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Research Article

Positive Solutions for Boundary Value Problems of Fractional Differential Equation with Integral Boundary Conditions

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By using two fixed-point theorems on cone, we discuss the existence results of positive solutions for the following boundary value problem of fractional differential equation with integral boundary conditions: $D_{0^+}^{\alpha}x(t)+a(t)f(t,x(t))=0, t\in(0,1), x(0)=x'(0)=0$, and $x(1)=\int_0^1 x(t)dA(t)$.

1. Introduction

Boundary value problem for fractional differential equation has aroused much attention in the past few years; many professors devoted themselves to the solvability of fractional differential equations, especially to the study of the existence of solutions for boundary value problems of fractional differential equation (see [1–28]). For example, Wang et al. [19] studied the existence of positive solutions for the following problem:

$$D_{0^+}^{\alpha}u(t)+\lambda f(t,u(t))=0,\quad t\in(0,1), \tag{1}$$

with the boundary conditions

$$u(0)=u'(0)=u(1)=0, \tag{2}$$

where $D_{0^+}^{\alpha}$ is the Riemann-Liouville differential operator of $2<\alpha\le 3$, λ is a positive parameter, and f may be singular at $t=0,1$ and may change sign. And Ma [14] discussed the positive solutions of

$$\begin{gathered} D_{0^+}^{\alpha}u(t)+a(t)f(t,u(t))=0,\quad t\in(0,1), \\ u(0)=u'(0)=0, \\ u(1)=\sum_{i=1}^{m}\beta_i u(\xi_i), \end{gathered} \tag{3}$$

where $m\ge 1$ is integer and $\xi_i,\beta_i>0$.

There have already been lots of books and papers involving the positive solutions for boundary value problems of fractional differential equation; however, only a few papers cover that for fractional differential equation boundary value problems with integral boundary conditions. Motivated by [14], we shall investigate the positive solutions of the following boundary value problem:

$$\begin{gathered} D_{0^+}^{\alpha}x(t)+a(t)f(t,x(t))=0,\quad t\in(0,1), \\ x(0)=x'(0)=0, \\ x(1)=\int_0^1 x(t)\,dA(t), \end{gathered} \tag{4}$$

where $D_{0^+}^{\alpha}$ is the Riemann-Liouville differential operator of $2<\alpha<3$, $A(t)$ is right continuous on $[0,1)$, left continuous at $t=1$, and nondecreasing on $[0,1]$ with $A(0)=0$, and

$\int_0^1 u(t)dA(t)$ denotes the Riemann-Stieltjes integrals of u with respect to A. And $a(t)$, $f(t, x(t))$ satisfies the following conditions:

(H1) $a \in L[0,1]$ is nonnegative and not identically zero on any compact subset of $(0,1)$, $\sigma = \int_0^1 t^{\alpha-1} dA(t) < 1$.

(H2) $f : [0,1] \times [0,+\infty) \to [0,+\infty)$ is continuous.

This paper consists of four sections. After the introduction, we recall some definitions, lemmas, and theorems in Section 2. And the main results of this paper are stated in Section 3. In the last section, we give two examples of the main results.

2. Preliminaries

Firstly, for convenience we recall some definitions, lemmas, and theorems.

Definition 1 (see [29, 30]). Let $f \in L^1(R^+)$ define the Riemann-Liouville fractional integral of order $\alpha > 0$ for f as

$$I_{0^+}^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t f(s)(t-s)^{\alpha-1} ds, \tag{5}$$

where $\Gamma(\alpha)$ is Euler gamma function.

Definition 2 (see [29, 30]). Define the Riemann-Liouville fractional derivative of order $\alpha > 0$ for f as

$$D_{0^+}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^{(n)} \int_0^t \frac{f(s)}{(t-s)^{\alpha+1-n}} ds, \quad n = [\alpha] + 1, \tag{6}$$

where f has absolutely continuous derivatives up to order $(n-1)$ on R^+.

