Hindawi
Journal of Function Spaces
Volume 2020, Article ID 2914269, 9 pages
https://doi.org/10.1155/2020/2914269

Research Article

Existence Theorems for Fractional Semilinear Integrodifferential Equations with Noninstantaneous Impulses and Delay

Bo Zhu [1] **and Minhui Zhu** [2]

[1] *School of Mathematic and Quantitative Economics, Shandong University of Finance and Economics, Jinan, 250014 Shandong, China*
[2] *School of Mathematics and Statistics, Qilu University of Technology, Jinan, 250353 Shandong, China*

Correspondence should be addressed to Bo Zhu; zhubo207@163.com

Received 29 May 2020; Accepted 13 July 2020; Published 4 August 2020

Guest Editor: Chuanjun Chen

In this paper, we consider a class of fractional semilinear integrodifferential equations with noninstantaneous impulses and delay. By the semigroup theory and fixed point theorems, we establish various theorems for the existence of mild solutions for the problem. An example involving partial differential equations with noninstantaneous impulses is given to show the application of our main results.

1. Introduction

Over the last couple of decades, fractional differential equations have been applied successfully to model many phenomena in physics, engineering, chemistry, financial, biology, etc. Consequently, the subject of fractional differential equations has attracted more and more attention worldwide and for more details, see for example [1–19] and the references therein.

Meanwhile, differential equations with impulsive effects have been used widely as mathematical models for the study of many phenomena in physical, biology, optimal control model of economics, etc. Much attention has been paid to the existence of solutions for the differential equations with impulses in abstract space. For details, see [7, 20–28].

In [26], the author initially studied the differential equations with noninstantaneous impulsive effects as follows:

$$\begin{cases} u'(t) = Au(t) + f(t, u(t)), & t \in (s_k, t_{k+1}],\ k = 0, 1, \cdots, N, \\ u(t) = g_k(t, u(t)), & t \in (t_k, s_k],\ k = 1, 2, \cdots, N, \\ u(0) = u_0, \end{cases} \tag{1}$$

where $A : D(A) \subset E \to E$ is the generator of a C_0-semigroup of bounded operators $\{T(t)\}_{t\geq 0}$ defined on a Banach space E.

In [20], the author studied the following integer order integrodifferential equations with instantaneous impulses in a Banach space E:

$$\begin{cases} u'(t) = Au(t) + f\left(t, u(\tau_1(t)), u(\tau_2(t)), \cdots, u(\tau_n(t)), \displaystyle\int_0^t h(t, s, u(\tau_{n+1}(s)))ds\right), \\ t \in [0, b], \quad t \neq t_k,\ k = 1, 2, \cdots, m, \\ u(0) + g(u) = u_0, \\ \Delta u|_{t=t_k} = I_k(u(t_k)), \quad k = 1, 2, \cdots, m, \end{cases} \tag{2}$$

where for any $t \geq 0$, the linear operator A is the infinitesimal generator of a compact, analytic semigroup, and the nonlinear term is Lipschitz continuous. The existence of mild solutions has been proved.

In this paper, we investigate the following fractional semilinear integrodifferential equations with noninstantaneous impulses and delay:

$$\begin{cases} {}^cD_t^{\beta}u(t) = A(t)u(t) + f\left(t, u(\tau_1(t)), u(\tau_2(t)), \cdots, u(\tau_n(t)), \int_0^t h(t, s, u(\tau_{n+1}(s)))ds\right), \\ t \in (s_k, t_{k+1}], \quad k = 0, 1, \cdots, N, \\ u(t) = l_k(t, u(t)), \quad t \in (t_k, s_k], \; k = 1, 2, \cdots, N, \\ u(0) + g(u) = u_0, \end{cases} \tag{3}$$

where $\beta \in (0, 1]$, ${}^cD_t^{\beta}$ is the Caputo's fractional derivative of order β, $A(t)$ is a closed and linear operator with domain $D(A)$ defined on a Banach space E, and the fixed points s_i and t_i satisfying $0 = s_0 < t_1 \leq s_1 < t_2 \leq \cdots < t_N \leq s_N < t_{N+1} = T_0$ are prefixed numbers. f, l_k, h, g $(k = 1, 2, \cdots, N)$ and τ_i $(i = 1, 2, \cdots, n + 1)$ are to be specified later.

Inspired by the results mentioned above, by the semigroup theory and fixed point theorems, we consider the existence of mild solutions for the fractional semilinear integrodifferential equations with noninstantaneous impulses and delay (3). In [7], the authors discussed the existence of solutions for the fractional ordinary differential equation with a generalized impulsive term. In [20–23], the authors discussed the integer or fractional differential equations with instantaneous impulses and the linear operator A is independent of t. In [26, 29, 30], the authors discussed the integer-order differential equations with noninstantaneous impulses and the linear operator A is independent of t. In [31–33], the authors discussed the fractional differential equations with noninstantaneous impulses and the linear operator A is also independent of t. In this paper, we consider the fractional semilinear integrodifferential equations with noninstantaneous impulses and delay, and the linear operator $A(t)$ is assumed to be dependent on t. Therefore, the mentioned results above are special cases of the problem investigated in this paper. Our results improve and generalize the results in References [7, 20–23, 26, 29–33].

The rest of this paper is organized as follows. In Section 2, we present the basic notation and preliminary results. In Section 3, we prove the existence of mild solutions. In Section 4, we give an illustrative example, followed by the conclusion of this paper in Section 5.

2. Preliminaries

Let $(E, \|\cdot\|)$ be a Banach space, $J = [0, T_0]$, $C(J, E) = \{u : J \longrightarrow E \text{ is continuous}\}$, $PC(J, E) = \{u : J \longrightarrow E : u \in C((s_k, t_{k+1}], E) \text{ and there exist } u(t_k^-) \text{ and } u(t_k^+) \text{ with } u(t_k^-) = u(t_k), k = 1, 2, \cdots, N\}$ with the PC-norm $\|u\|_{PC} = \sup\{\|u(t)\| : t \in J\}$.

Definition 1 [34]. The Riemann-Liouville fractional integral of order $\alpha > 0$ of a function $f : (0,\infty) \rightarrow R$ is given by

$$I_{0+}^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t - s)^{\alpha-1} f(s)ds, \tag{4}$$

provided that the right-hand side is pointwise defined on $(0, \infty)$.

