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Research Article

New Approach for Common Fixed Point Theorems via C-Class Functions in G_p-Metric Spaces

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We prove a new approach for some common fixed point results in complete G_p-metric spaces for weakly increasing self-mappings satisfying (ψ, φ)-contractions via the concept of C-class functions. An example is also provided.

1. Introduction and Mathematical Preliminaries

In 1922, Banach [1] proved his classical theorem which asserts suitable conditions ensuring the existence and uniqueness of fixed point of the underlying mapping. Over the last several decades, this theorem has been generalized and improved in various spaces (e.g., [2–6]). In 1994, Matthews [7] introduced the notion of a partial metric space and established the Banach contraction theorem in the class of partial metric spaces. Notably, in partial metric spaces, the distance from a point to itself need not be zero. In recent years, several authors proved variant (common) fixed point theorems in partial metric spaces. For more details, see [8–17].

For the sake of completeness, we recall the definition of a partial metric space (in short PMS) which runs as follows.

Definition 1 (see [7]). A partial metric on a nonempty set X is a function $p : X \times X \to R^+$, $R^+ := [0, \infty)$, such that for all $x, y, z \in X$

(p^1) $x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$,

(p^2) $p(x, x) \le p(x, y)$,

(p^3) $p(x, y) = p(y, x)$,

(p^4) $p(x, y) \le p(x, z) + p(z, y) - p(z, z)$.

A partial metric space is a pair (X, p) such that X is a nonempty set and p is a partial metric on X.

On the other hand, the notation of generalized metric spaces (in short G-metric spaces) was introduced by Mustafa and Sims [18] who presented and improved the Banach contraction principle in the class of G-metric spaces. The definition of a G-metric space is introduced as follows.

Definition 2 (see [18]). Let X be a nonempty set. Suppose that $G : X \times X \times X \to R^+$ satisfies

(a) $G(x, y, z) = 0$ if $x = y = z$,

(b) $G(x, y, z) > 0$, $\forall x, y, z \in X$, $x \ne y$,

(c) $G(x, x, y) \le G(x, y, z)$, $\forall x, y, z \in X$, $y \ne z$,

(d) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \cdots$ (symmetry in all three variables),

(e) $G(x, y, z) \le G(x, a, a) + G(a, y, z)$, $\forall x, y, z, a \in X$.

Then G is called a G-metric on X and (X, G) is called a G-metric space.

Recently, as a unification between partial metric spaces and G-metric spaces, Zand and Nezhad [19] defined the concept of a G_p-metric space in the following way.

Definition 3 (see [19]). Let X be a nonempty set. Suppose that $G_p : X \times X \times X \to R^+$ satisfies

(a) $x = y = z$ if $G_p(x,x,x) = G_p(y,y,y) = G_p(z,z,z)$ $\forall x,y,z \in X$;

(b) $G_p(x,x,x) \le G_p(x,x,y) \le G_p(x,y,z)$, $\forall x,y,z \in X$;

(c) $G_p(x,y,z) = G_p(x,z,y) = G_p(y,z,x) = \cdots$ (symmetry in all three variables);

(d) $G_p(x,y,z) \le G_p(x,a,a) + G_p(a,y,z) - G_p(a,a,a)$, $\forall x,y,z,a \in X$.

Then G_p is called a G_p-metric on X and (X,G_p) is called a G_p-metric space.

Example 4 (see [19]). Let $X = [0,\infty)$ and define $G_p(x,y,z) = \max\{x,y,z\}$ for all $x,y,z \in X$. Then (X,G_p) is a G_p-metric space. Note that (X,G_p) is not a G-metric space.

Proposition 5 (see [19]). *Let (X,G_p) be a G_p-metric space. Then for any $x,y,z \in X$ and $a \in X$, one has*

(i) $G_p(x,y,z) \le G_p(x,x,y) + G_p(x,x,z) - G_p(x,x,x)$;

(ii) $G_p(x,y,y) \le 2G_p(x,x,y) - G_p(x,x,x)$;

(iii) $G_p(x,y,z) \le G_p(x,a,a) + G_p(y,a,a) + G_p(z,a,a) - 2G_p(a,a,a)$;

(iv) $G_p(x,y,z) \le G_p(x,a,z) + G_p(a,y,z) - G_p(a,a,a)$.

Proposition 6 (see [19]). *Every G_p-metric space (X,G_p) defines a metric space (X,D_{G_p}), where*

$$D_{G_p}(x,y) = G_p(x,y,y) + G_p(y,x,x) - G_p(x,x,x) - G_p(y,y,y), \quad (1)$$

for all $x,y \in X$.

Definition 7 (see [19]). Let (X,G_p) be a G_p-metric space. A sequence $\{x_n\}$ is G_p-convergent to $x \in X$ if $\lim_{n,m\to\infty} G_p(x,x_m,x_n) = G_p(x,x,x)$.

We may write the above as $x_n \to x$.

Thus if $x_n \to x$ in a G_p-metric space (X,G_p), then for any $\epsilon > 0$, there exists $l \in \mathbb{N}$ such that $|G_p(x,x_n,x_m) - G_p(x,x,x)| < \epsilon$ for all $n,m > l$.

Proposition 8 (see [19]). *Let (X,G_p) be a G_p-metric space. Take a sequence $\{x_n\}$ in X and a point $x \in X$. The following are equivalent:*

(i) *$\{x_n\}$ is G_p-convergent to x;*

(ii) *$G_p(x_n,x_n,x) \to G_p(x,x,x)$ as $n \to \infty$;*

(iii) *$G_p(x_n,x,x) \to G_p(x,x,x)$ as $n \to \infty$.*

Definition 9 (see [19]). Let (X,G_p) be a G_p-metric space.

(i) A sequence $\{x_n\}$ is called a G_p-Cauchy if and only if $\lim_{m,n\to\infty} G_p(x_n,x_m,x_m)$ exists (and is finite).

(ii) A G_p-metric space (X,G_p) is said to be G_p-complete if and only if every G_p-Cauchy sequence in X is G_p-convergent to $x \in X$; that is, $G_p(x,x,x) = \lim_{m,n\to\infty} G_p(x_n,x_m,x_m)$.

