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Research Article

Fixed Point Theorems for New Type Contractive Mappings

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In this paper, we introduce some new concepts of contractions called $\gamma-$contractions and $\gamma-$weak contractions. We prove some fixed point theorems for mappings providing $\gamma-$contractions and $\gamma-$weak contractions unlike known results in the literature. Also, we present a few examples to illustrate the validity of the results obtained in the paper.

1. Introduction and Preliminaries

The fixed point theory is very important concept in mathematics. In 1922, Banach created a famous result called Banach contraction principle in the concept of the fixed point theory [1]. Later, most of the authors intensively introduced many works regarding the fixed point theory in various of spaces. The concept of a fuzzy metric space was introduced in different ways by some authors (see [2, 3]). Gregori and Sapena [4] introduced the notion of fuzzy contractive mappings and gave some fixed point theorems for complete fuzzy metric spaces in the sense of George and Veeramani, and also for Kramosil and Michalek's fuzzy metric spaces which are complete in Grabiec's sense. Mihet [5] developed the class of fuzzy contractive mappings of Gregori and Sapena, considered these mappings in non-Archimedean fuzzy metric spaces in the sense of Kramosil and Michalek, and obtained a fixed point theorem for fuzzy contractive mappings. At the same time, there are lots of different types of fixed point theorems presented by many authors by expanding the Banach's result in the literature (see [5–12]).

In this work, using a mapping $\gamma : [0,1) \longrightarrow \mathbb{R}$ we introduce some new types of contractions called $\gamma-$contractions and $\gamma-$weak contractions. Later, we prove some fixed point theorems for mappings providing $\gamma-$contractions and $\gamma-$weak contractions. Some examples are supplied in order to support the usability of our results. Our main results are substantially different and useful compared to some known results in the existing literature.

Definition 1 ([13]). A binary operation $* : [0,1] \times [0,1] \longrightarrow [0,1]$ is called a continuous triangular norm (in short, continuous $t-$norm) if it satisfies the following conditions:

(TN-1) $*$ is commutative and associative,

(TN-2) $*$ is continuous,

(TN-3) $*(a,1) = a$ for every $a \in [0,1]$,

(TN-4) $*(a,b) \le *(c,d)$ whenever $a \le c, b \le d$ and $a,b,c,d \in [0,1]$.

An arbitrary $t-$norm $*$ can be extended (by associativity) in a unique way to a nary operator taking for $(x_1, x_2, \dots, x_n) \in [0,1]^n$, $n \in \mathbb{N}$; the value $*(x_1, x_2, \dots, x_n)$ is defined, in [14], by $*_{i=1}^{0} x_i = 1$, $*_{i=1}^{n} x_i = *(*_{i=1}^{n-1} x_i, x_n) = *(x_1, x_2, \dots, x_n)$.

Definition 2 ([15]). A fuzzy metric space is an ordered triple $(X, M, *)$ such that X is a nonempty set, $*$ is a continuous t-norm, and M is a fuzzy set on $X^2 \times (0,\infty)$, satisfying the following conditions, for all $x, y, z \in X$, $s, t > 0$:

(FM-1) $M(x,y,t) > 0$,

(FM-2) $M(x,y,t) = 1$ iff $x = y$,

(FM-3) $M(x,y,t) = M(y,x,t)$,

(FM-4) $M(x,z,t+s) \ge M(x,y,t) * M(y,z,s)$,

(FM-5) $M(x,y,\cdot) : (0,\infty) \longrightarrow [0,1]$ is continuous.

If, in the above definition, the triangular inequality (FM-4) is replaced by

(NA) $M(x,z,\max\{t,s\}) \geq M(x,y,t) * M(y,z,s)$ for all $x,y,z \in X$, $s,t > 0$, or equivalently,

$$M(x,z,t) \geq M(x,y,t) * M(y,z,t) \tag{1}$$

and then the triple $(X,M,*)$ is called a non-Archimedean fuzzy metric space [16].

Definition 3. Let $(X,M,*)$ be a fuzzy metric space. Then see the following.

(i) A sequence $\{x_n\}$ in X is said to converge to x in X, denoted by $x_n \longrightarrow x$, if and only if $\lim_{n\longrightarrow\infty} M(x_n,x,t) = 1$ for all $t > 0$; i.e., for each $r \in (0,1)$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $M(x_n,x,t) > 1 - r$ for all $n \geq n_0$ [3, 17].

