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Research Article

Existence of Homoclinic Orbits for a Singular Differential Equation Involving p-Laplacian

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The efficient conditions guaranteeing the existence of homoclinic solutions to second-order singular differential equation with p-Laplacian $(\phi_p(x'(t)))' + f(x'(t)) + g(x(t)) + (h(t)/1 - x(t)) = e(t)$ are established in the paper. Here, $\phi_p(s) = |s|^{p-2}s$, $p > 1, f, g, h, e \in C(\mathbb{R}, \mathbb{R})$ with $h(t+T) = h(t)$. The approach is based on the continuation theorem for coincidence degree theory.

1. Introduction

In the last years, homoclinic solutions for Hamiltonian systems and differential and difference systems have been studied by several authors. Based on variational methods and critical points theory, Rabinowitz [1] has given fundamental contributions to homoclinic solutions for Hamiltonian systems. Carriãro and Miyagaki [2] obtained the existence of homoclinic orbits for second-order time-dependent Hamiltonian systems. Izydorek and Janczewska [3] obtained that a homoclinic orbit is obtained as a limit of $2kT$-periodic solutions of a certain sequence of the second-order differential equations. By means of an extension of Mawhin's continuation theorem, Lu et al. [4] obtained the existence of homoclinic solutions for a class of second-order neutral functional differential systems. Ding and Guo [5] showed that there exists at least one homoclinic solution for the anomalous diffusion system. For more results about homoclinic solutions, see, e.g., [6–10] and relevant references.

In recent years, homoclinic solution problems of second-order singular differential equation have raised concerns. Bonheure and Torres [11] studied the existence of homoclinic solutions for the model scalar second-order boundary value problem

$$-u'' + c(x)u' + a(x)u = \frac{b(x)}{u^p(x)}, \quad (1)$$

where $a, b, c \in C(\mathbb{R}, \mathbb{R}), p > 0$. When $c = 0$, Equation (1) has a good variational structure and can be studied by variational method for Equation (1), see [1, 12, 13]. When $c \neq 0$, variational method cannot be used to study Equation (1) because of the no good variational structure. Hence, based on the method of the upper and lower solutions and fixed point theorem on cones, the authors obtained the existence of homoclinic solutions for Equation (1) which is different from the variational methods used in [14–16].

Motivated by the above work, this paper is devoted to the study of the existence of homoclinic solutions to second-order singular differential equation with p-Laplacian:

$$\left(\phi_p\left(x'(t)\right)\right)' + f\left(x'(t)\right) + g(x(t)) + \frac{h(t)}{1 - x(t)} = e(t), \quad (2)$$

where $\phi_p(s)=|s|^{p-2}s, p>1, f, g, h, e\in C(\mathbb{R},\mathbb{R})$ with $h(t+T)=h(t)>0$. As in the literature, a solution $u(t)$ of Equation (2) is called a homoclinic solution if $u(t)\longrightarrow 0$ as $|t|\longrightarrow+\infty$. When such a solution satisfies in addition to $u'(t)\longrightarrow 0$ as $|t|\longrightarrow+\infty$, it is usually called a homoclinic solution or a pulse, although here, 0 is not a stationary solution of Equation (2). Since Equation. (2) is a strongly nonlinear equation, the traditional methods (including fixed point theorem and lower and upper solutions) are no longer applicable to study homoclinic solutions to Equation (2), so a new continuation theorem due to Mana'sevich and Mawhin will be developed for studying Equation (2).

The distinctive contributions of this paper are outlined as follows:

(1) The problem (2) is a more general form compared with existing problems (see [1, 11–13]). Hence, the results of this paper can be extended to other more specific problems

(2) Due to singularity, it is very difficult for estimating priori bound. In order to overcome this difficulty, we develop a new technique introduced in [17] for continuation theorem

(3) A unified framework is established to handle second-order equations with singularity term and p-Laplacian operator

The following sections are organized as follows: In Section 2, some useful lemmas and notations are given. In Section 3, sufficient conditions are established for the existence of homoclinic solutions of (2). In Section 4, two examples are given to show the feasibility of our results. Finally, Section 5 concludes the paper.

