even. Let $\{x_n\}$ be a ν-Cauchy sequence such that x_n are all different. Then, $\{x_n\}$ is 2-Cauchy.*

Proof. Fix $\varepsilon > 0$. Then, there exists $\ell \in \mathbb{N}$ such that

$$d\left(x_n, x_{n+1+j\nu}\right) < \varepsilon \tag{7}$$

for any $j \in \mathbb{N} \cup \{0\}$ and $n \in \mathbb{N}$ with $n \geq \ell$. Fix $j \in \mathbb{N} \cup \{0\}$ and $n \in \mathbb{N}$ with $n \geq \ell$. Then, as in the proof of Proposition 7, by induction, we can show

$$d\left(x_n, x_{n+1+j\nu+2k}\right) < (k\nu + 1)\,\varepsilon \tag{8}$$

for $k = 0, 1, \ldots, \nu/2 - 1$. Therefore, we obtain

$$d\left(x_n, x_{n+1+2j}\right) < \left(\frac{\nu^2}{2} - \nu + 1\right)\varepsilon \tag{9}$$

for any $j \in \mathbb{N} \cup \{0\}$. So $\{x_n\}$ is 2-Cauchy. □

Lemma 9. *Let (X, d) be a ν-generalized metric space and let $\{x_n\}$ be a sequence in X such that x_n are all different and $\sum_{n=1}^{\infty} d(x_n, x_{n+1}) < \infty$. Then, $\{x_n\}$ is ν-Cauchy.*

Remark 10. Example 1 in [7] tells that there exists some sequence $\{x_n\}$ in a 2-generalized metric space such that $\sum_{n=1}^{\infty} d(x_n, x_{n+1}) < \infty$ and $\{x_n\}$ is not Cauchy.

Proof. Fix $\varepsilon > 0$. Then, there exists $\ell \in \mathbb{N}$ such that $\sum_{i=\ell}^{\infty} d(x_i, x_{i+1}) < \varepsilon$. Fix $n \in \mathbb{N}$ with $n \geq \ell$. We will show

$$d\left(x_n, x_{n+1+j\nu}\right) \leq \sum_{i=n}^{n+j\nu} d\left(x_i, x_{i+1}\right) \tag{10}$$

by induction. It is obvious that (10) holds for $j = 0$. We assume that (10) holds for some $j \in \mathbb{N} \cup \{0\}$. Then, by (N3), we have

$$\begin{aligned} d\left(x_n, x_{n+1+(j+1)\nu}\right) &\le d\left(x_n, x_{n+1+j\nu}\right) \\ &\quad + \sum_{i=n+1+j\nu}^{n+(j+1)\nu} d\left(x_i, x_{i+1}\right) \\ &\le \sum_{i=n}^{n+(j+1)\nu} d\left(x_i, x_{i+1}\right). \end{aligned} \tag{11}$$

Therefore, (10) holds for $j := j + 1$. By induction, (10) holds for any $j \in \mathbb{N} \cup \{0\}$. Hence,

$$\begin{aligned} d\left(x_n, x_{n+1+j\nu}\right) &\le \sum_{i=n}^{n+j\nu} d\left(x_i, x_{i+1}\right) \\ &< \sum_{i=\ell}^{\infty} d\left(x_i, x_{i+1}\right) < \varepsilon \end{aligned} \tag{12}$$

holds. Therefore, $\{x_n\}$ is ν-Cauchy. □

Lemma 11. *Let (X, d) be a ν-generalized metric space and let $\{x_n\}$ be a sequence in X such that x_n are all different, $\lim_n d(x_n, x_{n+1}) = 0$, and $\{x_n\}$ converges to some $z \in X$. Then, $\{x_n\}$ converges only to $z \in X$.*

Proof. Arguing by contradiction, we assume that $\{x_n\}$ converges to w which differs from z. Since x_n are all different, $x_n \ne z$ and $x_n \ne w$ for sufficiently large $n \in \mathbb{N}$. By (N3), we have

$$\begin{aligned} &d(w, z) \\ &\quad \le \lim_{n\to\infty}\left(d(w, x_n) + \sum_{i=n}^{n+\nu-2} d\left(x_i, x_{i+1}\right) + d\left(x_{n+\nu-1}, z\right)\right) \\ &\quad = 0, \end{aligned} \tag{13}$$

where we define $\sum_{i=n}^{n-1} d(x_i, x_{i+1}) = 0$. By (N1), we obtain $w = z$. This is a contradiction. □

Lemma 12. *Let (X, d) be a ν-generalized metric space satisfying either of the following:*

(i) *ν is odd and X is complete;*

(ii) *ν is even and X is 2-complete.*

Let $\{x_n\}$ be a sequence in X such that x_n are all different and $\sum_{n=1}^{\infty} d(x_n, x_{n+1}) < \infty$. Then, there exists $z \in X$ such that $\{x_n\}$ converges only to z.