Take $\Psi = \{\psi : [0,\infty) \to [0,\infty)$ is continuous, nondecreasing, and $\psi^{-1}(\{0\}) = \{0\}\}$, and $\Phi = \{\varphi : \varphi : [0,\infty) \to [0,\infty)$ is lower semicontinuous, nondecreasing, and $\varphi^{-1}(\{0\}) = \{0\}\}$.

Definition 10 (see [20]). Let $(X,\preceq)$ be a partially ordered set. Two maps $f,g : X \to X$ are said to be weakly increasing if $fx \preceq gfx$ and $gx \preceq fgx$ for all $x \in X$.

Definition 11. Let (X,G_p) be a G_p-metric space endowed with a partial order $\preceq$. Let $\{x_n\}$ and z be in X. $(X,G_p,\preceq)$ is said to be regular if $x_n \to z$ and $\{x_n\}$ is nondecreasing; then $x_n \preceq z$ for all $n \in \mathbb{N}$.

Lemma 12 (see [21]). *Let (X,G_p) be a G_p-metric space. One has the following.*

(i) *If $G_p(x,y,z) = 0$, then $x = y = z$.*

(ii) *If $x \ne y$, then $G_p(x,y,y) > 0$.*

We rewrite the continuity of mappings in G_p-metric spaces.

Definition 13. Let (X,G_p) be a G_p-metric space and let $T : X \to X$ be a given mapping. One says that T is continuous at $u \in X$ if for every sequence $\{x_n\}$ converging to u in X, the sequence $\{Tx_n\}$ converges to Tu in X. If T is continuous at each point $u \in X$, then one says that T is continuous on X.

Ansari [22] introduced the class of C-functions which covers a large class of contractive conditions.

Definition 14 (see [22]). A mapping $F : [0,\infty)^2 \to \mathbb{R}$ is called a C-function if it is continuous and satisfies the following axioms:

(1) $F(s,t) \le s$ for all $s,t \in [0,\infty)$;

(2) $F(s,t) = s$ implies that either $s = 0$ or $t = 0$.

Mention that any C-function F verifies $F(0,0) = 0$. We denote by $\mathcal{C}$ the set of C-class functions.

Example 15 (see [22]). The following functions $F : [0,\infty)^2 \to \mathbb{R}$ are elements of $\mathcal{C}$. For all $s,t \in [0,\infty)$, consider

(1) $F(s,t) = s - t$;

(2) $F(s,t) = ms$, where $0 < m < 1$;

(3) $F(s,t) = s/(1+t)^r$, where $r \in (0,\infty)$;

(4) $F(s,t) = s\beta(s)$, where $\beta : [0,\infty) \to [0,1)$ is continuous;

(5) $F(s,t) = s - \varphi(s)$, where $\varphi : [0,\infty) \to [0,\infty)$ is a continuous function such that $\varphi(t) = 0 \Leftrightarrow t = 0$.

Abbas et al. [20] proved the following result.

Theorem 16 (see [20]). *Let $(X, \preceq)$ be a partially ordered set and let f and g be weakly increasing self-mappings on a complete G-metric space X. Suppose that there exist $\psi \in \Psi$ and $\varphi \in \Phi$ such that*

$$\psi(G(fx, gy, gy)) \leq \psi(M(x, y, y)) - \varphi(M(x, y, y)) \quad (2)$$

for all comparable $x, y \in X$, where

$$\begin{aligned} M(x, y, y) = {} & a_1 G(x, y, y) + a_2 G(x, fx, fx) \\ & + a_3 G(y, gy, gy) \\ & + a_4 [G(x, gy, gy) + G(y, fx, fx)], \end{aligned} \quad (3)$$

where $a_i > 0$ for $i = \{1, 2, 3, 4\}$ with $a_1 + a_2 + a_3 + 2a_4 \leq 1$. Assume either f or g is continuous, or $(X, G_p, \preceq)$ is regular. Then f and g have a common fixed point.

Very recently, Barakat and Zidan [13] extended Theorem 16 to the class of G_p-metric spaces where a general contractive condition is considered.

Theorem 17 (see [13]). *Let $(X, \preceq)$ be a partially ordered set. Let f and g be weakly increasing self-mappings on a complete G_p-metric space X satisfying*

$$\begin{aligned} \psi\left(G_p(fx, gy, gy)\right) \leq {} & \psi(M(x, y, y)) \\ & - \varphi(M(x, y, y)) \end{aligned} \quad (4)$$

for all comparable $x, y \in X$, where $\psi \in \Psi$, $\varphi \in \Phi$, and

$$\begin{aligned} M(x, y, y) = \max \Big\{ & G_p(x, y, y), G_p(x, fx, fx), \\ & G_p(y, gy, gy), \\ & \frac{1}{2}\left[G_p(x, gy, gy) + G_p(y, fx, fx)\right] \Big\}. \end{aligned} \quad (5)$$

Assume either f or g is continuous or $(X, G_p, \preceq)$ is regular. Then f and g have a common fixed point.

In this paper, we prove a common fixed point result in complete G_p-metric spaces for weakly increasing self-mappings satisfying (ψ, φ)-contractions via the concept of C-class functions. Some corollaries are also presented for particular cases of the C-function. For a given C-function, Theorem 17 is reached.

2. Main Results

First, we introduce an auxiliary lemma as follows.

Lemma 18. *Let (X, G_p) be a G_p-metric space and let $\{x_n\}$ be a sequence in X such that $\{G_p(x_n, x_n, x_{n+1})\}$ is decreasing and*

$$\lim_{n\to\infty} G_p(x_n, x_n, x_{n+1}) = 0. \quad (6)$$

If $\{x_{2n}\}$ is not a G_p-Cauchy sequence, then there exist an $\epsilon > 0$ and $\{m_k\}, \{n_k\}$ of positive integers such that the following sequences $\{G_p(x_{2m_k}, x_{2n_k}, x_{2n_k})\}$, $\{G_p(x_{2m_k}, x_{2n_k+1}, x_{2n_k+1})\}$, $\{G_p(x_{2m_k-1}, x_{2n_k}, x_{2n_k})\}$, $\{G_p(x_{2m_k-1}, x_{2n_k+1}, x_{2n_k+1})\}$, and $\{G_p(x_{2m_k}, x_{2n_k+2}, x_{2n_k+2})\}$ tend to ϵ when $k \to \infty$.