(ii) A sequence $\{x_n\}$ is a M-Cauchy sequence if and only if for all $\varepsilon \in (0,1)$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $M(x_n,x_m,t) \geq 1-\varepsilon$ for all $m > n \geq n_0$ [15, 17]. A sequence $\{x_n\}$ is a G-Cauchy sequence if and only if $\lim_{n\longrightarrow\infty} M(x_n,x_{n+p},t) = 1$ for any $p > 0$ and $t > 0$ [4, 14, 18].

(iii) The fuzzy metric space $(X,M,*)$ is called M-complete (G-complete) if every M-Cauchy (G-Cauchy) sequence is convergent.

2. Main Results

Definition 4. Let $\gamma : [0,1) \longrightarrow \mathbb{R}$ be a strictly increasing, continuous mapping and for each sequence $\{a_n\}_{n\in\mathbb{N}}$ of positive numbers $\lim_{n\longrightarrow\infty} a_n = 1$ if and only if $\lim_{n\longrightarrow\infty}\gamma(a_n) = +\infty$. Let Γ be the family of all γ functions.

A mapping $T : X \longrightarrow X$ is said to be a γ−contraction if there exists a $\delta \in (0,1)$ such that

$$\begin{gathered} M(Tx,Ty,t) < 1 \Longrightarrow \\ \gamma(M(Tx,Ty,t)) \geq \gamma(M(x,y,t)) + \delta \end{gathered} \tag{2}$$

for all $x, y \in X$ and $\gamma \in \Gamma$.

Example 5. Let $\gamma \in \Gamma$. The different types of the mapping γ are as follows:

(γ_1) $1/(1-x)$,

(γ_2) $1/(1-x) + x$,

(γ_3) $1/(1-x^2)$,

(γ_4) $1/\sqrt{1-x}$.

Remark 6. From (γ_1) and (2) it is easy to conclude that every γ− contraction T is a contractive mapping, that is,

$$M(Tx,Ty,t) > M(x,y,t) \tag{3}$$

for all $x, y \in X$, such that $Tx \neq Ty$. Thus every γ-contraction is a continuous mapping.

Theorem 7. *Let $(X,M,*)$ be a non-Archimedean fuzzy metric space and let $T : X \longrightarrow X$ be a γ−contraction. Then T has a unique fixed point in X.*

Proof. Let $x_0 \in X$ be arbitrary and fixed. Define sequence $\{x_n\}$ by

$$Tx_n = x_{n+1} \quad \text{for all } n \in \mathbb{N}. \tag{4}$$

If $x_n = x_{n+1}$ then x_{n+1} is a fixed point of T; then the proof is finished. Suppose that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. Therefore by (2), we get

$$\gamma(M(Tx_{n-1},Tx_n,t)) \geq \gamma(M(x_{n-1},x_n,t)) + \delta. \tag{5}$$

Repeating this process, we get

$$\begin{aligned} \gamma(M(Tx_{n-1},Tx_n,t)) &\geq \gamma(M(x_{n-1},x_n,t)) + \delta \\ &= \gamma(M(Tx_{n-2},\mathrm{T}x_{n-1},t)) + \delta \\ &\geq \gamma(M(x_{n-2},x_{n-1},t)) + 2\delta \cdots \\ &\geq \gamma(M(x_0,x_1,t)) + n\delta. \end{aligned} \tag{6}$$

Letting $n \longrightarrow \infty$, from (6) we get

$$\lim_{n\longrightarrow\infty}\gamma(M(Tx_{n-1},Tx_n,t)) = +\infty. \tag{7}$$

Then, we have

$$\lim_{n\longrightarrow\infty} M(Tx_{n-1},Tx_n,t) = 1. \tag{8}$$

Now, we want to show that $\{x_n\}$ is a Cauchy sequence. Suppose to the contrary, we assume that $\{x_n\}$ is not a Cauchy sequence. Then there are $\varepsilon \in (0,1)$ and $t_0 > 0$ such that for all $k \in \mathbb{N}$ there exist $n(k), m(k) \in \mathbb{N}$ with $n(k) > m(k) > k$ and

$$M(x_{n(k)},x_{m(k)},t_0) \leq 1-\varepsilon. \tag{9}$$

Assume that $m(k)$ is the least integer exceeding $n(k)$ satisfying inequality (9). Then, we have

$$M(x_{m(k)-1},x_{n(k)},t_0) > 1-\varepsilon \tag{10}$$

and so, for all $k \in \mathbb{N}$, we get

$$\begin{aligned} 1-\varepsilon &\geq M(x_{n(k)},x_{m(k)},t_0) \\ &\geq M(x_{m(k)-1},x_{m(k)},t_0) * M(x_{m(k)-1},x_{n(k)},t_0) \\ &\geq M(x_{m(k)-1},x_{m(k)},t_0) * (1-\varepsilon). \end{aligned} \tag{11}$$