Proof. By Lemma 9, $\{x_n\}$ is ν-Cauchy. By Propositions 7 and 8, the following hold.

(i) If ν is odd, then $\{x_n\}$ is Cauchy.

(ii) If ν is even, then $\{x_n\}$ is 2-Cauchy.

From the assumption on the completeness of x, $\{x_n\}$ converges to some point $z \in X$. By Lemma 11, $\{x_n\}$ converges only to z. □

3. Fixed Point Theorems

The following is a generalization of Subrahmanyam's fixed point theorem [8]; see [9–11].

Theorem 13. *Let (X, d) be as in Lemma 12. Let T be a sequentially continuous mapping on X. Assume that there exists $r \in [0, 1)$ satisfying*

$$d\left(Tx, T^2x\right) \le r d(x, Tx) \tag{14}$$

for all $x \in X$. Then, for any $x \in X$, $\{T^n x\}$ converges only to a fixed point of T.

Proof. Define a sequence $\{u_j\}$ in X by $u_j = T^j x$ for $j \in \mathbb{N}$. We prove the conclusion, dividing the following three cases.

(i) There exists $n \in \mathbb{N}$ such that $u_{n+1} = u_n$.

(ii) $u_{j+1} \ne u_j$ for all $j \in \mathbb{N}$ and there exist $m, n \in \mathbb{N}$ such that $m + 2 \le n$ and $u_m = u_n$.

(iii) $u_1, u_2, \ldots$ are all different.

In the first case, u_n is a fixed point of T. By (N1), $\{u_j\}$ converges only to u_n. In the second case, since $u_{m+1} = u_{n+1}$, we have

$$\begin{aligned} d\left(u_m, u_{m+1}\right) &= d\left(u_n, u_{n+1}\right) \\ &\le r d\left(u_{n-1}, u_n\right) \le \cdots \le r^{n-m} d\left(u_m, u_{m+1}\right), \end{aligned} \tag{15}$$

which implies $d(u_m, u_{m+1}) = 0$. This is a contradiction. Thus, the second case cannot be possible. In the third case, we have

$$\begin{aligned} \sum_{j=1}^{\infty} d\left(u_j, u_{j+1}\right) &\le \sum_{j=1}^{\infty} r^{j-1} d\left(u_1, u_2\right) \\ &= \frac{d\left(u_1, u_2\right)}{1 - r} < \infty. \end{aligned} \tag{16}$$

So by Lemma 12, there exists $z \in X$ such that $\{u_j\}$ converges only to z. We note that $\{Tu_j\} = \{u_{j+1}\}$ also converges only to z. Since T is sequentially continuous, we obtain $Tz = z$. □

A function f from X into $(-\infty, +\infty]$ is *proper* iff $\{x \in X : f(x) \in \mathbb{R}\}$ is nonempty.

The following is a generalization of Caristi's fixed point theorem [12, 13].

Theorem 14. *Let (X, d) be as in Lemma 12. Let T be a mapping on X. Let f be a proper, sequentially lower semicontinuous function from X into $(-\infty, +\infty]$ bounded from below. Assume that*

$$f(Tx) + d(x, Tx) \le f(x) \tag{17}$$

for all $x \in X$. Then, T has a fixed point.

Remark 15. This theorem is connected with Theorem 2 in [14]. See Section 4.