2. Preliminary and Some Lemmas

In this section, we give some notations and lemmas which will be used in this paper. The set of all positive integers is denoted by N. Let

$$C_T=\{x\mid x\in C(\mathbb{R},\mathbb{R}), x(t+T)\equiv x(t),\forall t\in\mathbb{R}\} \tag{3}$$

with the norm $|\varphi|_0=\max_{t\in[0,T]}|\varphi(t)|, \forall\varphi\in C_T$. When $p\neq 2$, p-Laplacian $(\phi_p(s'))'$ in (2) is a nonlinear operator, the famous Mawlin's continuation theorem [18] cannot be directly applied to (2). In order to generalize Mawlin's continuation theorem, Mana'sevich and Mawhin [17] obtained the following continuation theorem for nonlinear systems with p-Laplacian-like operators:

(1) For each $\lambda\in(0,1)$, the problem

$$\left(\phi_p\left(u'\right)\right)=\lambda f\left(t,u,u'\right), u(0)=u(T), u'(0)=u'(T), \tag{4}$$

has no solution on $\partial\Omega$

(2) The equation

$$\mathscr{F}(a)=\frac{1}{T}\int_0^T f(t,a,0)dt=0, \tag{5}$$

has no solution on $\partial\Omega\cap\mathbb{R}^N$

(3) The Brouwer degree

$$d_B\left(\mathscr{F},\Omega\cap\mathbb{R}^N,0\right)\neq 0 \tag{6}$$

Theorem 1. *Assume that Ω is an open bounded set in C_T such that the above conditions (4)–(6) hold.*

Then, problem

$$\left(\phi_p\left(u'\right)\right)=f\left(t,u,u'\right), u(0)=u(T), u'(0)=u'(T), \tag{7}$$

has a solution in $\bar{\Omega}$.

Lemma 2 (see [19]). *If $u\in C(\mathbb{R},\mathbb{R})$, $a>0, \mu_1,\mu_2>1$ are constants, then for every $t\in\mathbb{R}$, the following inequality holds:*

$$u(t)\leq(2a)^{-1/\mu_1}\left(\int_{t-a}^{t+a}|u(s)|^{\mu_1}ds\right)^{1/\mu_1}+a(2a)^{-1/\mu_2}\left(\int_{t-a}^{t+a}\left|u'(s)\right|^{\mu_2}ds\right)^{1/\mu_2}. \tag{8}$$

Lemma 3 (see [4]). *Let $\{u_k\}\in C^1_{2kT}$ be a sequence of $2kT$-periodic functions, such that for each $k\in N$, u_k satisfies*

$$|u_k|_0\leq A_0, \left|u_k'\right|_0\leq A_1, \tag{9}$$

where A_0, A_1 are constants independent of $k\in N$. Then, there exist $u_0\in C(\mathbb{R},\mathbb{R})$ and a subsequence $\{u_{k_j}\}$ of $\{u_k\}$ such that for each $j\in N$

$$\max_{t\in[-jT,jT]}|u_{k_i}(t)-u_0(t)|\longrightarrow 0 \text{ as } i\longrightarrow+\infty. \tag{10}$$

For investigating the existence of homoclinic solutions to (2), for each $k\in N$, we firstly consider the existence of $2kT$-periodic solutions $u_k(t)$ for the following equation:

$$\left(\phi_p\left(x'(t)\right)\right)'+f\left(x'(t)\right)+g(x(t))+\frac{h(t)}{1-x(t)}=e_k(t), \tag{11}$$

where $e_k : \mathbb{R} \longrightarrow \mathbb{R}$ is a $2kT$-periodic extension such that

$$e_k(t) = \begin{cases} e(t), & t \in [-kT, kT - \varepsilon_0), \\ e(kT - \varepsilon_0) + \dfrac{h(-kT) - h(kT - \varepsilon_0)}{\varepsilon_0}(t - kT + \varepsilon_0), & t \in [kT - \varepsilon_0, kT], \end{cases} \tag{12}$$

here $\varepsilon_0 \in (0, T/2)$ is a constant.