By taking $k \longrightarrow \infty$ in (11) and using (8), we obtain

$$\lim_{k\longrightarrow\infty} M(x_{n(k)},x_{m(k)},t_0) = 1-\varepsilon. \tag{12}$$

From (FM-4), we get

$$\begin{aligned} &M(x_{m(k)+1},x_{n(k)+1},t_0) \\ &\quad\geq M(x_{m(k)+1},x_{m(k)},t_0) * M(x_{m(k)},x_{n(k)},t_0) \\ &\quad * M(x_{n(k)1},x_{n(k)+1},t_0). \end{aligned} \tag{13}$$

Taking the limit as $k \longrightarrow \infty$ in (13), we obtain

$$\lim_{k\longrightarrow\infty} M(x_{n(k)+1},x_{m(k)+1},t_0) = 1-\varepsilon. \tag{14}$$

By applying inequality (2) with $x = x_{m(k)}$ and $y = x_{n(k)}$

$$\begin{aligned}&\gamma\left(M\left(x_{n(k)+1}, x_{m(k)+1}, t\right)\right)\\&\quad \geq \gamma\left(M\left(x_{n(k)}, x_{m(k)}, t\right)\right) + \delta.\end{aligned} \tag{15}$$

Taking the limit as $k \longrightarrow \infty$ in (15), applying (2), from (12), (14), and continuity of γ, we obtain

$$\gamma(1-\varepsilon) \geq \gamma(1-\varepsilon) + \delta \tag{16}$$

which is a contradiction. Thus $\{x_n\}$ is a Cauchy sequence in X. From the completeness of $(X, M, *)$ there exists $z \in X$ such that

$$\lim_{n \longrightarrow \infty} x_n = z. \tag{17}$$

Finally, the continuity of T yields $M(Tz, z, t) = \lim_{n\longrightarrow\infty} M(Tx_n, x_n, t) = \lim_{n\longrightarrow\infty} M(x_{n+1}, x_n, t) = 1$. Now, we show that T has a unique fixed point. Suppose that z_1 and z_2 are two fixed points of T. Indeed, if for $z_1, z_2 \in X$, $Tz_1 = z_1 \neq z_2 = Tz_2$, then we get

$$\gamma\left(M\left(z_1, z_2, t\right)\right) \geq \gamma\left(M\left(z_1, z_2, t\right)\right) + \delta \tag{18}$$

which is a contradiction. Thus, T has a unique fixed point. Hence, the proof is completed. □

Example 8. Let $X = [0, 1)$, $a * b = \min\{a, b\}$,

$$M(x, y, t) = \begin{cases} \dfrac{1}{(1 + \max\{x, y\})}, & x \neq y, \\ 1, & x = y, \end{cases} \tag{19}$$

for all $t > 0$. Let $\gamma : [0, 1) \longrightarrow \mathbb{R}$ such that $\gamma(x) = 1/(1-x)$ for all $x \in [0, 1)$ and define $T : X \longrightarrow X$ by $T(x) = 2x^2/5$ for all $x \in X$. Clearly, $(X, M, *)$ is a non-Archimedean fuzzy metric space.