Lemma 3. *Let $y \in C[0,1]$, $2 < \alpha \le 3$; then the boundary value problem*

$$\begin{gathered} D_{0^+}^{\alpha} x(t) + y(t) = 0, \quad t \in (0,1), \\ x(0) = x'(0) = 0, \\ x(1) = \int_0^1 x(t)\, dA(t) \end{gathered} \tag{7}$$

has the unique solution $x(t) = \int_0^1 G_1(t,s) y(s) ds$, where

$$G_1(t,s) = G(t,s) + \frac{t^{\alpha-1}}{1-\sigma} \int_0^1 G(\tau, s)\, dA(\tau), \tag{8}$$

$$G(t,s) = \begin{cases} \dfrac{t^{\alpha-1}(1-s)^{\alpha-1} - (t-s)^{\alpha-1}}{\Gamma(\alpha)}, & 0 \le s \le t \le 1; \\ \dfrac{t^{\alpha-1}(1-s)^{\alpha-1}}{\Gamma(\alpha)}, & 0 \le t \le s \le 1. \end{cases} \tag{9}$$

Proof. The boundary value problem can be converted to an equivalent integral equation:

$$x(t) = -I_{0^+}^{\alpha} y(t) + c_1 t^{\alpha-1} + c_2 t^{\alpha-2} + c_3 t^{\alpha-3}, \quad c_1, c_2, c_3 \in R. \tag{10}$$

Then the solution is

$$x(t) = -\frac{1}{\Gamma(\alpha)} \int_0^t y(s)(t-s)^{\alpha-1} ds + c_1 t^{\alpha-1} + c_2 t^{\alpha-2} + c_3 t^{\alpha-3}. \tag{11}$$

It follows from the boundary conditions $x(0) = x'(0) = 0$ that $c_3 = c_2 = 0$ and

$$c_1 = \frac{1}{\Gamma(\alpha)} \int_0^1 y(s)(1-s)^{\alpha-1} ds + \int_0^1 x(s)\, dA(s). \tag{12}$$

Thus we get

$$\begin{aligned} x(t) &= -\frac{1}{\Gamma(\alpha)} \int_0^t y(s)(t-s)^{\alpha-1} ds \\ &\quad + \frac{1}{\Gamma(\alpha)} \int_0^1 y(s)(1-s)^{\alpha-1} ds \cdot t^{\alpha-1} \\ &\quad + \int_0^1 x(s)\, dA(s) \cdot t^{\alpha-1} \\ &= \int_0^1 G(t,s) y(s) ds + \int_0^1 x(s)\, dA(s) \cdot t^{\alpha-1}. \end{aligned} \tag{13}$$

Then we can obtain

$$\begin{aligned} \int_0^1 x(s)\, dA(s) &= \int_0^1 \left[\int_0^1 G(s,\tau) y(\tau) d\tau \right. \\ &\quad \left. + \int_0^1 x(\tau)\, dA(\tau) s^{\alpha-1} \right] dA(s) \\ &= \int_0^1 \int_0^1 G(s,\tau) y(\tau) d\tau\, dA(s) \\ &\quad + \int_0^1 x(s)\, dA(s) \cdot \int_0^1 t^{\alpha-1} dA(t), \end{aligned} \tag{14}$$

which means

$$\int_0^1 x(s)\, dA(s) = \frac{\int_0^1 \int_0^1 G(s,\tau) y(\tau) d\tau\, dA(s)}{1 - \int_0^1 t^{\alpha-1} dA(t)}. \tag{15}$$

So

$$\begin{aligned} x(t) &= \int_0^1 G(t,s)\, y(s)\, ds \\ &\quad + \frac{\int_0^1 \int_0^1 G(s,\tau)\, y(\tau)\, d\tau\, dA(s)}{1-\int_0^1 t^{\alpha-1} dA(s)} t^{\alpha-1} \\ &= \int_0^1 \left[G(t,s) + \frac{t^{\alpha-1}}{1-\sigma} \int_0^1 G(\tau,s)\, dA(\tau) \right] y(s)\, ds \\ &= \int_0^1 G_1(t,s)\, y(s)\, ds. \end{aligned} \tag{16}$$

□

Lemma 4 (see [20]). *$G(t,s)$ defined in (9) has the following properties:*

(i) $G(t,s) > 0$, $t,s \in (0,1)$.

(ii) $G(t,s) = G(1-s, 1-t)$, $t,s \in [0,1]$.

(iii) $k(1-t)k(s) \le \Gamma(\alpha)G(t,s) \le (\alpha-1)k(s)$, $t,s \in [0,1]$ *where* $k(t) = t(1-t)^{\alpha-1}$.

Lemma 5. *If (H2) is satisfied, then $G_1(t,s)$ defined in (8) has the following properties:*

$$k(1-t)\, k(s) \le \Gamma(\alpha)\, G_1(t,s) \le Lk(s), \quad t,s \in [0,1], \tag{17}$$

where $L = (\alpha-1)(1+\int_0^1 dA(s)/(1-\sigma))$.