In the present paper, we list the following assumptions:

(H_1). $f : \mathbb{R} \longrightarrow \mathbb{R}$ is a continuous bounded nonnegative function

(H_2). $g : \mathbb{R} \longrightarrow \mathbb{R}$ is strictly monotone increasing and there are positive constants σ and n such that

$$xg(x) \geq \sigma |x|^{n+1} \text{ for } x \in \mathbb{R} \tag{13}$$

(H_3).

$$\sup_{t\in\mathbb{R}} |e(t)| = \rho_1 < +\infty, \int_{\mathbb{R}} |e(t)|^{\frac{n+1}{n}} dt = \rho_2 < +\infty, n > 0 \tag{14}$$

3. Main Results

Let $y(t) = 1 - x(t)$, then (11) is changed into the following form:

$$\left(\phi_p\left(-y'(t)\right)\right)' + f\left(-y'(t)\right) + g(1 - y(t)) + \frac{h(t)}{y(t)} = e_k(t). \tag{15}$$

Obviously, the existence of $2kT$-periodic solutions to (2) is a transfer to the existence of $2kT$-periodic solutions to (15). For (15), consider the corresponding parameter equation:

$$\begin{aligned} &\left(\phi_p\left(-y'(t)\right)\right)' + \lambda f\left(-y'(t)\right) \\ &\quad + \lambda g(1 - y(t)) + \lambda \frac{h(t)}{y(t)} = \lambda e_k(t), \lambda \in (0, 1]. \end{aligned} \tag{16}$$

Here, we give the main results of the present paper in the following theorem.

Theorem 4. *Assume that the assumptions (H_1)–(H_3) hold. Then, Equation (2) has at least one positive T -periodic solution, if $\rho_1 > f(0)$, $h_l/\rho_1 - f(0) < 1$, $h_l = \min_{t\in\mathbb{R}} h(t)$.*

Proof. Let

$$\begin{aligned} \Omega = \Big\{ x : &\left(\phi_p\left(-y'(t)\right)\right)' + \lambda f\left(-y'(t)\right) + \lambda g(1 - y(t)) + \lambda \frac{h(t)}{y(t)} \\ &= \lambda e_k(t), \lambda \in (0, 1], x(t + 2kT) \\ &= x(t), x(t) > 0, t \in [-kT, kT], k \in N \Big\} \end{aligned} \tag{17}$$

with the norm

$$|\varphi|_\infty = \max_{t\in[-kT,kT]} \left\{ |\varphi|_0, |\varphi'|_0 \right\}. \tag{18}$$

Let $u \in \Omega$, then u satisfies

$$\left(\phi_p\left(-u'(t)\right)\right)' + \lambda f\left(-u'(t)\right) + \lambda g(1 - u(t)) + \lambda \frac{h(t)}{u(t)} = \lambda e_k(t). \tag{19}$$

There exist $t_1, t_2 \in [-kT, kT]$ such that

$$u(t_1) = \max_{t\in[-kT,kT]} u(t), u(t_2) = \min_{t\in[-kT,kT]} u(t). \tag{20}$$

This implies that

$$u'(t_1) = u'(t_2) = 0. \tag{21}$$

By (19), we have

$$f(0) + g(1 - u(t_1)) + \frac{h(t_1)}{u(t_1)} = e_k(t_1), \tag{22}$$

$$g(1 - u(t_1)) \geq -|e_k|_0 - f(0) = -\rho_1 - f(0). \tag{23}$$

In view of monotonicity of g, it follows by (22) that

$$u(t_1) < 1 - g^{-1}(-\rho_1 - f(0)) := A_1. \tag{24}$$

On the other hand,

$$f(0) + g(1 - u(t_2)) + \frac{h(t_2)}{u(t_2)} = e_k(t_2). \tag{25}$$

We claim that

$$u(t_2) > \frac{h_l}{\rho_1 - f(0)} := A_2. \tag{26}$$

In fact, if (26) is not true, then

$$u(t_2) \leq \frac{h_l}{\rho_1 - f(0)} < 1. \tag{27}$$

By (25), we have

$$g(1-u(t_2)) \le \rho_1 - f(0) - \frac{h_l}{u(t_2)}, \le \rho_1 - f(0) - (\rho_1 - f(0)), = 0. \tag{28}$$