Proof. Assume that $\{x_{2n}\}$ is not a G_p-Cauchy sequence. So there exist $\epsilon > 0$, and $\{m_k\}$ and $\{n_k\}$ of positive integers such that

$$\begin{aligned} & G_p\left(x_{2m_k}, x_{2n_k-2}, x_{2n_k-2}\right) < \epsilon, \\ & G_p\left(x_{2m_k}, x_{2n_k}, x_{2n_k}\right) > \epsilon, \\ & n_k > m_k > k \end{aligned} \quad (7)$$

for all $k \in \mathbb{N}$. Then

$$\begin{aligned} \epsilon & \leq G_p\left(x_{2m_k}, x_{2n_k}, x_{2n_k}\right) \\ & \leq G_p\left(x_{2m_k}, x_{2n_k-2}, x_{2n_k-2}\right) \\ & \quad + G_p\left(x_{2n_k-2}, x_{2n_k-1}, x_{2n_k-1}\right) \\ & \quad + G_p\left(x_{2n_k-1}, x_{2n_k}, x_{2n_k}\right) \\ & \quad - G_p\left(x_{2n_k-2}, x_{2n_k-2}, x_{2n_k-2}\right) \\ & \quad - G_p\left(x_{2n_k-1}, x_{2n_k-1}, x_{2n_k-1}\right) \\ & < \epsilon + G_p\left(x_{2n_k-2}, x_{2n_k-1}, x_{2n_k-1}\right) \\ & \quad + G_p\left(x_{2n_k-1}, x_{2n_k}, x_{2n_k}\right). \end{aligned} \quad (8)$$

By taking the limit in above inequalities and using (6), we get

$$\lim_{k\to\infty} G_p\left(x_{2m_k}, x_{2n_k}, x_{2n_k}\right) = \epsilon. \quad (9)$$

On the other hand

$$\begin{aligned} G_p\left(x_{2m_k}, x_{2n_k}, x_{2n_k}\right) & \leq G_p\left(x_{2m_k}, x_{2n_k+1}, x_{2n_k+1}\right) \\ & \quad + G_p\left(x_{2n_k+1}, x_{2n_k}, x_{2n_k}\right) \\ & \quad - G_p\left(x_{2n_k+1}, x_{2n_k+1}, x_{2n_k+1}\right) \\ & \leq G_p\left(x_{2m_k}, x_{2n_k+1}, x_{2n_k+1}\right) \\ & \quad + 2G_p\left(x_{2n_k}, x_{2n_k+1}, x_{2n_k+1}\right) \\ & \quad - 2G_p\left(x_{2n_k+1}, x_{2n_k+1}, x_{2n_k+1}\right) \\ & \leq G_p\left(x_{2m_k}, x_{2n_k+1}, x_{2n_k+1}\right) \\ & \quad + 2G_p\left(x_{2n_k}, x_{2n_k+1}, x_{2n_k+1}\right) \end{aligned}$$

$$\leq G_p\left(x_{2m_k}, x_{2n_k}, x_{2n_k}\right) + G_p\left(x_{2n_k}, x_{2n_k+1}, x_{2n_k+1}\right) + 2G_p\left(x_{2n_k}, x_{2n_k+1}, x_{2n_k+1}\right). \tag{10}$$

Letting $k \to \infty$, again using (6) and (9), we obtain

$$\lim_{k\to\infty} G_p\left(x_{2m_k}, x_{2n_k+1}, x_{2n_k+1}\right) = \epsilon. \tag{11}$$

Similarly, we can prove that the remaining sequences tend to ϵ as $k \to \infty$. □

Now, we state and prove our main result in the following way.

Theorem 19. *Let $(X, \preceq)$ be a partially ordered set. Let f and g be weakly increasing self-mappings on a complete G_p-metric space X. Assume there exist $\psi, \varphi \in \Psi$ and $F \in \mathcal{C}$ such that*

$$\psi\left(G_p\left(fx, gy, gy\right)\right) \leq F\left(\psi\left(M\left(x, y, y\right)\right), \varphi\left(M\left(x, y, y\right)\right)\right) \tag{12}$$

for all comparable $x, y \in X$, where

$$M\left(x, y, y\right) = \max\left\{G_p\left(x, y, y\right), G_p\left(x, fx, fx\right), G_p\left(y, gy, gy\right), \frac{\left[G_p\left(x, gy, gy\right) + G_p\left(y, fx, fx\right)\right]}{2}\right\}. \tag{13}$$

Suppose that one of the following two cases is satisfied:

(i) *f or g is continuous;*

(ii) *$(X, G_p, \preceq)$ is regular.*

Then the maps f and g have a common fixed point.

Proof. Assume that u is a fixed point of f. Taking $x = y = u$ in (12), we have

$$\psi\left(G_p\left(u, gu, gu\right)\right) = \psi\left(G_p\left(fu, gu, gu\right)\right) \leq F\left(\psi\left(M\left(u, u, u\right)\right), \varphi\left(M\left(u, u, u\right)\right)\right), \tag{14}$$

where

$$\begin{aligned} M\left(u, u, u\right) &= \max\left\{G_p\left(u, u, u\right), G_p\left(u, fu, fu\right), G_p\left(u, gu, gu\right), \frac{\left[G_p\left(u, gu, gu\right) + G_p\left(u, fu, fu\right)\right]}{2}\right\} \\ &= \max\left\{G_p\left(u, u, u\right), G_p\left(u, u, u\right), G_p\left(u, gu, gu\right), \frac{\left[G_p\left(u, gu, gu\right) + G_p\left(u, u, u\right)\right]}{2}\right\} \\ &= \max\left\{G_p\left(u, u, u\right), G_p\left(u, gu, gu\right)\right\} = G_p\left(u, gu, gu\right). \end{aligned} \tag{15}$$

Hence we get

$$\psi\left(G_p\left(u, gu, gu\right)\right) = \psi\left(G_p\left(fu, gu, gu\right)\right) \leq F\left(\psi\left(G_p\left(u, gu, gu\right)\right), \varphi\left(G_p\left(u, gu, gu\right)\right)\right) \leq \psi\left(G_p\left(u, gu, gu\right)\right). \tag{16}$$

