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Research Article

On Coupled Common Fixed Point Theorems for Nonlinear Contractions with the Mixed Weakly Monotone Property in Partially Ordered S-Metric Spaces

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The main aim of this paper is to establish some coupled common fixed point theorems under a Geraghty-type contraction using mixed weakly monotone property in partially ordered S-metric space. Also, we give some sufficient conditions for the uniqueness of a coupled common fixed point. Some examples are provided to demonstrate the validity of our results.

1. Introduction and Preliminaries

One of the most important results in fixed point theory is the Banach Contraction Principle (*BCP* for short) proposed by Banach [1]. After that, there were many authors who have studied and proved the results for fixed point theory by generalizing the Banach Contraction Principle in several directions. One of the celebrated results was given by Geraghty [2].

For the sake of convenience, we recall Geraghty's theorem. Let $\mathfrak{F}$ be the family of all functions $\beta : [0,\infty) \mapsto [0,1)$ satisfying the condition:

$$\lim_{n\to\infty} \beta(t_n) = 1 \Longrightarrow \lim_{n\to\infty} t_n = 0. \tag{1}$$

Geraghty [2] proved the following unique fixed point theorem in complete metric spaces.

Theorem 1 (see [2]). *Let (X,d) be a complete metric space and let $T : X \mapsto X$ be an operator. Suppose that there exists $\beta \in \mathfrak{F}$ such that*

$$d(Tx, Ty) \le \beta(d(x,y))\, d(x,y), \tag{2}$$

for all $x, y \in X$. Then T has a unique fixed point $x^* \in X$.

Later, Amini-Harandi and Emami [3] generalized this result to the setting of partially ordered metric spaces as follows.

Theorem 2 (see [3]). *Let (X,d) be a complete partially ordered metric space and let $f : X \mapsto X$ be an increasing self-mapping such that there exists $x_0 \in X$ such that $x_0 \preceq fx_0$. Suppose that there exists $\beta \in \mathfrak{F}$ such that*

$$d(fx, fy) \le \beta(d(x,y))\, d(x,y), \tag{3}$$

for all $x, y \in X$ satisfying $x \preceq y$ or $x \succeq y$. Then, in each of the following two cases, the mapping f has at least one fixed point in X:

(1) *f is continuous or,*

(2) *for any nondecreasing sequence* $\{x_n\}$ *in* X, *if* $x_n \to x \in X$ *as* $n \to \infty$, *then* $x_n \preceq x$ *for all* $n \geq 1$.

If, moreover, for all $x, y \in X$, *there exists* $z \in X$ *comparable with* x *and* y, *then the fixed point of* f *is unique.*

For more generalizations of Theorems 1 and 2, see [4–7].

On the other hand, several authors have studied fixed point theory in generalized metric spaces. For details, we refer readers to [8–13]. In 2012, Sedghi et al. [14] have introduced the notion of an S-metric space and proved that this notion is a generalization of a metric space. Also, they have proved some properties of S-metric spaces and some fixed point theorems for a self-map on an S-metric space. An interesting work is that we can naturally transport certain results in metric spaces and known generalized metric spaces to S-metric spaces. After that, Sedghi and Dung [15] proved a general fixed point theorem in S-metric spaces which is a generalization of [14, Theorem 3.1] and obtained many analogues of fixed point theorems in metric spaces for S-metric spaces. In [16], Gordji et al. have introduced the concept of a mixed weakly monotone pair of maps and proved some coupled common fixed point theorems for contractive-type maps using the mixed weakly monotone property in partially ordered metric spaces. These results are of particular interest to state coupled common fixed point theorems for maps with mixed weakly monotone property in partially ordered S-metric spaces. In 2013, Dung [17] used the notion of a mixed weakly monotone pair of maps to state a coupled common fixed point theorem for maps on partially ordered S-metric spaces and generalized the main results of [16–18] into the structure of S-metric spaces.

In this paper, motivated by the developments discussed above, we state some coupled common fixed point theorems for a pair of mappings with the mixed weakly monotone property satisfying a generalized contraction by using the ideas of Geraghty [2] in partially ordered S-metric spaces. Also, we give some sufficient conditions for the uniqueness of a coupled common fixed point. Some examples are provided to illustrate our main theorems.

In the sequel, the letters $\mathbb{R}$, $\mathbb{R}^+$, and $\mathbb{N}$ will denote the set of all real numbers, the set of all nonnegative real numbers, and the set of all positive integers, respectively.

Let $(X, \preccurlyeq)$ be a partially ordered set. Then $X \times X$ is a partially ordered set with partial order $\preccurlyeq$ defined by

$$\begin{gathered} (x, y) \preccurlyeq (u, v) \Longleftrightarrow \\ x \preccurlyeq u, v \preccurlyeq y, \\ \forall x, y, u, v \in X. \end{gathered} \tag{4}$$

Definition 3 ([14, Definition 2.1]). Let X be a nonempty set. An S-metric on X is a function $S : X^3 \mapsto [0, \infty)$ that satisfies the following conditions for all $x, y, z, a \in X$:

(1) $S(x, y, z) = 0$ if and only if $x = y = z$.

(2) $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$.

The pair (X, S) is called an S-metric space.

The following is an intuitive geometric example for S-metric spaces.

Example 4 ([14, Example 2.4]). Let $X = \mathbb{R}^2$ and let d be an ordinary metric on X. Put

$$S(x, y, z) = d(x, y) + d(x, z) + d(y, z) \tag{5}$$

for all $x, y, z \in \mathbb{R}^2$; that is, S is the perimeter of the triangle given by x, y, z. Then S is an S-metric on X.