Proof. In the case where $\nu = 1$, Theorem 14 becomes the original Caristi fixed point theorem. So we assume $\nu \geq 2$. Arguing by contradiction, we assume that T does not have a fixed point. Then, we note $f(Tx) < f(x)$ for every $x \in X$ with $f(x) < \infty$. By induction, we define a sequence $\{u_j\}$ in X satisfying the following:

$$\begin{gathered} u_{j+1} \neq u_j \quad \text{for any } j \in \mathbb{N}, \\ f\left(u_{j+1}\right) + d\left(u_j, u_{j+1}\right) \leq f\left(u_j\right) < \infty \\ \text{for any } j \in \mathbb{N}, \\ f\left(u_{j+1}\right) < \inf\left\{f(x) : f(x) + d\left(u_j, x\right) \leq f\left(u_j\right)\right\} \\ + \frac{1}{j} \quad \text{for any } j \in \mathbb{N}. \end{gathered} \tag{18}$$

Fix $u_1 \in X$ with $f(u_1) < \infty$. We assume that u_j is defined for some $j \in \mathbb{N}$. Then, we put

$$S_j = \left\{x \in X : f(x) + d\left(u_j, x\right) \leq f\left(u_j\right)\right\}. \tag{19}$$

Since $Tu_j \in S_j$, S_j is nonempty. Since $Tu_j \neq u_j$, we can define u_{j+1} satisfying (18). By induction, we have defined $\{u_j\}$. We note that u_j are all different because $f(u_{j+1}) < f(u_j)$ for any $j \in \mathbb{N}$. Since f is bounded from below, $\{f(u_j)\}$ converges. We have

$$\begin{aligned} \sum_{j=1}^{\infty} d\left(u_j, u_{j+1}\right) &\leq \sum_{j=1}^{\infty}\left(f\left(u_j\right) - f\left(u_{j+1}\right)\right) \\ &= f(u_1) - \lim_{j\to\infty} f\left(u_j\right) \\ &\leq f(u_1) - \inf\{f(x) : x \in X\} < \infty. \end{aligned} \tag{20}$$

So by Lemma 12, there exists $z \in X$ such that $\{u_j\}$ converges only to z. Since f is sequentially lower semicontinuous, we have

$$\begin{gathered} f(z) \leq \liminf_{j\to\infty} f\left(u_j\right) \quad \text{and hence } f(z) < f\left(u_j\right) \\ \text{for any } j \in \mathbb{N}. \end{gathered} \tag{21}$$

We note that $z \neq u_j$ for any $j \in \mathbb{N}$. Since $f(Tz) < f(z) < f(u_j)$, $Tz \neq u_j$ for any $j \in \mathbb{N}$. Fix $m \in \mathbb{N}$ with $1/m < f(z) - f(Tz)$. Then, for any $k \in \mathbb{N}$, we have

$$\begin{aligned} & f(Tz) + d\left(u_m, Tz\right) \\ &\leq f(Tz) + \sum_{j=m}^{m+\nu-2} d\left(u_j, u_{j+1}\right) + d\left(u_{m+\nu-1}, z\right) \\ &\qquad + d(z, Tz) \\ &\leq f(z) + \sum_{j=m}^{m+\nu-2} d\left(u_j, u_{j+1}\right) + d\left(u_{m+\nu-1}, z\right) \\ &\leq f(z) + \sum_{j=m}^{m+2\nu-2} d\left(u_j, u_{j+1}\right) + d\left(u_{m+2\nu-1}, z\right) \\ &\leq \cdots \leq f(z) + \sum_{j=m}^{m+k\nu-2} d\left(u_j, u_{j+1}\right) + d\left(u_{m+k\nu-1}, z\right) \\ &< f\left(u_{m+k\nu-1}\right) + \sum_{j=m}^{m+k\nu-2}\left(f\left(u_j\right) - f\left(u_{j+1}\right)\right) \\ &\qquad + d\left(u_{m+k\nu-1}, z\right) \\ &= f\left(u_m\right) + d\left(u_{m+k\nu-1}, z\right). \end{aligned} \tag{22}$$

As k tends to infinity, we obtain $f(Tz) + d(u_m, Tz) \leq f(u_m)$ and hence $Tz \in S_m$. Then, we have

$$\begin{aligned} f\left(u_{m+1}\right) &< \inf\{f(x) : x \in S_m\} + \frac{1}{m} \\ &< f(Tz) + f(z) - f(Tz) = f(z). \end{aligned} \tag{23}$$

This is a contradiction. □

4. Counterexample

Kirk and Shahzad in [7] gave a counterexample to Theorem 2 in [14]. In this section, we give another example.

Lemma 16 (see [4]). *Let (X, ρ) be a bounded metric space and let M be a real number satisfying*

$$\sup\{\rho(x, y) : x, y \in X\} \leq M. \tag{24}$$

Let A and B be two subsets of X with $A \cap B = \varnothing$. Define a function d from $X \times X$ into $[0, \infty)$ by

$$\begin{aligned} & d(x, x) = 0 \\ & d(x, y) = d(y, x) = \rho(x, y) \quad \text{if } x \in A,\ y \in B \\ & d(x, y) = M \text{ otherwise.} \end{aligned} \tag{25}$$

Then, (X, d) is a 2-generalized metric space.