Thus, we have $u(t_2) \ge 1$ which is a contradiction to (27). From (24) and (26), we have

$$A_2 < u(t) < A_1 \text{ for all } t \in [-kT, kT]. \tag{29}$$

Now, we estimate the bound of $u'(t)$. For $i \in \{-k, -k+1, \cdots, k-1\}$, there exists $t_i \in [iT, (i+1)T]$ such that

$$u'(t_i) = \frac{1}{T}\int_{iT}^{(i+1)T} u'(s)ds. \tag{30}$$

It follows from (29) and (30) that

$$|u'(t_i)| = \left|\frac{1}{T}\int_{iT}^{(i+1)T} u'(s)ds\right|, = \frac{1}{T}|u(iT) - u((i+1)T)| < \frac{2A_1}{T}. \tag{31}$$

Integrating (19) over $[t_i, t]$, we have

$$\begin{aligned}
\phi_p\left(-u'(t)\right) &= \phi_p\left(-u'(t_i)\right) - \lambda\int_{t_i}^{t} f\left(-u'(s)\right)ds, \\
&\quad -\lambda\int_{t_i}^{t} g(1-u(s))ds - \lambda\int_{t_i}^{t}\frac{h(s)}{u(s)}ds \\
&\quad + \lambda\int_{t_i}^{t} e_k(s)ds, \\
\left|u'(t)\right|^{p-1} &\le \left(\frac{2A_1}{T}\right)^{p-1} + \int_{iT}^{(i+1)T} |f\left(-u'(s)\right)|ds, \\
&\quad + \int_{iT}^{(i+1)T}|g(1-u(s))|ds + \int_{iT}^{(i+1)T}\left|\frac{h(s)}{u(s)}\right|ds \\
&\quad + \int_{iT}^{(i+1)T}|e_k(s)|ds, \le \left(\frac{2A_1}{T}\right)^{p-1} \\
&\quad + Tf_m + Tg_{A_2,A_1} + \frac{Th_m}{A_2} + T\rho_1 := A_3,
\end{aligned} \tag{32}$$

where $g_{A_2,A_1} = \max_{A_2 \le u \le A_1} |g(1-u)|$, $f_m = \max_{s\in\mathbb{R}}|f(s)|$, $h_m = \max_{s\in\mathbb{R}}|h(s)|$. Thus,

$$\left|u'\right|_0 = \max_{t\in[-kT,kT]}|u'(t)| \le (A_3)^{1/(p-1)}. \tag{33}$$

Let $|u|_\infty = \max\{A_1, A_2, (A_3)^{1/(p-1)}\} + 1$ for $u \in \Omega$. Then, condition (1) of Theorem 1 holds. Next, let

$$\mathscr{F}(a) = f(0) + g(1-a) + \frac{\bar{h}}{a} - \bar{e_k} = 0, a \in \mathbb{R}. \tag{34}$$

Clearly, equation (34) has no solution on $\partial\Omega \cap \mathbb{R}$. Hence, condition (2) of Theorem 1 holds. Furthermore, by $\bar{h} > 0$ and (29), we have the following inequalities:

$$\begin{aligned}
&f(0) + g(1-u) + \frac{\bar{h}}{u} - \bar{e_k} > 0 \text{ for } u \in (0, A_2], \\
&f(0) + g(1-u) + \frac{\bar{h}}{u} - \bar{e_k} < 0 \text{ for } u \in [A_1, +\infty).
\end{aligned} \tag{35}$$

