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Research Article

A Note on the Fractional Generalized Higher Order KdV Equation

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We obtain exact solutions to the fractional generalized higher order Korteweg-de Vries (KdV) equation using the complex method. It has showed that the applied method is very useful and is practically well suited for the nonlinear differential equations, those arising in mathematical physics.

1. Introduction

Nonlinear fractional differential equations (NFDEs) are universally applied in signal processing, electrical networks, acoustics, fluid dynamics, biology, chemistry, physics, etc. For example, the singular behaviours [1–9] and impulsive phenomena [10–19] often exhibit some blow-up properties [20–25] which occur in a lot of complex physical processes. NFDEs have been attracted extensive attention and have been widely investigated [26–38]. Exact solutions of NFDEs play an important role in the study of mathematical physics phenomena. Therefore, seeking exact solutions of NFDEs is an interesting and significant subject.

The fractional generalized higher order KdV equation is a useful model. Applying the generalized exp(-$\Phi(\zeta)$)-expansion method, Lu *et al.* [39] obtained exact solutions of this equation. In this article, we would like to utilize the complex method [40–43] to seek exact solutions to the fractional generalized higher order KdV equation.

2. Preliminaries

Let $f : R \longrightarrow R$, $x \longrightarrow f(x)$ be a continuous function and $\nu > 0$ denote a constant discrete span. Define the operator $FW(\nu)$ as follows:

$$FW(\nu) f(x) := f(x+\nu), \tag{1}$$

then the fractional difference of $f(x)$ of order μ can be expressed as

$$\begin{aligned}\Delta^{\mu} f(x) &= (FW-1)^{\mu} f(x) \\ &= \sum_{n=0}^{\infty} (-1)^n \binom{\mu}{n} f[x+(\mu-n)\nu],\end{aligned} \tag{2}$$

where $0 < \mu \le 1$, and its fractional derivative of order μ can be expressed as

$$f^{(\mu)}(x) = \lim_{\nu \longrightarrow 0} \left(\frac{\sum_{n=0}^{\infty} (-1)^n \binom{\mu}{n} f[x+(\mu-n)\nu]}{\nu^{\mu}} \right). \tag{3}$$

The above is expressed as

$$\begin{aligned} &\frac{1}{\Gamma(-\mu)} \int_0^x (x-z)^{-\mu-1} f(z)\,dz, \quad \mu < 0, \\ &\frac{1}{\Gamma(1-\mu)} \frac{d}{d_x} \int_0^x (x-z)^{-\mu} [f(z) - f(0)]\,dz, \\ &\qquad\qquad 0 < \mu < 1, \\ &\left(f^{(\mu-n)}(x)\right)^{(n)}, \quad n \le \mu \le n+1,\ n \ge 1. \end{aligned} \tag{4}$$

Further, Jumarie's modified Riemann-Liouville derivative [44, 45] is given by

$$D_x^{\mu} x^{\gamma} = \frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-\mu)} x^{(\gamma-\mu)}, \quad \gamma > 0, \tag{5}$$

then its related NFDE is given by

$$f\left(u, u_t, u_x, D_t^{\mu} u, D_x^{\mu} u, D_t^{\gamma} u, D_x^{\gamma} u, \ldots\right)=0, \quad 0<\mu \leq 1. \tag{6}$$

Let $m \in \mathbb{N}:=\{1,2,3,\ldots\}$, $r_j \in\{0,1,2,\ldots\}$, $j=0,1,\ldots,m$, $r=\left(r_0, r_1, \ldots, r_m\right)$, and

$$K_r[w](z):=\prod_{j=0}^m\left[w^{(j)}(z)\right]^{r_j}, \tag{7}$$

then the degree of $K_r[w]$ is denoted by $d(r):=\sum_{j=0}^m r_j$. We define the differential polynomial as

$$F\left(w, w', \ldots, w^{(m)}\right):=\sum_{r \in H} a_r K_r[w], \tag{8}$$

in which H is a finite index set and a_r are constants. The degree of $F(w, w', \ldots, w^{(m)})$ can be denoted by $\operatorname{deg} F(w, w', \ldots, w^{(m)}):=\max_{r \in H}\{d(r)\}$.