Case 1. We assume that $x < y$ for all $x, y \in (0, 1)$. Since $x^2 < x$ and $y^2 < y$, then $\max\{x, y\} > \max\{Tx, Ty\}$. So, there exists $\delta \in (0, 1)$ such that

$$\frac{1}{\max\{Tx, Ty\}} + 1 \geq \frac{1}{\max\{x, y\}} + 1 + \delta. \tag{20}$$

Therefore

$$\frac{1 + \max\{Tx, Ty\}}{\max\{Tx, Ty\}} \geq \frac{1 + \max\{x, y\}}{\max\{x, y\}} + \delta, \tag{21}$$

and so

$$\frac{1 + \max\{Tx, Ty\}}{1 + \max\{Tx, Ty\} - 1} \geq \frac{1 + \max\{x, y\}}{1 + \max\{x, y\} - 1} + \delta, \tag{22}$$

and, then, we get

$$\frac{1}{1 - 1/(1 + \max\{Tx, Ty\})} \geq \frac{1}{1 + 1/\max\{x, y\}} + \delta, \tag{23}$$

which implies

$$\frac{1}{1 - M(Tx, Ty, t)} \geq \frac{1}{1 + M(x, y, t)} + \delta. \tag{24}$$

That is,

$$\gamma\left(M(Tx, Ty, t)\right) \geq \gamma\left(M(x, y, t)\right) + \delta. \tag{25}$$

Case 2. Let $x = 0$ and $y \in (0, 1)$. Since $x^2 = 0$, $y^2 < y$, then $\max\{x, y\} = y > y^2 = \max\{Tx, Ty\}$. Hence, we have

$$\begin{aligned}M(Tx, Ty, t) &= \frac{1}{1 + \max\{Tx, Ty\}} > \frac{1}{1 + \max\{x, y\}}\\&= M(x, y, t).\end{aligned} \tag{26}$$

So, there exists $\delta \in (0, 1)$ such that

$$\frac{1}{1 - M(Tx, Ty, t)} \geq \frac{1}{1 - M(x, y, t)} + \delta. \tag{27}$$

That is,

$$\gamma\left(M(Tx, Ty, t)\right) \geq \gamma\left(M(x, y, t)\right) + \delta. \tag{28}$$

Therefore, T is a $\gamma-$ contraction. Then all the conditions of Theorem 7 hold and T has a unique fixed point $x = 0$.

Definition 9. Let $(X, M, *)$ be a non-Archimedean fuzzy metric space. A mapping $T : X \longrightarrow X$ is said to be a $\gamma-$weak contraction if there exists $\delta \in (0, 1)$ such that

$$\begin{aligned}&M(Tx, Ty, t) < 1 \Longrightarrow\\&\gamma\left(M(Tx, Ty, t)\right)\\&\quad \geq \gamma\left(\min\left\{M(x, y, t), M(x, Tx, t), M(y, Ty, t)\right\}\right)\\&\qquad + \delta\end{aligned} \tag{29}$$

for all $x, y \in X$ and $\gamma \in \Gamma$.

Remark 10. Every $\gamma-$contraction is a $\gamma-$weak contraction. But the converse is not true.

Example 11. Let $X = A \cup B$ where $A = \{1/10, 1/2, 1, 2, 3\}$, $B = [4, 6]$. $a * b = \min\{a, b\}$ and $M(x, y, t) = \min\{x, y\}/\max\{x, y\}$ for all $t > 0$. Clearly, $(X, M, *)$ is a complete non-Archimedean fuzzy metric space. Let $\gamma : [0, 1) \longrightarrow \mathbb{R}$ such that $\gamma(x) = 1/\sqrt{1-x}$ for all $x \in [0, 1)$ and define $T : X \longrightarrow X$ by

$$T(x) = \begin{cases} \dfrac{1}{10}, & x \in A, \\ \dfrac{1}{2}, & x \in B. \end{cases} \tag{30}$$

Since T is not continuous, T is not $\gamma-$ contraction by Remark 6.

Now, we show that T is a γ−weak contraction for all $x \in A$ and $y \in B$.

Case 1. Let $x = 1$ and $y \in B$,

$$M(Tx, Ty, t) = \frac{1}{5} > \frac{1}{2y} = \min\left\{\frac{x}{y}, \frac{1}{10x}, \frac{1}{2y}\right\}$$
$$= \min\{M(x, y, t), M(x, Tx, t), M(y, Ty, t)\}). \tag{31}$$

Then we have

$$\gamma\left(\frac{1}{\sqrt{1-1/5}}\right) > \gamma\left(\frac{1}{\sqrt{1-1/2y}}\right). \tag{32}$$

So, there exists $\delta \in (0, 1)$ such that

$$\gamma(M(Tx, Ty, t)) \geq \gamma(\min\{M(x, y, t), M(x, Tx, t), M(y, Ty, t)\}) + \delta. \tag{33}$$