Lemma 5 ([17, Lemma 1.4]). *Let* (X, S) *be an S-metric space. Then*

$$\begin{aligned} S(x, x, z) &\leq 2S(x, x, y) + S(y, y, z), \\ S(x, x, z) &\leq 2S(x, x, y) + S(z, z, y), \end{aligned} \tag{6}$$

for all $x, y, z \in X$.

Lemma 6 ([14, Lemma 2.5]). *Let* (X, S) *be an S-metric space. Then* $S(x, x, y) = S(y, y, x)$, *for all* $x, y \in X$.

Lemma 7 (see [16]). *Let* (X, d) *be a metric space. Then* $X \times X$ *is a metric space with metric* D_d *given by*

$$D_d((x, y), (u, v)) = d(x, u) + d(y, v), \tag{7}$$

for all $x, y, u, v \in X$.

Lemma 8. *Let* (X, S) *be an S-metric space. Then* $X \times X$ *is an S-metric space with S-metric* D_s *given by*

$$D_s((x, y), (u, v), (w, s)) = S(x, u, w) + S(y, v, t), \tag{8}$$

for all $x, y, u, v, w, t \in X$.

Proof. For all $x, y, u, v, w, t \in X$, it is obvious that the first condition of S-metric for D_s holds true.

We only need to check the second condition of S-metric:

$$\begin{aligned} D_s&((x, y), (u, v), (w, t)) = S(x, u, w) + S(y, v, t) \\ &\leq S(x, x, a) + S(u, u, a) + S(w, w, a) + S(y, y, b) \\ &\quad + S(v, v, b) + S(t, t, b) \\ &= D_s((x, y), (x, y), (a, b)) \\ &\quad + D_s((u, v), (u, v), (a, b)) \\ &\quad + D_s((w, t), (w, t), (a, b)). \end{aligned} \tag{9}$$

By the above, D_s is an S-metric on $X \times X$. □

Definition 9 ([16, Definition 1.5]). Let $(X, \preceq)$ be a partially ordered set and let $f, g : X \times X \mapsto X$ be two maps. We say the pair (f, g) has the mixed weakly monotone property on X if for all $x, y \in X$, we have

$$\begin{aligned} & x \preceq f(x, y), \quad f(y, x) \preceq y \\ & \text{implies } f(x, y) \\ & \quad \preceq g(f(x, y), f(y, x)), g(f(y, x), f(x, y)) \\ & \quad \preceq f(y, x), \\ & x \preceq g(x, y), \quad g(y, x) \preceq y \\ & \text{implies } g(x, y) \\ & \quad \preceq f(g(x, y), g(y, x)), f(g(y, x), g(x, y)) \\ & \quad \preceq g(y, x). \end{aligned} \tag{10}$$

Example 10 ([16, Example 1.6]). Let $f, g : \mathbb{R} \times \mathbb{R} \mapsto \mathbb{R}$ be two functions given by

$$\begin{aligned} f(x, y) &= x - 2y, \\ g(x, y) &= x - y. \end{aligned} \tag{11}$$

Then the pair (f, g) has the mixed weakly monotone property.

Definition 11 ([16, Definition 1.1]). Let $(X, \preceq)$ be a partially ordered set and let $f : X \times X \mapsto X$ be a map. We say the pair f has the mixed monotone property on X if for all $x, y \in X$, we have

$$\begin{aligned} & x_1, x_2 \in X, \quad x_1 \preceq x_2, \\ & \text{implies } f(x_1, y) \preceq f(x_2, y), \\ & y_1, y_2 \in X, \quad y_1 \preceq y_2, \\ & \text{implies } f(x, y_1) \succeq f(x, y_2). \end{aligned} \tag{12}$$

Remark 12 ([17, Remark 1.20]). Let $(X, \preceq)$ be a partially ordered set; let $f : X \times X \mapsto X$ be a map with the mixed monotone property on X. Then, for all $n \in \mathbb{N}$, the pair (f^n, f^n) has the mixed weakly monotone property on X.

Definition 13. An element $(x, y) \in X \times X$ is called a

(1) coupled fixed point of a mapping $f : X \times X \mapsto X$ if $x = f(x, y)$ and $y = f(y, x)$;

(2) coupled common fixed point of two mappings $f, g : X \times X \mapsto X$ if $x = f(x, y) = g(x, y)$ and $y = f(y, x) = g(y, x)$.

2. Main Results

In this section, we establish some coupled common fixed point theorems by considering mappings on generalized metric spaces endowed with partial order. Before proceeding further, first, we define the following function which will be used in our results.

Let $\{x_n\}$ and $\{y_n\}$ be any two sequences of nonnegative real numbers. Define with Θ the set of all functions $\theta : [0, \infty)^2 \mapsto [0, 1)$ which, satisfying $\theta(x_n, y_n) \to 1$, implies $x_n, y_n \to 0$.

Some examples of such a function are as follows.

Example 14. Let $\theta : [0, \infty)^2 \mapsto [0, 1)$ be defined by

$$\theta(x, y) = \begin{cases} \dfrac{\sin(k_1 x + k_2 y)}{k_1 x + k_2 y}, & x > 0 \text{ or } y > 0, \ k_1, k_2 \in (0, 1); \\ l \in [0, 1), & x = y = 0. \end{cases} \tag{13}$$

Example 15. Let $\theta : [0, \infty)^2 \mapsto [0, 1)$ be defined by

$$\theta(x, y) = \begin{cases} \dfrac{\ln(1 + k_1 x + k_2 y)}{k_1 x + k_2 y}, & x > 0 \text{ or } y > 0, \ k_1, k_2 \in (0, 1); \\ l \in [0, 1), & x = y = 0. \end{cases} \tag{14}$$