Proof. The proof can be easily accomplished by Lemma 4, so we omitted it. □

Theorem 6 (see [31]). *Let E be a Banach space and $P \subseteq E$ be a cone in E. Suppose that Ω_1, Ω_2 are two bounded open sets of E with $\overline{\Omega_1} \subset \Omega_2$. Assume that $T : P \cap (\overline{\Omega_2} \setminus \Omega_1) \to P$ is a completely continuous operator such that either*

(i) *$\|Tx\| \le \|x\|$ for any $x \in P \cap \partial\Omega_1$ and $\|Tx\| \ge \|x\|$ for any $x \in P \cap \partial\Omega_2$ or*

(ii) *$\|Tx\| \ge \|x\|$ for any $x \in P \cap \partial\Omega_1$ and $\|Tx\| \le \|x\|$ for any $x \in P \cap \partial\Omega_2$.*

Then T has a fixed point in $P \cap (\overline{\Omega_2} \setminus \Omega_1)$.

Theorem 7 (see [32]). *Let P be a cone of a Banach space E. $P_c = \{x \in P : \|x\| < c\}$. θ is a nonnegative continuous concave function on P, such that, for any $x \in \overline{P_c}$, $\theta(x) \le \|x\|$, and $P(\theta,b,d) = \{x \in P : \theta(x) \ge b,\ \|x\| \le d\}$. Assume that $T : \overline{P_c} \to \overline{P_c}$ is completely continuous, and there exist constants $a < b < d \le c$ such that*

(c1) *$\{x \in P(\theta,b,d) : \theta(x) > b\} \ne \emptyset$ and $\theta(Tx) > b$ for $x \in P(\theta,b,d)$;*

(c2) *$\|Tx\| < a$ for $x \in \overline{P_a}$;*

(c3) *$\theta(Tx) > b$ for any $x \in P(\theta,b,c)$ with $\|Tx\| > d$.*

Then T has at least three fixed points x_1, x_2, x_3 with $\|x_1\| < a$, $b < \theta(x_2)$, $\|x_3\| > a$, and $\theta(x_3) < b$.

Let $E = C[0,1]$ be a Banach space with the maximum norm $\|x\| = \max_{0\le t\le 1}|x(t)|$; define the cone $P \subseteq E$ as

$$P = \{x \in E : x(t) \ge 0,\ t \in [0,1]\}. \tag{18}$$

Define a continuous operator $T : P \to E$ as

$$(Tx)(t) = \int_0^1 G_1(t,s)\, a(s)\, f(s, x(s))\, ds. \tag{19}$$

Lemma 8. *Assume that (H1) and (H2) hold; then $T : P \to P$ is a completely continuous operator.*

Proof. The lemma can be easily proven, so we omitted it. □

3. Main Results

We define the following notation: given $\delta \in (0, 1/2)$, take

$$\begin{aligned} h &= \min_{\delta\le t\le 1-\delta} k(1-t) = \min_{\delta\le t\le 1-\delta} (1-t)\, t^{\alpha-1}, \\ M &= \frac{\Gamma(\alpha)}{L\int_0^1 k(s)\, a(s)\, ds}, \\ N &= \frac{\Gamma(\alpha)}{h\int_\delta^{1-\delta} k(s)\, a(s)\, ds}. \end{aligned} \tag{20}$$

Now we can obtain the following theorems.

Theorem 9. *Suppose that (H1) and (H2) are satisfied; there exist two positive constants $r_2 > r_1 > 0$ such that*

(H3) *$f(t,x) \le Mr_2$, $(t,x) \in [0,1] \times [0,r_2]$;*

(H4) *$f(t,x) \ge Nr_1$, $(t,x) \in [0,1] \times [0,r_1]$.*

Then boundary value problem (4) has at least one positive solution $x \in P$ such that $r_1 \le \|x\| \le r_2$.

Proof. The solution of boundary value problem (4) is equivalent to the fixed point of operator T. Let $\Omega_2 = \{x \in P : \|x\| < r_2\}$; when $x \in \partial\Omega_2$, for any $t \in [0,1]$ we have $0 \le x(t) \le r_2$. By Lemma 5 and (H3) we get

$$\begin{aligned} (Tx)(t) &= \int_0^1 G_1(t,s)\, a(s)\, f(s,x(s))\, ds \\ &\le \frac{LMr_2}{\Gamma(\alpha)} \int_0^1 k(s)\, a(s)\, ds = r_2 = \|x\|, \end{aligned} \tag{21}$$

which means when $x \in \partial\Omega_2$, $\|Tx\| \le \|x\|$.