The ordinary differential equation (ODE) is given by

$$F\left(w, w', \ldots, w^{(m)}\right)=c w^n+d, \tag{9}$$

where $c \neq 0, d$ are constants, $n \in \mathbb{N}$.

Suppose that the meromorphic solutions w of (9) have at least one pole. Let $p, q \in \mathbb{N}$ and insert the Laurent series

$$w(z)=\sum_{\tau=-q}^{\infty} \beta_\tau z^\tau, \quad \beta_{-q} \neq 0,\ q>0, \tag{10}$$

into (9); if it is determined p different Laurent singular parts:

$$\sum_{\tau=-q}^{-1} \beta_\tau z^\tau, \tag{11}$$

then (9) is said to satisfy the weak $\langle p, q\rangle$ condition.

Give two complex numbers ν_1, ν_2 such that $\operatorname{Im}(\nu_1/\nu_2)>0$, let L be the discrete subset $L[2\nu_1, 2\nu_2]:=\{\nu \mid \nu=2c_1\nu_1+2c_2\nu_2,\ c_1, c_2 \in \mathbb{Z}\}$, and L is isomorphic to $\mathbb{Z} \times \mathbb{Z}$. Let the discriminant $\Delta=\Delta(b_1, b_2):=b_1^3-27 b_2^2$ and

$$h_n:=h_n(L):=\sum_{\nu \in L \backslash\{0\}} \frac{1}{\nu^n}. \tag{12}$$

A meromorphic function $\wp(z):=\wp(z, g_2, g_3)$ with double periods $2\nu_1, 2\nu_2$, which satisfies the equation

$$\left(\wp'(z)\right)^2=4 \wp^3(z)-g_2 \wp(z)-g_3, \tag{13}$$

in which $\Delta(g_2, g_3) \neq 0$, $g_2=60 h_4$, and $g_3=140 h_6$, is called the Weierstrass elliptic function.

If a meromorphic function ς is a rational function of z, an elliptic function, or a rational function of $e^{\alpha z}$, $\alpha \in \mathbb{C}$, then we say that ς belongs to the class W.

In 2009, Eremenko *et al.* [46] considered the following m-order Briot-Bouquet equation (BBEq):

$$F\left(w, w^{(m)}\right)=\sum_{j=0}^n F_j(w)\left(w^{(m)}\right)^j=0, \tag{14}$$

where $m \in \mathbb{N}$ and $F_j(w)$ are constant coefficient polynomials. For the m-order BBEq, we have the lemma as follows.

Lemma 1 (see [41, 47, 48]). *Let $m, s, n, p \in \mathbb{N}$, and $\operatorname{deg} F(w, w^{(m)})<n$, and the m-order BBEq*

$$F\left(w, w^{(m)}\right)=c w^n+d \tag{15}$$

satisfies weak $\langle p, q\rangle$ condition, then the meromorphic solutions $w(z)$ belong to the class W. Suppose that for some values of parameters such solutions w exist, then other meromorphic solutions should form one-parametric family $(z-z_0)$, $z_0 \in \mathbb{C}$. Then each elliptic solution with a pole at $z=0$ can be expressed as

$$\begin{aligned} w(z)= & \sum_{i=1}^{s-1} \sum_{j=2}^q \frac{(-1)^j \beta_{-ij}}{(j-1)!} \frac{d^{j-2}}{d z^{j-2}} \\ & \cdot\left(\frac{1}{4}\left[\frac{\wp'(z)+C_i}{\wp(z)-D_i}\right]^2-\wp(z)\right)+\sum_{i=1}^{s-1} \frac{\beta_{-i1}}{2} \\ & \cdot \frac{\wp'(z)+C_i}{\wp(z)-D_i}+\sum_{j=2}^q \frac{(-1)^j \beta_{-sj}}{(j-1)!} \frac{d^{j-2}}{d z^{j-2}} \wp(z) \\ & +\beta_0, \end{aligned} \tag{16}$$

in which β_{-ij} are determined by (10), $\sum_{i=1}^s \beta_{-i1}=0$, and $C_i^2=4 D_i^3-g_2 D_i-g_3$.