Thus

$$\left(f(0) + g(1-A_2) + \frac{\bar{h}}{A_2} - \bar{e_k}\right)\left(f(0) + g(1-A_1) + \frac{\bar{h}}{A_1} - \bar{e_k}\right) < 0, \tag{36}$$

and $d_B(\mathscr{F}, (\gamma_0, M_1) \cap \mathbb{R}, 0) \ne 0$, i.e., condition (3) of Theorem 1 holds. By using Theorem 1, we see that Equation (15) exists at least one positive $2kT$-periodic solution $u_k(t)$ such that

$$A_2 \le u_k(t) \le A_1, |u'_k(t)| \le (A_3)^{1/p-1}, k \in N. \tag{37}$$

Since $y(t) = 1 - x(t)$, there exist positive constants B_1, B_2, and B_3 such that

$$B_1 \le \omega_k(t) \le B_2, |\omega'_k(t)| \le B_3, k \in N, \tag{38}$$

where $\omega_k(t)$ is $2kT$-periodic solution to (11). Thus,

$$\left(\phi_p\left(\omega_k{}'(t)\right)\right)' + f\left(\omega_k{}'(t)\right) + g(\omega_k(t)) + \frac{h(t)}{1-\omega_k(t)} = e_k(t), \tag{39}$$

In view of Lemma 3, there exist $\omega_0 \in C^1(\mathbb{R}, \mathbb{R})$ and a subsequence $\{\omega_{k_i}\}$ of $\{\omega_k\}$ such that

$$\begin{aligned}
&\max_{t\in[-jT,jT]}|\omega_{k_i}(t) - \omega_0(t)| \longrightarrow 0, \\
&\max_{t\in[-jT,jT]}|\omega_{k_i}{}'(t) - \omega_0{}'(t)| \longrightarrow 0 \text{ as } t \longrightarrow +\infty, j \in N.
\end{aligned} \tag{40}$$

From (39), (40), and the standard argument, $\omega_0(t)$ is a solution of (11), i.e.,

$$\begin{aligned}
&\left(\phi_p\left(\omega_0{}'(t)\right)\right)' + f\left(\omega_0{}'(t)\right) \\
&\quad + g(\omega_0(t)) + \frac{h(t)}{1-\omega_0(t)} = e(t), t \in \mathbb{R}.
\end{aligned} \tag{41}$$

Now, we will show

$$\omega_0(t) \longrightarrow 0, \quad \omega_0{}'(t) \longrightarrow 0 \text{ as } |t| \longrightarrow +\infty. \tag{42}$$

Multiplying (39) by $\omega_k(t)$ and integrating it over $[-kT, KT]$, we have

$$\begin{aligned}&\int_{-kT}^{kT} \left|\omega_k{}'(t)\right|^p + \int_{-kT}^{kT} f\left(\omega_k{}'(s)\right)\omega_k(s)ds \\ &\quad + \int_{-kT}^{kT} g(\omega_k(s))\omega_k(s)ds + \int_{-kT}^{kT} \frac{h(s)\omega_k(s)}{1-\omega_k(s)}ds \\ &= \int_{-kT}^{kT} e_k(s)\omega_k(s)ds.\end{aligned} \tag{43}$$

From (43), assumptions (H_1) and (H_2), we have

$$\begin{aligned}&\int_{-kT}^{kT} \left|\omega_{k'}(t)\right|^p + \sigma\int_{-kT}^{kT} \left|\omega_k(s)\right|^{n+1} ds \\ &\leq \int_{-kT}^{kT} |e_k(s)\omega_k(s)|ds \\ &\leq \left(\int_{-kT}^{kT} |e_k(s)|^{n+1/n}ds\right)^{n/n+1} \left(\int_{-kT}^{kT} |\omega_k(s)|^{n+1}ds\right)^{1/n+1}.\end{aligned} \tag{44}$$