Let $\Omega_1 = \{x \in P : \|x\| < r_1\}$; when $x \in \partial\Omega_1$, for any $t \in [0,1]$ we have $0 \le x(t) \le r_1$. By Lemma 5 and (H4) we get

$$\begin{aligned}(Tx)(t) &= \int_0^1 G_1(t,s)\,a(s)\,f(s,x(s))\,ds \\ &\ge k(1-t)\frac{Nr_1}{\Gamma(\alpha)}\int_0^1 k(s)\,a(s)\,ds \\ &\ge \min_{\delta\le t\le 1-\delta} k(1-t)\frac{Nr_1}{\Gamma(\alpha)}\int_\delta^{1-\delta} k(s)\,a(s)\,ds \\ &= r_1 = \|x\|,\end{aligned} \tag{22}$$

which means when $x \in \partial\Omega_1$, $\|Tx\| \ge \|x\|$.

It follows from Theorem 6 that we know that T has at least one fixed point in $(\overline{\Omega_2} \setminus \Omega_1)$, which means that the boundary value problem (4) has at least one solution. □

Theorem 10. *Suppose that (H1) and (H2) are satisfied; there exist four positive constants a, b, c, d with $0 < a < b < (h/L)d < d < c$, such that*

(H5) $f(t,x) < Ma$, $(t,x) \in [0,1]\times[0,a]$;

(H6) $f(t,x) > Nb$, $(t,x) \in [\delta,1-\delta]\times[b,d]$;

(H7) $f(t,x) \le Mc$, $(t,x) \in [0,1]\times[0,c]$.

Then boundary value problem (4) has at least three positive solutions x_1, x_2, x_3, such that

$$\begin{aligned}&\max_{0\le t\le 1}|x_1(t)| < a, \\ &b < \min_{\delta\le t\le 1-\delta}|x_2(t)| < \max_{0\le t\le 1}|x_2(t)| \le c, \\ &a < \max_{0\le t\le 1}|x_3(t)| \le c, \\ &\min_{\delta\le t\le 1-\delta}|x_3(t)| < b.\end{aligned} \tag{23}$$

Proof. Define a nonnegative continuous concave function θ on P as

$$\theta(x) = \min_{\delta\le t\le 1-\delta} x(t). \tag{24}$$

If $x \in \overline{P_c} = \{x \in P : \|x\| \le c\}$, then $\|x\| \le c$; it follows from (H7) that $f(t,x) \le Mc$; hence

$$\begin{aligned}\|Tx\| &\le \frac{L}{\Gamma(\alpha)}\int_0^1 k(s)\,a(s)\,f(s,x(s))\,ds \\ &\le \frac{LMc}{\Gamma(\alpha)}\int_0^1 k(s)\,a(s)\,ds = c.\end{aligned} \tag{25}$$

Thus, $T(\overline{P_c}) \subseteq \overline{P_c}$. It follows from Lemma 8 that T is completely continuous. In the same way, let $x \in \overline{P_a}$; it follows from (H5) that $f(t,x) < Ma$ for any $t \in [0,1]$, which shows that condition (c2) of Theorem 7 is fulfilled.

Let $x(t) = (b+d)/2$; it is easy to know that $x \in P(\theta,b,d)$ and $\{x \in P(\theta,b,d) : \theta(x) > b\} \ne \emptyset$. If $x \in P(\theta,b,d)$, we have $b \le x(t) \le d$ for any $t \in [\delta,1-\delta]$. We know $f(t,x(t)) > Nb$ for $\delta \le t \le 1-\delta$ by (H6). So we get

$$\begin{aligned}\theta(Tx) &= \min_{\delta\le t\le 1-\delta} Tx(t) \\ &= \min_{\delta\le t\le 1-\delta}\int_0^1 G_1(t,s)\,a(s)\,f(s,x(s))\,ds \\ &\ge \min_{\delta\le t\le 1-\delta}\int_0^1 \frac{k(1-t)\,k(s)}{\Gamma(\alpha)}a(s)\,f(s,x(s))\,ds \\ &> \frac{Nhb}{\Gamma(\alpha)}\int_\delta^{1-\delta} k(s)\,a(s)\,ds = b.\end{aligned} \tag{26}$$

So condition (c1) of Theorem 7 holds.