Each rational function solution $R(z)$ can be expressed as

$$R(z)=\sum_{i=1}^s \sum_{j=1}^q \frac{\beta_{ij}}{(z-z_i)^j}+\beta_0, \tag{17}$$

and it has $s(\leq p)$ distinct poles of multiplicity q.

Each simply periodic solution $R(\eta)$ is a rational function of $\eta=e^{\alpha z}(\alpha \in \mathbb{C})$ and can be expressed as

$$R(\eta)=\sum_{i=1}^s \sum_{j=1}^q \frac{\beta_{ij}}{(\eta-\eta_i)^j}+\beta_0, \tag{18}$$

and it has $s(\leq p)$ distinct poles of multiplicity q.

Lemma 2 (see [48, 49]). *Weierstrass elliptic functions $\wp(z)$ have the following addition formula:*

$$\begin{aligned} \wp(z-z_0)= & -\wp(z_0)-\wp(z) \\ & +\frac{1}{4}\left[\frac{\wp'(z)+\wp'(z_0)}{\wp(z)-\wp(z_0)}\right]^2. \end{aligned} \tag{19}$$

When $g_2=g_3=0$, it can be degenerated to rational functions according to

$$\wp(z, 0,0)=\frac{1}{z^2}. \tag{20}$$

When $\Delta(g_2, g_3)=0$, it can also be degenerated to simple periodic functions according to

$$\wp\left(z, 3 d^2,-d^3\right)=2 d-\frac{3 d}{2} \coth^2 \sqrt{\frac{3 d}{2}} z. \tag{21}$$

3. Main Results

The fractional generalized higher order KdV equation [39] is given as

$$D_t^{\mu}+uu_x-uu_{xxx}+u_{xxxxx}=0. \quad (22)$$

Substituting traveling wave transform

$$u(x,t)=w(z),\quad z=k\left(x+\frac{\lambda t^{\mu}}{\Gamma(1+\mu)}\right), \quad (23)$$

into (22), we get

$$k\lambda w'+kww'-k^3ww'''+k^5w^{(5)}=0, \quad (24)$$

Integrating (24) yields

$$\lambda w+\frac{w^2}{2}+k^2ww''-\frac{k^2}{2}(w')^2+k^4w^{(4)}+\delta=0, \quad (25)$$

where k and λ are constants and δ is the integral constant.

Theorem 3. *If $k\neq 0$, then the meromorphic solutions $w(z)$ of (25) have the following forms.*

(I) *The rational function solutions*

$$w_r(z)=\frac{-30k^2}{(z-z_0)^2}+\frac{5}{2}, \quad (26)$$

where $\lambda=-5/2$, $\delta=25/8$, and $z_0\in\mathbb{C}$.

(II) *The simply periodic solutions*

$$w_s(z)=-\frac{15k^2\alpha^2}{2}\coth^2\frac{\alpha(z-z_0)}{2}+\frac{10k^2\alpha^2+5}{2}, \quad (27)$$

where $\lambda=(3k^4\alpha^4-5)/2$, $\delta=(30k^6\alpha^6-55k^4\alpha^4+25)/8$, and $z_0\in\mathbb{C}$.

(III) *The elliptic function solutions*

$$\begin{aligned}w_d(z)=&-30k^2\left(-\wp(z)+\frac{1}{4}\left(\frac{\wp'(z)+D}{\wp(z)-C}\right)^2\right)\\&+30k^2C+\frac{5}{2},\end{aligned} \quad (28)$$

where $C^2=4D^3-g_2D-g_3$, $g_2=(2\lambda+5)/36k^4$, and $g_3=-(100+55\lambda+12\delta)/9720k^6$.