Case 2. Let $x \in \{2, 3\}$ and $y \in B$,

$$M(Tx, Ty, t) = \frac{1}{5} > \frac{1}{10x} = \min\left\{\frac{x}{y}, \frac{1}{10x}, \frac{1}{2y}\right\}$$
$$= \min\{M(x, y, t), M(x, Tx, t), M(y, Ty, t)\}). \tag{34}$$

Then we have

$$\gamma\left(\frac{1}{\sqrt{1-1/5}}\right) > \gamma\left(\frac{1}{\sqrt{1-1/10x}}\right). \tag{35}$$

So, there exists $\delta \in (0, 1)$ such that

$$\gamma(M(Tx, Ty, t)) \geq \gamma(\min\{M(x, y, t), M(x, Tx, t), M(y, Ty, t)\}) + \delta. \tag{36}$$

Case 3. Let $x \in \{1/10, 1/2\}$ and $y \in B$,

$$M(Tx, Ty, t) = \frac{1}{5} > \frac{x}{y} = \min\left\{\frac{x}{y}, \frac{1}{10x}, \frac{1}{2y}\right\}$$
$$= \min\{M(x, y, t), M(x, Tx, t), M(y, Ty, t)\}). \tag{37}$$

Then we have

$$\gamma\left(\frac{1}{\sqrt{1-1/5}}\right) > \gamma\left(\frac{1}{\sqrt{1-x/y}}\right). \tag{38}$$

So, there exists $\delta \in (0, 1)$ such that

$$\gamma(M(Tx, Ty, t)) \geq \gamma(\min\{M(x, y, t), M(x, Tx, t), M(y, Ty, t)\}) + \delta. \tag{39}$$

Therefore, T is a γ−weak contraction.

Theorem 12. *Let $(X, M, *)$ be a non-Archimedean fuzzy metric space and let $T : X \longrightarrow X$ be a γ−weak contraction. Then T has a unique fixed point in X.*

Proof. Let $x_0 \in X$ be arbitrary and fixed. Define sequence $\{x_n\}$ by

$$Tx_n = x_{n+1} \quad \text{for all } n \in \mathbb{N}. \tag{40}$$

If $x_n = x_{n+1}$ then x_{n+1} is a fixed point of T; then the proof is finished. Suppose that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. Therefore by (29), we get

$$\begin{aligned}\gamma(M(Tx_{n-1}, Tx_n, t)) &\geq \gamma(\min\{M(x_{n-1}, x_n, t), M(x_{n-1}, Tx_{n-1}, t)), M(x_n, Tx_n, t)\}) + \delta \\ &= \gamma(\min\{M(x_{n-1}, x_n, t), M(x_{n-1}, x_n, t)), M(x_n, x_{n+1}, t)\}) + \delta = \gamma(\min\{M(x_{n-1}, x_n, t), M(x_n, x_{n+1}, t)\}) + \delta.\end{aligned} \tag{41}$$

If there exists $n \in \mathbb{N}$ such that

$$\min\{M(x_{n-1}, x_n, t), M(x_n, x_{n+1}, t)\} = M(x_n, x_{n+1}, t), \tag{42}$$

from (41) becomes

$$\gamma(M(Tx_{n-1}, Tx_n, t)) = M(x_n, x_{n+1}, t) \geq \gamma(M(x_n, x_{n+1}, t)) + \delta > M(x_n, x_{n+1}, t) \tag{43}$$

which is a contradiction, therefore,

$$\min\{M(x_{n-1}, x_n, t), M(x_n, x_{n+1}, t)\} = M(x_{n-1}, x_n, t) \tag{44}$$

for all $n \in \mathbb{N}$. That is, from property of γ, (41), and (44), we get

$$M(x_n, x_{n+1}, t) > M(x_{n-1}, x_n, t). \tag{45}$$

Thus, from (41), we have

$$\gamma(M(x_n, x_{n+1}, t)) \geq \gamma(M(x_{n-1}, x_n, t)) + \delta \tag{46}$$

for all $n \in \mathbb{N}$. It implies that

$$\gamma(M(x_n, x_{n+1}, t)) \geq \gamma(M(x_{n-1}, x_n, t)) + n\delta. \tag{47}$$

By taking $n \longrightarrow \infty$ in (47), we get

$$\lim_{n\longrightarrow\infty} \gamma(M(x_n, x_{n+1}, t)) = \lim_{n\longrightarrow\infty} \gamma(M(Tx_{n-1}, Tx_n, t)) = +\infty. \tag{48}$$

Then, we have

$$\lim_{n\longrightarrow\infty} M(Tx_{n-1}, Tx_n, t) = 1. \tag{49}$$

The proof that $\{x_n\}$ is a Cauchy sequence can be shown as in Theorem 7. From the completeness of $(X, M, *)$ there exists $z \in X$ such that

$$\lim_{n \longrightarrow \infty} x_n = z. \tag{50}$$

Now, we show that z is a fixed point of T. Since γ is continuous, there are two cases.