When $x \in P(\theta,b,c)$ with $\|Tx\| > d$, noting that

$$\|Tx\| \le \frac{L}{\Gamma(\alpha)}\int_0^1 k(s)\,a(s)\,f(s,x(s))\,ds, \tag{27}$$

thus

$$\begin{aligned}(Tx)(t) &\ge \frac{k(1-t)}{\Gamma(\alpha)}\int_0^1 k(s)\,a(s)\,f(s,x(s))\,ds \\ &\ge \frac{k(1-t)}{L}\|Tx\|,\end{aligned} \tag{28}$$

so we obtain

$$\begin{aligned}\theta(Tx) = \min_{\delta\le t\le 1-\delta} Tx(t) &\ge \frac{\min_{\delta\le t\le 1-\delta} k(1-t)}{L}\|Tx\| \\ &> \frac{h}{L}d > b.\end{aligned} \tag{29}$$

That is to say, (c3) is satisfied.

All conditions of Theorem 7 are satisfied, so T has at least three fixed points x_1, x_2, x_3, which means that the boundary value problem (4) has at least three positive solutions x_1, x_2, x_3, such that

$$\begin{aligned}&\max_{0\le t\le 1}|x_1(t)| < a, \\ &b < \min_{\delta\le t\le 1-\delta}|x_2(t)| < \max_{0\le t\le 1}|x_2(t)| \le c, \\ &a < \max_{0\le t\le 1}|x_3(t)| \le c, \\ &\min_{\delta\le t\le 1-\delta}|x_3(t)| < b.\end{aligned} \tag{30}$$

The proof of this theorem is finished. □

4. Some Examples

Now we present two examples to illustrate our main results.

Example 1. Let us see the following problem:

$$\begin{aligned}&D_{0^+}^{5/2}x(t) + \frac{1}{4}\sin t + x + 3 = 0, \quad t \in (0,1), \\ &x(0) = x'(0) = 0, \\ &x(1) = \int_0^1 x(t)\,dt.\end{aligned} \tag{31}$$

Choose $\delta = 1/3$; we obtain that $L = 4$, $h = 2/\sqrt{243}$, $M = 105\sqrt{\pi}/64 \approx 2.91$, and $N = 76545\sqrt{\pi}/(704\sqrt{2} - 256) \approx 183.48$. Then for any $(t, x) \in [0, 1] \times [0, 2]$, we have $f(t, x) = (1/4)\sin t + x + 3 \leq 5.25 \leq Mr_2 \approx 5.82$, and for any $(t, x) \in [0, 1] \times [0, 0.01]$, we get $f(t, x) = (1/4)\sin t + x + 3 \geq 3 \geq Nr_1 \approx 1.83$.

Then the boundary value problem has at least one positive solution $x \in P$ such that $0.01 \leq \|x\| \leq 2$.

Example 2. We now study the following problem:

$$\begin{gathered} D_{0^+}^{5/2} x(t) + f(t, x) = 0, \quad t \in (0, 1), \\ x(0) = x'(0) = 0, \\ x(1) = \int_0^1 x(t)\, dt, \end{gathered} \tag{32}$$

where

$$f(t, x) = \begin{cases} \dfrac{t}{3} + x^2, & x \leq 1; \\ 183 + \dfrac{t}{3} + x, & x > 1. \end{cases} \tag{33}$$

Choose $\delta = 1/3$; we have $M \approx 2.91$, $N \approx 183.48$, $L = 4$, and $h \approx 0.1283$. Let $a = 1/2$, $b = 1$, $c = 100$, and $d = 35$; then for any $(t, x) \in [0, 1] \times [0, 1/2]$, we have $f(t, x) = t/3 + x^2 \leq 0.59 < Ma \approx 1.46$, for any $(t, x) \in [1/3, 2/3] \times [1, 35]$, we have $f(t, x) = 183 + t/3 + x \geq 184.11 > Nb \approx 183.48$, and for any $(t, x) \in [0, 1] \times [0, 100]$, we have $f(t, x) = 183 + t/3 + x \leq 283.34 < Mc \approx 291$. Then by Theorem 10, we conclude that this boundary value problem has at least three positive solutions x_1, x_2, x_3, such that

$$\begin{gathered} \max_{0 \leq t \leq 1} |x_1(t)| < \frac{1}{2}, \\ 1 < \min_{\delta \leq t \leq 1-\delta} |x_2(t)| < \max_{0 \leq t \leq 1} |x_2(t)| \leq 100, \\ \frac{1}{2} < \max_{0 \leq t \leq 1} |x_3(t)| \leq 100, \\ \min_{\delta \leq t \leq 1-\delta} |x_3(t)| < 1. \end{gathered} \tag{34}$$

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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