Proof. Substituting (10) into (25) we have $p=1$, $q=2$, $\beta_{-2}=-30k^2$, $\beta_{-1}=0$, $\beta_0=5/2$, $\beta_1=0$, $\beta_2=-(2\lambda+5)/24k^2$, and β_3 is an arbitrary constant.

Therefore, (25) satisfies the weak $\langle 1,2\rangle$ condition. In the following, we will show the meromorphic solutions of (25).

By (17), we infer that the indeterminant rational solutions of (25) are

$$R_1(z)=\frac{\beta_{11}}{z^2}+\frac{\beta_{12}}{z}+\beta_{10}, \quad (29)$$

with pole at $z=0$.

Inserting $R_1(z)$ into (25), we have

$$\begin{aligned}&\frac{1}{2}{\beta_{10}}^2+\lambda\beta_{10}+\delta+\frac{\lambda\beta_{11}+\beta_{11}\beta_{10}}{z}\\&+\frac{2\beta_{12}\beta_{10}+{\beta_{11}}^2+2\lambda\beta_{12}}{2z^2}\\&+\frac{2k^2\beta_{10}\beta_{11}+\beta_{12}\beta_{11}}{z^3}\\&+\frac{3k^2{\beta_{11}}^2+12k^2\beta_{10}\beta_{12}+{\beta_{12}}^2}{2z^4}\\&+\frac{24k^4\beta_{11}+6k^2\beta_{11}\beta_{12}}{z^5}+\frac{120k^4\beta_{12}+4k^2{\beta_{12}}^2}{z^6}\\&=0,\end{aligned} \quad (30)$$

then we get $\beta_{12}=-30k^2$, $\beta_{11}=0$, and $\beta_{10}=5/2$.

So we can determine that

$$R_1(z)=\frac{-30k^2}{z^2}+\frac{5}{2}, \quad (31)$$

where $\lambda=-5/2$ and $\delta=25/8$.

Therefore the rational solutions of (25) are

$$w_r(z)=\frac{-30k^2}{(z-z_0)^2}+\frac{5}{2}, \quad (32)$$

where $\lambda=-5/2$, $\delta=25/8$, and $z_0\in\mathbb{C}$.

Let $\eta=e^{\alpha z}$. To derive simply periodic solutions, we substitute $w=R(\eta)$ into (25) to yield

$$\begin{aligned}&k^4\alpha^4\left(R^{(4)}\eta^4+6R'''\eta^3+7R''\eta^2+R'\eta\right)-\frac{k^2}{2}(\alpha R'\eta)^2\\&+k^2\alpha^2R\left(\eta R'+\eta^2R''\right)+\frac{R^2}{2}+\lambda R+\delta=0.\end{aligned} \quad (33)$$

Substituting

$$R_2(z)=\frac{\beta_{21}}{(\eta-1)^2}+\frac{\beta_{22}}{(\eta-1)}+\beta_{20} \quad (34)$$

into (33), we obtain that

$$R_2(z)=-\frac{30k^2\alpha^2}{(\eta-1)^2}-\frac{30k^2\alpha^2}{(\eta-1)}-\frac{5k^2\alpha^2}{2}+\frac{5}{2}, \quad (35)$$

where $\lambda=(3k^4\alpha^4-5)/2$ and $\delta=(30k^6\alpha^6-55k^4\alpha^4+25)/8$.

Substituting $\eta=e^{\alpha z}$ into (35), we achieve simply periodic solutions to (25) with pole at $z=0$

$$\begin{aligned}w_{s0}(z)&=-\frac{30k^2\alpha^2}{(e^{\alpha z}-1)^2}-\frac{30k^2\alpha^2}{(e^{\alpha z}-1)}-\frac{5k^2\alpha^2}{2}+\frac{5}{2}\\&=-30k^2\alpha^2\frac{e^{\alpha z}}{(e^{\alpha z}-1)^2}-\frac{5k^2\alpha^2}{2}+\frac{5}{2}\\&=-\frac{15k^2\alpha^2}{2}\coth^2\frac{\alpha z}{2}+\frac{10k^2\alpha^2+5}{2},\end{aligned} \quad (36)$$

where $\lambda=(3k^4\alpha^4-5)/2$ and $\delta=(30k^6\alpha^6-55k^4\alpha^4+25)/8$.