Example 16. Let $\theta : [0, \infty)^2 \mapsto [0, 1)$ be defined by

$$\theta(x, y) = \begin{cases} \dfrac{\ln(1 + \max\{k_1 x, k_2 y\})}{\max\{k_1 x, k_2 y\}}, & x > 0 \text{ or } y > 0, \ k_1, k_2 \in (0, 1); \\ l \in [0, 1), & x = y = 0. \end{cases} \tag{15}$$

Example 17. Let $\theta : [0, \infty)^2 \mapsto [0, 1)$ be defined by

$$\theta(x, y) = \begin{cases} 1 - k(x + y), & x > 0 \text{ or } y > 0, \ k(x + y) \leq 1; \\ l \in [0, 1), & k(x + y) > 1. \end{cases} \tag{16}$$

Theorem 18. *Let (X, S) be a partially ordered S-metric space; let $f, g : X \times X \mapsto X$ be two maps such that*

(1) *X is complete;*

(2) *the pair (f, g) has the mixed weakly monotone property on X:*

$$\begin{aligned} & x_0 \preceq f(x_0, y_0), \\ & f(y_0, x_0) \preceq y_0 \\ & \quad \text{or } x_0 \preceq g(x_0, y_0), \\ & g(y_0, x_0) \preceq y_0 \\ & \qquad \text{for some } x_0, y_0 \in X; \end{aligned} \tag{17}$$

(3) *assume that there exists $\theta \in \Theta$ such that*

$$\begin{aligned} & S(f(x, y), f(x, y), g(u, v)) \\ & \quad + S(f(y, x), f(y, x), g(v, u)) \\ & \quad \leq \theta(S(x, x, u), S(y, y, v)) [S(x, x, u) + S(y, y, v)], \end{aligned} \tag{18}$$

for all $x, y, u, v \in X$ with $x \preceq u$, $y \succeq v$;

(4) *f or g is continuous.*

Then f and g have a coupled common fixed point in X.

Proof.

Step 1. We construct two Cauchy sequences in X.

Let $x_0, y_0 \in X$ be such that $x_0 \preceq f(x_0, y_0)$, $y_0 \succeq f(y_0, x_0)$.

Put $x_1 = f(x_0, y_0)$, $y_1 = f(y_0, x_0)$, $x_2 = g(x_1, y_1)$, $y_2 = g(y_1, x_1)$.

From the choice of x_0, y_0 and the fact that (f, g) has mixed weakly monotone property we have

$$\begin{aligned} x_1 &= f(x_0, y_0) \preceq g(f(x_0, y_0), f(y_0, x_0)) \\ &= g(x_1, y_1) = x_2 \Longrightarrow \\ x_1 &\preceq x_2, \\ x_2 &= g(x_1, y_1) \preceq f(g(x_1, y_1), g(y_1, x_1)) \\ &= f(x_2, y_2) = x_3 \Longrightarrow \\ x_2 &\preceq x_3. \end{aligned} \tag{19}$$

Thus,

$$\begin{aligned} y_1 &= f(y_0, x_0) \succeq g(f(y_0, x_0), f(x_0, y_0)) \\ &= g(y_1, x_1) = y_2 \Longrightarrow \\ y_1 &\succeq y_2, \\ y_2 &= g(y_1, x_1) \succeq f(g(y_1, x_1), g(x_1, y_1)) \\ &= f(y_2, x_2) = y_3 \Longrightarrow \\ y_2 &\succeq y_3. \end{aligned} \tag{20}$$

Continuing this way, we obtain

$$\begin{aligned} x_{2k+1} &= f(x_{2k}, y_{2k}), \\ y_{2k+1} &= f(y_{2k}, x_{2k}), \\ x_{2k+2} &= g(x_{2k+1}, y_{2k+1}), \\ y_{2k+1} &= g(y_{2k+1}, x_{2k+1}), \end{aligned} \tag{21}$$

for all $k \in \mathbb{N}$.

Therefore, the sequences $\{x_n\}$ and $\{y_n\}$ are monotone:

$$\begin{aligned} &x_0 \preceq x_1 \preceq \cdots \preceq x_n \preceq \cdots, \\ &y_0 \succeq y_1 \succeq \cdots \succeq y_n \succeq \cdots. \end{aligned} \tag{22}$$

Assume that there exists a nonnegative integer n such that

$$S(x_{n+1}, x_{n+1}, x_n) + S(y_{n+1}, y_{n+1}, y_n) = 0. \tag{23}$$

It follows that

$$S(x_{n+1}, x_{n+1}, x_n) = S(y_{n+1}, y_{n+1}, y_n) = 0. \tag{24}$$

From the definition of S-metric space, we have $x_{n+1} = x_n$, $y_{n+1} = y_n$. It follows from (21) that (x_n, y_n) is a coupled common fixed point of f and g.

Now, we suppose that for all nonnegative n

$$S(x_{n+1}, x_{n+1}, x_n) + S(y_{n+1}, y_{n+1}, y_n) \neq 0. \tag{25}$$

Using (18) and (21), for $n = 2k + 1$, we have

$$\begin{aligned} &S(x_{2k+1}, x_{2k+1}, x_{2k+2}) + S(y_{2k+1}, y_{2k+1}, y_{2k+2}) \\ &\quad = S(f(x_{2k}, y_{2k}), f(x_{2k}, y_{2k}), g(x_{2k+1}, y_{2k+1})) \\ &\quad + S(f(y_{2k}, x_{2k}), f(y_{2k}, x_{2k}), g(y_{2k+1}, x_{2k+1})) \\ &\quad \leq \theta(S(x_{2k}, x_{2k}, x_{2k+1}), S(y_{2k}, y_{2k}, y_{2k+1})) \\ &\quad \cdot (S(x_{2k}, x_{2k}, x_{2k+1}) + S(y_{2k}, y_{2k}, y_{2k+1})), \end{aligned} \tag{26}$$

which implies that

$$\begin{aligned} &S(x_{2k+1}, x_{2k+1}, x_{2k+2}) + S(y_{2k+1}, y_{2k+1}, y_{2k+2}) \\ &\quad < S(x_{2k}, x_{2k}, x_{2k+1}) + S(y_{2k}, y_{2k}, y_{2k+1}). \end{aligned} \tag{27}$$