We deduce $F(\psi(G_p(u, gu, gu))), \varphi(G_p(u, gu, gu)) \leq \psi(G_p(u, gu, gu))$. By a property of the C-class F, we get $\psi(G_p(u, gu, gu)) = 0$ or $\varphi(G_p(u, gu, gu)) = 0$. The functions ψ and φ are in Ψ, so $G_p(u, gu, gu) = 0$; that is, $u = gu$; that is, u is a common fixed point of f and g. Now, if u is a fixed point of g, similarly we get that u is also fixed point of f. Let x_0 be an arbitrary point of X. The pair (f, g) is weakly increasing, so we construct a sequence $\{x_n\}$ in X as follows:

$$\begin{aligned} x_1 &= fx_0 \preceq gfx_0 = gx_1 = x_2, \\ x_2 &= gx_1 \preceq fgx_1 = fx_2 = x_3, \\ &\vdots \\ x_{2n+1} &= fx_{2n} \preceq x_{2n+2} = gx_{2n+1}. \end{aligned} \tag{17}$$

We have $x_n \preceq x_{n+1}$ for all $n \geq 0$.

Now, suppose that $G_p(x_{2n}, x_{2n+1}, x_{2n+1}) = 0$ for some $n \geq 0$. Then $x_{2n} = x_{2n+1} = fx_{2n}$; that is, x_{2n} is a fixed point of f. Proceeding similarly, we get that x_{2n} is a fixed point of g.

From now on, we suppose that $G_p(x_{2n}, x_{2n+1}, x_{2n+1}) > 0$ for every $n \in \mathbb{N}$. Since x_{2n} and x_{2n+1} are comparable, by (12),

$$\begin{aligned} \psi\left(G_p\left(x_{2n+1}, x_{2n+2}, x_{2n+2}\right)\right) &= \psi\left(G_p\left(fx_{2n}, gx_{2n+1}, gx_{2n+1}\right)\right) \\ &\leq F\left(\psi\left(M\left(x_{2n}, x_{2n+1}, x_{2n+1}\right)\right), \varphi\left(M\left(x_{2n}, x_{2n+1}, x_{2n+1}\right)\right)\right), \end{aligned} \tag{18}$$

where

$$\begin{aligned}
M(x_{2n},x_{2n+1},x_{2n+1}) &= \max\Big\{G_p(x_{2n},x_{2n+1},x_{2n+1}), G_p(x_{2n},fx_{2n},fx_{2n}), G_p(x_{2n+1},gx_{2n+1},gx_{2n+1}),\\
&\frac{[G_p(x_{2n},gx_{2n+1},gx_{2n+1})+G_p(x_{2n+1},fx_{2n},fx_{2n})]}{2}\Big\} = \max\Big\{G_p(x_{2n},x_{2n+1},x_{2n+1}), G_p(x_{2n},x_{2n+1},x_{2n+1}), G_p(x_{2n+1},\\
&x_{2n+2},x_{2n+2}), \frac{[G_p(x_{2n},x_{2n+2},x_{2n+2})+G_p(x_{2n+1},x_{2n+1},x_{2n+1})]}{2}\Big\} \le \max\Big\{G_p(x_{2n},x_{2n+1},x_{2n+1}), G_p(x_{2n+1},x_{2n+2},\\
&x_{2n+2}), \frac{[G_p(x_{2n},x_{2n+1},x_{2n+1})+G_p(x_{2n+1},x_{2n+2},x_{2n+2})-G_p(x_{2n+1},x_{2n+1},x_{2n+1})+G_p(x_{2n+1},x_{2n+1},x_{2n+1})]}{2}\Big\}\\
&\le \max\Big\{G_p(x_{2n},x_{2n+1},x_{2n+1}), G_p(x_{2n+1},x_{2n+2},x_{2n+2}), \frac{1}{2}[G_p(x_{2n},x_{2n+1},x_{2n+1})+G_p(x_{2n+1},x_{2n+2},x_{2n+2})]\Big\}\\
&= \max\{G_p(x_{2n},x_{2n+1},x_{2n+1}), G_p(x_{2n+1},x_{2n+2},x_{2n+2})\}.
\end{aligned} \tag{19}$$

If $G_p(x_{2n+1},x_{2n+2},x_{2n+2}) \ge G_p(x_{2n},x_{2n+1},x_{2n+1})$ for some $n \ge 0$, then $M(x_{2n},x_{2n+1},x_{2n+1}) = G_p(x_{2n+1},x_{2n+2},x_{2n+2})$. Using (18), we have

$$\begin{aligned}
&\psi(G_p(x_{2n+1},x_{2n+2},x_{2n+2}))\\
&\quad\le F(\psi(G_p(x_{2n+1},x_{2n+2},x_{2n+2})),\\
&\quad\varphi(G_p(x_{2n+1},x_{2n+2},x_{2n+2}))).
\end{aligned} \tag{20}$$

By a property of F, this implies that $\psi(G_p(x_{2n+1},x_{2n+2},x_{2n+2}))$ or $\varphi(G_p(x_{2n+1},x_{2n+2},x_{2n+2})) = 0$, which is a contradiction. Therefore, for all $n \ge 0$, $G_p(x_{2n+1},x_{2n+2},x_{2n+2}) < G_p(x_{2n},x_{2n+1},x_{2n+1})$. Similarly, we may show that $G_p(x_{2n},x_{2n+1},x_{2n+1}) < G_p(x_{2n-1},x_{2n},x_{2n})$ for all $n \ge 0$. We deduce that for all $n \ge 0$

$$G_p(x_{n+1},x_{n+2},x_{n+2}) < G_p(x_n,x_{n+1},x_{n+1}). \tag{21}$$

So the sequence $\{G_p(x_{n+1},x_{n+2},x_{n+2})\}$ is decreasing. Then there exists $L \ge 0$, such that $\lim_{n\to\infty} G_p(x_{n+1},x_{n+2},x_{n+2}) = L$. We claim that $L = 0$. We have

$$\lim_{n\to\infty} M(x_n,x_{n+1},x_{n+1}) = L. \tag{22}$$

Recall that

$$\begin{aligned}
&\psi(G_p(x_{n+1},x_{n+2},x_{n+2}))\\
&\quad\le F(\psi(M(x_n,x_{n+1},x_{n+1})),\\
&\quad\phi(M(x_n,x_{n+1},x_{n+1}))).
\end{aligned} \tag{23}$$