Case 1. For each $n \in \mathbb{N}$, there exists $i_n \in \mathbb{N}$ such that $x_{i_n+1} = Tz$ and $i_n > i_{n-1}$, where $i_0 = 1$. Then, we get

$$z = \lim_{n \longrightarrow \infty} x_{i_n+1} = \lim_{n \longrightarrow \infty} Tz = Tz. \tag{51}$$

This proves that z is a fixed point of T.

Case 2. There exists $n_0 \in \mathbb{N}$ such that $x_{n+1} \neq Tz$ for all $n \geq n_0$. That is, $M(Tx_n, Tz, t) < 1$ for all $n \geq n_0$. It follows from (29), property of γ,

$$\begin{aligned} \gamma\left(M\left(x_{n+1}, Tz, t\right)\right) &= \gamma\left(M\left(Tx_n, Tz, t\right)\right) \\ &\geq \gamma\left(\min\left\{M\left(x_n, z, t\right), M\left(x_n, Tx_n, t\right),\right.\right. \\ &\left.\left. M\left(z, Tz, t\right)\right\}\right) + \delta = \gamma\left(\min\left\{M\left(x_n, z, t\right),\right.\right. \\ &\left.\left. M\left(x_n, x_{n+1}, t\right), M\left(z, Tz, t\right)\right\}\right) + \delta. \end{aligned} \tag{52}$$

If $M(z, Tz, t) < 1$, then we have

$$\lim_{n \longrightarrow \infty} M\left(x_n, z, t\right) = 1, \tag{53}$$

and there exists $n_1 \in \mathbb{N}$ such that for all $n \geq n_1$, we get

$$\begin{aligned} &\min\left\{M\left(x_n, z, t\right), M\left(x_n, x_{n+1}, t\right), M\left(z, Tz, t\right)\right\} \\ &\quad = M\left(z, Tz, t\right). \end{aligned} \tag{54}$$

From (52), we have

$$\gamma\left(M\left(x_{n+1}, Tz, t\right)\right) \geq \gamma\left(M\left(z, Tz, t\right)\right) + \delta \tag{55}$$

for all $n \geq \max\{n_0, n_1\}$. Since γ is continuous, taking the limit as $n \longrightarrow \infty$ in (55), we obtain

$$\gamma\left(M\left(z, Tz, t\right)\right) \geq \gamma\left(M\left(z, Tz, t\right)\right) + \delta \tag{56}$$

which is a contradiction. Therefore, $M(z, Tz, t) = 1$; that is, z is a fixed point of T.

Now, we prove that the fixed point of T is unique. Let z_1, z_2 be two fixed points of T. Suppose that $z_1 \neq z_2$; then we have $Tz_1 \neq Tz_2$. It follows from (29) that we have

$$\begin{aligned} \gamma\left(M\left(z_1, z_2, t\right)\right) &= \gamma\left(M\left(Tz_1, Tz_2, t\right)\right) \\ &\geq \gamma\left(\min\left\{M\left(z_1, z_2, t\right), M\left(z_1, Tz_1, t\right),\right.\right. \\ &\left.\left. M\left(z_2, Tz_2, t\right)\right\}\right) + \delta = \gamma\left(\min\left\{M\left(z_1, z_2, t\right),\right.\right. \\ &\left.\left. M\left(z_1, z_1, t\right), M\left(z_2, z_2, t\right)\right\}\right) + \delta = \gamma\left(M\left(z_1, z_2, t\right)\right) \\ &+ \delta \end{aligned} \tag{57}$$

which is a contradiction. Then, $M(z_1, z_2, t) = 1$, that is, $z_1 = z_2$. Therefore, the fixed point of T is unique. □

Example 13. Let $(X, M, *)$ be the non-Archimedean fuzzy metric space and let T be considered in Example 11. Let $\gamma : [0, 1) \longrightarrow \mathbb{R}$ such that $\gamma(x) = 1/(1 - x^2)$ for all $x \in [0, 1)$. So, T is a $\gamma-$weak contraction. Therefore, Theorem 12 can be applicable to T and the unique fixed point of T is $1/10$.