For all $k \in \mathbb{N}$, write

$$\begin{aligned} \gamma_{2k+1} = {} & S(x_{2k+1}, x_{2k+1}, x_{2k+2}) \\ & + S(y_{2k+1}, y_{2k+1}, y_{2k+2}), \end{aligned} \tag{28}$$

and then the sequence $\{\gamma_{2k+1}\}$ is monotone decreasing. Therefore, there exists $\gamma \geq 0$ such that

$$\begin{aligned} \lim_{k \to \infty} \gamma_{2k+1} = {} & \lim_{k \to \infty} [S(x_{2k+1}, x_{2k+1}, x_{2k+2}) \\ & + S(y_{2k+1}, y_{2k+1}, y_{2k+2})] = \gamma. \end{aligned} \tag{29}$$

We claim that $\gamma = 0$. On the contrary, suppose that $\gamma > 0$, and we have from (26) that

$$\begin{aligned} &\frac{S(x_{2k+1}, x_{2k+1}, x_{2k+2}) + S(y_{2k+1}, y_{2k+1}, y_{2k+2})}{S(x_{2k}, x_{2k}, x_{2k+1}) + S(y_{2k}, y_{2k}, y_{2k+1})} \\ &\quad \leq \theta(S(x_{2k}, x_{2k}, x_{2k+1}), S(y_{2k}, y_{2k}, y_{2k+1})) < 1. \end{aligned} \tag{30}$$

Letting $k \to \infty$, we get

$$\theta(S(x_{2k}, x_{2k}, x_{2k+1}), S(y_{2k}, y_{2k}, y_{2k+1})) \longrightarrow 1. \tag{31}$$

Using the property of the function θ, we have

$$\begin{aligned} &S(x_{2k}, x_{2k}, x_{2k+1}), S(y_{2k}, y_{2k}, y_{2k+1}) \longrightarrow 0 \\ &\qquad \text{as } k \longrightarrow \infty. \end{aligned} \tag{32}$$

So, we have

$$\begin{aligned} &S(x_{2k}, x_{2k}, x_{2k+1}) + S(y_{2k}, y_{2k}, y_{2k+1}) \longrightarrow 0 \\ &\qquad \text{as } k \longrightarrow \infty, \end{aligned} \tag{33}$$

which contradicts the assumption $\gamma > 0$. Thus, $\gamma = 0$.

Analogously to $n = 2k + 2$, we also have

$$\lim_{k\to\infty}\left[S\left(x_{2k+2},x_{2k+2},x_{2k+3}\right)+S\left(y_{2k+2},y_{2k+2},y_{2k+3}\right)\right] = 0. \tag{34}$$

Thus, we have

$$\lim_{n\to\infty}\left[S\left(x_n,x_n,x_{n+1}\right)+S\left(y_n,y_n,y_{n+1}\right)\right]=0. \tag{35}$$

Now, we have to prove that $\{x_n\}$ and $\{y_n\}$ are two Cauchy sequences in the S-metric space (X, S).

For all $n, m \in \mathbb{N}$ with $n \le m$, by using Lemma 5, we have that

$$\begin{aligned}
&S\left(x_{2n+1},x_{2n+1},x_{2m+1}\right)+S\left(y_{2n+1},y_{2n+1},y_{2m+1}\right)\\
&\quad\le\left(2S\left(x_{2n+1},x_{2n+1},x_{2n+2}\right)\right.\\
&\quad\left.+2S\left(y_{2n+1},y_{2n+1},y_{2n+2}\right)\right)\\
&\quad+\left(S\left(x_{2n+2},x_{2n+2},x_{2m+1}\right)\right.\\
&\quad\left.+S\left(y_{2n+2},y_{2n+2},y_{2m+1}\right)\right)\\
&\quad\le\left(2S\left(x_{2n+1},x_{2n+1},x_{2n+2}\right)\right.\\
&\quad\left.+2S\left(y_{2n+1},y_{2n+1},y_{2n+2}\right)\right)\\
&\quad+\left(2S\left(x_{2n+2},x_{2n+2},x_{2n+3}\right)\right.\\
&\quad\left.+2S\left(y_{2n+2},y_{2n+2},y_{2n+3}\right)\right)+\cdots\\
&\quad+\left(2S\left(x_{2m-1},x_{2m-1},x_{2m}\right)\right.\\
&\quad\left.+2S\left(y_{2m-1},y_{2m-1},y_{2m}\right)\right)+\left(S\left(x_{2m},x_{2m},x_{2m+1}\right)\right.\\
&\quad\left.+S\left(y_{2m},y_{2m},y_{2m+1}\right)\right)=2\gamma_{2n+1}+2\gamma_{2n+2}+\cdots\\
&\quad+2\gamma_{2m-1}+\gamma_{2m}.
\end{aligned} \tag{36}$$

Taking the limit as $n, m \to \infty$ and using (35), we obtain

$$S\left(x_{2n+1},x_{2n+1},x_{2m+1}\right)+S\left(y_{2n+1},y_{2n+1},y_{2m+1}\right)\longrightarrow 0. \tag{37}$$

Therefore,

$$S\left(x_{2n+1},x_{2n+1},x_{2m+1}\right),S\left(y_{2n+1},y_{2n+1},y_{2m+1}\right)\longrightarrow 0. \tag{38}$$