As $n \to \infty$, by continuity of F, ψ, and φ, we get

$$\psi(L) \le F(\psi(L),\varphi(L)) \le \psi(L). \tag{24}$$

By a property of F, we get $\psi(L) = 0$ or $\varphi(L) = 0$; that is, $L = 0$. We conclude that

$$\lim_{n\to\infty} G_p(x_{n+1},x_{n+2},x_{n+2}) = 0. \tag{25}$$

We shall show that $\{x_n\}$ is a G_p-Cauchy sequence. Suppose that $\{x_{2n}\}$ is not a G_p-Cauchy sequences. By (12),

$$\begin{aligned}
&\psi(G_p(x_{2m_k+1},x_{2n_k+2},x_{2n_k+2}))\\
&\quad= \psi(G_p(fx_{2m_k},gx_{2n_k+1},gx_{2n_k+1}))\\
&\quad\le F(\psi(M(x_{2m_k},x_{2n_k+1},x_{2n_k+1})),\\
&\quad\varphi(M(x_{2m_k},x_{2n_k+1},x_{2n_k+1}))),
\end{aligned} \tag{26}$$

where

$$\begin{aligned}
&M(x_{2m_k},x_{2n_k+1},x_{2n_k+1})\\
&\quad= \max\Big\{G_p(x_{2m_k},x_{2n_k+1},x_{2n_k+1}),\\
&\quad G_p(x_{2m_k},fx_{2m_k},fx_{2m_k}),\\
&\quad G_p(x_{2n_k+1},gx_{2n_k+1},gx_{2n_k+1}),\\
&\quad \frac{1}{2}[G_p(x_{2m_k},gx_{2n_k+1},gx_{2n_k+1})\\
&\quad + G_p(x_{2n_k+1},fx_{2m_k},gx_{2m_k})]\Big\}\\
&\quad= \max\Big\{G_p(x_{2m_k},x_{2n_k+1},x_{2n_k+1}),\\
&\quad G_p(x_{2m_k},x_{2m_k+1},x_{2m_k+1}),\\
&\quad G_p(x_{2n_k+1},x_{2n_k+2},x_{2n_k+2}),\\
&\quad \frac{1}{2}[G_p(x_{2m_k},x_{2n_k+2},x_{2n_k+2})\\
&\quad + G_p(x_{2n_k+1},x_{2m_k+1},x_{2m_k+1})]\Big\}.
\end{aligned} \tag{27}$$

By taking the limit as $k \to \infty$, from Lemma 18, we have

$$\lim_{k\to\infty} M\left(x_{2m_k}, x_{2n_k+1}, x_{2n_k+1}\right) = \max\{\epsilon, 0, 0, \epsilon\} = \epsilon\,(>0). \tag{28}$$

Hence $\psi(\epsilon) \le F(\psi(\epsilon), \phi(\epsilon))$. We deduce that $\psi(\epsilon) = 0$ or $\phi(\epsilon) = 0$, which is a contradiction. This proves that $\{x_{2n}\}$ is a G_p-Cauchy sequence and hence $\{x_n\}$ is a G_p-Cauchy sequence. By G_p-completeness of X, there exists $z \in X$ such that $\{x_n\}$ converges to z as $n \to \infty$.

Now, we will distinguish the cases (i) and (ii) of Theorem 19.

(i) Without loss of generality, suppose that g is continuous. Since $x_{2n+1} \to z$, we obtain that $x_{2n+2} = g(x_{2n+1}) \to gz$. But as $x_{2n+2} \to z$ (as a subsequence of $\{x_n\}$), it follows that $gz = z$. From the beginning of the proof, we get $gz = z = fz$.

The case that f is continuous is treated similarly.

(ii) Suppose that $(X, G_p, \preceq)$ is regular. We know that sequence $\{x_n\}$ is nondecreasing and $x_n \to z$ in X; then by regularity of $(X, G_p, \preceq)$, $x_{2n+1} \preceq z$ for all $n \in \mathbb{N}$. By (12)

$$\begin{aligned}\psi\left(G_p\left(x_{2n+1}, gz, gz\right)\right) &= \psi\left(G_p\left(fx_{2n}, gz, gz\right)\right)\\ &\le F\left(\psi\left(M\left(x_{2n}, z, z\right)\right), \varphi\left(M\left(x_{2n}, z, z\right)\right)\right),\end{aligned} \tag{29}$$

where

$$\begin{aligned}M\left(x_{2n}, z, z\right) = \max\Bigg\{&G_p\left(x_{2n}, z, z\right), G_p\left(x_{2n}, fx_{2n}, fx_{2n}\right), G_p\left(z, gz, gz\right),\\ &\frac{\left[G_p\left(x_{2n}, gz, gz\right) + G_p\left(z, fx_{2n}, fx_{2n}\right)\right]}{2}\Bigg\}\\ = \max\Bigg\{&G_p\left(x_{2n}, z, z\right), G_p\left(x_{2n}, x_{2n+1}, x_{2n+1}\right), G_p\left(z, gz, gz\right),\\ &\frac{\left[G_p\left(x_{2n}, gz, gz\right) + G_p\left(z, x_{2n+1}, x_{2n+1}\right)\right]}{2}\Bigg\}.\end{aligned} \tag{30}$$

By taking the limit as $n \to \infty$, we have $\lim_{n\to\infty} M\left(x_{2n}, z, z\right) = G_p(z, gz, gz)$. Thus

$$\begin{aligned}\psi\left(G_p\left(z, gz, gz\right)\right) &= \lim_{n\to\infty}\sup \psi\left(G_p\left(fx_{2n}, gz, gz\right)\right)\\ &\le \lim_{n\to\infty}\sup\left[F\left(\psi\left(M\left(x_{2n}, z, z\right)\right), \varphi\left(M\left(x_{2n}, z, z\right)\right)\right)\right]\\ &\le F\left(\psi\left(G_p\left(z, gz, gz\right)\right), \varphi\left(G_p\left(z, gz, gz\right)\right)\right)\\ &\le \psi\left(G_p\left(z, gz, gz\right)\right).\end{aligned} \tag{31}$$

Similarly, we may get $G_p(z, gz, gz) = 0$ and so $z = fz = gz$. □

Now, we provide some corollaries from our obtained result given by Theorem 19. First, putting $\psi(t) = t$ in Theorem 19, we obtain the following.