Remark 17. We assume $X = A \cup B$ in Lemma 4 in [4]. However, we do not use this assumption in the proof.

Example 18. Let $X = \{0\} \cup \{1/n : n \in \mathbb{N}\}$ and define a metric ρ on X as usual. Define two subsets A and B of X by

$$A = \left\{\frac{1}{n} : n \in \mathbb{N},\ n \text{ is odd}\right\},$$
$$B = \left\{\frac{1}{n} : n \in \mathbb{N},\ n \text{ is even}\right\}. \tag{26}$$

Define a function d from $X \times X$ into $[0, 1]$ as in Lemma 16 with $M = 1$. Define a mapping T on X by

$$Tx = \begin{cases} 1 & \text{if } x = 0 \\ \dfrac{1}{(n+1)} & \text{if } x = \dfrac{1}{n} \text{ for some } n \in \mathbb{N}. \end{cases} \tag{27}$$

And define a function f from X into $[0, 2]$ by

$$f(x) = \begin{cases} 2 & \text{if } x = 0 \\ x & \text{otherwise.} \end{cases} \tag{28}$$

Then, (X, d) is a complete, 2-generalized metric space, f is sequentially continuous with respect to d, and (17) holds. However, T does not have a fixed point.

Proof. By Lemma 16, (X, d) is a 2-generalized metric space. We will show that X is complete. Let $\{x_j\}$ be a Cauchy sequence in X. We consider the following three cases.

(i) $\#\{j : x_j = 0\} = \infty$, where $\#\{j : x_j = 0\}$ is the number of the elements of $\{j : x_j = 0\}$.

(ii) $\#\{j : x_j = 1/n\} = \infty$ for some $n \in \mathbb{N}$.

(iii) $\#\{j : x_j = 0\} < \infty$ and $\#\{j : x_j = 1/n\} < \infty$ for any $n \in \mathbb{N}$.

In the first case, since $0 \notin A \cup B$, $x_j = 0$ holds for sufficiently large $j \in \mathbb{N}$. Thus, $\{x_j\}$ converges to 0. In the second case, since

$$\inf\left\{d\left(x, \frac{1}{n}\right) : x \in X \setminus \left\{\frac{1}{n}\right\}\right\} = \left|\frac{1}{n} - \frac{1}{(n+1)}\right| > 0, \tag{29}$$

$x_j = 1/n$ holds for sufficiently large $j \in \mathbb{N}$. Thus, $\{x_j\}$ converges to $1/n$. We consider the third case. Since $\{x_j\}$ is Cauchy, there exists $\ell_1 \in \mathbb{N}$ such that

$$\sup\left\{d\left(x_j, x_k\right) : k > j\right\} < 1 \tag{30}$$

for any $j \in \mathbb{N}$ with $j \geq \ell_1$. We note that $x_{\ell_1} \in A \cup B$ because of the definition of d. Without loss of generality, we may assume that $x_{\ell_1} \in A$. There exists $\ell_2 \in \mathbb{N}$ such that $\ell_2 > \ell_1$ and $x_j \neq x_{\ell_1}$ for $j \in \mathbb{N}$ with $j \geq \ell_2$. Then, $x_{\ell_2} \in B$ clearly holds. Also, there exists $\ell_3 \in \mathbb{N}$ such that $\ell_3 > \ell_2$ and $x_{\ell_3} \neq x_{\ell_2}$. Then, $x_{\ell_3} \in A$ and $x_{\ell_3} \neq x_{\ell_1}$ clearly hold. We obtain $d(x_{\ell_1}, x_{\ell_3}) < 1$. This is a contradiction. Therefore, the third case cannot be possible. We have shown that (X, d) is complete. We next show that f is sequentially continuous. Let a sequence $\{y_j\}$ in X converge to some y. Then, from the definition of d, there exists $\ell \in \mathbb{N}$ such that $y_j = y$ for $j \in \mathbb{N}$ with $j \geq \ell$. This fact implies that f is sequentially continuous. For any $x \in X$, $f(Tx) + d(x, Tx) = f(x)$ holds. So (17) is satisfied. However, it is clear that T does not have a fixed point. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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