By interchanging the roles of f and g and proceeding along the arguments discussed above, we also obtain that

$$\begin{aligned}
S\left(x_{2n},x_{2n},x_{2m+1}\right),S\left(y_{2n},y_{2n},y_{2m+1}\right)&\longrightarrow 0,\\
S\left(x_{2n},x_{2n},x_{2m}\right),S\left(y_{2n},y_{2n},y_{2m}\right)&\longrightarrow 0,\\
S\left(x_{2n+1},x_{2n+1},x_{2m}\right),S\left(y_{2n+1},y_{2n+1},y_{2m}\right)&\longrightarrow 0.
\end{aligned} \tag{39}$$

Hence, for all $n, m \in \mathbb{N}$ with $n \le m$, we get

$$\lim_{n,m\to\infty}\left[S\left(x_n,x_n,x_m\right)+S\left(y_n,y_n,y_m\right)\right]=0. \tag{40}$$

It implies that

$$\lim_{n,m\to\infty}S\left(x_n,x_n,x_m\right)=\lim_{n,m\to\infty}S\left(y_n,y_n,y_m\right)=0. \tag{41}$$

Therefore, $\{x_n\}$ and $\{y_n\}$ are two Cauchy sequences in the S-metric space (X, S). Since (X, S) is a complete S-metric space, hence $\{x_n\}$ and $\{y_n\}$ are S-convergent. Then there exist $x, y \in X$ such that $x_n \to x$ and $y_n \to y$, respectively.

Step 2. We prove that (x, y) is a coupled common fixed point of f and g.

We consider the following two cases.

Case 1 (f is continuous). We have

$$\begin{aligned}
x&=\lim_{n\to\infty}x_n=\lim_{n\to\infty}f\left(x_n,y_n\right)\\
&=f\left(\lim_{n\to\infty}x_n,\lim_{n\to\infty}y_n\right)=f\left(x,y\right),\\
y&=\lim_{n\to\infty}y_n=\lim_{n\to\infty}f\left(y_n,x_n\right)\\
&=f\left(\lim_{n\to\infty}y_n,\lim_{n\to\infty}x_n\right)=f\left(y,x\right).
\end{aligned} \tag{42}$$

Now using (18), we have

$$\begin{aligned}
&S\left(f\left(x,y\right),f\left(x,y\right),g\left(x,y\right)\right)\\
&\quad+S\left(f\left(y,x\right),f\left(y,x\right),g\left(y,x\right)\right)\\
&\quad\le\theta\left(S\left(x,x,x\right),S\left(y,y,y\right)\right)\left[S\left(x,x,x\right)+S\left(y,y,y\right)\right].
\end{aligned} \tag{43}$$

That is,

$$\begin{aligned}
&S\left(x,x,g\left(x,y\right)\right)+S\left(y,y,g\left(y,x\right)\right)\\
&\quad\le\theta\left(S\left(x,x,x\right),S\left(y,y,y\right)\right)\\
&\quad\cdot\left[S\left(x,x,x\right)+S\left(y,y,y\right)\right].
\end{aligned} \tag{44}$$

Since $S(x,x,x) = S(y,y,y) = 0$, we get $S(x,x,g(x,y)) = S(y,y,g(y,x)) = 0$; that is, $g(x,y) = x$, $g(y,x) = y$.

Therefore, (x, y) is a coupled common fixed point of f and g.

Case 2 (g is continuous). We also prove that (x, y) is a coupled common fixed point of f and g similarly as in Case 1. □

Theorem 19. *Let (X, S) be a partially ordered S-metric space; let $f, g : X \times X \mapsto X$ be two maps such that*

(1) *X is complete;*

(2) *the pair (f, g) has the mixed weakly monotone property on X:*

$$\begin{aligned}
x_0&\preceq f\left(x_0,y_0\right),\\
f\left(y_0,x_0\right)&\preceq y_0\\
\text{or } x_0&\preceq g\left(x_0,y_0\right),\\
g\left(y_0,x_0\right)&\preceq y_0\\
&\text{for some } x_0,y_0\in X;
\end{aligned} \tag{45}$$

(3) *assume that there exists* $\theta \in \Theta$ *such that*

$$S(f(x,y), f(x,y), g(u,v)) + S(f(y,x), f(y,x), g(v,u)) \leq \theta(S(x,x,u), S(y,y,v))[S(x,x,u) + S(y,y,v)], \tag{46}$$

for all $x, y, u, v \in X$ *with* $x \preceq u$, $y \succeq v$;

(4) X *has the following properties:*

(a) *If* $\{x_n\}$ *is an increasing sequence with* $x_n \to x$, *then* $x_n \preceq x$ *for all* $n \in \mathbb{N}$.

(b) *If* $\{x_n\}$ *is a decreasing sequence with* $x_n \to x$, *then* $x \preceq x_n$ *for all* $n \in \mathbb{N}$.

Then, f *and* g *have a coupled common fixed point in* X.