Definition 2 [34]. The Caputo fractional derivative of order $\alpha > 0$ of a function $f : (0,\infty) \longrightarrow R$ is given by

$${}^cD_t^{\alpha} f(t) = \frac{1}{\Gamma(n - \alpha)} \int_0^t (t - s)^{n-\alpha-1} f^{(n)}(s)ds, \tag{5}$$

where $\Gamma(\cdot)$ denotes the Gamma function, α is a fractional number, $n = [\alpha] + 1$, provided that the right-hand side is pointwise defined on R^+.

Definition 3 [35, 36]. Let $A(t)$ be a closed and linear operator with domain $D(A)$ defined on a Banach space E and $\beta > 0$. Let $\rho[A(t)]$ be the resolvent set of $A(t)$, we call $A(t)$ the generator of an β-resolvent family if there exist $\omega \geq 0$ and a strongly continuous function $U_{\beta} : R_+^2 \longrightarrow B(E)$ such that $\{\lambda^{\beta} : \text{Re } \lambda > \omega\} \subset \rho(A)$ and

$$\left(\lambda^{\beta} I - A(s)\right)^{-1} u = \int_0^{\infty} e^{-\lambda(t-s)} U_{\beta}(t, s)u dt, \quad \text{Re }(\lambda) > \omega, u \in E. \tag{6}$$

In this case, $U_{\beta}(t, s)$ is called the β-resolvent family generated by $A(t)$.

Definition 4. A function $u \in PC(J, E)$ is called a mild solution of the problem (3), if $u(0) + g(u) = u_0$, and

$$u(t) = \begin{cases} U_{\beta}(t, 0)(u_0 - g(u)) + \int_0^t U_{\beta}(t, s) f\left(s, u(\tau_1(s)), u(\tau_2(s)), \cdots, u(\tau_n(s)), \int_0^s h(s, \sigma, u(\tau_{n+1}(\sigma)))d\sigma\right)ds, \; t \in [0, t_1], \\ l_k(t, u(t)), \quad t \in (t_k, s_k], \; k = 1, 2, \cdots, N, \\ U_{\beta}(t, s_k) l_k(s_k, u(s_k)) + \int_{s_k}^t U_{\beta}(t, s) f\left(s, u(\tau_1(s)), u(\tau_2(s)), \cdots, u(\tau_n(s)), \int_0^s h(s, \sigma, u(\tau_{n+1}(\sigma)))d\sigma\right)ds, \; t \in (s_k, t_{k+1}], \; k = 1, 2, \cdots, N. \end{cases} \tag{7}$$

Lemma 5. *(Sadovskii fixed point theorem). Let B be a convex, closed, and bounded subset of a Banach space E and $Q : B \longrightarrow B$ be a condensing map. Then, Q has one fixed point in B.*

3. Main Results

For convenience in presentation, we give here the basic assumptions to be used later throughout the paper.

(H_1) The function $f : J \times E^{n+1} \longrightarrow E$ is continuous, and for all $r > 0$, there exist nonnegative Lebesgue integrable functions $L_i \in L^1(J, R^+)$ $(i = 1, 2, \cdots, n+1)$ such that for any $t \in J, x_i, y_i \in T_r = \{x \in E : \|x\| \le r\}$ $(i = 1, 2, \cdots, n+1)$, we have

$$\|f(t, x_1, x_2, \cdots, x_{n+1}) - f(t, y_1, y_2, \cdots, y_{n+1})\| \le \sum_{i=1}^{n+1} L_i(t)\|x_i - y_i\|. \tag{8}$$

(H_2) The function $h : J \times J \times E \longrightarrow E$ is continuous, and for all $r > 0$, there exists a nonnegative Lebesgue integrable function $L_h(t, s) \in L^1(J \times J, R^+)$ such that for any $t \in J, x, y \in T_r$, we have

$$\|h(t, s, x) - h(t, s, y)\| \le L_h(t, s)\|x - y\|. \tag{9}$$

(H_3) $\tau_i : J \longrightarrow J$ are continuous such that $0 \le \tau_i(t) \le t$ $(i = 1, 2, \cdots, n+1)$ for all $t \in J$.

(H_4) The functions $l_k : J \times E \longrightarrow E$ and $g : \mathrm{PC}(J, E) \longrightarrow E$ are continuous, and for all $r > 0$, there exist nonnegative constants L_{l_k}, L_g such that for any $t \in J, x, y \in T_r, u, v \in \mathrm{PC}(J, E)$, we have

$$\begin{aligned} &\|l_k(t, x) - l_k(t, y)\| \le L_{l_k}\|x - y\|, k = 1, 2, \cdots, N, \\ &\|g(u) - g(v)\| \le L_g\|u - v\|_{\mathrm{PC}}. \end{aligned} \tag{10}$$

(H_4') There is a function $\phi_k : J \longrightarrow R^+$ $(k = 1, 2, 3, \cdots, N)$ such that $\|l_k(t, x)\| \le \phi_k(t)$ for every $t \in [t_k, s_k]$ and for every $x \in T_r$. Let $M_k \coloneqq \sup_{t \in [t_k, s_k]} \phi_k(t) < \infty$, $k = 1, 2, 3, \cdots, N$.

(H_5) The function $f : J \times E^{n+1} \longrightarrow E$ satisfies the Carathéodory condition, i.e., $f(\cdot, x_1, x_2, \cdots, x_{n+1})$ is measurable for all $x_i \in E$, and $f(t, \cdot, \cdot, \cdots, \cdot)$ is continuous for a.e. $t \in J$, and for all $r > 0$, there exists $m_{f_i} \in L^1(J, R^+)$ $(i = 1, 2, \cdots, n+1)$ such that $\|f(t, x_1, x_2, \cdots, x_{n+1})\| \le \sum_{i=1}^{n+1} m_{f_i}(t)\|x_i\|$ for $t \in J$, $x_i \in T_r$.

(H_6) The function $h : J \times J \times E \rightarrow E$ is continuous, and for all $r > 0$, there exists $m_h \in L^1(J \times J, R^+)$ such that $\|h(t, s, x)\| \le m_h(t, s)\|x\|$ for $s, t \in J, x \in T_r$.