Conclusion 14. In this paper, we introduce new contraction types in non-Archimedean fuzzy metric spaces and presented new fixed point results. Our results can be expanded and solutions to new problems can be produced in this way. Also, a new more general contraction can be achieved or common fixed point theorems for a class of mappings can be obtained using $\gamma-$contractions and also our result can be extended to other spaces.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares no conflicts of interest.

References

[1] S. Banach, "Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales," *Fundamenta Mathematicae*, vol. 3, pp. 133–181, 1922.

[2] Z. Deng, "Fuzzy pseudo-metric spaces," *Journal of Mathematical Analysis and Applications*, vol. 86, no. 1, pp. 74–95, 1982.

[3] I. Kramosil and J. Michalek, "Fuzzy metrics and statistical metric spaces," *Kybernetika*, vol. 11, no. 5, pp. 326–334, 1975.

[4] V. Gregori and A. Sapena, "On fixed-point theorems in fuzzy metric spaces," *Fuzzy Sets and Systems*, vol. 125, no. 2, pp. 245–252, 2002.

[5] D. Miheţ, "Fuzzy Ψ-contractive mappings in non-Archimedean fuzzy metric spaces," *Fuzzy Sets and Systems*, vol. 159, no. 6, pp. 739–744, 2008.

[6] S. Chauhan, S. Bhatnagar, and S. Radenović, "Common fixed point theorems for weakly compatible mappings in fuzzy metric spaces," *Le Matematiche*, vol. 68, no. 1, pp. 87–98, 2013.

[7] D. Mihet, "A class of contractions in fuzzy metric spaces," *Fuzzy Sets and Systems*, vol. 161, no. 8, pp. 1131–1137, 2010.

[8] P. Salimi, C. Vetro, and P. Vetro, "Some new fixed point results in non-Archimedean fuzzy metric spaces," *Nonlinear Analysis: Modelling and Control*, vol. 18, no. 3, pp. 344–358, 2013.

[9] M. Sangurlu and D. Turkoglu, "Fixed point theorems for $(\psi \circ \varphi)$-contractions in a fuzzy metric spaces," *Journal of Nonlinear Sciences and Applications. JNSA*, vol. 8, no. 5, pp. 687–694, 2015.

[10] S. Sedghi, N. Shobkolaei, T. Došenović, and S. Radenović, "Suzuki-type of common fixed point theorems in fuzzy metric spaces," *Mathematica Slovaca*, vol. 68, no. 2, pp. 451–462, 2018.

[11] D. Turkoglu and M. Sangurlu, "Fixed point theorems for fuzzy (ψ)-contractive mappings in fuzzy metric spaces," *Journal of Intelligent & Fuzzy Systems. Applications in Engineering and Technology*, vol. 26, no. 1, pp. 137–142, 2014.

[12] C. Vetro, "Fixed points in weak non-Archimedean fuzzy metric spaces," *Fuzzy Sets and Systems*, vol. 162, pp. 84–90, 2011.

[13] B. Schweizer and A. Sklar, "Statistical metric spaces," *Pacific Journal of Mathematics*, vol. 10, pp. 313–334, 1960.

[14] M. Grabiec, "Fixed points in fuzzy metric spaces," *Fuzzy Sets and Systems*, vol. 27, no. 3, pp. 385–389, 1988.

[15] A. George and P. Veeramani, "On some results in fuzzy metric spaces," *Fuzzy Sets and Systems*, vol. 64, no. 3, pp. 395–399, 1994.

[16] V. Istratescu, *An Introduction to Theory of Probabilistic Metric Spaces with Applications*, Politehnica University of Bucharest, Bucharest, Romania, 1974.

[17] B. Schweizer and A. Sklar, *Probabilistic Metric Spaces*, Dover Publications, Amsterdam, Netherlands, 1983.

[18] R. Vasuki and P. Veeramani, "Fixed point theorems and Cauchy sequences in fuzzy metric spaces," *Fuzzy Sets and Systems*, vol. 135, no. 3, pp. 415–417, 2003.