Corollary 20. *Let $(X, \preceq)$ be a partially ordered set. Let f and g be weakly increasing self-mappings on a complete G_p-metric space X satisfying*

$$G_p\left(fx, gy, gy\right) \le F\left(M\left(x, y, y\right), \varphi\left(M\left(x, y, y\right)\right)\right) \tag{32}$$

for all comparable $x, y \in X$, where $\varphi \in \Psi$, $F \in \mathscr{C}$, and

$$\begin{aligned}M\left(x, y, y\right) = \max\Bigg\{&G_p\left(x, y, y\right), G_p\left(x, fx, fx\right), G_p\left(y, gy, gy\right),\\ &\frac{\left[G_p\left(x, gy, gy\right) + G_p\left(y, fx, fx\right)\right]}{2}\Bigg\}.\end{aligned} \tag{33}$$

Assume either f or g is continuous, or $(X, G_p, \preceq)$ is regular. Then f and g have a common fixed point.

Proceeding as Theorem 19, we have the following.

Corollary 21. *Let $(X, \preceq)$ be a partially ordered set. Let f and g be weakly increasing self-mappings on a complete G_p-metric space X. Assume that that there exist $\psi, \varphi \in \Psi$ and $F \in \mathscr{C}$ such that*

$$\psi\left(G_p\left(fx, gy, gy\right)\right) \le F\left(\psi\left(M\left(x, y, y\right)\right), \varphi\left(M\left(x, y, y\right)\right)\right) \tag{34}$$

for all comparable $x, y \in X$, where

$$\begin{aligned}M\left(x, y, y\right) &= a_1 G_p\left(x, y, y\right) + a_2 G_p\left(x, fx, fx\right)\\ &\quad + a_3 G_p\left(y, gy, gy\right)\\ &\quad + a_4\left[G_p\left(x, gy, gy\right) + G_p\left(y, fx, fx\right)\right],\end{aligned} \tag{35}$$

where $a_i > 0$ for $i = \{1, 2, 3, 4\}$ with $a_1 + a_2 + a_3 + a_4 \le 1$. Assume either f or g is continuous or $(X, G_p, \preceq)$ is regular. Then f and g have a common fixed point.

The above corollary is the G_p-metric space version of Theorem 16 via the C-class function F, except that φ is taken in addition to the fact that it is continuous in $[0, \infty)$ (with respect to the conditions on φ in Theorem 16). If we set $\psi(t) = t$ in Corollary 21, we get the following.

Corollary 22. *Let $(X, \preceq)$ be a partially ordered set. Let f and g be weakly increasing self-mappings on a complete G_p-metric space X satisfying*

$$G_p\left(fx, gy, gy\right) \le F\left(M\left(x, y, y\right), \varphi\left(M\left(x, y, y\right)\right)\right) \tag{36}$$

for all comparable $x, y \in X$, where $\varphi \in \Psi$, $F \in \mathscr{C}$, and

$$\begin{aligned} M(x, y, y) &= a_1 G_p(x, y, y) + a_2 G_p(x, fx, fx) \\ &\quad + a_3 G_p(y, gy, gy) \\ &\quad + a_4 \left[G_p(x, gy, gy) + G_p(y, fx, fx)\right], \end{aligned} \tag{37}$$

where $a_i > 0$ for $i = 1, 2, 3, 4$ with $a_1 + a_2 + a_3 + 2a_4 \le 1$. Assume either f or g is continuous or $(X, G_p, \preceq)$ is regular. Then f and g have a common fixed point.

Taking $F(s, t) = ms$ with $0 < m < 1$ ($\psi(t) = \varphi(t) = t$) in Theorem 19, we state the following.

Corollary 23. *Let $(X, \preceq)$ be a partially ordered set. Let f and g be weakly increasing self-mappings on a complete G_p-metric space X satisfying*

$$\begin{aligned} G_p(fx, gy, gy) \le m \cdot \max\Big\{&G_p(x, y, y), \\ &G_p(x, fx, fx), G_p(y, gy, gy), \\ &\frac{\left[G_p(x, gy, gy) + G_p(y, fx, fx)\right]}{2}\Big\}, \end{aligned} \tag{38}$$

for all comparable $x, y \in X$. Assume either f or g is continuous or $(X, G_p, \preceq)$ is regular. Then f and g have a common fixed point.

Proceeding similarly as Theorem 19, we have the following.

Corollary 24. *Let $(X, \preceq)$ be a partially ordered set. Let f and g be weakly increasing self-mappings on a complete G_p-metric space X satisfying*

$$\begin{aligned} &\psi\left(G_p(fx, gy, gy)\right) \\ &\quad \le F\left(\psi\left(G_p(x, y, y)\right), \varphi\left(G_p(x, y, y)\right)\right) \end{aligned} \tag{39}$$

for all comparable $x, y \in X$, where $\psi, \varphi \in \Psi$ and $F \in \mathscr{C}$.

Assume either f or g is continuous or $(X, G_p, \preceq)$ is regular. Then f and g have a common fixed point.

Taking $F(s, t) = s/(1 + t)$ in Theorem 19, we have the following.

Corollary 25. *Let $(X, \preceq)$ be a partially ordered set. Let f and g be weakly increasing self-mappings on a complete G_p-metric space X satisfying*

$$G_p(fx, gy, gy) \le \frac{k G_p(x, y, y)}{1 + G_p(x, y, y)} \tag{40}$$

for all comparable $x, y \in X$, where $0 < k < 1$. Assume either f or g is continuous or $(X, G_p, \preceq)$ is regular. Then f and g have a common fixed point.

We provide the following example illustrating Theorem 19.