Proof. Proceeding along the same steps as in Theorem 18, we obtain a nondecreasing sequence $\{x_n\}$ converging to x and a nonincreasing sequence $\{y_n\}$ converging to y, for some $x, y \in X$. If $x_n = x$ and $y_n = y$ for all $n \geq 0$, then by construction, $x_{n+1} = x$, $y_{n+1} = y$. Thus, (x, y) is a coupled common fixed point of f and g. So we assume either $x_n \neq x$ or $y_n \neq y$ for $n \geq 0$. Then by using (18) and Lemma 5, we have

$$\begin{aligned} S(x,x,f(x,y)) + S(y,y,f(y,x)) &\leq 2S(x,x,x_{2k+2}) \\ &+ S(x_{2k+2}, x_{2k+2}, f(x,y)) + 2S(y,y,y_{2k+2}) \\ &+ S(y_{2k+2}, y_{2k+2}, f(y,x)) = 2S(x,x,x_{2k+2}) \\ &+ S(g(x_{2k+1}, y_{2k+1}), g(x_{2k+1}, y_{2k+1}), f(x,y)) \\ &+ 2S(y,y,y_{2k+2}) \\ &+ S(g(y_{2k+1}, x_{2k+1}), g(y_{2k+1}, x_{2k+1}), f(y,x)) \\ &\leq 2S(x,x,x_{2k+2}) + 2S(y,y,y_{2k+2}) \\ &+ \theta(S(x_{2k+1}, x_{2k+1}, x), S(y_{2k+1}, y_{2k+1}, y)) \\ &\cdot [S(x_{2k+1}, x_{2k+1}, x) + S(y_{2k+1}, y_{2k+1}, y)] \\ &< 2S(x,x,x_{2k+2}) + 2S(y,y,y_{2k+2}) \\ &+ S(x_{2k+1}, x_{2k+1}, x) + S(y_{2k+1}, y_{2k+1}, y). \end{aligned} \tag{47}$$

Letting $n \to \infty$ in the above inequality, we get

$$S(x,x,f(x,y)) + S(y,y,f(y,x)) = 0. \tag{48}$$

Thus, $x = f(x,y)$, $y = f(y,x)$. By interchanging the roles of f and g and using the same method mentioned above, we also get $x = g(x,y)$, $y = g(y,x)$.

Hence, (x, y) is a coupled common fixed point of f and g. □

Corollary 20. *Let* $(X, \preceq)$ *be a partially ordered set and let* S *be an* S*-metric on* X *such that* (X, S) *is a complete* S*-metric space. Suppose that* $f, g : X \times X \mapsto X$ *are two maps having the mixed weakly monotone property and assume that there exists* $\mu \in \Theta$ *such that*

$$S(f(x,y), f(x,y), g(u,v)) \leq \frac{1}{2} \cdot \mu(S(x,x,u), S(y,y,v))[S(x,x,u) + S(y,y,v)] \tag{49}$$

for all $x, y, u, v \in X$ *with* $x \preceq u$, $y \succeq v$.

Suppose that either

(1) f *or* g *is continuous;*

(2) X *has the following property:*

(a) *If* $\{x_n\}$ *is an increasing sequence with* $x_n \to x$, *then* $x_n \preceq x$ *for all* $n \in \mathbb{N}$.

(b) *If* $\{x_n\}$ *is a decreasing sequence with* $x_n \to x$, *then* $x \preceq x_n$ *for all* $n \in \mathbb{N}$.

If there exist $x_0, y_0 \in X$ *such that* $x_0 \preceq f(x_0, y_0)$, $f(y_0, x_0) \preceq y_0$ *or* $x_0 \preceq g(x_0, y_0)$, $g(y_0, x_0) \preceq y_0$, *then* f *and* g *have a coupled common fixed point in* X.

Proof. For all $x, y, u, v \in X$, write

$$S(f(y,x), f(y,x), g(v,u)) \leq \frac{1}{2} \cdot \mu(S(y,y,v), S(x,x,u)) \cdot [S(y,y,v) + S(x,x,u)]. \tag{50}$$

Adding (49) and (50), we get

$$\begin{aligned} &S(f(x,y), f(x,y), g(u,v)) + S(f(y,x), f(y,x), g(v,u)) \\ &\leq \frac{1}{2}[\mu(S(x,x,u), S(y,y,v)) + \mu(S(y,y,v), S(x,x,u))][S(x,x,u) \\ &+ S(y,y,v) = \theta(S(x,x,u), S(y,y,v))[S(x,x,u) + S(y,y,v)], \end{aligned} \tag{51}$$

where $\theta(\beta_1, \beta_2) = (1/2)[\mu(\beta_1, \beta_2) + \mu(\beta_2, \beta_1)]$, for all $\beta_1, \beta_2 \in [0, \infty)$.

It is easy to verify that $\theta \in \Theta$. Applying Theorems 18 and 19, we get desired result. □

Remark 21. Taking $\mu(\beta_1, \beta_2) = k$ in Corollary 20 for all $\beta_1, \beta_2 \in [0, \infty)$ and $k \in [0, 1)$, we get the following corollary coinciding with [17, Corollary 2.4].

Corollary 22. *In addition to the hypotheses of Corollary 20, suppose that for all* $x, y, u, v \in X$ *with* $x \preceq u$, $y \succeq v$, *and some* $k \in [0, 1)$, *inequality (49) in Corollary 20 is replaced by*

$$S(f(x,y), f(x,y), g(u,v)) \leq \frac{k}{2}[S(x,x,u) + S(y,y,v)]. \tag{52}$$

Then f *and* g *have a coupled common fixed point in* X.

By choosing $f = g$ in Theorems 18 and 19 and using Remark 12, we get coupled fixed point theorem of f written by the following corollary.

Corollary 23. *Let $(X, \preceq)$ be a partially ordered S-metric space and let $f : X \times X \mapsto X$ be a map such that*

(1) *X is complete;*

(2) *f has the mixed monotone property on X;*

$$x_0 \preceq f(x_0, y_0), \quad f(y_0, x_0) \preceq y_0 \quad \text{for some } x_0, y_0 \in X; \tag{53}$$

(3) *assume that there exists $\theta \in \Theta$ such that*

$$\begin{aligned} &S(f(x, y), f(x, y), f(u, v)) \\ &+ S(f(y, x), f(y, x), f(v, u)) \\ &\leq \theta(S(x, x, u), S(y, y, v))[S(x, x, u) + S(y, y, v)], \end{aligned} \tag{54}$$

for all $x, y, u, v \in X$ with $x \preceq u$, $y \succeq v$;

(4) *f is continuous or X has the following properties:*

(a) *If $\{x_n\}$ is an increasing sequence with $x_n \to x$, then $x_n \preceq x$ for all $n \in \mathbb{N}$.*

(b) *If $\{x_n\}$ is a decreasing sequence with $x_n \to x$, then $x \preceq x_n$ for all $n \in \mathbb{N}$.*

Then f has a coupled common fixed point in X.