Theorem 6. *Assume $\rho = \max\{\Delta, M(\Delta + N_1 + N_2)\} < 1$ and conditions (H_1)–(H_4) hold, where $M = \max_{0 \le s < t \le T_0} \|U_\beta(t, s)\| < +\infty$, $\Delta = \max\{L_g, L_{l_k}, k = 1, 2, \cdots, N\}$,*

$$\begin{aligned} N_1 &= \max\left\{\int_0^{t_1} \sum_{i=1}^{n} L_i(s)ds, \int_{s_k}^{t_{k+1}} \sum_{i=1}^{n} L_i(s)ds, k = 1, 2, \cdots, N\right\}, \\ N_2 &= \max\left\{\int_0^{t_1} L_{n+1}(s)\int_0^{t_1} L_h(s, \sigma)d\sigma ds, \right. \\ &\qquad \left. \int_{s_k}^{t_{k+1}} L_{n+1}(s)\int_0^{t_{k+1}} L_h(s, \sigma)d\sigma ds, k = 1, 2, \cdots, N\right\}. \end{aligned} \tag{11}$$

Then, there exists a unique mild solution $u^*(t)$ of the problem (3) on $\mathrm{PC}(J, E)$ and $\|u^*\|_{PC} \le r$, where

$$\begin{aligned} r \ge \max &\left\{\frac{\|l_k(t, 0)\|}{1 - L_{l_k}}, \frac{M\left(\|g(0)\|_{\mathrm{PC}} + \int_0^{t_1} \|f(s, 0, 0, \cdots, 0)\|ds + \int_0^{t_1}\int_0^s \|h(s, \sigma, 0)\|d\sigma ds\right)}{1 - M\left(L_g + \int_0^{t_1} \sum_{i=1}^{n} L_i(s)ds + \int_0^{t_1} L_{n+1}(s)\int_0^s L_h(s, \sigma)d\sigma ds\right)}, \right. \\ &\left. \frac{M\left(\|l_k(s_k, 0)\| + \int_{s_k}^{t_{k+1}} \|f(s, 0, 0, \cdots, 0)\|ds + \int_{s_k}^{t_{k+1}}\int_0^s \|h(s, \sigma, 0)\|d\sigma ds\right)}{1 - M\left(L_{l_k} + \int_{s_k}^{t_{k+1}} \sum_{i=1}^{n} L_i(s)ds + \int_{s_k}^{t_{k+1}} L_{n+1}(s)\int_0^s L_h(s, \sigma)d\sigma ds\right)}, \quad k = 1, 2, 3, \cdots, N\right\}. \end{aligned} \tag{12}$$

Proof. Define an operator $Q : \mathrm{PC}(J.E) \longrightarrow \mathrm{PC}(J.E)$ by

$$(Qu)(t) = \begin{cases} U_\beta(t, 0)(u_0 - g(u)) + \int_0^t U_\beta(t, s)f\left(s, u(\tau_1(s)), \cdots, u(\tau_n(s)), \int_0^s h(s, \sigma, u(\tau_{n+1}(\sigma)))d\sigma\right)ds, & t \in [0, t_1], \\ l_k(t, u(t)), \quad t \in (t_k, s_k], k = 1, 2, \cdots, N, & \\ U_\beta(t, s_k)l_k(s_k, u(s_k)) + \int_{s_k}^t U_\beta(t, s)f\left(s, u(\tau_1(s)), \cdots, u(\tau_n(s)), \int_0^s h(s, \sigma, u(\tau_{n+1}(\sigma)))d\sigma\right)ds, & t \in (s_k, t_{k+1}], k = 1, 2, \cdots, N. \end{cases} \tag{13}$$

Let $B_r = \{u \in \mathrm{PC}(J, E): \|u\|_{\mathrm{PC}} \le r\}$. We will prove $Q(B_r) \subset B_r$. Let $u \in B_r$, for $t \in [0, t_1]$, we have

$$\begin{aligned}\|(Qu)(t)\| &\le M\left(L_g\|u\|_{\mathrm{PC}} + \|g(0)\|_{\mathrm{PC}}\right) + M\int_0^t \sum_{i=1}^n L_i(s)\|u(\tau_i \\ &\cdot (s))\| + \|f(s,0,0,\cdots,0)\|ds + M\int_0^t L_{n+1}(s) \\ &\cdot \left(\int_0^s L_h(s,\sigma)\|u(\tau_{n+1}(\sigma))\|d\sigma \right.\\ &+ \int_0^s \|h(s,\sigma,0)\|d\sigma)ds \le M\left(\|g(0)\|_{\mathrm{PC}}\right.\\ &+ \int_0^{t_1}\|f(s,0,0,\cdots,0)\|ds + \int_0^{t_1}\int_0^s \|h(s,\sigma,0) \\ &\cdot \|d\sigma ds) + M\left(L_g + \int_0^{t_1}\sum_{i=1}^n L_i(s)ds\right.\\ &+ \int_0^{t_1} L_{n+1}(s)\int_0^s L_h(s,\sigma)d\sigma ds)r \le r.\end{aligned} \tag{14}$$

For all $t \in (s_k, t_{k+1}]$ $(k = 1, 2, \cdots, N)$, we have

$$\begin{aligned}\|(Qu)(t)\| &\le M\left(L_{l_k}r + \|l_k(s_k,0)\|\right) + M\int_{s_k}^{t_{k+1}} r\sum_{i=1}^n L_i(s) \\ &+ \|f(s,0,0,\cdots,0)\|ds + M\int_{s_k}^{t_{k+1}} rL_{n+1}(s) \\ &\cdot \left(\int_0^s L_h(s,\sigma)d\sigma + \int_0^s \|h(s,\sigma,0)\|d\sigma\right)ds \\ &\le M\left(\|l_k(s_k,0)\| + \int_{s_k}^{t_{k+1}}\|f(s,0,0,\cdots,0)\|ds\right.\\ &+ \int_{s_k}^{t_{k+1}}\int_0^s \|h(s,\sigma,0)\|d\sigma ds) + M\left(L_{l_k}\right.\\ &+ \int_{s_k}^{t_{k+1}}\sum_{i=1}^n L_i(s)ds + \int_{s_k}^{t_{k+1}} L_{n+1}(s) \\ &\cdot \int_0^s L_h(s,\sigma)d\sigma ds)r \le r.\end{aligned} \tag{15}$$