Example 26. Let $F(s, t) = s/(1 + t)$ for all $s, t \ge 0$. Let $X = [0, 1]$ be a set endowed with the partial order $x \preceq y \Leftrightarrow y \le x$. Let $G_p(x, y, z) = \max\{x, y, z\}$ be a G_p-metric on X, given $f, g : X \to X$ as $f(x) = x/12$ and

$$g(x) = \begin{cases} \dfrac{x}{6}; & x \in \left[0, \dfrac{1}{2}\right), \\ \dfrac{x^2}{2}; & x \in \left[\dfrac{1}{2}, 1\right]. \end{cases} \tag{41}$$

It is clear that f is continuous on (X, G_p) and the pair (f, g) is weakly increasing. Take $\psi(t) = t^2$ and $\varphi(t) = t$ for all $t \ge 0$. We shall prove that, for all $x, y \in [0, 1]$, we have

$$\psi\left(G_p(fx, gy, gy)\right) \le \frac{\psi(M(x, y, y))}{1 + \varphi(M(x, y, y))}. \tag{42}$$

First, for $x \in [0, 1/2)$ we have

$$\begin{aligned} \psi\left(G_p(fx, gy, gy)\right) &= \psi\left(\max\{fx, gy, gy\}\right) \\ &= \psi\left(\max\left\{\frac{x}{12}, \frac{y}{6}, \frac{y}{6}\right\}\right). \end{aligned} \tag{43}$$

Now, we will discuss the following two cases.

Case 1. Suppose that $x/12 < y/6$. We have

$$\psi\left(G_p(fx, gy, gy)\right) = \psi\left(\frac{y}{6}\right) = \frac{y^2}{36}. \tag{44}$$

Moreover

$$\begin{aligned} M(x, y, y) = \max\Big\{&G_p(x, y, y), G_p(x, fx, fx), \\ &G_p(y, gy, gy), \frac{\left[G_p(x, gy, gy) + G_p(y, fx, fx)\right]}{2}\Big\} \\ = \max\Big\{&\max\{x, y, y\}, \max\{x, fx, fx\}, \\ &\max\{y, gy, gy\}, \frac{1}{2}\left[\max\{x, gy, gy\}\right. \\ &\left.+ \max\{y, fx, fx\}\right]\Big\} = \max\Big\{x, x, y, \\ &\frac{1}{2}\left[x + \max\left\{y, \frac{x}{12}\right\}\right]\Big\} = x. \end{aligned} \tag{45}$$

Thus

$$\frac{\psi(M(x, y, y))}{1 + \varphi(M(x, y, y))} = \frac{\psi(x)}{1 + \varphi(x)} = \frac{x^2}{1 + x}. \tag{46}$$

Therefore

$$\psi\left(G_p\left(fx,gy,gy\right)\right)=\frac{y^2}{36}\le\frac{x^2}{36}\le\frac{x^2}{1+x}=\frac{\psi\left(M\left(x,y,y\right)\right)}{1+\varphi\left(M\left(x,y,y\right)\right)}. \tag{47}$$

Thus (42) holds.

Case 2. Suppose that $x/12\ge y/6$; hence

$$\psi\left(G_p\left(fx,gy,gy\right)\right)=\psi\left(\frac{x}{12}\right)=\frac{x^2}{144}. \tag{48}$$

Moreover

$$\begin{aligned}M\left(x,y,y\right)&=\max\left\{G_p\left(x,y,y\right),G_p\left(x,fx,fx\right),G_p\left(y,gy,gy\right),\frac{\left[G_p\left(x,gy,gy\right)+G_p\left(y,fx,fx\right)\right]}{2}\right\}\\&=\max\left\{\max\left\{x,y,y\right\},\max\left\{x,fx,fx\right\},\max\left\{y,gy,gy\right\},\frac{1}{2}\left[\max\left\{x,gy,gy\right\}+\max\left\{y,fx,fx\right\}\right]\right\}=\max\left\{x,x,y,\frac{1}{2}\left[x+\max\left\{y,\frac{x}{12}\right\}\right]\right\}=x.\end{aligned} \tag{49}$$

Therefore

$$\frac{\psi\left(M\left(x,y,y\right)\right)}{1+\varphi\left(M\left(x,y,y\right)\right)}=\frac{\psi\left(x\right)}{1+\varphi\left(x\right)}=\frac{x^2}{1+x}. \tag{50}$$

Thus

$$\psi\left(G_p\left(fx,gy,gy\right)\right)=\frac{x^2}{144}\le\frac{x^2}{1+x}=\frac{\psi\left(M\left(x,y,y\right)\right)}{1+\varphi\left(M\left(x,y,y\right)\right)}; \tag{51}$$

that is, (42) holds.

Second, for $x\in[1/2,1]$

$$\psi\left(G_p\left(fx,gy,gy\right)\right)=\psi\left(\max\left\{fx,gy,gy\right\}\right)=\psi\left(\max\left\{\frac{x}{12},y^2,y^2\right\}\right). \tag{52}$$

Let us discuss the following two cases.

Case 1. Suppose that $x/12<y^2/2$. Hence

$$\psi\left(G_p\left(fx,gy,gy\right)\right)=\psi\left(\frac{y^2}{2}\right)=\frac{y^4}{4}. \tag{53}$$

Moreover

$$\begin{aligned}M\left(x,y,y\right)&=\max\left\{G_p\left(x,y,y\right),G_p\left(x,fx,fx\right),G_p\left(y,gy,gy\right),\frac{\left[G_p\left(x,gy,gy\right)+G_p\left(y,fx,fx\right)\right]}{2}\right\}\\&=\max\left\{\max\left\{x,y,y\right\},\max\left\{x,fx,fx\right\},\max\left\{y,gy,gy\right\},\frac{1}{2}\left[\max\left\{x,gy,gy\right\}+\max\left\{y,fx,fx\right\}\right]\right\}=\max\left\{x,x,y,\frac{1}{2}\left[x+\max\left\{y,\frac{x}{12}\right\}\right]\right\}=x.\end{aligned} \tag{54}$$

Therefore

$$\frac{\psi\left(M\left(x,y,y\right)\right)}{1+\varphi\left(M\left(x,y,y\right)\right)}=\frac{\psi\left(x\right)}{1+\varphi\left(x\right)}=\frac{x^2}{1+x}. \tag{55}$$

We deduce

$$\psi\left(G_p\left(fx,gy,gy\right)\right)=\frac{y^4}{4}\le\frac{x^2}{4}\le\frac{x^2}{1+x}=\frac{\psi\left(M\left(x,y,y\right)\right)}{1+\varphi\left(M\left(x,y,y\right)\right)}; \tag{56}$$

that is, (42) holds.