Theorem 24. *In addition to the hypotheses of Theorem 18, suppose that, for all $(x, y), (s, t) \in X \times X$, there exists $(p, q) \in X \times X$ that is comparable with (x, y) and (s, t). Then f and g have a unique coupled fixed point in X.*

Proof. By Theorem 18, f and g have a coupled common fixed point (x, y). Let (s, t) be another coupled common fixed point of f and g.

By assumption, there exists $(p, q) \in X \times X$ that is comparable to (x, y) and (s, t). Put $p = p_0, q = q_0$ and choose $p_1, q_1 \in X$ such that $p_1 = f(p_0, q_0)$, $q_1 = f(q_0, p_0)$. Using the same construction as in the proof of Theorem 18, we have two sequences $\{p_n\}$ and $\{q_n\}$ such that

$$\begin{aligned} p_{2k+1} &= f(p_{2k}, q_{2k}), \\ q_{2k+1} &= f(q_{2k}, p_{2k}), \\ p_{2k+2} &= g(p_{2k+1}, q_{2k+1}), \\ q_{2k+2} &= g(q_{2k+1}, p_{2k+1}), \end{aligned} \tag{55}$$

satisfying

$$\begin{aligned} p_0 \preceq p_1 \preceq \cdots \preceq p_n \preceq \cdots, \\ q_0 \succeq q_1 \succeq \cdots \succeq q_n \succeq \cdots. \end{aligned} \tag{56}$$

Since (p, q) is comparable to (x, y), we can assume that $(x, y) \succeq (p, q) = (p_0, q_0)$.

Then it is easy to show that (p_n, q_n) and (x, y) are comparable; that is, $(x, y) \succeq (p_n, q_n)$, for all $n \in \mathbb{N}$.

For $n = 2k + 1$, from (18) we have

$$\begin{aligned} &S(p_{2k+1}, x, x) + S(q_{2k+1}, y, y) \\ &= S(f(p_{2k}, q_{2k}), f(x, y), f(x, y)) \\ &+ S(f(q_{2k}, p_{2k}), f(y, x), f(y, x)) \\ &\leq \theta(S(p_{2k}, x, x), S(q_{2k}, y, y)) \\ &\cdot [S(p_{2k}, x, x) + S(q_{2k}, y, y)] \end{aligned} \tag{57}$$

which implies

$$\begin{aligned} &S(p_{2k+1}, x, x) + S(q_{2k+1}, y, y) \\ &< S(p_{2k}, x, x) + S(q_{2k}, y, y). \end{aligned} \tag{58}$$

We see that the sequence $\{S(p_{2k}, x, x) + S(q_{2k}, y, y)\}$ is decreasing, and there exist $\xi \geq 0$ such that

$$S(p_{2k}, x, x) + S(q_{2k}, y, y) \longrightarrow \xi, \quad \text{as } k \longrightarrow \infty. \tag{59}$$

Now, we have to show that $\xi = 0$. On the contrary, suppose that $\xi > 0$. Following the same arguments as in the proof of Theorem 18, we obtain

$$\theta(S(p_{2k}, x, x), S(q_{2k}, y, y)) \longrightarrow 1, \quad \text{as } k \longrightarrow \infty. \tag{60}$$

It follows that

$$S(p_{2k}, x, x), S(q_{2k}, y, y) \longrightarrow 0, \quad \text{as } k \longrightarrow \infty. \tag{61}$$

This implies

$$S(p_{2k}, x, x) + S(q_{2k}, y, y) \longrightarrow 0, \quad \text{as } k \longrightarrow \infty, \tag{62}$$

which is not possible in virtue of (30). Hence, $\xi = 0$. Therefore, (59) becomes

$$S(p_{2k}, x, x) + S(q_{2k}, y, y) \longrightarrow 0, \quad \text{as } k \longrightarrow \infty. \tag{63}$$

Similarly, we can get that

$$S(p_{2k}, s, s) + S(q_{2k}, t, t) \longrightarrow 0, \quad \text{as } k \longrightarrow \infty. \tag{64}$$

Using (63)-(64), the second condition of S-metric, and taking the limit $k \to \infty$, we obtain that

$$S(s, x, x) + S(t, y, y) = 0. \tag{65}$$

Thus, we conclude that $x = s$, $y = t$.

Analogous to $n = 2k + 1$, by interchanging the roles of f and g, (65) holds true for $n = 2k + 2$.

Therefore, we conclude that f and g have a unique coupled common fixed point. □

Similarly, we can prove the following theorem.

Theorem 25. *In addition to the hypotheses of Theorem 19, suppose that, for all $(x, y), (s, t) \in X \times X$, there exists $(p, q) \in X \times X$ that is comparable with (x, y) and (s, t). Then f and g have a unique coupled fixed point in X.*

Finally, we give some examples to demonstrate the validity of our results.

Example 26. Let $X = [0, \pi/4)$, with the S-metric defined by $S(x, y, z) = |x - z| + |y - z|$ and the natural ordering of real numbers $\leq$. Then X is a totally ordered, complete S-metric space.