For all $t \in (t_k, s_k](k = 1, 2, \cdots, N)$, we have

$$\|(Qu)(t)\| \le L_{l_k}r + \|l_k(t,0)\| \le r. \tag{16}$$

From the above inequalities, we have that $Qu \in B_r$. Next, we prove that Q is a contraction map on B_r. For all $u, v \in \mathrm{PC}(J, E)$ and $t \in [0, t_1]$, we have

$$\begin{aligned}\|(Qu)(t) - (Qv)(t)\| &\le \|U_\beta(t,0)(g(u) - g(v))\| \\ &+ \int_0^t (t-s)^{\alpha-1}\|U_\beta(t,s)\| \\ &\times \|f(s,u(\tau_1(s)),u(\tau_2(s)),\cdots, \\ &u(\tau_n(s)),\int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))d\sigma) \\ &- f(s,v(\tau_1(s)),v(\tau_2(s)),\cdots,v(\tau_n(s)), \\ &\int_0^s h(s,\sigma,v(\tau_{n+1}(\sigma)))d\sigma)\|ds \le ML_g\|u \\ &- v\|_{\mathrm{PC}} + M\int_0^t \sum_{i=1}^n L_i(s)\|u(\tau_i(s)) \\ &- v(\tau_i(s))\|ds + M\int_0^t L_{n+1}(s) \\ &\cdot \left\|\int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))d\sigma\right.\\ &- \int_0^s h(s,\sigma,v(\tau_{n+1}(\sigma)))d\sigma\|ds \\ &\le ML_g\|u - v\|_{\mathrm{PC}} + M\int_0^t \sum_{i=1}^n L_i(s)ds\|u \\ &- v\|_{\mathrm{PC}} + M\int_0^t L_{n+1}(s)\int_0^s L_h(s,\sigma)\|(u \\ &\cdot (\tau_{n+1}(\sigma)) - v(\tau_{n+1}(\sigma)))\|d\sigma ds \\ &\le ML_g\|u - v\|_{\mathrm{PC}} + M\int_0^t \sum_{i=1}^n L_i(s)ds\|u \\ &- v\|_{\mathrm{PC}} + M\int_0^t L_{n+1}(s)\int_0^s L_h(s,\sigma) \\ &\cdot d\sigma ds\|u - v\|_{\mathrm{PC}} \le M\left(L_g\right.\\ &+ \int_0^{t_1}\sum_{i=1}^n L_i(s)ds + \int_0^{t_1} L_{n+1}(s)\int_0^{t_1} L_h \\ &\cdot (s,\sigma)d\sigma ds)\|u - v\|_{\mathrm{PC}} \le M\left(L_g + N_1\right.\\ &+ N_2)\|u - v\|_{\mathrm{PC}}.\end{aligned} \tag{17}$$

For all $t \in (s_k, t_{k+1}]$ $(k = 1, 2, \cdots, N)$, we have

$$\begin{aligned}\|(Qu)(t) - (Qv)(t)\| &\le \|U_\beta(t,s_k)(l_k(s_k,u(s_k)) - l_k(s_k,v(s_k)))\| \\ &+ M\int_{s_k}^t \|f(s,u(\tau_1(s)),u(\tau_2(s)),\cdots, \\ &u(\tau_n(s)),\int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))d\sigma) \\ &- f(s,v(\tau_1(s)),v(\tau_2(s)),\cdots,v(\tau_n(s)), \\ &\int_0^s h(s,\sigma,v(\tau_{n+1}(\sigma)))d\sigma)\|ds \\ &\le ML_{l_k}\|u - v\|_{\mathrm{PC}} + M\int_{s_k}^t \sum_{i=1}^n L_i(s) \\ &\cdot \|u(\tau_i(s)) - v(\tau_i(s))\|ds\end{aligned}$$

$$+M\int_{s_k}^{t} L_{n+1}(s)\left\|\int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))d\sigma - \int_0^s h(s,\sigma,v(\tau_{n+1}(\sigma)))d\sigma\right\|ds \le ML_{l_k}\|u-v\|_{PC} + M\int_{s_k}^{t}\sum_{i=1}^{n} L_i(s)ds\|u-v\|_{PC} + M\int_{s_k}^{t} L_{n+1}(s)\int_0^s L_h(s,\sigma) \cdot\|(u(\tau_{n+1}(\sigma))-v(\tau_{n+1}(\sigma)))\|d\sigma ds \le ML_{l_k}\|u-v\|_{PC} + M\int_{s_k}^{t_{k+1}}\sum_{i=1}^{n} L_i(s)ds\|u-v\|_{PC} + M\int_{s_k}^{t_{k+1}} L_{n+1}(s)\int_0^{t_{k+1}} L_h(s,\sigma) \cdot d\sigma ds\|u-v\|_{PC} \le M\left(L_{l_k}+N_1+N_2\right)\|u-v\|_{PC}. \tag{18}$$

For all $t\in(t_k,s_k](k=1,2,\cdots,N)$, we have

$$\|(Qu)(t)-(Qv)(t)\| = \|l_k(t,u(t))-l_k(t,v(t))\| \le L_{l_k}\|u-v\|_{PC}. \tag{19}$$

From the above results, for all $u,v\in PC(J,E)$, we have

$$\|Qu-Qv\|_{PC} \le \rho\|u-v\|_{PC}. \tag{20}$$

Therefore, $Q: B_r\to B_r$ is a contraction map and there exists a unique mild solution u^* of the problem (3) in B_r and $\|u^*\|_{PC}\le r$.

Theorem 7. *Assume that the conditions (H_3), (H_4), (H_4'), (H_5), and (H_6) hold, $U_\beta(t,s)$ is compact for $t,s>0$ and*

$$\varrho := \max\left\{L_{l_k}, ML_{l_k}, M\left(L_g+\int_0^{t_1}\sum_{i=1}^{n} m_{f_i}(s)ds + \int_0^{t_1} m_{f_{n+1}}(s)\int_0^{t_1} m_h(s,\sigma)d\sigma ds\right), M\left(M_k+\int_{s_k}^{t_{k+1}}\sum_{i=1}^{n} m_{f_i}(s)ds+\int_{s_k}^{t_{k+1}} m_{f_{n+1}}(s)\int_0^{t_{k+1}} m_h(s,\sigma)d\sigma ds\right), k=1,2,\cdots,N\right\} < 1. \tag{21}$$

Then, the problem (3) has a mild solution $u^*\in PC(J,E)$ and $\|u^*\|_{PC}\le r$, where

$$r \ge \max\left\{M_k, \frac{M(\|u_0\|+\|g(0)\|)}{1-M\left(L_g+\int_0^{t_1}\sum_{i=1}^{n} m_{f_i}(s)ds+\int_0^{t_1} m_{f_{n+1}}(s)\int_0^{t_1} m_h(s,\sigma)d\sigma ds\right)}, \frac{MM_k}{1-M\left(\int_{s_k}^{t_{k+1}}\sum_{i=1}^{n} m_{f_i}(s)ds+\int_{s_k}^{t_{k+1}} m_{f_{n+1}}(s)\int_0^{t_{k+1}} m_h(s,\sigma)d\sigma ds\right)}, \quad k=1,2,3,\cdots,N\right\}. \tag{22}$$