Case 2. Suppose that $x/12\ge y^2/2$. Then

$$\psi\left(G_p\left(fx,gy,gy\right)\right)=\psi\left(\frac{x}{12}\right)=\frac{x^2}{144}. \tag{57}$$

Moreover

$$\begin{aligned}M\left(x,y,y\right)&=\max\left\{G_p\left(x,y,y\right),G_p\left(x,fx,fx\right),G_p\left(y,gy,gy\right),\frac{\left[G_p\left(x,gy,gy\right)+G_p\left(y,fx,fx\right)\right]}{2}\right\}\\&=\max\left\{\max\left\{x,y,y\right\},\max\left\{x,fx,fx\right\},\max\left\{y,gy,gy\right\},\frac{1}{2}\left[\max\left\{x,gy,gy\right\}+\max\left\{y,fx,fx\right\}\right]\right\}=\max\left\{x,x,y,\frac{1}{2}\left[x+\max\left\{y,\frac{x}{12}\right\}\right]\right\}=x.\end{aligned} \tag{58}$$

Therefore

$$\frac{\psi\left(M\left(x,y,y\right)\right)}{1+\varphi\left(M\left(x,y,y\right)\right)}=\frac{\psi\left(x\right)}{1+\varphi\left(x\right)}=\frac{x^2}{1+x}. \tag{59}$$

Thus

$$\psi\left(G_p\left(fx, gy, gy\right)\right) = \frac{x^2}{144} \leq \frac{x^2}{1+x} = \frac{\psi\left(M\left(x, y, y\right)\right)}{1+\varphi\left(M\left(x, y, y\right)\right)}. \quad (60)$$

Then (42) holds. All the conditions of Theorem 19 are satisfied and 0 is the common fixed point for f and g.

On the other hand, the contractive condition of Theorem 17 is not satisfied. Indeed, for $x = 1$ and $y = 1/3$, we have

$$\psi\left(G_p\left(fx, gy, gy\right)\right) = \frac{x^2}{144} = \frac{1}{144} > 0 = x^2 - x = \psi\left(M\left(x, y, y\right)\right) - \varphi\left(M\left(x, y, y\right)\right). \quad (61)$$

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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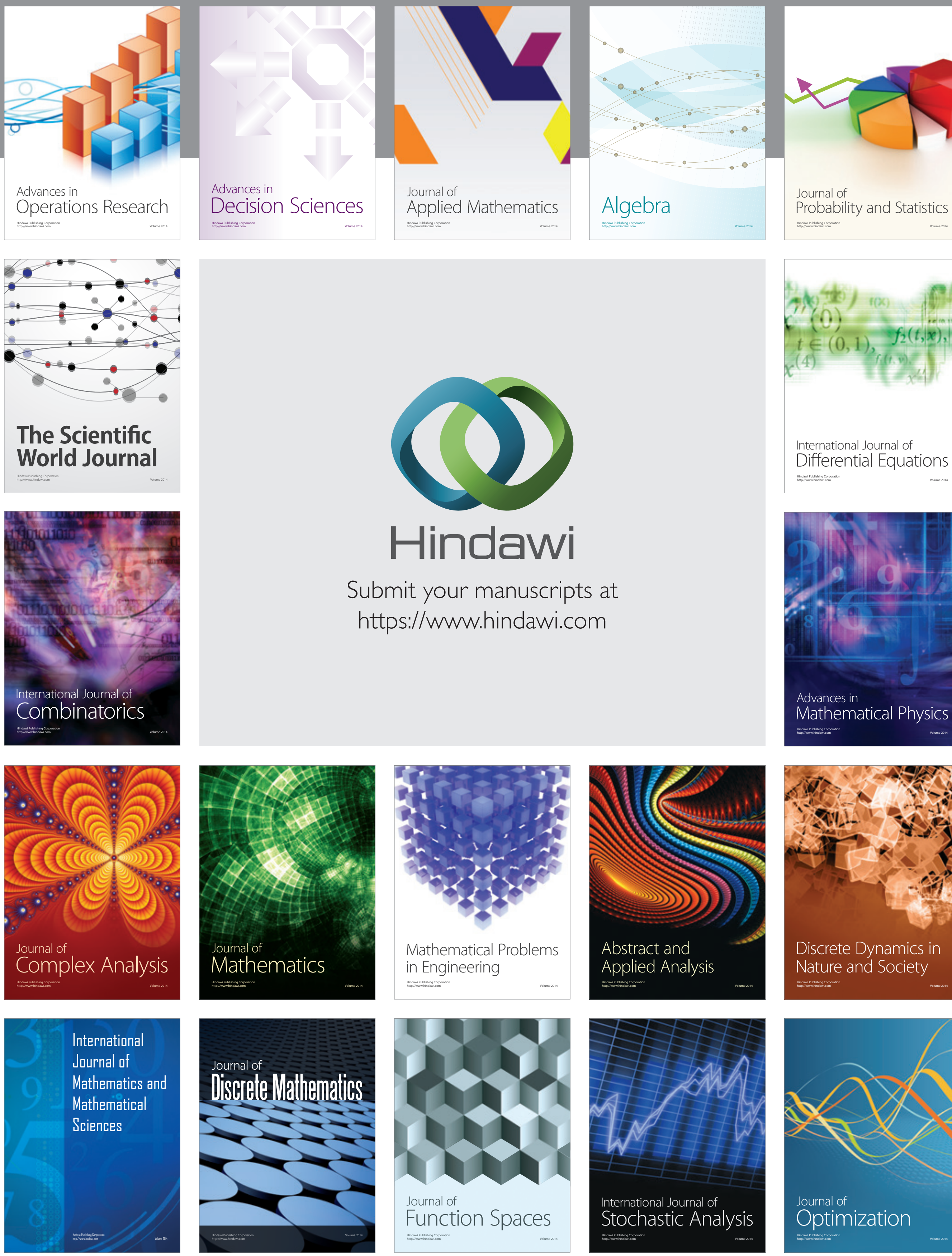
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