In view of (H_3) and (12), we have

$$\begin{aligned}&\int_{-kT}^{kT} |e_k(s)|^{(n+1)/n}ds \\ &= \int_{-kT}^{kT-\varepsilon_0} |e_k(s)|^{(n+1)/n}ds + \int_{kT-\varepsilon_0}^{kT} |e_k(s)|^{(n+1)/n}ds \\ &\leq \int_{-kT}^{kT-\varepsilon_0} |e(s)|^{(n+1)/n}ds + \varepsilon_0\rho_1^{(n+1)/n} \\ &\leq \int_{\mathbb{R}} |e(s)|^{(n+1)/n}ds + \varepsilon_0\rho_1^{(n+1)/n} \\ &\leq \rho_2 + \varepsilon_0\rho_1^{(n+1)/n} \coloneqq C_1.\end{aligned} \tag{45}$$

In view of (44) and (45), then

$$\sigma\int_{-kT}^{kT} |\omega_k(s)|^{n+1}ds \leq C_1^{n/(n+1)}\left(\int_{-kT}^{kT} |\omega_k(s)|^{n+1}ds\right)^{1/(n+1)}, \tag{46}$$

$$\int_{-kT}^{kT} \left|\omega_k{}'(t)\right|^p \leq C_1^{n/(n+1)}\left(\int_{-kT}^{kT} |\omega_k(s)|^{n+1}ds\right)^{1/(n+1)}. \tag{47}$$

It follows by (46) that

$$\int_{-kT}^{kT} |\omega_k(s)|^{n+1}ds \leq C_1\sigma^{-(n+1)/n}, k \in N. \tag{48}$$

In view of (47) and (48), then

$$\int_{-kT}^{kT} \left|\omega_k{}'(t)\right|^p \leq C_1\sigma^{-1/n}, k \in N. \tag{49}$$

From (48), (49) and standard limit analysis, we have

$$\int_{-\infty}^{+\infty} \left|\omega_0(s)\right|^{n+1}ds + \int_{-\infty}^{+\infty} \left|\omega_{0'}(s)\right|^p ds \leq C_1\sigma^{-(n+1)/n} + C_1\sigma^{-1/n}, \tag{50}$$

$$\int_{|t|>r} \left(|\omega_0(s)|^{n+1} + |\omega_{0'}(s)|^p\right)ds \longrightarrow 0 \text{ as } r \longrightarrow +\infty, \tag{51}$$

which together with Lemma 2 yields that

$$\begin{aligned}|\omega_0(t)| &\leq (2a)^{-1/(n+1)}\left(\int_{t-a}^{t+a} |\omega_0(s)|^{n+1}ds\right)^{1/(n+1)} \\ &\quad + a(2a)^{-1/p}\left(\int_{t-a}^{t+a} \left|\omega_0{}'(s)\right|^p ds\right)^{1/p} \\ &\leq \max\left\{(2a)^{-1/(n+1)}, a(2a)^{-1/p}\right\} \\ &\quad \cdot \left[\left(\int_{t-a}^{t+a} |\omega_0(s)|^{n+1}ds\right)^{1/(n+1)} \right. \\ &\quad \left. + \left(\int_{t-a}^{t+a} \left|\omega_0{}'(s)\right|^p ds\right)^{1/p}\right] \longrightarrow 0 \text{ as } |t| \longrightarrow +\infty.\end{aligned} \tag{52}$$

Thus,

$$|\omega_0(t)| \longrightarrow 0 \text{ as } |t| \longrightarrow +\infty. \tag{53}$$

Next, we prove

$$|\omega_0{}'(t)| \longrightarrow 0 \text{ as } |t| \longrightarrow +\infty. \tag{54}$$