Proof. We introduce the decomposition $Q=Q_1+Q_2$, where $Q_j: PC(J,E)\to PC(J,E)$ $(j=1,2)$ are defined by

$$(Q_1u)(t) = \begin{cases} U_\beta(t,0)(u_0-g(u)), & t\in[0,t_1], \\ l_k(t,u(t)), & t\in(t_k,s_k],\ k=1,2,\cdots,N, \\ U_\beta(t,s_k)l_k(s_k,u(s_k)), & t\in(s_k,t_{k+1}],\ k=1,2,\cdots,N, \end{cases}$$

$$(Q_2u)(t) = \begin{cases} \int_0^t U_\beta(t,s)f\left(s,u(\tau_1(s)),\cdots,u(\tau_n(s)),\int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))d\sigma\right)ds, & t\in[0,t_1], \\ 0, & t\in(t_k,s_k],\ k=1,2,\cdots,N, \\ \int_{s_k}^t U_\beta(t,s)f\left(s,u(\tau_1(s)),\cdots,u(\tau_n(s)),\int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))d\sigma\right)ds, & t\in(s_k,t_{k+1}],\ k=1,2,\cdots,N. \end{cases} \tag{23}$$

Let $B_r=\{u\in PC(J,E): \|u\|_{PC}\le r\}$, we then prove that the operator $Q=Q_1+Q_2$ is a condensing map on B_r. It is easy to see that B_r is a closed, bounded, and convex subset of $PC(J,E)$.

Step 1. We prove $Q(B_r) \subset B_r$. Let $u \in B_r$. For $t \in [0, t_1]$, we have

$$\begin{aligned}\|(Qu)(t)\| &\le \|U_\beta(t,0)(u_0 - g(u))\| + \int_0^t \|U_\beta(t,s)\| \\ &\quad \times \|f(s, u(\tau_1(s)), u(\tau_2(s)), \cdots, u(\tau_n(s)), \\ &\quad \int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma))))\| ds \le M(\|u_0\| + \|g(0)\| \\ &\quad + L_g\|u\|_{PC}) + M\int_0^t \sum_{i=1}^n m_{f_i}(s)\|u(\tau_i(s))\| ds \\ &\quad + M\int_0^t m_{f_{n+1}}(s)\left\|\int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))d\sigma\right\| ds \\ &\le M(\|u_0\| + \|g(0)\| + L_g r) + M\int_0^t \sum_{i=1}^n m_{f_i}(s) ds \|u\|_{PC} \\ &\quad + M\int_0^t m_{f_{n+1}}(s)\int_0^s m_h(s,\sigma) d\sigma ds \|u\|_{PC} \\ &\le M(\|u_0\| + \|g(0)\| + L_g r) + M\left(\int_0^{t_1} \sum_{i=1}^n m_{f_i}(s) ds \right. \\ &\quad \left. + \int_0^{t_1} m_{f_{n+1}}(s)\int_0^{t_1} m_h(s,\sigma) d\sigma ds\right) r \le r. \end{aligned} \tag{24}$$

For all $t \in (t_k, s_k]$ $(k = 1, 2, \cdots, N)$, we have

$$\|(Qu)(t)\| = \|(Q_1u)(t)\| = \|l_k(t, u(t))\| \le M_k \le r. \tag{25}$$

For all $t \in (s_k, t_{k+1}]$ $(k = 1, 2, \cdots, N)$, we have

$$\begin{aligned}\|(Qu)(t)\| &\le \|U_\beta(t,s_k) l_k(s_k, u(s_k))\| \\ &\quad + \int_{s_k}^t \left\| U_\beta(t,s) f(s, u(\tau_1(s)), u(\tau_2(s)), \cdots, u(\tau_n(s)), \right. \\ &\quad \left. \int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))d\sigma) \right\| ds \\ &\le MM_k + M\int_{s_k}^t \sum_{i=1}^n m_{f_i}(s)\|u(\tau_i(s))\| ds \\ &\quad + M\int_{s_k}^t m_{f_{n+1}}(s)\left\|\int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))d\sigma\right\| ds \\ &\le MM_k + M\int_{s_k}^t \sum_{i=1}^n m_{f_i}(s) ds \|u\|_{PC} \\ &\quad + M\int_{s_i}^t m_{f_{n+1}}(s)\int_0^s m_h(s,\sigma)\|u(\tau_{n+1}(\sigma))\| d\sigma ds \\ &\le MM_k + M\int_{s_k}^{t_{k+1}} \sum_{i=1}^n m_{f_i}(s) ds \|u\|_{PC} \\ &\quad + M\int_{s_k}^{t_{k+1}} m_{f_{n+1}}(s)\int_0^{t_{k+1}} m_h(s,\sigma) d\sigma ds \|u\|_{PC} \\ &\le MM_k + M\left(\int_{s_k}^{t_{k+1}} \sum_{i=1}^n m_{f_i}(s) ds \right. \\ &\quad \left. + \int_{s_k}^{t_{k+1}} m_{f_{n+1}}(s)\int_0^{t_{k+1}} m_h(s,\sigma) d\sigma ds\right) r \le r. \end{aligned} \tag{26}$$

Hence, $Q(B_r) \subset B_r$.

Step 2. We prove that Q_1 is a contraction on B_r. For any u, $v \in B_r$ and $t \in [0, t_1]$, we have

$$\begin{aligned}\|(Q_1u)(t) - (Q_1v)(t)\| &\le \|U_\beta(t,0)(g(u) - g(v))\| \\ &\le ML_g\|u - v\|_{PC}. \end{aligned} \tag{27}$$

For all $t \in (t_k, s_k]$ $(k = 1, 2, \cdots, N)$, we have

$$\|(Q_1u)(t) - (Q_1v)(t)\| \le L_{l_k}\|u - v\|_{PC}. \tag{28}$$

For all $t \in (s_k, t_{k+1}]$ $(k = 1, 2, \cdots, N)$, we have

$$\begin{aligned}\|(Q_1u)(t) - (Q_1v)(t)\| &\le \|U_\beta(t,s_k)(l_k(s_k,u(s_k)) \\ &\quad - l_k(s_k, v(s_k)))\| \le ML_{l_k}\|u - v\|_{PC}. \end{aligned} \tag{29}$$

Thus, since $\varrho < 1$, we get that Q_1 is a contraction on B_r.