Let $\theta : [0, \pi/4)^2 \mapsto [0, 1)$ be defined by

$$\theta(x, y) = \begin{cases} 1 - \dfrac{\sin(x + y)}{2}; & x + y > 0, \\ 0; & x = y = 0. \end{cases} \tag{66}$$

For all $x, y \in X$, put $f(x, y) = g(x, y) = (2x - y + 15)/16$.

The pair (f, g) has the mixed weakly monotone property and $\forall x, y, u, v \in X$ with $x \leq u$, $y \geq v$, we have that

$$\begin{aligned} & S(f(x, y), f(x, y), g(u, v)) \\ & \quad + S(f(y, x), f(y, x), g(v, u)) \\ & \quad = 2|f(x, y) - g(u, v)| + 2|f(y, x) - g(v, u)| \\ & \quad = 2\left|\frac{2x - y + 15}{16} - \frac{2u - v + 15}{16}\right| \\ & \quad + 2\left|\frac{2y - x + 15}{16} - \frac{2v - u + 15}{16}\right| \\ & \quad = \frac{1}{8}|2(x - u) + (v - y)| + \frac{1}{8}|2(y - v) + (u - x)| \\ & \quad \leq \frac{1}{8}(2|x - u| + |v - y|) \\ & \quad + \frac{1}{8}(|2(y - v)| + |u - x|) \leq \frac{1}{2}(|x - u| + |v - y|) \\ & \quad \leq \left(1 - \frac{\sin(|x - u| + |v - y|)}{2}\right)(|x - u| + |v - y|) \\ & \quad = \theta(S(x, x, u), S(y, y, v)) \\ & \quad \cdot [S(x, x, u) + S(y, y, v)]. \end{aligned} \tag{67}$$

Then the contractive condition (18) in Theorem 18 holds, and $(1, 1)$ is the unique coupled common fixed point.

Example 27. Let $X = [0, 1]$, with the S-metric defined by $S(x, y, z) = (1/2)(|x - z| + |y - z|)$ and the natural ordering of real numbers $\leq$. Then X is a totally ordered, complete S-metric space.

Let $\theta : [0, \infty)^2 \mapsto [0, 1)$ be defined by

$$\theta(x, y) = \begin{cases} \dfrac{\ln(1 + \max\{x, y\})}{\max\{x, y\}}, & x > 0 \text{ or } y > 0; \\ l \in [0, 1), & x = y = 0. \end{cases} \tag{68}$$

For all $x, y \in X$, put $f(x, y) = g(x, y) = (1/8)[\ln(1 + x) - 2\ln(1 + y)]$.

The pair (f, g) has the mixed weakly monotone property. $\forall x, y, u, v \in X$ with $x \leq u$, $y \geq v$, we have that

$$\begin{aligned} & S(f(x, y), f(x, y), g(u, v)) \\ & \quad + S(f(y, x), f(y, x), g(v, u)) \\ & \quad = \left|\frac{\ln(1 + x) - 2\ln(1 + y)}{8}\right. \\ & \quad - \left.\frac{\ln(1 + u) - 2\ln(1 + v)}{8}\right| \\ & \quad + \left|\frac{\ln(1 + y) - 2\ln(1 + x)}{8}\right. \\ & \quad - \left.\frac{\ln(1 + v) - 2\ln(1 + u)}{8}\right| \\ & \quad = \frac{1}{8}|(\ln(1 + x) - \ln(1 + u)) \\ & \quad + 2(\ln(1 + v) - \ln(1 + y))| \\ & \quad + \frac{1}{8}|(\ln(1 + y) - \ln(1 + v)) \\ & \quad + 2(\ln(1 + u) - \ln(1 + x))| \leq \frac{1}{8}|\ln(1 + x) \\ & \quad - \ln(1 + u)| + \frac{1}{4}|\ln(1 + v) - \ln(1 + y)| \\ & \quad + \frac{1}{8}|\ln(1 + y) - \ln(1 + v)| + \frac{1}{4}|\ln(1 + u) \\ & \quad - \ln(1 + x)| \leq \frac{1}{2}|\ln(1 + u) - \ln(1 + x)| \\ & \quad + \frac{1}{2}|\ln(1 + y) - \ln(1 + v)| = \frac{1}{2}\left|\ln\left(\frac{1 + u}{1 + x}\right)\right| \\ & \quad + \frac{1}{2}\left|\ln\left(\frac{1 + y}{1 + v}\right)\right| = \frac{1}{2}\left|\ln\left(1 + \frac{u - x}{1 + x}\right)\right| \\ & \quad + \frac{1}{2}\left|\ln\left(1 + \frac{y - v}{1 + v}\right)\right| \leq \frac{1}{2}|\ln(1 + (u - x))| \\ & \quad + \frac{1}{2}|\ln(1 + (y - v))| = \frac{1}{2}\ln(1 + |u - x|) + \frac{1}{2} \\ & \quad \cdot \ln(1 + |y - v|) \leq \ln(1 + \max\{|x - u|, |y - v|\}) \\ & \quad = \frac{\ln(1 + \max\{|x - u|, |y - v|\})}{\max\{|x - u|, |y - v|\}} \\ & \quad \cdot \max\{|x - u|, |y - v|\} \\ & \quad \leq \frac{\ln(1 + \max\{|x - u|, |y - v|\})}{\max\{|x - u|, |y - v|\}}(|x - u| \\ & \quad + |y - v|) = \theta(S(x, x, u), S(y, y, v))[S(x, x, u) \\ & \quad + S(y, y, v)]. \end{aligned} \tag{69}$$

Then the contractive condition (18) in Theorem 18 holds, and $(0, 0)$ is the unique coupled common fixed point.

Conflict of Interests

The authors declare that they have no competing interests.

Authors' Contribution

Both authors contributed equally and significantly to writing this paper. All authors read and approved the final paper.

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