Furthermore, by (38) we have

$$B_1 \leq \omega_0(t) \leq B_2, |\omega'_0(t)| \leq B_3, t \in \mathbb{R} \tag{55}$$

which together with (41) yields that

$$|\left(\phi_p\left(\omega_0{}'(t)\right)\right)'| \leq f_{B_3} + g_{B_1,B_2} + \frac{h_m}{B_1 - 1} + |e|_0 \coloneqq C_2, \tag{56}$$

where $f_{B_3} = \max_{|s|\le B_3} |f(s)|$, $g_{B_1,B_2} = \max_{B_1\le s\le B_2} |g(s)|$. If (54) does not hold. Then, there are constant $\delta \in (0, 1/2)$ and a sequence $\{t_k\}$ with $|t_1| < |t_2| < \cdots$ such that

$$\begin{aligned} |\phi_p(\omega_0{}'(t_k))| &\ge 2\delta, k = 1, 2, \cdots, \\ |\phi_p\left(\omega_0{}'(t)\right)| &= |\phi_p(\omega_0{}'(t_k)) + \int_{t_k}^{t} \left(\phi_p(\omega_0{}'(s))\right)' ds|, \\ &\ge |\phi_p(\omega_0{}'(t_k))| - \int_{t_k}^{t} |\left(\phi_p(\omega_0{}'(s))\right)'| ds, \\ &\ge \delta \text{ for } t \in \left[t_k, t_k + \frac{\delta}{1 + C_2}\right], \\ \int_{-\infty}^{+\infty} |\phi_p(\omega_0{}'(t))| dt &\ge \sum_{k=1}^{+\infty} \int_{t_k}^{t_k+(\delta/(1+C_2))} |\phi_p(\omega_0{}'(t))| dt = +\infty, \end{aligned} \tag{57}$$

which contradicts to (50). It is easy to see that (54) holds. Thus, $\omega_0(t)$ is just a homoclinic solution to Eq. (2).

4. Examples

This section presents two examples that demonstrate the validity of our theoretical results.

Example 5. Consider the following equation:

$$\begin{aligned} &\left(\left|u'(t)\right|^2 u'(t)\right)' + 1 + 2x'^2(t) + 2x^3(t) \\ &+ \frac{(3 - (1/2)\cos t)}{(1 - x(t))} = 5 + \frac{1}{2}\sin^2 t, \end{aligned} \tag{58}$$

where

$$\begin{aligned} &p = 4, f\left(x'(t)\right) = 1 + 2x'^2(t), g(x(t)) = 2x^3(t), \\ &h(t) = 3 - \frac{1}{2}\cos t, e(t) = 5 + \frac{1}{2}\sin^2 t. \end{aligned} \tag{59}$$

Obviously, $\rho_1 = 5.5, f(0) = 1, h_l = 2.5, h_l/(\rho_1 - f(0)) = 5/7 < 1$. We also easily check that assumptions (H_1)-(H_3) hold. Based on Theorem 4, Equation (58) has at least one nontrivial homoclinic solution.

Example 6. Consider the following equation:

$$u''(t) + 3 + 2x'^2(t) + 4x^3(t) + \frac{(5 - (1/2)\sin t)}{(1 - x(t))} = 8 + \frac{1}{2}\cos^2 t, \tag{60}$$

where

$$\begin{aligned} &p = 2, f\left(x'(t)\right) = 3 + 2x'^2(t), g(x(t)) = 4x^3(t), \\ &h(t) = 5 - \frac{1}{2}\sin t, e(t) = 8 + \frac{1}{2}\cos^2 t. \end{aligned} \tag{61}$$

Obviously, $\rho_1 = 8.5, f(0) = 3, h_l = 4.5, h_l/(\rho_1 - f(0)) = 9/11 < 1$. We also easily check that assumptions (H_1)-(H_3) hold. Based on Theorem 4, Equation (60) has at least one nontrivial homoclinic solution.

5. Conclusions

In this paper, we study a class of second-order singular equation with p-Laplacian. By employing some analytic techniques and continuation theorem due to Mana'sevich and Mawhin, we have presented some new sufficient criteria for the existence of homoclinic solutions for the above singular equation. These criteria possess adjustable parameters which are important in some applied fields. Finally, two examples are given to demonstrate the effectiveness of the obtained theoretical results. However, there exist many problems for further study such as heteroclinic orbits of second-order singular equations.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no competing interests.

Authors' Contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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