Step 3. We prove that Q_2 is completely continuous on B_r. Firstly, we prove that the operator Q_2 is continuous. Let $\{u_m\}_0^\infty \subset B_r$, and $u_m \to u \in B_r$. For any $t \in [0, t_1]$, by (H_3), we have

$$\begin{aligned}\|u_m(\tau_i(t)) - u(\tau_i(t))\| \le \|u_m - u\|_{PC} \to 0 \ (m \to \infty), \\ i = 1, 2, \cdots, n+1. \end{aligned} \tag{30}$$

By (H_5),

$$\begin{aligned} &f\left(t, u_m(\tau_1(t)), u_m(\tau_2(t)), \cdots, u_m(\tau_n(t)), \int_0^s h(s,\sigma,u_m(\tau_{n+1}(\sigma)))d\sigma\right) \\ &\quad \to f\left(t, u(\tau_1(t)), u(\tau_2(t)), \cdots, u(\tau_n(t)), \int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))d\sigma\right), \\ &\quad (m \to \infty), \quad \text{a.e. } t \in [0, t_1], \end{aligned}$$

$$\begin{aligned} &\left\| f\left(t, u_m(\tau_1(t)), \cdots, u_m(\tau_n(t)), \int_0^s h(s,\sigma,u_m(\tau_{n+1}(\sigma)))d\sigma\right) \right. \\ &\quad \left. - f\left(t, u(\tau_1(t)), u(\tau_2(t)), \cdots, u(\tau_n(t)), \int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))d\sigma\right) \right\| \\ &\quad \le 2r\left(\sum_{i=1}^n m_{f_i}(t) + m_{f_{n+1}}(t)\int_0^t m_h(t,s) ds\right), \quad \text{a.e. } t \in [0, t_1]. \end{aligned} \tag{31}$$

By the Lebesgue dominated convergence theorem, we have

$$\|(Q_2u_m)(t)-(Q_2u)(t)\| \le M\int_0^t \|f(t,u_m(\tau_1(t)),\cdots,u_m(\tau_n(t)),\int_0^s h(s,\sigma,u_m(\tau_{n+1}(\sigma)))d\sigma) - f(t,u(\tau_1(t)),u(\tau_2(t)),\cdots,u(\tau_n(t)),\int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))d\sigma)\|ds \to 0, \tag{32}$$

as $m\to\infty$. Therefore, $\|Q_2u_m - Q_2u\|_{PC}\to 0$, as $m\to\infty$. For $t\in(s_k,t_{k+1}]$ $(k=1,2,\cdots,N)$, the proof is similar to that for $t\in[0,t_1]$. Hence, Q_2 is continuous. By the proof of Step 1, for any $t\in J$, it is easy to see that Q_2 maps a bounded set into a bounded set in B_r.

Secondly, we prove that $Q_2(B_r)$ is equicontinuous.

Case 1. For $[0,t_1]$, $0\le e_1<e_2\le t_1$, $u\in B_r$,

$$\begin{aligned}\|(Q_2u)(e_2)-(Q_2u)(e_1)\| \le{}& \left\|\int_0^{e_1}\left(U_\beta(e_2,s)-U_\beta(e_1,s)\right)\times f(s,u(\tau_1(s)),u(\tau_2(s)),\cdots,u(\tau_n(s)),\int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))d\sigma)\cdot ds\right\| \\ &+\left\|\int_{e_1}^{e_2}U_\beta(e_2,s)\times f(s,u(\tau_1(s)),u(\tau_2(s)),\cdots,u(\tau_n(s)),\int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))d\sigma)ds\right\| \\ \coloneqq{}& I_1+I_2.\end{aligned} \tag{33}$$

By (H_5), we have

$$\begin{aligned} I_1 &\le \left(\int_0^{t_1}\sum_{i=1}^n m_{f_i}(s)ds + \int_0^{t_1} m_{f_{n+1}}(s)\int_0^{t_1} m_h(s,\sigma)d\sigma ds\right)\cdot r\sup_{s\in[0,t_1]}\|U_\beta(e_2,s)-U_\beta(e_1,s)\|, \\ I_2 &\le M\left(\int_{e_1}^{e_2}\sum_{i=1}^n m_{f_i}(s)ds + \int_{e_1}^{e_2} m_{f_{n+1}}(s)\int_0^s m_h(s,\sigma)d\sigma ds\right)r. \end{aligned} \tag{34}$$

Further, the compactness of $U_\beta(t,s)$ for $t,s>0$ implies that the $U_\beta(t,s)$ for $t,s>0$ is continuous in the sense of uniformly operator topology. Thus, $I_1\to 0$ as $e_2\to e_1$. It is easy to see that $I_2\to 0$ as $e_2-e_1\to 0$ since the functions $m_{f_i}\in L^1(J,R^+)$ $(i=1,2,\cdots,n+1)$ and $m_h(t,s)\in L^1(J\times J,R^+)$. Therefore, $\|(Q_2u)(e_2)-(Q_2u)(e_1)\|\to 0$ as $e_2-e_1\to 0$.

Case 2. For $(s_k,t_{k+1}]$ $(k=1,2,\cdots,N)$, $s_k\le e_1<e_2\le t_{k+1}$, $u\in B_r$,

$$\begin{aligned}\|(Q_2u)(e_2)-(Q_2u)(e_1)\| \le{}& \left\|\int_{s_k}^{e_1}\left(U_\beta(e_2,s)-U_\beta(e_1,s)\right)\times f(s,u(\tau_1(s)),u(\tau_2(s)),\cdots,u(\tau_n(s)),\int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))\cdot d\sigma)ds\right\| \\ &+\left\|\int_{e_1}^{e_2}U_\beta(e_2,s)\times f(s,u(\tau_1(s)),u(\tau_2(s)),\cdots,u(\tau_n(s)),\int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))\cdot d\sigma)ds\right\| \coloneqq I_3+I_4.\end{aligned} \tag{35}$$

Similar to the proof for Case 1, we get $\|(Q_2u)(e_2)-(Q_2u)(e_1)\|\to 0$ as $e_2-e_1\to 0$. Therefore, $Q_2(B_r)$ is equicontinuous.