Thurs, simply periodic solutions of (25) are

$$w_s(z) = -\frac{15k^2\alpha^2}{2}\coth^2\frac{\alpha(z-z_0)}{2} + \frac{10k^2\alpha^2+5}{2}, \quad (37)$$

where $\lambda = (3k^4\alpha^4 - 5)/2$, $\delta = (30k^6\alpha^6 - 55k^4\alpha^4 + 25)/8$, and $z_0 \in \mathbb{C}$.

From (16) of Lemma 1, the elliptic solutions of (25) are expressed as

$$w_{d0}(z) = \beta_{-2}\wp(z) + \beta_{30}, \quad (38)$$

with pole at $z = 0$.

Putting $w_{d0}(z)$ into (25), we get

$$w_{d0}(z) = -30k^2\wp(z) + \frac{5}{2}, \quad (39)$$

where $g_2 = (2\lambda+5)/36k^4$ and $g_3 = -(100+55\lambda+12\delta)/9720k^6$.

So, the elliptic solutions of (25) are

$$w_d(z) = -30k^2\wp(z-z_0) + \frac{5}{2}, \quad (40)$$

where $z_0 \in \mathbb{C}$.

We can apply the addition formula to rewrite it as

$$\begin{aligned} w_d(z) = {} & -30k^2\left(-\wp(z) + \frac{1}{4}\left(\frac{\wp'(z)+D}{\wp(z)-C}\right)^2\right) \\ & + 30k^2C + \frac{5}{2}, \end{aligned} \quad (41)$$

where $C^2 = 4D^3 - g_2D - g_3$, $g_2 = (2\lambda+5)/36k^4$, and $g_3 = -(100+55\lambda+12\delta)/9720k^6$.

Substitute traveling wave transform into the meromorphic solutions $w(z)$ of (25) to get traveling wave exact solutions to the fractional generalized higher order KdV equation. So we obtain Theorem 4 as follows. □

Theorem 4. *If $k \neq 0$, then traveling wave solutions of (25) have the following forms.*

(I) *The rational function solutions*

$$w_r(x,t) = w_r\left(kx - \frac{5kt^\mu}{2\Gamma(1+\mu)}\right), \quad (42)$$

where $\lambda = -5/2$, $\delta = 25/8$, and $z_0 \in \mathbb{C}$.

(II) *The simply periodic solutions*

$$w_s(x,t) = w_s\left(kx + \frac{(3k^5\alpha^4 - 5k)t^\mu}{2\Gamma(1+\mu)}\right), \quad (43)$$

where $\lambda = (3k^4\alpha^4 - 5)/2$, $\delta = (30k^6\alpha^6 - 55k^4\alpha^4 + 25)/8$, and $z_0 \in \mathbb{C}$.

(III) *The elliptic function solutions*

$$w_d(x,t) = w_d\left(kx + \frac{k\lambda t^\mu}{\Gamma(1+\mu)}\right), \quad (44)$$

where $C^2 = 4D^3 - g_2D - g_3$, $g_2 = (2\lambda+5)/36k^4$, and $g_3 = -(100+55\lambda+12\delta)/9720k^6$.

4. Conclusions

In this note, we have used the complex method to construct exact solutions to the mentioned NFDE. Although we do not show that the meromorphic solutions of the fractional generalized higher order KdV equation belong to the class W, we can still obtain the meromorphic solutions to this NFDE and then get its traveling wave exact solutions. The results demonstrate that the applied method is direct and efficient method, which allows us to do tedious and complicated algebraic calculation. We can utilize these ideas to other NFDEs.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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