Finally, we prove that $Q_2(B_r)$ is precompact. For every fixed t $(0<t\le t_1)$ and $0<\varepsilon<t$, let $u\in B_r$ and define

$$(Q_{2,\varepsilon}u)(t) = \int_0^{t-\varepsilon} U_\beta(t,s)f(s,u(\tau_1(s)),u(\tau_2(s)),\cdots,u(\tau_n(s)),\int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))d\sigma)ds. \tag{36}$$

Due to the compactness of $U(t,s)$ for $t,s>0$, the set $Y_\varepsilon(t)=\{(Q_{2,\varepsilon}u)(t)\colon u\in B_r\}$ is relatively compact in E for any ε $(0<\varepsilon<t)$. For any $u\in B_r$, it follows from

$$\begin{aligned}\|(Q_2u)(t)-(Q_{2,\varepsilon}u)(t)\| \le{}& \left\|\int_{t-\varepsilon}^t U_\beta(t,s)f(s,u(\tau_1(s)),\cdots,u(\tau_n(s)),\int_0^s h(s,\sigma,u(\tau_{n+1}(\sigma)))d\sigma)\cdot ds\right\| \\ \le{}& M\left(\int_{t-\varepsilon}^t\sum_{i=1}^n m_{f_i}(s)ds + \|m_h\|\int_{t-\varepsilon}^t m_{f_{n+1}}(s)\int_0^{t_1} m_h(s,\sigma)d\sigma ds\right)r,\end{aligned} \tag{37}$$

that $Y(t)=\{(Q_2u)(t)\colon u\in B_r\}$ is totally bounded. Thus, $Y(t)$ is relatively compact in E. Similarly, we can prove that $Q_2(B_r)(t)\subset E$, $t\in(s_k,t_{k+1}]$ $(k=1,2,\cdots,N)$ is precompact. Using the Arzelà-Ascoli theorem, we have that $Q_2:\mathrm{PC}[J,E]\to\mathrm{PC}[J,E]$ is completely continuous. Hence, the operator $Q=Q_1+Q_2$ is a condensing operator on B_r. By Lemma 5, Q has a fixed point u^* in B_r, which is a mild solution of the problem (3).

Remark 8. In this paper, we consider the fractional semilinear integrodifferential equations with noninstantaneous impulses and delay, and the linear operator $A(t)$ is assumed to be dependent on t. In conditions (H_1) and (H_2), $L_i \in L^1(J, R^+)$ $(i = 1, 2, \cdots, n+1)$ and $L_h(t, s) \in L^1(J \times J, R^+)$ are Lebesgue integrable functions, not constants. The mentioned results above are special cases of the problem investigated in this paper.

4. An Application

In order to show the application of the main results of this paper, we consider the following problem:

$$\begin{cases} {}^cD_t^\alpha u(x,t) = t\dfrac{\partial^2}{\partial x^2}u(x,t) + \dfrac{t}{1+t^2}u(x,\sin t) + \dfrac{1}{1+t^2}\left(\displaystyle\int_0^t u(x,\sin s)ds\right)^{1/2}, \\ (t,x) \in \bigcup_{i=1}^{N}[s_i, t_{i+1}] \times [0,\pi], \\ u(x,t) = G_i(t, u(x,t)), \quad x \in [0,\pi],\ t \in (t_i, s_i], \\ u(0,t) = u(\pi,t) = 0, \quad 0 \le t \le 1, \\ u(x,0) + \dfrac{1}{3}u\cos u = u_0(x), \quad 0 \le x \le \pi, \end{cases} \tag{38}$$

where $0 = s_0 < t_1 \le s_1 < \cdots < t_N \le s_N < t_{N+1} = 1, 0 < \alpha \le 1$, $u_0(x) \in E = L^2[0,\pi]$ and the operator $A : D(A) \subset E \to E$ is defined by $A(t)(z) = t(\partial^2 u/\partial x^2)$ with

$$D(A) = \left\{u \in E : u'' \in E, u(0) = u(\pi) = 0\right\}. \tag{39}$$

It is well known that $A(t)$ generates a β-resolvent family on E.

Let

$$\begin{aligned} u(t) &= u(\cdot, t), \\ f(t,x,y) &= \frac{t}{1+t^2}x\sin t + \frac{1}{1+t^2}y^{1/2}, \\ l_i(t,u) &= G_i(\mathrm{t}, u(\cdot,t)), \\ g(u) &= \frac{1}{3}u\cos u. \end{aligned} \tag{40}$$

Then, (38) takes the abstract form (3). Assume that all the conditions of Theorems 6 and 7 are satisfied, then the problem (38) has a mild solution on $[0, 1]$.

5. Conclusions

This paper investigates the existence of mild solutions for a class of fractional semilinear integrodifferential equations with noninstantaneous impulses and delay by the fixed point theorem and the semigroup theory. In [7, 20–23, 26, 29–33], the linear operator A is independent of t, while in this paper, the linear operator $A(t)$ is allowed to be dependent on t. Therefore, the differential equations in [7, 20–23, 26, 29–33] are special cases of the problem investigated in this paper. Hence, our results improve and generalize the results of References [7, 20–23, 26, 29–33].

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest

Authors' Contributions

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

Acknowledgments

This work was supported by the Project of Shandong Province Higher Educational Science and Technology Program (No. J16LI14) and by the National Natural Science Foundation of China (No. 11871302).

References

[1] G. Wang, "Twin iterative positive solutions of fractional q-difference Schrödinger equations," *Applied Mathematics Letters*, vol. 76, pp. 103–109, 2018.

[2] B. Zhu and B. Han, "Existence and uniqueness of mild solutions for fractional partial integro-differential equations," *Mediterranean Journal of Mathematics*, vol. 17, no. 4, 2020.

[3] X. Zhang, L. Liu, and Y. Wu, "Multiple positive solutions of a singular fractional differential equation with negatively perturbed term," *Mathematical and Computer Modelling*, vol. 55, no. 3-4, pp. 1263–1274, 2012.

[4] C. Chen, H. Liu, X. Zheng, and H. Wang, "A two-grid MMOC finite element method for nonlinear variable-order time-fractional mobile/immobile advection–diffusion equations," *Computers and Mathematics with Applications*, vol. 79, no. 9, pp. 2771–2783, 2020.

[5] P. Chen, X. Zhang, and Y. Li, "Study on fractional non-autonomous evolution equations with delay," *Computers & Mathematics with Applications*, vol. 73, no. 5, pp. 794–803, 2017.

[6] X. Zhang, L. Liu, Y. Wu, and B. Wiwatanapataphee, "Nontrivial solutions for a fractional advection dispersion equation in anomalous diffusion," *Applied Mathematics Letters*, vol. 66, pp. 1–8, 2017.

[7] Z. Lin, J. R. Wang, and W. Wei, "Multipoint BVPs for generalized impulsive fractional differential equations," *Applied Mathematics and Computation*, vol. 258, pp. 608–616, 2015.

[8] B. Zhu, L. Liu, and Y. Wu, "Local and global existence of mild solutions for a class of nonlinear fractional reaction-diffusion equations with delay," *Applied Mathematics Letters*, vol. 61, pp. 73–79, 2016.

[9] X. Zhang and Q. Zhong, "Triple positive solutions for nonlocal fractional differential equations with singularities both on time and space variables," *Applied Mathematics Letters*, vol. 80, pp. 12–19, 2018.

[10] J. He, X. Zhang, L. Liu, Y. Wu, and Y. Cui, "A singular fractional Kelvin–Voigt model involving a nonlinear operator and their convergence properties," *Boundary Value Problems*, vol. 2019, no. 1, 2019.

[11] B. Zhu, L. Liu, and Y. Wu, "Local and global existence of mild solutions for a class of semilinear fractional integro-differential equations," *Fractional Calculus and Applied Analysis*, vol. 20, no. 6, pp. 1338–1355, 2017.

[12] P. Chen, X. Zhang, and Y. Li, "Cauchy problem for fractional non-autonomous evolution equations," *Banach Journal of Mathematical Analysis*, vol. 14, no. 2, pp. 559–584, 2020.

[13] P. Chen, X. Zhang, and Y. Li, "Existence and approximate controllability of fractional evolution equations with nonlocal conditions via resolvent operators," *Fractional Calculus & Applied Analysis*, vol. 23, no. 1, pp. 268–291, 2020.

[14] P. Chen, X. Zhang, and Y. Li, "Approximate controllability of nonautonomous evolution system with nonlocal conditions," *Journal of Dynamical and Control Systems*, vol. 26, no. 1, pp. 1–16, 2020.

[15] P. Chen, X. Zhang, and Y. Li, "Approximation technique for fractional evolution equations with nonlocal integral conditions," *Mediterranean Journal of Mathematics*, vol. 14, no. 6, pp. 1–16, 2017.

[16] P. Chen, X. Zhang, and Y. Li, "Non-autonomous parabolic evolution equations with non-instantaneous impulses governed by noncompact evolution families," *Journal of Fixed Point Theory and Applications*, vol. 21, no. 3, p. 84, 2019.

[17] P. Chen, X. Zhang, and Y. Li, "Non-autonomous evolution equations of parabolic type with non-instantaneous impulses," *Mediterranean Journal of Mathematics*, vol. 16, no. 5, p. 118, 2019.

[18] P. Chen, X. Zhang, and Y. Li, "Fractional non-autonomous evolution equation with nonlocal conditions," *Journal of Pseudo-Differential Operators and Applications*, vol. 10, no. 4, pp. 955–973, 2019.

[19] P. Chen, Y. Li, X. Zhang, and Department of Mathematics, Northwest Normal University, Lanzhou 730070, China, "Cauchy problem for stochastic non-autonomous evolution equations governed by noncompact evolution families," *Discrete & Continuous Dynamical Systems - B*, vol. 22, no. 11, 2017.

[20] Z. Yan, "Existence of solutions for nonlocal impulsive partial functional integrodifferential equations via fractional operators," *Journal of Computational and Applied Mathematics*, vol. 235, no. 8, pp. 2252–2262, 2011.

[21] A. Chauhan and J. Dabas, "Local and global existence of mild solution to an impulsive fractional functional integro-differential equation with nonlocal condition," *Communications in Nonlinear Science and Numerical Simulation*, vol. 19, no. 4, pp. 821–829, 2014.

[22] F.-D. Ge, H.-C. Zhou, and C.-H. Kou, "Approximate controllability of semilinear evolution equations of fractional order with nonlocal and impulsive conditions via an approximating technique," *Applied Mathematics & Computation*, vol. 275, pp. 107–120, 2016.

[23] G. L. Zhang, M. H. Song, and M. Z. Liu, "Exponential stability of the exact solutions and the numerical solutions for a class of linear impulsive delay differential equations," *Journal of Computational and Applied Mathematics*, vol. 285, pp. 32–44, 2015.

[24] P. Chen, X. Zhang, Y. Li, and Department of Mathematics, Northwest Normal University, Lanzhou 730070, China, "A blowup alternative result for fractional nonautonomous evolution equation of Volterra type," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 5, pp. 1975–1992, 2018.

[25] H. Li and Y. Kao, "Mittag-Leffler stability for a new coupled system of fractional-order differential equations with impulses," *Applied Mathematics & Computation*, vol. 361, pp. 22–31, 2019.

[26] E. Hernandez and D. O'Regan, "On a new class of abstract impulsive differential equations," *Proceedings of the American Mathematical Society*, vol. 141, pp. 1641–1649, 2010.

[27] G. Arthi, J. H. Park, and H. Y. Jung, "Existence and exponential stability for neutral stochastic integrodifferential equations with impulses driven by a fractional Brownian motion," *Communications in Nonlinear Science and Numerical Simulation*, vol. 32, pp. 145–157, 2016.

[28] A. Chadha and D. N. Pandey, "Existence results for an impulsive neutral stochastic fractional integro- differential equation with infinite delay," *Nonlinear Analysis*, vol. 128, pp. 149–175, 2015.

[29] M. Pierri, D. O'Regan, and V. Rolnik, "Existence of solutions for semi-linear abstract differential equations with not instantaneous impulses," *Applied Mathematics and Computation*, vol. 219, no. 12, pp. 6743–6749, 2013.

[30] X. Yu and J. Wang, "Periodic boundary value problems for nonlinear impulsive evolution equations on Banach spaces," *Communications in Nonlinear Science and Numerical Simulation*, vol. 22, no. 1-3, pp. 980–989, 2015.

[31] J. R. Wang, A. G. Ibrahim, and D. O'Regan, "Topological structure of the solution set for fractional non-instantaneous impulsive evolution inclusions," *Journal of Fixed Point Theory and Applications*, vol. 20, no. 2, p. 59, 2018.

[32] A. Anguraj and S. Kanjanadevi, "Non-instantaneous impulsive fractional neutral differential equations with state-dependent delay," *Progress in Fractional Differentiation and Applications*, vol. 3, no. 3, pp. 207–218, 2017.

[33] P. Kumar, R. Haloi, D. Bahuguna, and D. Pandey, "Existence of solutions to a new class of abstract non-instantaneous impulsive fractional integro-differential equations," *Nonlinear Dynamics and Systems Theory*, vol. 16, pp. 73–85, 2016.

[34] S. Samko, A. Kilbas, and O. Marichev, *Fractional Integrals and Derivatives: Theory and Applications*, Gordon and Breach, Amsterdam, 1993.

[35] D. Araya and C. Lizama, "Almost automorphic mild solutions to fractional differential equations," *Nonlinear Analysis*, vol. 69, no. 11, pp. 3692–3705, 2008.

[36] A. Debbouche and D. Baleanu, "Controllability of fractional evolution nonlocal impulsive quasilinear delay integro-differential systems," *Computers & Mathematics with Applications*, vol. 62, no. 3, pp. 1442–1450, 2011.